

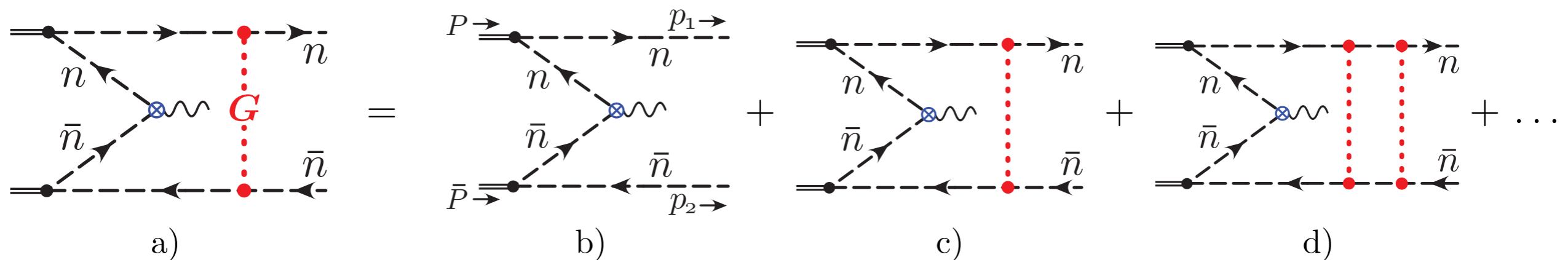
Effective Field Theory of Forward Scattering and its Applications

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(small x application Marzani, Niell, Pathak,
Stewart)

Near Forward scattering has been a significant challenge since the advent of modern QFT. The calculation of forward scattering cross section still lacks theoretical underpinning. Furthermore, even if there is a hard scattering interaction forward scattering sub-processes can reduce predictive power.



S-Matrix theory, String Theory, AdS/CFT have shed much light on the problem but it is perhaps true that there is still no systematic, first principles, QFT approach to the problem.

EFT approach, SCET, has been shown to be remarkably powerful tool

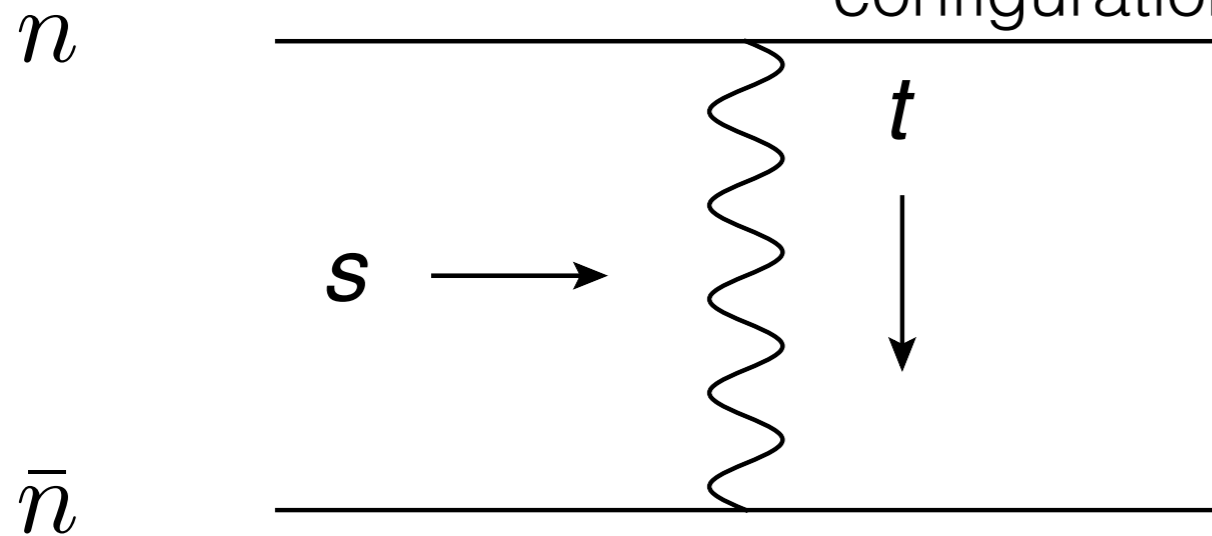
- B physics (inclusives as well as exclusives)
- Hard Scattering factorization theorems and resummations.
- Jet Physics
- Fermi Liquid Theory

However, almost all papers written on the subject have not addressed the issue of the problem of forward scattering.

Canonical Modes

- Soft $(\lambda, \lambda, \lambda)$ $(\lambda^2, \lambda^2, \lambda^2)$ $\lambda \sim E/Q$
- Collinear $(\lambda^2, 1, \lambda)$ $(1, \lambda^2, \lambda)$
- Hard $(1, 1, 1)$ (Integrate out)

However, there exists another mode in the theory, for which symmetry plays no role, that arise for exceptional external momentum configurations (near forward)



The Glauber mode

$$p_g^\mu \sim (\lambda^2, \lambda^2, \lambda)$$

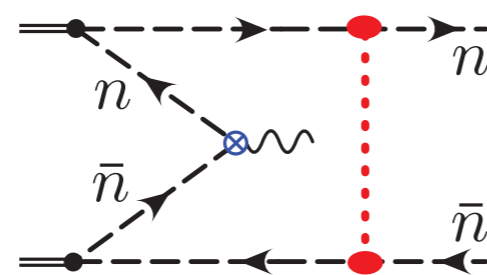


Shows up at leading power

$$O_g = \frac{g^2}{\vec{p}_\perp^2} \left(\bar{\xi}_n \frac{\not{n}}{2} \xi_n \right) \left(\xi_{\bar{n}} \frac{\not{\bar{n}}}{2} \xi_{\bar{n}} \right)$$

The Glauber gluon contributes at leading order in near forward scattering and builds up a coherent shock wave solution. Leading order Glauber contributions threaten factorization.

Primary challenge to factorization theorems



**couples n -collinear,
 \bar{n} -collinear, and
soft modes**

Instantaneous analogous to
Coulomb exchange

To prove factorization must either show that they're contributions are subsumed by other modes which factorize, or if not, that they cancel in the observable of interest.

Construction:

$\lambda \ll 1$

large Q

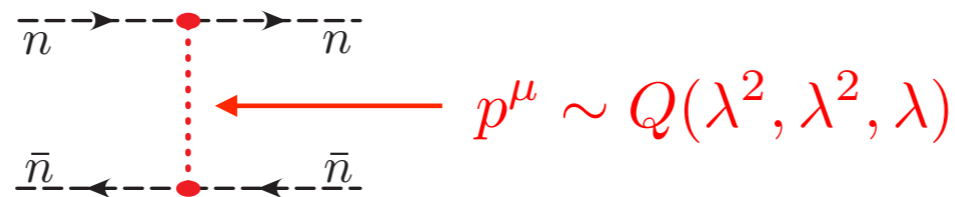
will do calculations with back-to-back collinear particles for simplicity

mode	fields	p^μ momentum scaling	physical objects	type
n -collinear	ξ_n, A_n^μ	$(n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n -collinear "jet"	onshell
\bar{n} -collinear	$\xi_{\bar{n}}, A_{\bar{n}}^\mu$	$(\bar{n} \cdot p, n \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	\bar{n} -collinear "jet"	onshell
soft	ψ_S, A_S^μ	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	ψ_{us}, A_{us}^μ	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	—	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda), a + b > 2$ (here $\{a, b\} = \{2, 2\}, \{2, 1\}, \{1, 2\}$)	forward scattering potential	offshell
hard	—	$p^2 \gtrsim Q^2$	hard scattering	offshell

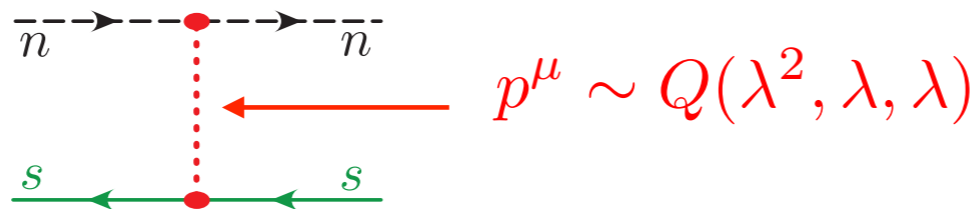
Integrate out

Need 3-types of Glauber momenta:

n - \bar{n}
fwd. scattering

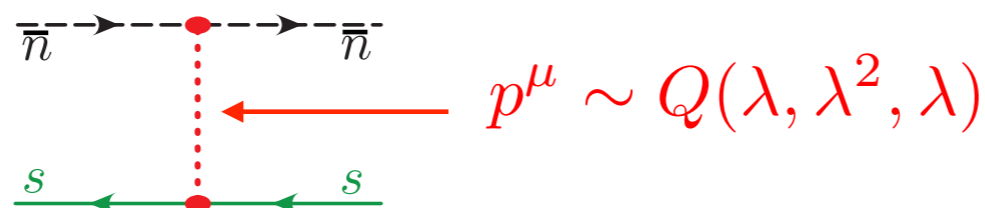


n - s
fwd. scattering



- $\frac{1}{k_\perp^2}$ potentials

\bar{n} - s
fwd. scattering



- instantaneous in x^+, x^- (t and z)

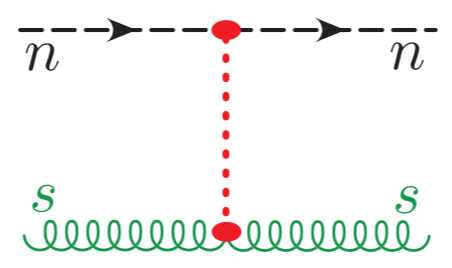
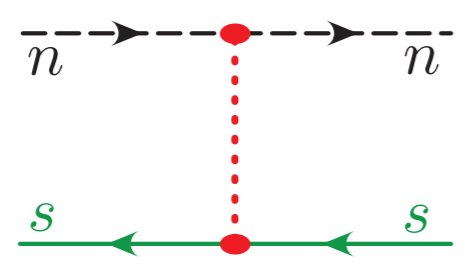
Goal: Write down an EFT which incorporates Glauber interactions into high energy scattering that will allow for a general analysis on their effects on observables

This abets:

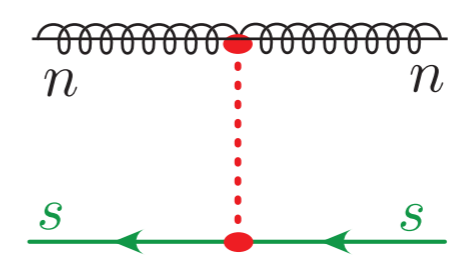
- 1) Generalize/Simplify factorization proofs.
- 2) Determine when and at what level
Glaubers contribute
- 3) Calculate systematically when Glaubers
do indeed contribute.

n - S fwd. scattering

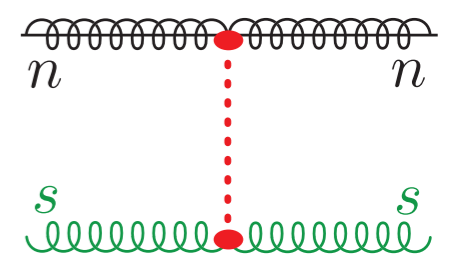
$$\lambda^2 = \frac{t}{s} \ll 1$$



$s \gg t$



integrated out



determine

$$\mathcal{O}(\lambda^3) : \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

$\lambda^2 \quad \lambda^{-2} \quad \lambda^3$

(2 rapidity sectors)

with bilinear octet operators

$$\psi_s^n = S_n^\dagger \psi_s$$

$$\mathcal{O}(\lambda^2) : \mathcal{O}_n^{qB} = \bar{\chi}_n T^B \frac{\not{n}}{2} \chi_n,$$

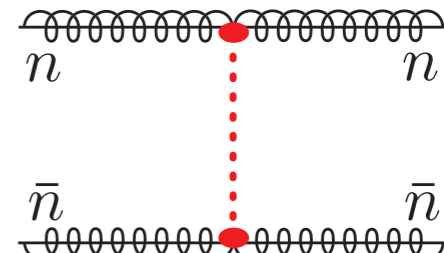
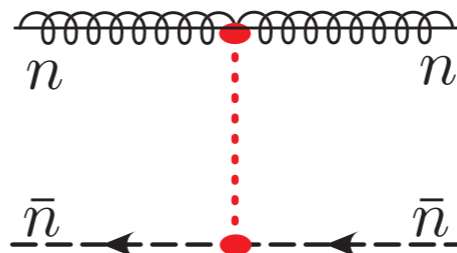
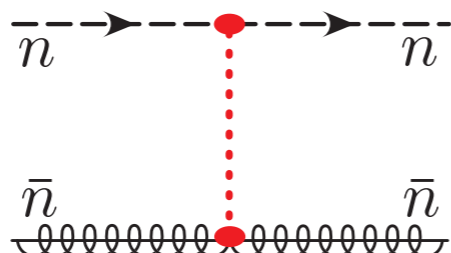
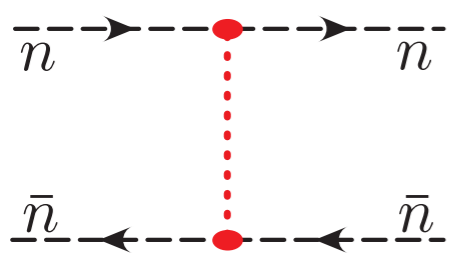
$$\mathcal{O}(\lambda^3) : \mathcal{O}_s^{q_n B} = 8\pi\alpha_s \left(\bar{\psi}_S^n T^B \frac{\not{n}}{2} \psi_S^n \right),$$

$$\mathcal{O}_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^C \frac{\bar{n}}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{n\perp}^{D\mu}$$

$$\mathcal{O}_s^{g_n B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{nC} \frac{n}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{nD\mu} \right)$$

Operators manifestly gauge invariant in all sectors

n - \bar{n} fwd. scattering $s \gg t$



might guess

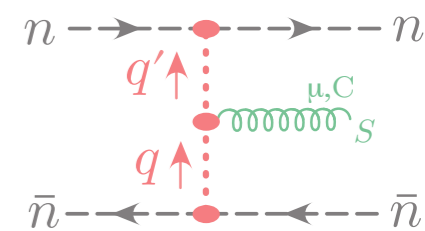
$$\mathcal{O}(\lambda^2) : \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jB}$$

actually $\mathcal{O}(\lambda^2) :$
(3 rapidity sectors)

$$\sum_{i,j=q,g} \underbrace{\mathcal{O}_n^{iB}}_{\lambda^2} \underbrace{\frac{1}{\mathcal{P}_\perp^2}}_{\lambda^{-2}} \underbrace{\mathcal{O}_s^{BC}}_{\lambda^2} \underbrace{\frac{1}{\mathcal{P}_\perp^2}}_{\lambda^{-2}} \underbrace{\mathcal{O}_{\bar{n}}^{jC}}_{\lambda^2}$$

must allow for soft emission from **between** the rapidity sectors:

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \mathcal{P}_\perp^2 \delta^{BC} (+ \dots)$$



$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \left\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g \tilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} - \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \tilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu} \mathcal{P}_\mu^\perp - g \tilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \hat{\mathcal{B}} \right. \\ \left. - \frac{n_\mu \bar{n}_\nu}{2} \mathcal{S}_n^T i g \tilde{G}_s^{\mu\nu} \mathcal{S}_{\bar{n}} \right\}^{BC} .$$

Match at two gluons

If Glaubers are contributing this implies that is a large hierarchy of scales and thus P.T. will in general breakdown necessitating resummations.

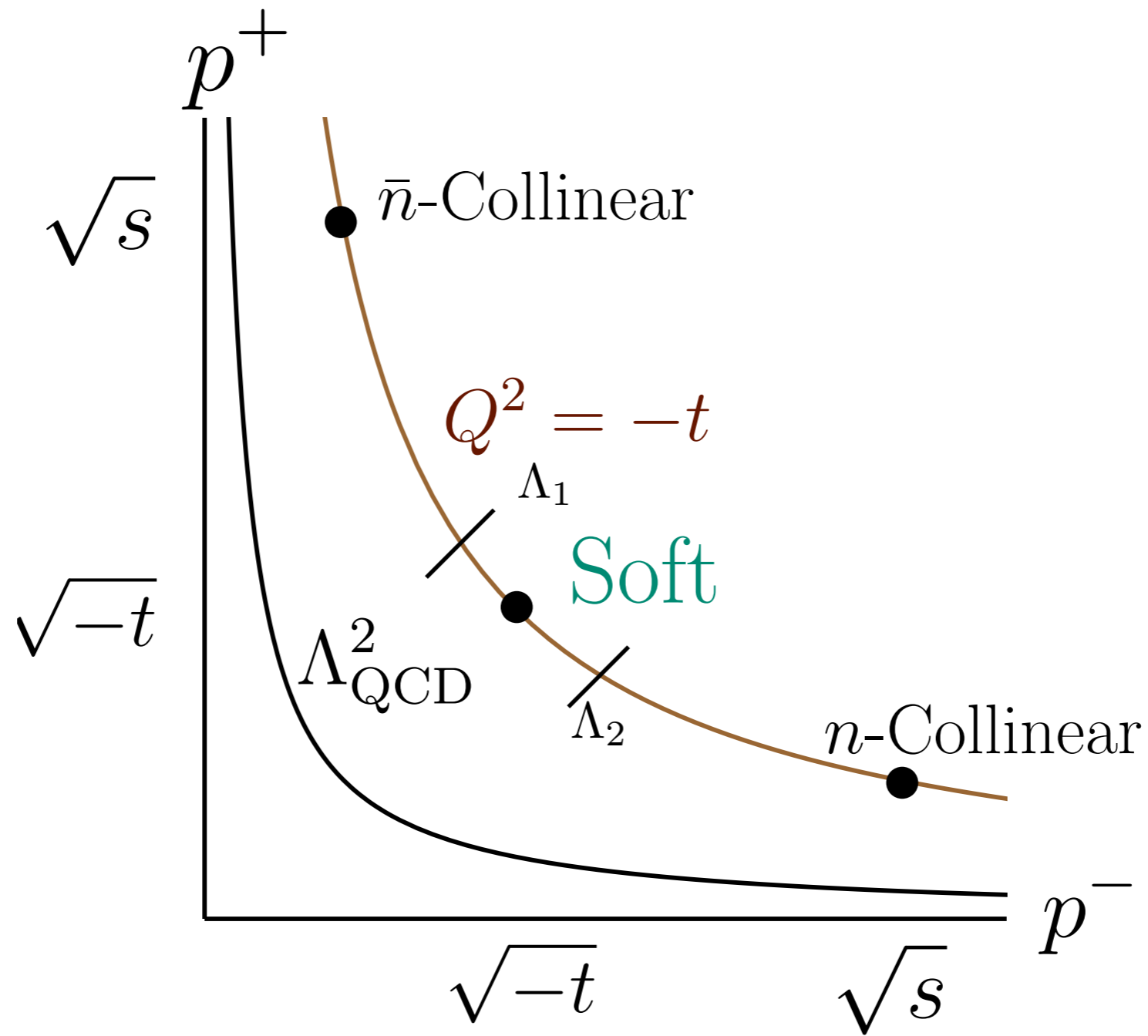
$$s \gg t \gg \Lambda$$

$Log^n(s/t), Log^m(t/\Lambda)$ Pollutes Perturbative Expansion

Canonical RG does not in any straight forward way sum these logs, as the invariant s does not flow thru any propagator (with no hard scattering), what actually shows up is

$$Log(P_-/\Lambda_1) + Log(\Lambda_1/\sqrt{t}) + Log(\Lambda_2/\sqrt{t}) + Log(P_+/\Lambda_2)$$

Λ_1, Λ_2 Rapidity Cut-Offs separated modes which live on same mass-shell hyperbola.



$$\lambda = \sqrt{\frac{-t}{s}}$$

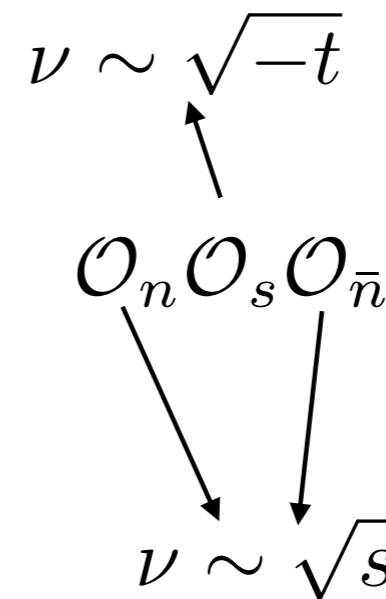
In EFT there exist integrals unregulated by dim. reg.
 These rapidity divergences cancel in the sum of diagrams.

Introduce a rapidity regulator with a rapidity scale ν
The rapidity logs are then summed by running collinear
and soft functions to their natural rapidity scale
“Rapidity Renormalization Group” (RRG)

Example Reggeization:

Each sector has a
natural rapidity scale

We have a choice to either run
collinear ops from $\nu \sim \sqrt{s}$ to
 $\nu \sim \sqrt{-t}$ or run the soft function.



There exists strong constraints on the rapidity anomalous dimensions due to the fact that there is **no hard matching coefficient**.

Consider running of single index operators

$$\nu \frac{d}{d\nu} \sum_{ij=q,g} \mathcal{O}_{ns}^{ij} = \nu \frac{d}{d\nu} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) \frac{1}{\mathcal{P}_\perp^2} (\mathcal{O}_s^{qnA} + \mathcal{O}_s^{gnA}) = 0$$

$$\nu \frac{d}{d\nu} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) = \gamma_{n\nu} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}), \quad \nu \frac{d}{d\nu} (\mathcal{O}_s^{qnA} + \mathcal{O}_s^{gnA}) = \gamma_{s\nu} (\mathcal{O}_s^{qnA} + \mathcal{O}_s^{gnA}).$$

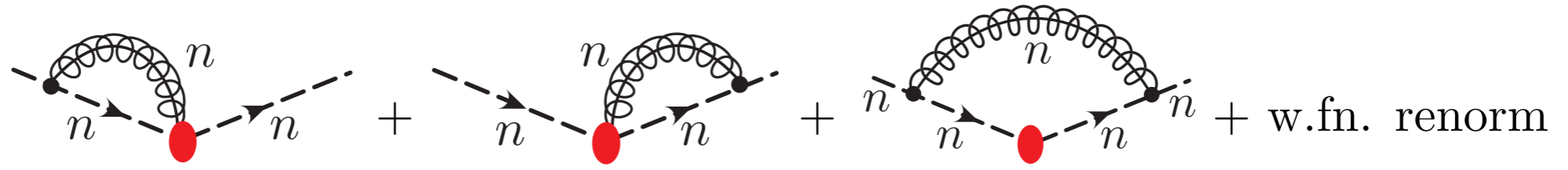
mixing leads to
universality

$$\gamma_{s\nu} \equiv \gamma_{s\nu}^{qq} + \gamma_{s\nu}^{gq} = \gamma_{s\nu}^{gg} + \gamma_{s\nu}^{qg}.$$

$$\gamma_{n\nu} \equiv \gamma_{n\nu}^{qq} + \gamma_{n\nu}^{gq} = \gamma_{n\nu}^{gg} + \gamma_{n\nu}^{qg},$$

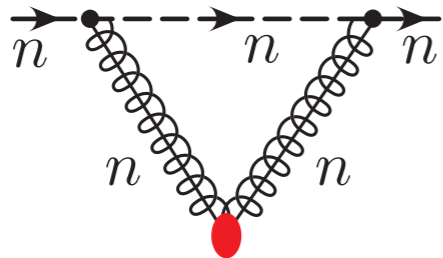
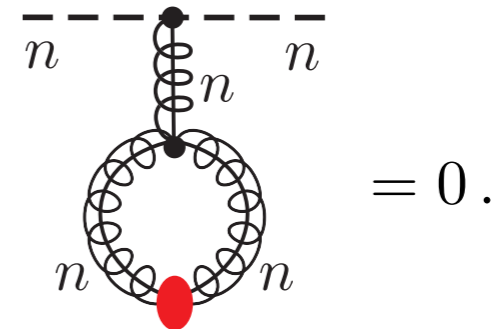
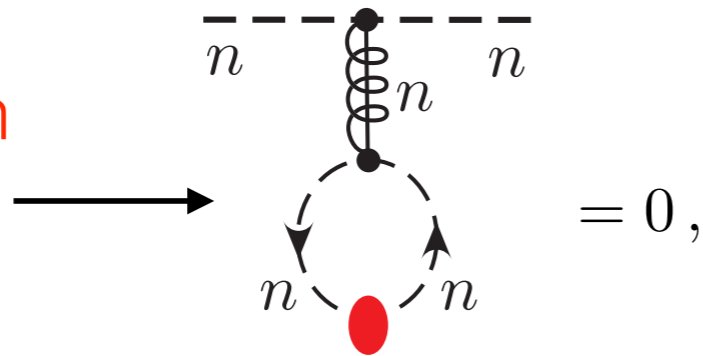
$$\gamma_{n\nu}^{qq} + \gamma_{n\nu}^{gq} = -\gamma_{s\nu}^{qq} - \gamma_{s\nu}^{gq}, \quad \text{or} \quad \gamma_{n\nu} = -\gamma_{s\nu}.$$

Collinear Runnings



$$= \mathcal{S}^{nq} \frac{\alpha_s C_A}{4\pi} \left[w^2 \frac{2h(\epsilon, \frac{\mu^2}{m^2})}{\eta} + w^2 \frac{2}{\epsilon} \ln \left(\frac{\nu}{\bar{n} \cdot p} \right) + \frac{3}{2\epsilon} \right] = \mathcal{S}^{nq} \delta V_n^{qq},$$

after zero-bin subtraction



$$= \mathcal{S}^{nq} \frac{\alpha_s C_A}{4\pi} \left[w^2 \frac{2g(\epsilon, \frac{\mu^2}{-t})}{\eta} - w^2 \frac{2}{\epsilon} \ln \left(\frac{\nu}{\bar{n} \cdot p} \right) - \frac{3}{2\epsilon} \right] = \mathcal{S}^{nq} \delta V_n^{gq},$$

Rapidity divergence

$$\nu \frac{\partial}{\partial \nu} w = -\frac{\eta}{2} w$$

$$\gamma_{n\nu}^{qq} = -\frac{\alpha_s C_A}{4\pi} \left[-2h(\epsilon, \mu^2/m^2) + \frac{2}{\epsilon} \right] = \frac{\alpha_s(\mu) C_A}{2\pi} \ln \left(\frac{\mu^2}{m^2} \right),$$

$$\gamma_{n\nu}^{gq} = -\frac{\alpha_s C_A}{4\pi} \left[-2g(\epsilon, \mu^2/(-t)) - \frac{2}{\epsilon} \right] = \frac{\alpha_s(\mu) C_A}{2\pi} \ln \left(\frac{-t}{\mu^2} \right),$$

$$\gamma_{n\nu}^{gg} = -\frac{\alpha_s C_A}{4\pi} \left[-2g(\epsilon, \mu^2/(-t)) - 2h(\epsilon, \mu^2/m^2) \right] = \frac{\alpha_s(\mu) C_A}{2\pi} \ln \left(\frac{-t}{m^2} \right),$$

$$\gamma_{n\nu}^{qg} = 0.$$

$$\gamma_{n\mu}^{qq} = \frac{\alpha_s(\mu) C_A}{2\pi} \left[2 \ln \left(\frac{\nu}{\bar{n} \cdot p} \right) + \frac{3}{2} \right],$$

$$\gamma_{n\mu}^{gq} = -\frac{\alpha_s(\mu) C_A}{2\pi} \left[2 \ln \left(\frac{\nu}{\bar{n} \cdot p} \right) + \frac{3}{2} \right],$$

$$\gamma_{n\mu}^{gg} = -\frac{2\alpha_s(\mu) n_F T_F}{3\pi},$$

$$\gamma_{n\mu}^{qg} = \frac{2\alpha_s(\mu) n_F T_F}{3\pi}.$$

Obeys Consistency Conditions

$$\gamma_{n\mu}^{qq} + \gamma_{n\mu}^{gq} = \gamma_{n\mu}^{gg} + \gamma_{n\mu}^{qg} = 0.$$

$$\gamma_{n\nu}^{qq} + \gamma_{n\nu}^{gq} = \gamma_{n\nu}^{gg} + \gamma_{n\nu}^{qg}$$

To sum logs all
we need is

$$\gamma_{n\nu} = \frac{\alpha_s C_A}{2\pi} \ln \left(\frac{-t}{m^2} \right).$$

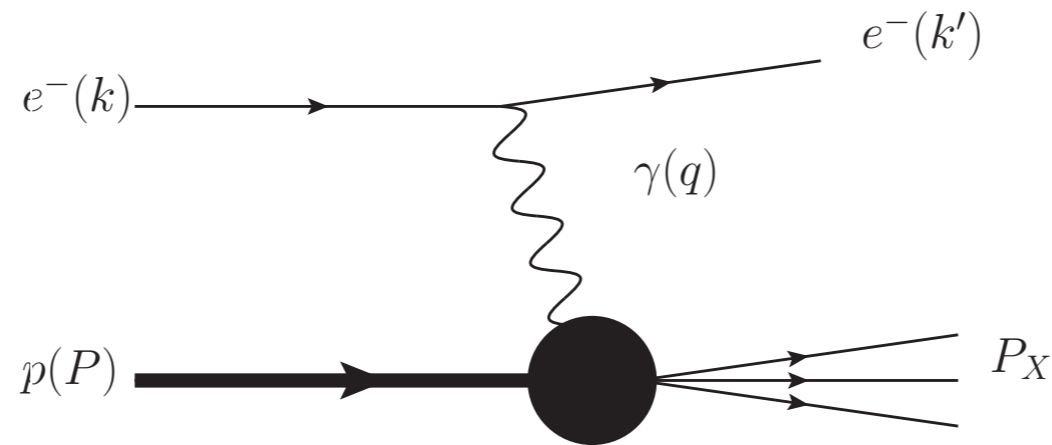
$$\frac{d}{d \log \nu} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) = \gamma_{n\nu} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})$$

$$(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu = \sqrt{-t}) = \left(\frac{s}{-t} \right)^{-\gamma_{n\nu}/2} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu = \sqrt{s}).$$

$$\begin{aligned} & (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu = \sqrt{-t}) \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{AB}(\nu = \sqrt{-t}) \frac{1}{\mathcal{P}_\perp^2} (\mathcal{O}_{\bar{n}}^{qB} + \mathcal{O}_{\bar{n}}^{gB})(\nu = \sqrt{-t}) \\ &= \left(\frac{s}{-t} \right)^{-\gamma_{n\nu}} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu = \sqrt{s}) \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{AB}(\nu = \sqrt{-t}) \frac{1}{\mathcal{P}_\perp^2} (\mathcal{O}_{\bar{n}}^{qB} + \mathcal{O}_{\bar{n}}^{gB})(\nu = \sqrt{s}), \end{aligned}$$

Reggeization (two loops?)

Small x - Physics

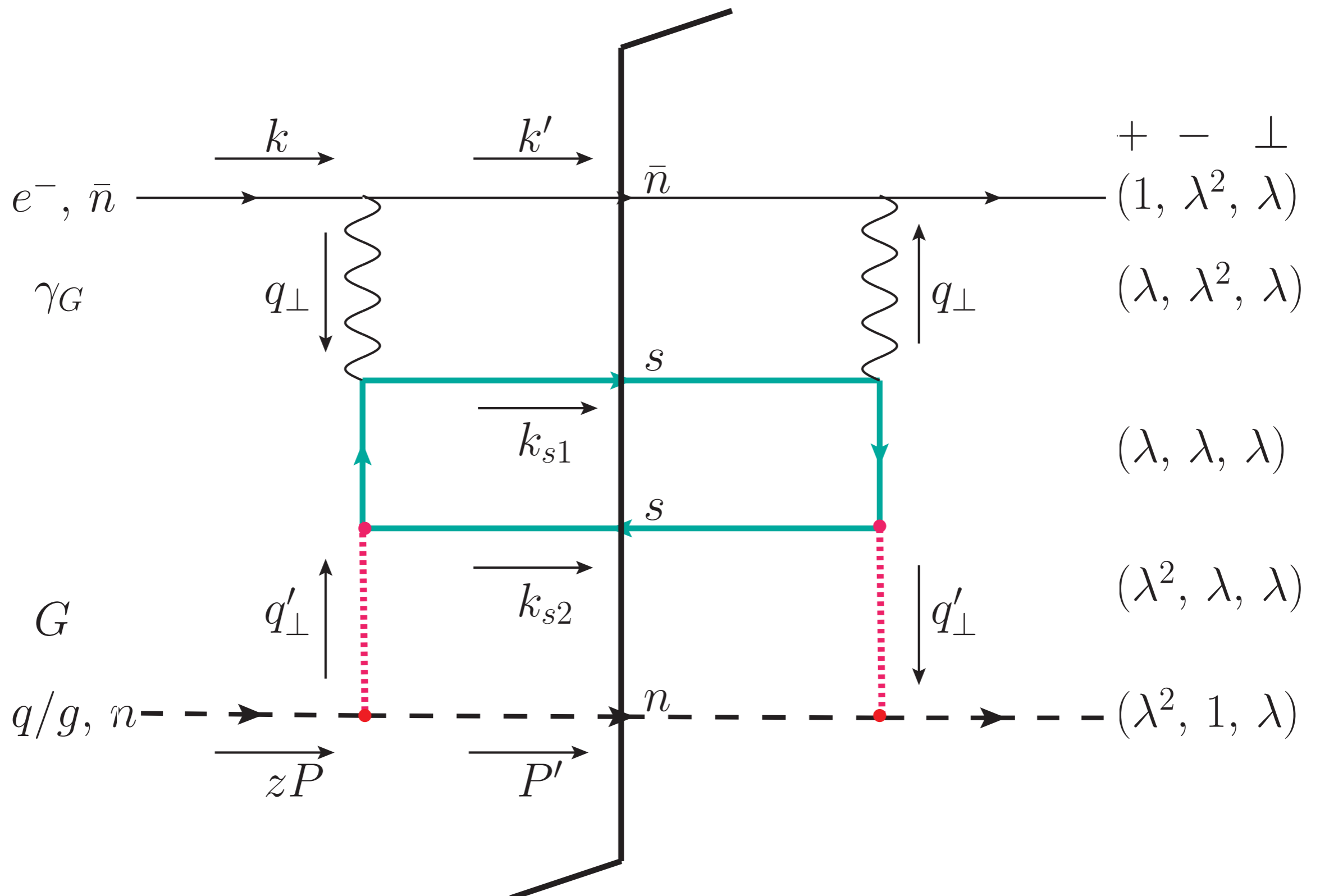


Splitting Functions
in small x limit

$$P_{ab}^{(n-1)}(x) \sim \frac{1}{x} [\ln^{n-1}(x) + \mathcal{O}(\ln^{n-2} x)]$$

Must resum logs in the anomalous dimension as
opposed to cusp which saturates

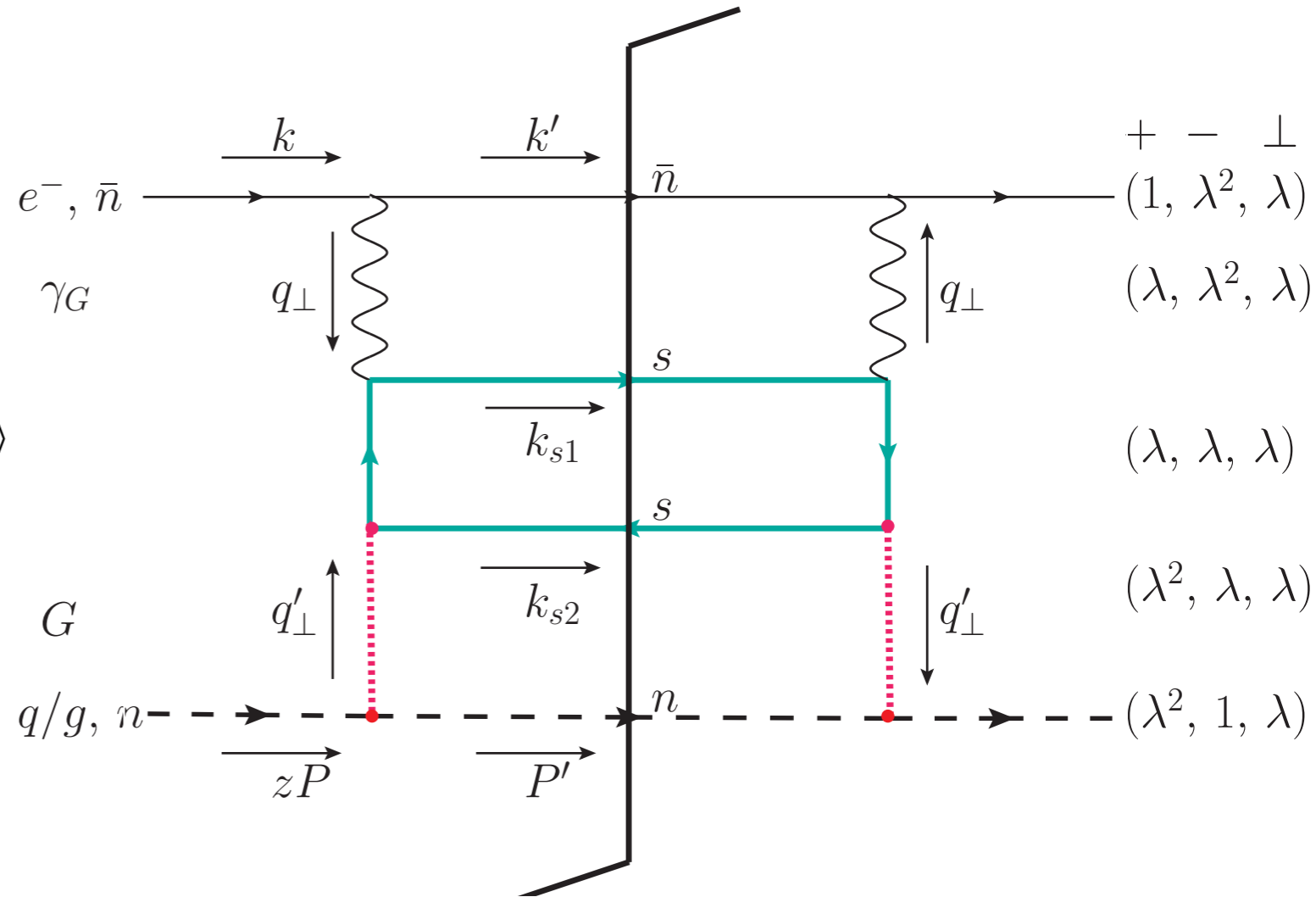
Leading Power: in EFT arises from TOP of Glauber operators on each side of the cut.



$$U_{(1,1)} = i \int [dx^\pm][dx'^\pm] \sum_{k^\pm} \int \frac{d^2 q_\perp}{q_\perp^2} \frac{d^2 q'_\perp}{q'^2_\perp} [\mathcal{O}_{n,k^-}^{qA}(q'_\perp) + \mathcal{O}_{n,k^-}^{gA}(q'_\perp)](\tilde{x}') [\mathcal{O}_{\bar{n},k^+}^e(q_\perp)](\tilde{x})$$

$$\times \mathcal{O}_{s(1,1),-k^\pm}^A(q_\perp, q'_\perp)(\tilde{x}, \tilde{x}'),$$

$$T_{(1,1)}^{\text{DIS}} = \frac{1}{V_4} \sum_X \langle ep | U_{(1,1)}^\dagger | e' X \rangle \langle e' X | U_{(1,1)} | ep \rangle$$



[Marzani, Neill, AP, Rothstein, Stewart]

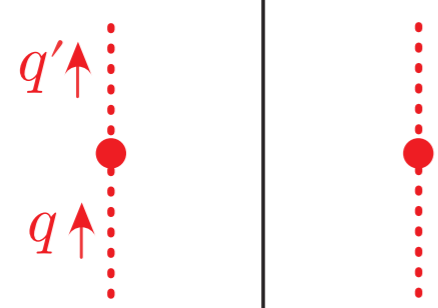
$$F(x, Q^2) = \int_0^1 \frac{dz}{z} d^2 q'_\perp C_n[q'_\perp, \nu/(zP_-)] S(q'_\perp, q_\perp, \nu/(xP_-))$$

Run soft function in rapidity to the scale $\nu = P_-$

In that way no large rapidity logs in collinear function.

Rapidity running for soft function is exactly BFKL:

Lowest order

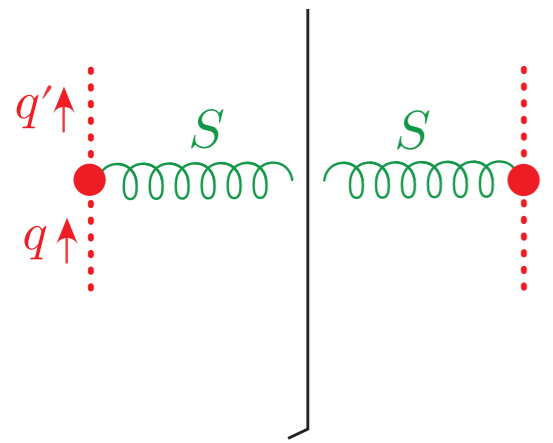


$$\langle 0 | O_{s(1,1)}^{AB}(q_\perp, q'_\perp) | 0 \rangle = -i 8\pi\alpha_s(\mu) \delta^{AB} \vec{q}_\perp^2 (2\pi)^2 \delta^2(\vec{q}_\perp + \vec{q}'_\perp).$$

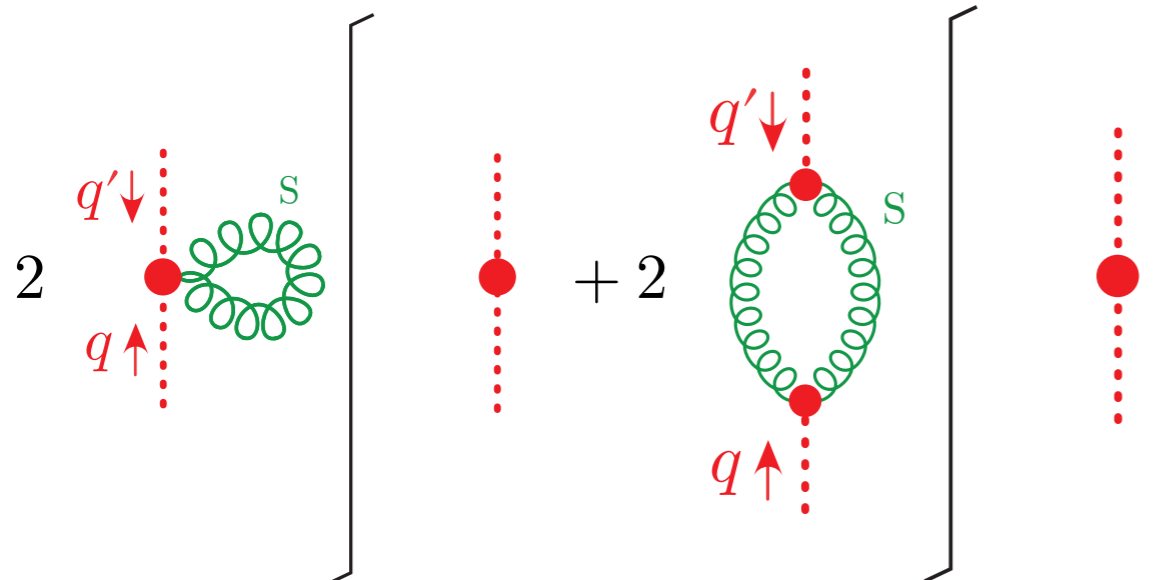
$$= S_G^{(0)}(q_\perp, q'_\perp) \equiv \frac{1}{V} \frac{1}{(\vec{q}_\perp^2 \vec{q}'_\perp^2)^2} \langle 0 | O_{s(1,1)}^{AB} | 0 \rangle \langle 0 | O_{s(1,1)}^{AB\dagger} | 0 \rangle$$

$$= \left(\frac{8\pi\alpha_s}{\vec{q}_\perp^2} \right)^2 \delta^{AA} (2\pi)^2 \delta^2(\vec{q}_\perp + \vec{q}'_\perp).$$

This form will allow to renormalize through a convolution



$$\begin{aligned}
&= (8\pi\alpha_s)^2 4\alpha_s C_A \delta^{AA} w^2 \Gamma\left(\frac{\eta}{2}\right) \int \frac{d^2 k_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2 \vec{q}_\perp^2 \vec{k}_\perp^2} (2\pi)^2 \delta^2(\vec{k}_\perp - \vec{q}_\perp - \vec{q}'_\perp) \\
&= (8\pi\alpha_s)^2 4\alpha_s C_A \delta^{AA} w^2 \Gamma\left(\frac{\eta}{2}\right) \int \frac{d^2 k_\perp}{\vec{k}_\perp^2 \vec{q}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} (2\pi)^2 \delta^2(\vec{k}_\perp + \vec{q}'_\perp) \\
&= \frac{C_A \alpha_s}{\pi^2} w^2 \Gamma\left(\frac{\eta}{2}\right) \int \frac{d^2 k_\perp \vec{k}_\perp^2}{(\vec{k}_\perp - \vec{q}_\perp)^2 \vec{q}_\perp^2} S_G^{(0)}(k_\perp, q'_\perp),
\end{aligned}$$



$$\begin{aligned}
&= -\frac{2(8\pi\alpha_s)^2 \alpha_s}{(\vec{q}_\perp^2)^3} C_A \delta^{AB} w^2 \Gamma\left(\frac{\eta}{2}\right) \int \frac{d^2 k_\perp (\vec{q}_\perp^2)^2}{\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} (2\pi)^2 \delta(\vec{q}_\perp + \vec{q}'_\perp) \\
&= -\frac{C_A \alpha_s}{2\pi^2} w^2 \Gamma\left(\frac{\eta}{2}\right) \int d^2 k_\perp \frac{\vec{q}_\perp^2}{\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} S_G^{(0)}(q_\perp, q'_\perp),
\end{aligned}$$

RRG

$$0 = \nu \frac{d}{d\nu} \tilde{S}_G^{\text{bare}}(q_\perp, q'_\perp) = \nu \frac{d}{d\nu} \int d^2 k_\perp Z^{-1}(q_\perp, k_\perp) \tilde{S}_G(k_\perp, q'_\perp, \nu).$$

extract anom.
dim

$$\gamma_{S_G}(q_\perp, q'_\perp) = \frac{2C_A\alpha_s(\mu)}{\pi^2} \left[\frac{1}{(\vec{q}_\perp - \vec{q}'_\perp)^2} - \delta^2(\vec{q}_\perp - \vec{q}'_\perp) \int d^2 k_\perp \frac{\vec{q}_\perp^2}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right]$$

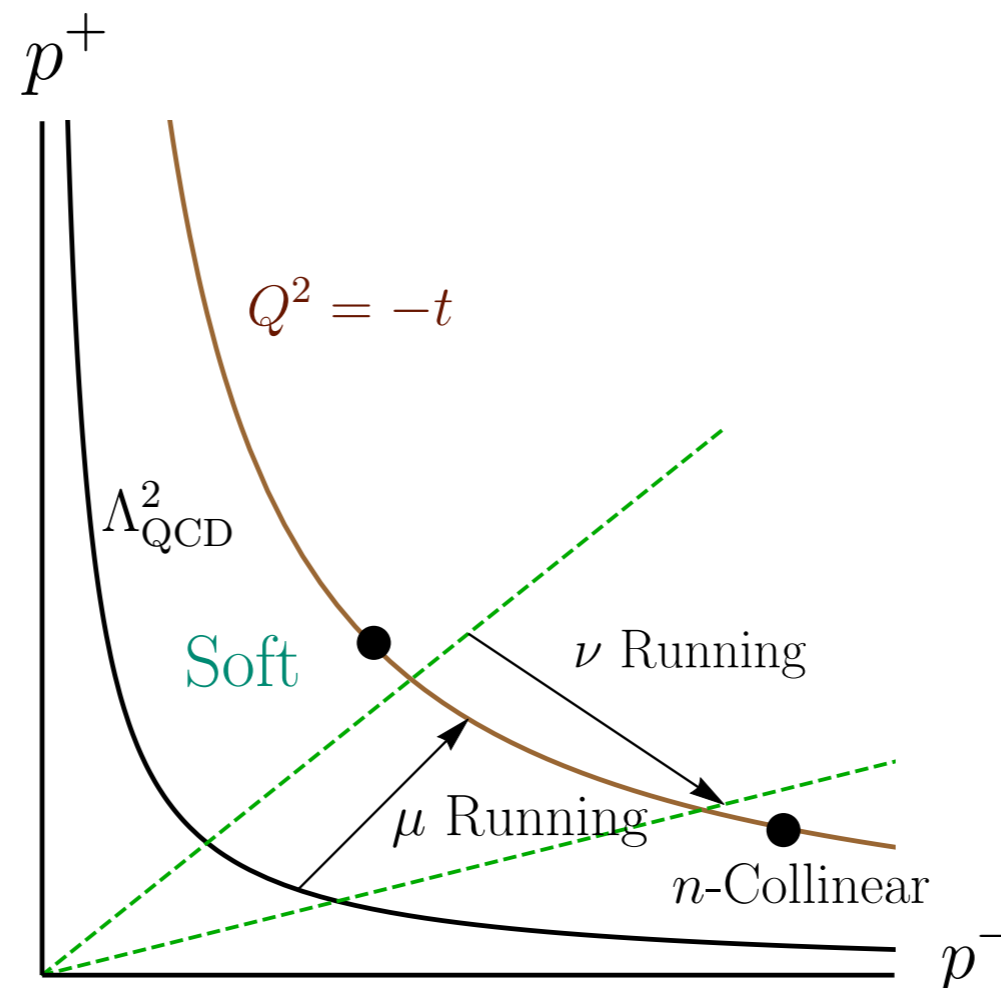
BFKL

$$\nu \frac{d}{d\nu} \tilde{S}_G(q_\perp, q'_\perp, \nu) = \frac{2C_A\alpha_s(\mu)}{\pi^2} \int d^2 k_\perp \left[\frac{\tilde{S}_G(k_\perp, q'_\perp, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^2 \tilde{S}_G(q_\perp, q'_\perp, \nu)}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right].$$

Integrate out modes with off-shellness of order t in collinear function and run

$$C_n\left(q'_\perp, \frac{\nu}{zP^-}\right) = \frac{1}{\vec{q}'_\perp{}^2} \int_z^1 \frac{d\xi}{\xi} H_n\left(\frac{z}{\xi}, \frac{q'_\perp}{\mu}, \frac{\nu}{zP^-}\right) f(\xi, \mu).$$

Final step is RG running to t .



Conclusions

Set up systematic EFT to address the question of Glauber Gluons (Completes SCET)

- Universal action applicable to all kinematic situations.
- Address factorization violation.
- When Glaubers do contribute allow for the resummations of logs via combination of RRG and RG.