Threshold logarithms in a parton shower

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work with Zoltan Nagy, DESY

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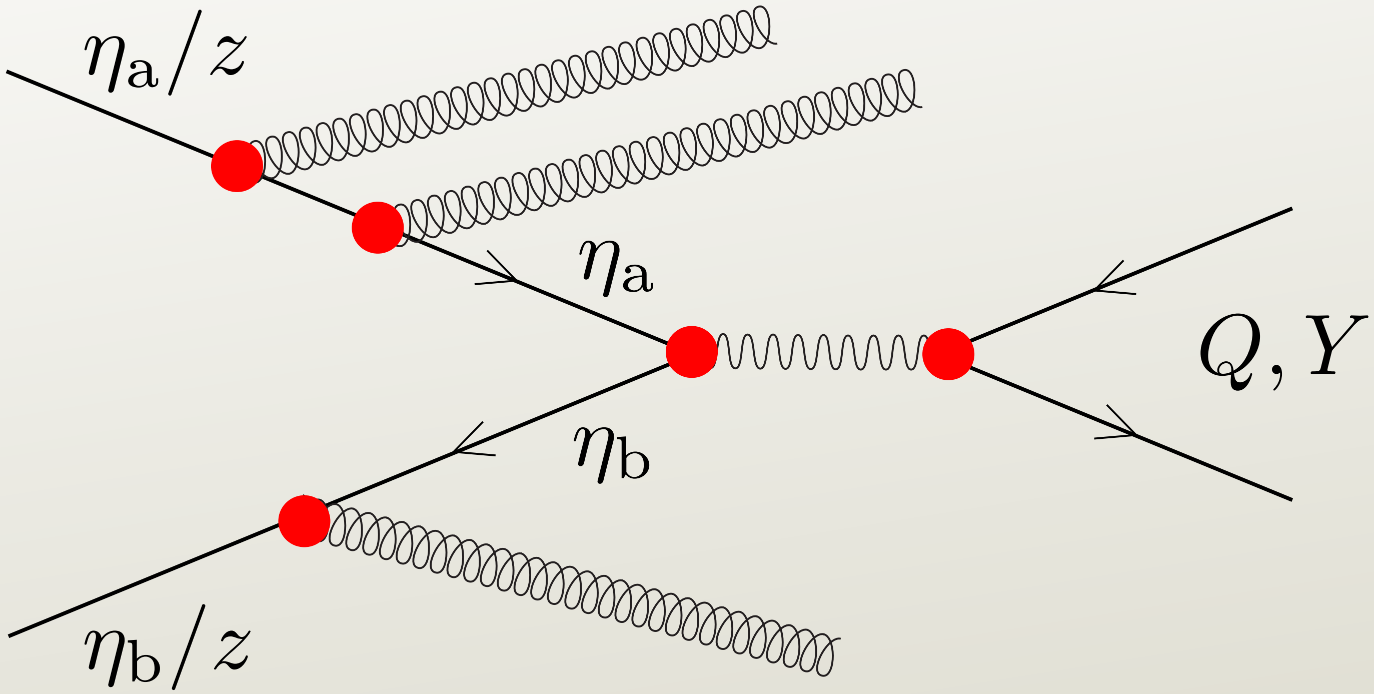
Threshold logarithms

- Consider the Drell-Yan process with dimuon rapidity Y and mass Q.



- There are logarithms of $(1 - z)$:

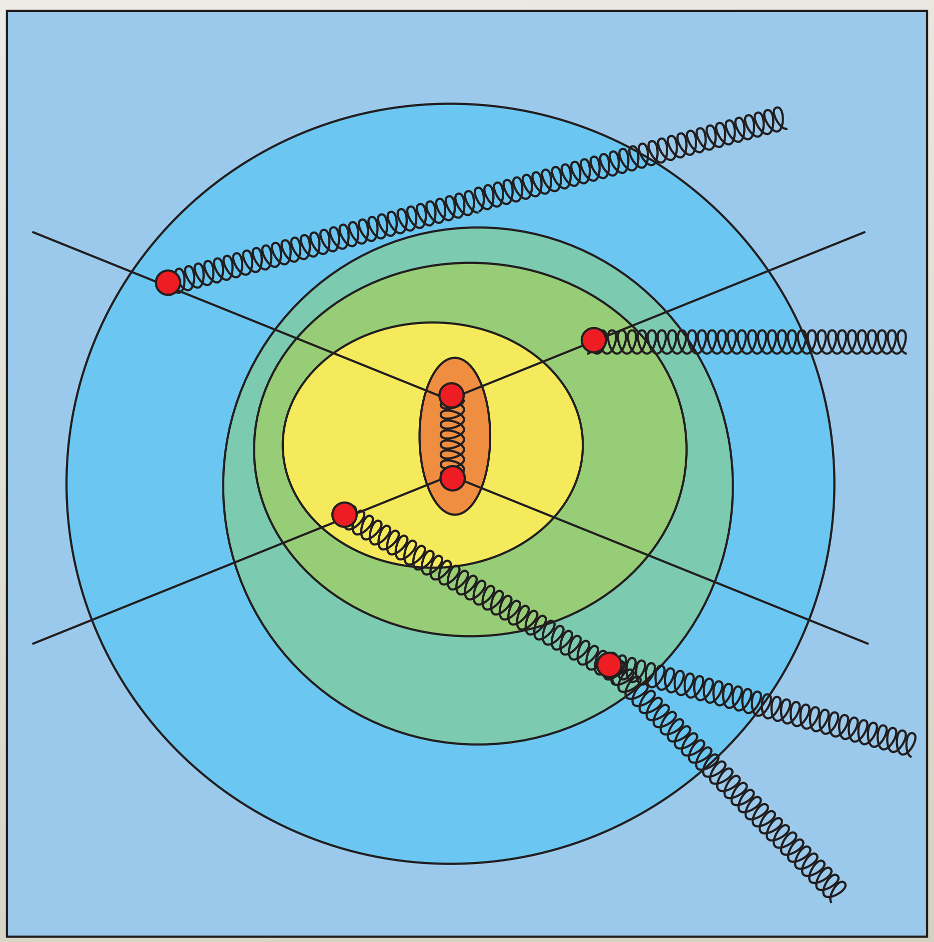
$$\int_0^1 dz\ f_{a/A}(\eta_{\rm a}/z, \mu_{\rm F}^2) \left\{ \delta(1-z) + C\alpha_s \left[\frac{\log(1-z)}{1-z} \right]_+ + \cdots \right\}$$

- There is a large literature on summing these logarithms starting with Sterman (1987).

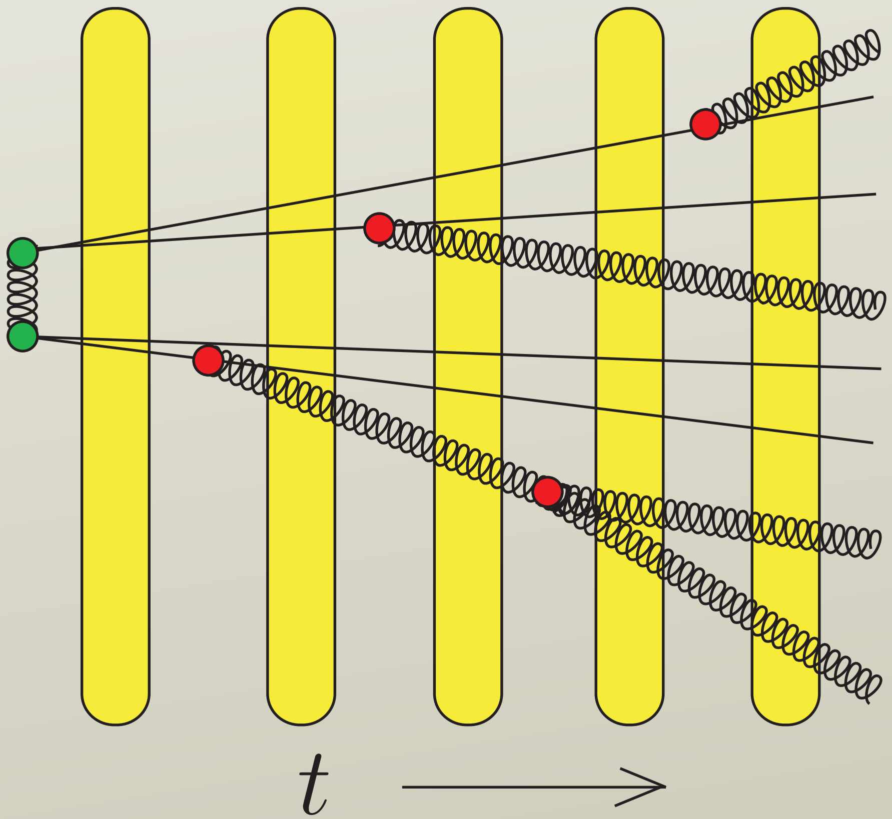
Deductor parton shower

- Zoltan Nagy (DESY) and I have a parton shower event generator, DEDUCTOR.
- http://pages.uoregon.edu/soper/deductor/
- Hadronization and underlying event not included.
- Dipole shower.
- In principle, uses quantum density matrix in color & spin.
- LC+ approximation for color.
- Z. Nagy and D. E. Soper,
  "Summing threshold logs in a parton shower,"
  JHEP **1610** (2016) 019.

- Showers develop in “shower time.”
- Hardest interactions first.



Real time picture



Shower time picture

Shower ordering variable

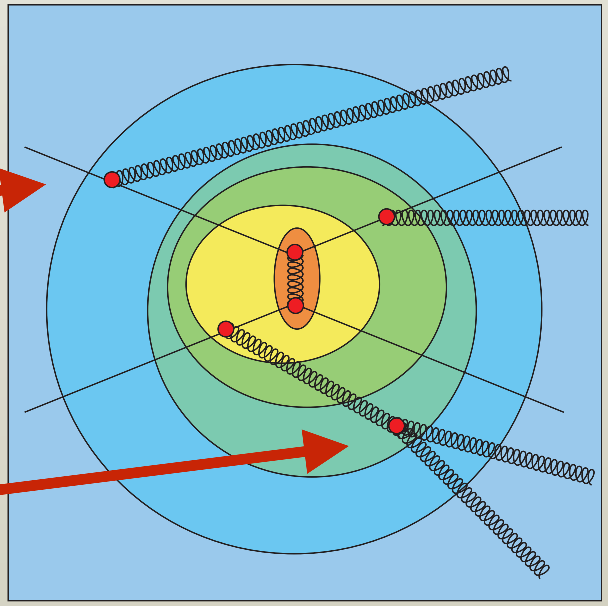
- Originally, PYTHIA used virtuality to order splittings.
- Now, PYTHIA and SHERPA use "k_T."
- DEDUCTOR uses Λ,

$$\Lambda^2 = -\frac{(\hat{p}_\mathrm{a} - \hat{p}_{m+1})^2}{2p_\mathrm{a}\cdot Q_0}\, Q_0^2$$

initial state

$$\Lambda^2 = \frac{(\hat{p}_l + \hat{p}_{m+1})^2}{2p_l\cdot Q_0}\, Q_0^2$$

final state



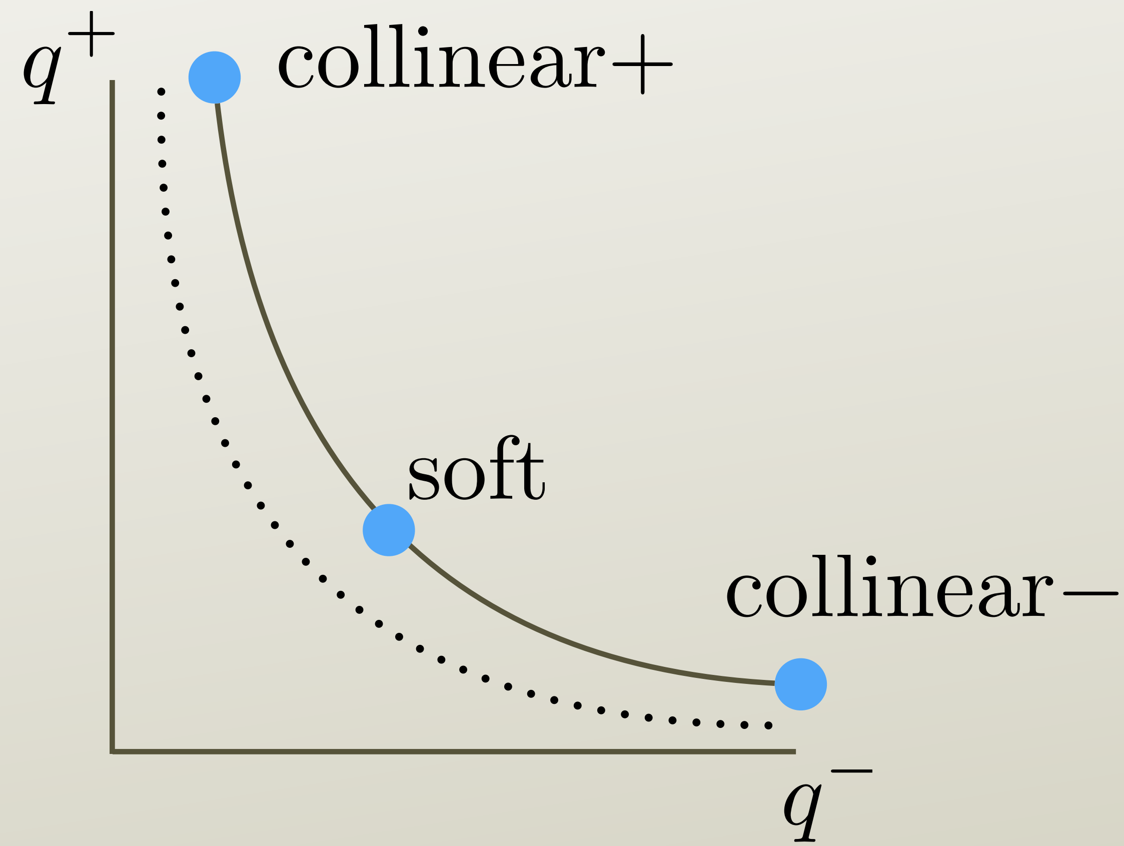
- Here Q_0 is a fixed timelike vector.

Contrast with SCET

- SCET distinguishes multiple regions.

$$(q^+, q^-, q_\perp) = (1, \lambda^2, \lambda)Q \quad \text{collinear+}$$
$$(q^+, q^-, q_\perp) = (\lambda^2, 1, \lambda)Q \quad \text{collinear}-$$
$$(q^+, q^-, q_\perp) = (\lambda, \lambda, \lambda)Q \quad \text{soft}$$

$$\lambda \ll 1$$



- A parton shower has just larger λ and smaller λ.

- Shower evolution from the renormalization group.

$$\frac{d}{d\lambda}$$

The shower state

- Here I am ignoring spin.

- To describe the state at shower time t based on an ensemble of runs of the program, use the density operator in color space

$$\rho(\{p,f\}_m,t) = \sum_{\{c\}_m,\{c'\}_m} \rho(\{p,f,c',c\}_m,t)\,\big|\{c\}_m\big\rangle\big\langle\{c'\}_m\big|$$

- Here $\rho(\{p,f,c',c\}_m,t)$ is the probability to find the system with momenta and flavors $\{p,f\}_m$ in this color state.

- Thus we use quantum statistical mechanics.

- Denote this function by $\big|\rho(t)\big)$.

Evolution equation

The shower state evolves in shower time.

$$\frac{d}{dt}\big|\rho(t)\big) = [\mathcal{H}_I(t) - \mathcal{S}(t)]\big|\rho(t)\big)$$

$\mathcal{H}_{\rm I}(t)$ = splitting operator

$\mathcal{S}(t)$ = no-splitting operator

Perturbative solution

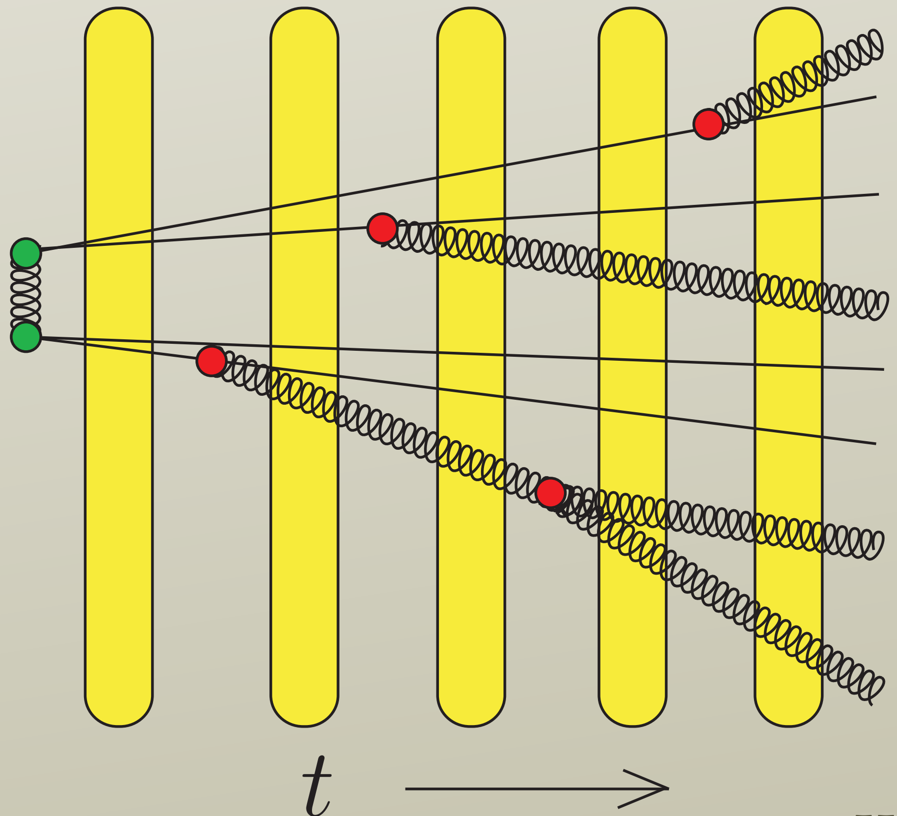
$$\big|\rho(t)\big) = \mathcal{U}_{\mathcal{S}}(t,t_0)\big|\rho(t_0)\big)$$

$$\frac{d}{dt}\mathcal{U}_{\mathcal{S}}(t,t') = [\mathcal{H}_I(t) - \mathcal{S}(t)]\mathcal{U}_{\mathcal{S}}(t,t')$$

$$\mathcal{U}_{\mathcal{S}}(t,t') = \mathcal{N}_{\mathcal{S}}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}_{\mathcal{S}}(t,\tau)\mathcal{H}_I(\tau)\mathcal{N}_{\mathcal{S}}(\tau,t')$$

where

$$\mathcal{N}_{\mathcal{S}}(\tau,t') = \mathbb{T}\exp\left[-\int_{t'}^{\tau} d\tau'\ \mathcal{S}(\tau')\right]$$



Role of parton distributions

- $\rho(t)$ contains a factor of parton distributions,

$$\big|\rho(t)\big) = \mathcal{F}(t)\big|\rho_{\rm pert}(t)\big)$$

- Here $\big|\rho_{\rm pert}(t)\big)$ is $\big|M\big\rangle\big\langle M\big|$ from Feynman diagrams.

- We include parton distribution functions at the current scale.

$$\begin{aligned}\mathcal{F}(t)\big|\{p,f,c',c\}_m\big) \\ &= \frac{f_{a/A}(\eta_{\rm a},\mu^2_{\rm a}(t))\,f_{b/B}(\eta_{\rm b},\mu^2_{\rm b}(t))}{4n_{\rm c}(a)n_{\rm c}(b)\,4\eta_{\rm a}\eta_{\rm b}p_{\rm B}\!\cdot\! p_{\rm B}}\;\big|\{p,f,c',c\}_m\big)\end{aligned}$$

Evolution for the perturbative state

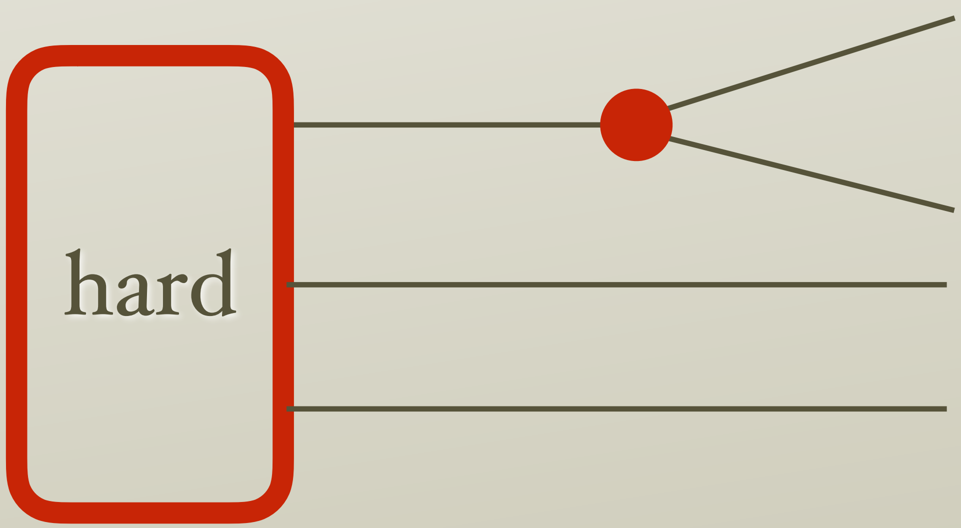
$$\big|\rho(t)\big) = \mathcal{F}(t)\big|\rho_{\rm pert}(t)\big)$$

$$\frac{d}{dt}\big|\rho_{\rm pert}(t)\big) = [\mathcal{H}_I^{\rm pert}(t) - \mathcal{S}^{\rm pert}(t)]\big|\rho_{\rm pert}(t)\big)$$

- Calcuate $\mathcal{H}_I^{\rm pert}(t)$ from Feynman diagrams.

- Then use

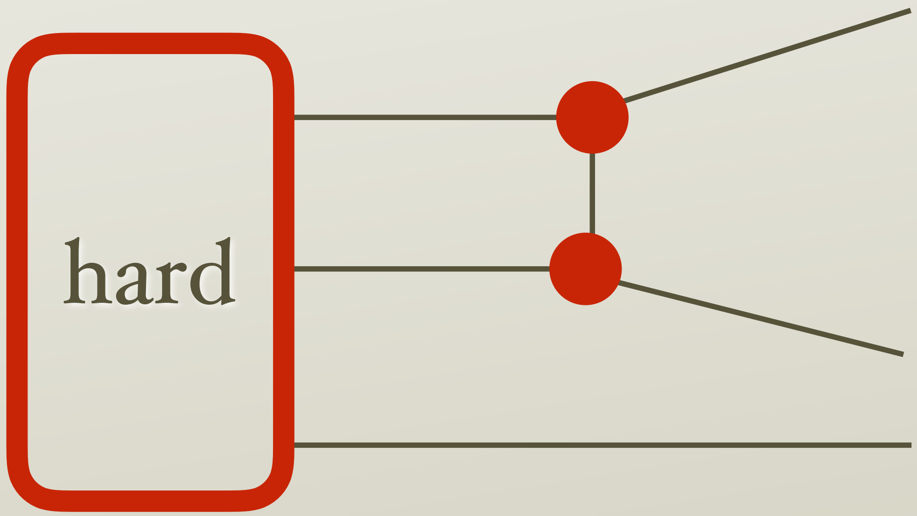
$$\mathcal{H}_I(t) = \mathcal{F}(t)\mathcal{H}_I^{\rm pert}(t)\mathcal{F}(t)^{-1}$$



$$\big|\rho(t)\big) = \mathcal{F}(t)\big|\rho_{\rm pert}(t)\big)$$

$$\frac{d}{dt}\big|\rho_{\rm pert}(t)\big) = [\mathcal{H}_I^{\rm pert}(t) - \mathcal{S}^{\rm pert}(t)]\big|\rho_{\rm pert}(t)\big)$$

- Calculate $\mathcal{S}^{\rm pert}$ from Feynman diagrams.



- Then use

$$\mathcal{S}(t) = \mathcal{S}^{\rm pert}(t) - \mathcal{F}(t)^{-1}\left[\frac{d}{dt}\,\mathcal{F}(t)\right]$$

The standard method

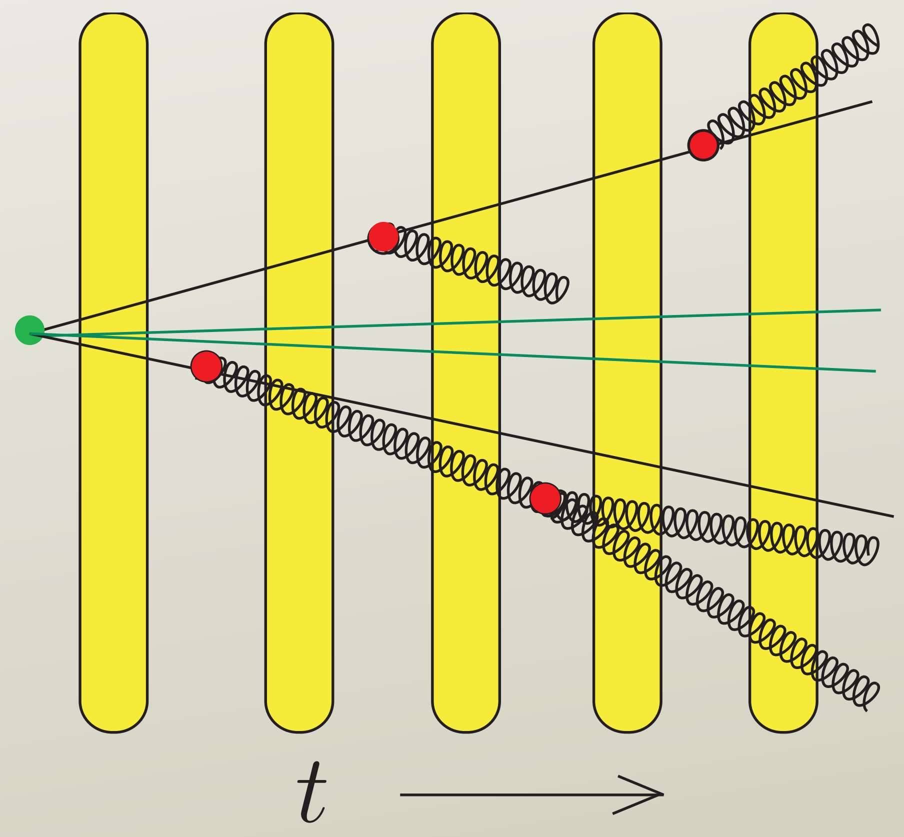
- Suppose that the shower state evolves according to

$$\big|\rho(t)\big) = \mathcal{U}_{\mathcal{V}}(t,t')\big|\rho(t')\big)$$

$$\frac{d}{dt}\,\mathcal{U}_{\mathcal{V}}(t,t') = [\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)]\,\mathcal{U}_{\mathcal{V}}(t,t')$$

$$\mathcal{H}_{\rm I}(t) = \text{splitting operator}$$

$$\mathcal{V}(t) = \text{no-splitting operator}$$



- We calculate $\mathcal{V}(t)$ from $\mathcal{H}_I(t)$ so that the inclusive cross section does not change during the shower.

Cf. the Deductor method

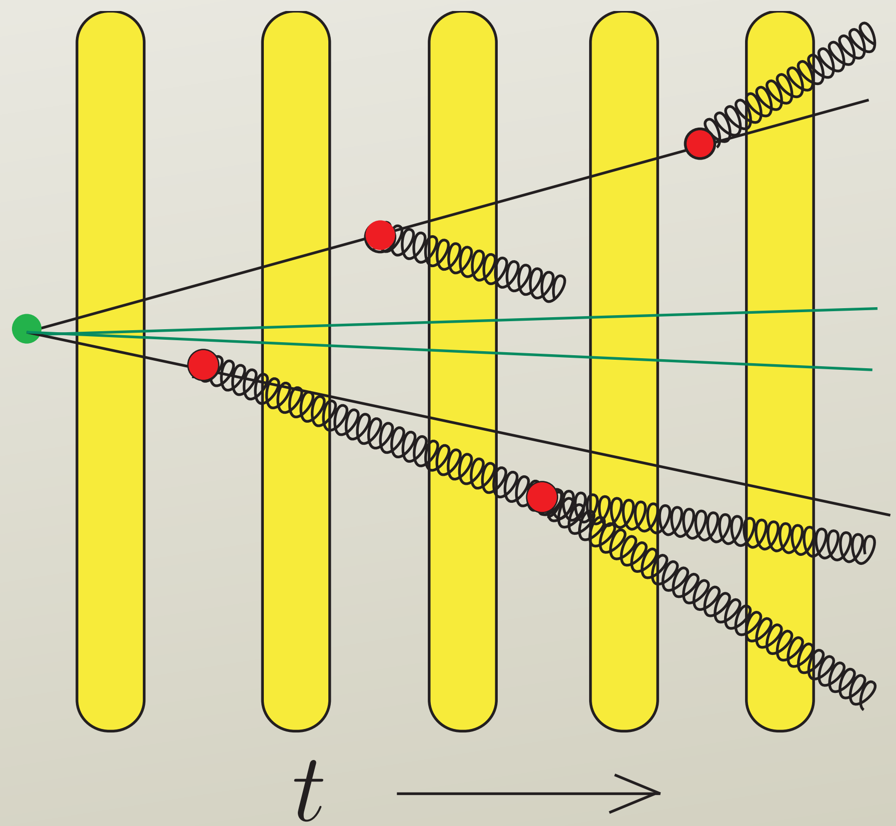
- The shower state evolves in shower time.

$$\big|\rho(t)\big) = \mathcal{U}_{\mathcal{S}}(t,t')\big|\rho(t')\big)$$

$$\frac{d}{dt}\,\mathcal{U}_{\mathcal{S}}(t,t') = [\mathcal{H}_{\rm I}(t) - \mathcal{S}(t)]\,\mathcal{U}_{\mathcal{S}}(t,t')$$

$\mathcal{H}_{\rm I}(t)$ = splitting operator

$\mathcal{S}(t)$ = virtual splitting and parton evolution



- We simply calculate $\mathcal{S}(t)$ from one loop virtual graphs plus parton evolution.

What happens

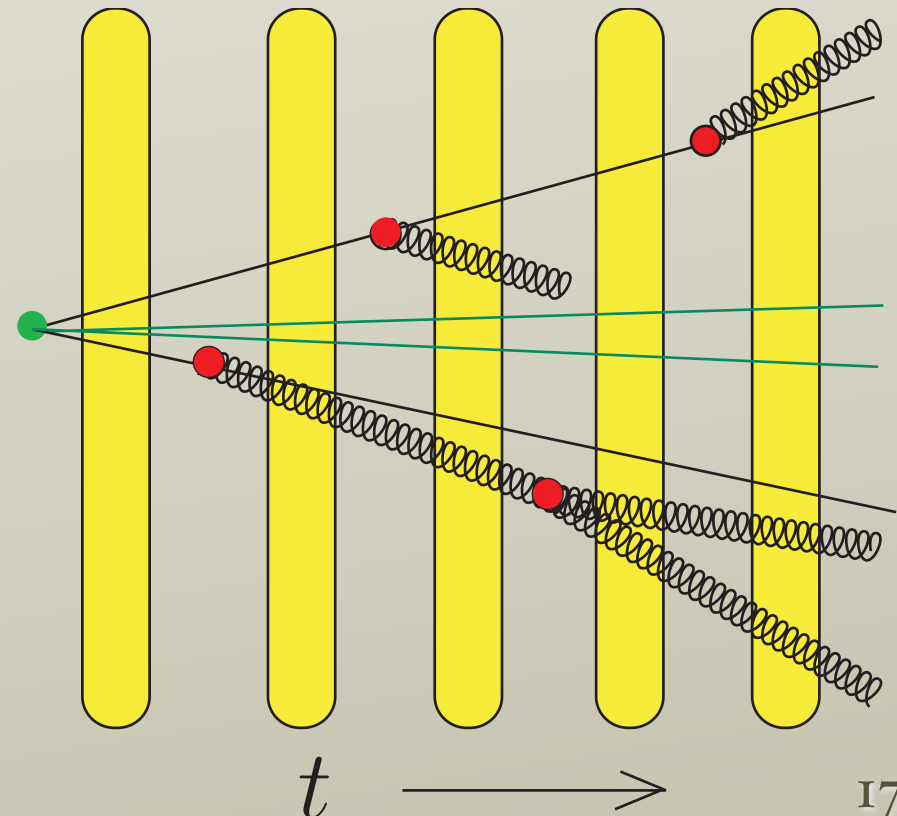
$$\mathcal{U}_{\mathcal{S}}(t,t_0) = \mathcal{N}_{\mathcal{S}}(t,t_0) + \int_{t_0}^{t} d\tau\, \mathcal{U}_{\mathcal{S}}(t,\tau)\mathcal{H}_I(\tau)\mathcal{N}_{\mathcal{S}}(\tau,t_0)$$

$$\mathcal{N}_{\mathcal{S}}(t_2,t_1) = \mathbb{T}\exp\left[\int_{t_1}^{t_2} d\tau\ [-\mathcal{V}(\tau) + (\mathcal{V}(\tau) - \mathcal{S}(\tau))]\right]$$

- Within the LC+ approximation, the operators commute.
- There is an extra factor

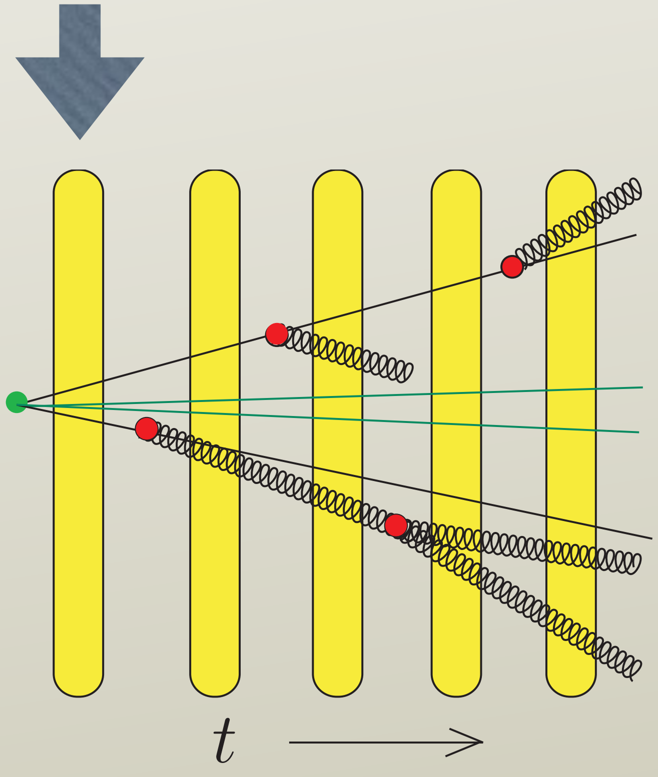
$$\exp\left[\int_{t_1}^{t_2} d\tau\ (\mathcal{V}(\tau) - \mathcal{S}(\tau))\right]$$

that changes the cross section.



The most important term

- Look at the Drell-Yan process.
- Look at the factor for line "a" just after the hard interaction.
- Assume that no real gluons have been emitted yet.



- Use y = dimensionless virtuality variable (with $y \ll 1$) and z = momentum fraction.

- Result: almost everything cancels.
- Two terms do not quite cancel.

$$
[\mathcal{V}_{\rm a}(t) - \mathcal{S}_{\rm a}(t)]\big|\{p,f,c',c\}_m\big) =
$$

$$
\Bigg\{\int_0^{1/(1+y)}\frac{dz}{z}\sum_{\hat a}\frac{\alpha_s}{2\pi}\left(\frac{f_{\hat a/A}(\eta_{\rm a}/z,Q^2y)}{f_{a/A}(\eta_{\rm a},Q^2y)}P_{a\hat a}(z) - \delta_{a\hat a}\frac{2C_a z}{1-z}\right)[1\otimes 1]
$$

$$
-\int_0^1\frac{dz}{z}\sum_{\hat a}\frac{\alpha_s}{2\pi}\left(\frac{f_{\hat a/A}(\eta_{\rm a}/z,Q^2y)}{f_{a/A}(\eta_{\rm a},Q^2y)}P_{a\hat a}(z) - \delta_{a\hat a}\frac{2C_a z}{1-z}\right)[1\otimes 1]
$$

$$
+\cdots\Bigg\}\big|\{p,f,c',c\}_m\big)
$$

- $z < 1/(1+y)$ comes from splitting kinematics.
- $z < 1$ comes from parton evolution.

- This leaves an integration over a tiny range of z:

$$
[\mathcal{V}_{\rm a}(t) - \mathcal{S}_{\rm a}(t)]\big|\{p,f,c',c\}_m) =
$$

$$
\Bigg\{ \int_{1/(1+y)}^1 \frac{dz}{z} \sum_{\hat a} \frac{\alpha_s}{2\pi} \left(\delta_{a\hat a} \frac{2C_a z}{1-z} - \frac{f_{\hat a/A}(\eta_{\rm a}/z, Q^2 y)}{f_{a/A}(\eta_{\rm a}, Q^2 y)} P_{a\hat a}(z) \right) [1 \otimes 1]
$$

$$
+ \cdots \Bigg\} \big|\{p,f,c',c\}_m)
$$

- for $(1-z) < y/(1+y) \ll 1$, use

$$
P_{a\hat a}(z) \approx \delta_{a\hat a} \frac{2C_a z}{1-z}
$$

- This gives

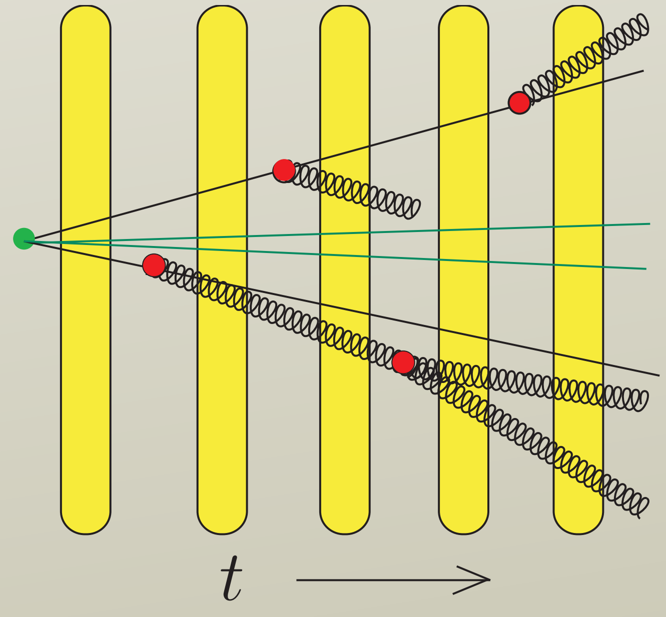
$$
\begin{aligned}
&[\mathcal{V}_{\rm a}(t) - \mathcal{S}_{\rm a}(t)]\big|\{p,f,c',c\}_m\big) = \\
&\Bigg\{\int_{1/(1+y)}^{1} dz\; \frac{\alpha_s}{2\pi}\,\frac{2C_a}{1-z}\left(1 - \frac{f_{a/A}(\eta_{\rm a}/z, Q^2 y)}{f_{a/A}(\eta_{\rm a}, Q^2 y)}\right)[1\otimes 1] \\
&\quad + \cdots \Bigg\}\big|\{p,f,c',c\}_m\big)
\end{aligned}
$$

- The $1/(1-z)$ factor creates the “threshold log.”
- For $y \ll 1$, this contribution is suppressed by a factor of y.
- But, the parton factor can be large, so we keep this.

Conclusion on threshold logs

- We find simple and intuitive leading order formulas.
- This is in the context of a leading order parton shower not "NLO," "NLL" or "NNLL."

- This is implemented as part of DEDUCTOR.
- The summation applies to all hard processes.
- The shower sums the threshold logs jointly with other large logs.



Some details

- There are more terms, some with non-trivial color.
- We need to account for switch to parton distributions based on virtuality instead of transverse momentum as the measure of hardness.

$$
\begin{aligned}
&\int_{t_1}^{t_2} d\tau\ (\mathcal{V}_{\rm a}(\tau) - \mathcal{S}_{\rm a}(\tau))\big|\{p,f,c',c\}_m\big) \\
&\quad = \int_{\mu_{\rm a}^2(t_2)}^{\mu_{\rm a}^2(t_1)} \frac{d\mu^2}{\mu^2} \left\{ \int_{1/(1+\mu^2/Q^2)}^{1} dz\ \frac{\alpha_{\rm s}(\lambda_{\rm R}(1-z)\mu^2)}{2\pi}\,\theta((1-z)\mu^2 > m_\perp^2(a)) \right. \\
&\qquad \times \left[1 - \frac{f_{a/A}(\eta_{\rm a}/z,\mu^2)}{f_{a/A}(\eta_{\rm a},\mu^2)}\right] \frac{2C_a}{1-z}\ [1\otimes 1] \\
&\qquad - \int_{1/(1+\mu^2/Q^2)}^{1} dz\ \frac{\alpha_{\rm s}(\lambda_{\rm R}\mu^2)}{2\pi}\,\theta((1-z)\mu^2 > m_\perp^2(a)) \\
&\qquad \times \sum_{\hat a} \frac{f_{\hat a/A}(\eta_{\rm a}/z,\mu_{\rm a}^2(t))}{z f_{a/A}(\eta_{\rm a},\mu_{\rm a}^2(t))}\, P_{a\hat a}^{\rm reg}(z)\ [1\otimes 1] \\
&\qquad - \int_{0}^{1/(1+\mu^2/Q^2)} dz\ \frac{\alpha_{\rm s}(\lambda_{\rm R}(1-z)\mu^2)}{2\pi}\,\theta((1-z)\mu^2 > m_\perp^2(a)) \\
&\qquad \times \left[1 - \frac{f_{a/A}(\eta_{\rm a}/z,\mu^2)}{f_{a/A}(\eta_{\rm a},\mu^2)}\right] \sum_{k\neq {\rm a,b}} \Delta_{{\rm a}k}(z,\mu^2/Q^2)\,\theta(\psi_{{\rm a}k} > \psi_{\rm min}) \\
&\qquad \times \big([(\boldsymbol{T}_{\rm a}\cdot \boldsymbol{T}_k)\otimes 1] + [1\otimes(\boldsymbol{T}_{\rm a}\cdot\boldsymbol{T}_k)]\big) \\
&\qquad \left. - \frac{\alpha_{\rm s}(\lambda_{\rm R}\mu^2)}{2\pi}\,{\rm i}\pi\,\big([(\boldsymbol{T}_{\rm a}\cdot\boldsymbol{T}_{\rm b})\otimes 1] - [1\otimes(\boldsymbol{T}_{\rm a}\cdot\boldsymbol{T}_{\rm b})]\big) \right\} \\
&\qquad \times \big|\{p,f,c',c\}_m\big)\ .
\end{aligned}
$$

← main

← $P^{\rm reg}$

← Δ with cut

← $\mathrm{i}\pi$

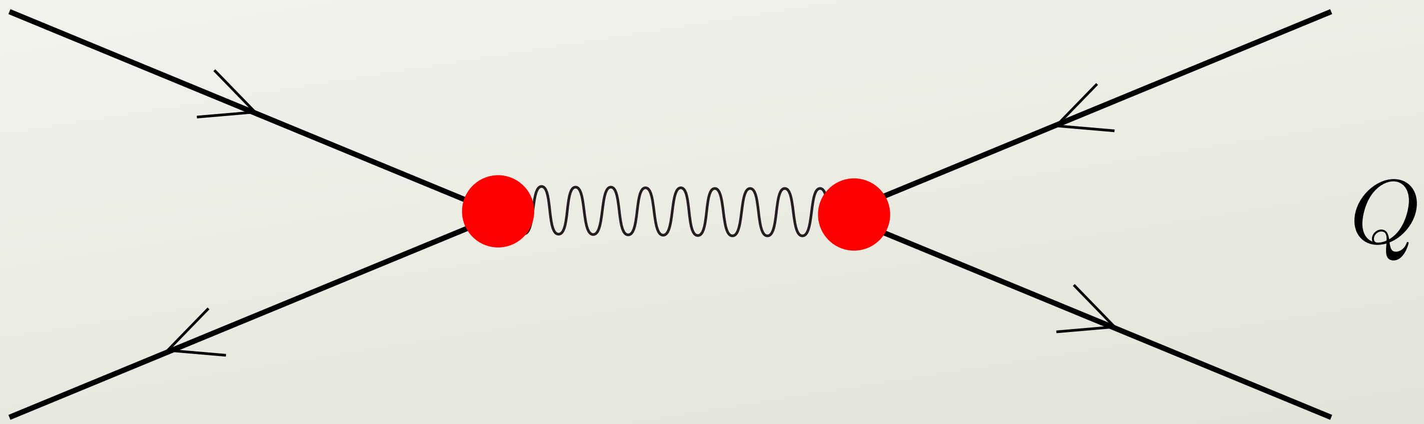
- Factors for changing the evolution of the parton distribution functions to reflect Λ ordering.

$$Z_a(\eta_{\rm a},\mu_0^2) = \exp\Bigg(\int_0^1 dz \int_{(1-z)\,\mu_0^2}^{\mu_0^2} \frac{d\mu_\perp^2}{\mu_\perp^2}\,\frac{\alpha_{\rm s}(\lambda_{\rm R}\mu_\perp^2)}{2\pi}\,\theta(\mu_\perp^2 > m_\perp^2(a)) \times \frac{2C_a}{1-z}\left\{1 - \frac{f_{a/A}(\eta_{\rm a}/z,\mu_0^2)}{f_{a/A}(\eta_{\rm a},\mu_0^2)}\right\}\Bigg)\,.$$

$$Z_b(\eta_{\rm b},\mu_0^2) = \exp\Bigg(\int_0^1 dz \int_{(1-z)\,\mu_0^2}^{\mu_0^2} \frac{d\mu_\perp^2}{\mu_\perp^2}\,\frac{\alpha_{\rm s}(\lambda_{\rm R}\mu_\perp^2)}{2\pi}\,\theta(\mu_\perp^2 > m_\perp^2(b)) \times \frac{2C_b}{1-z}\left\{1 - \frac{f_{b/B}(\eta_{\rm a}/z,\mu_0^2)}{f_{b/B}(\eta_{\rm b},\mu_0^2)}\right\}\Bigg)\,.$$

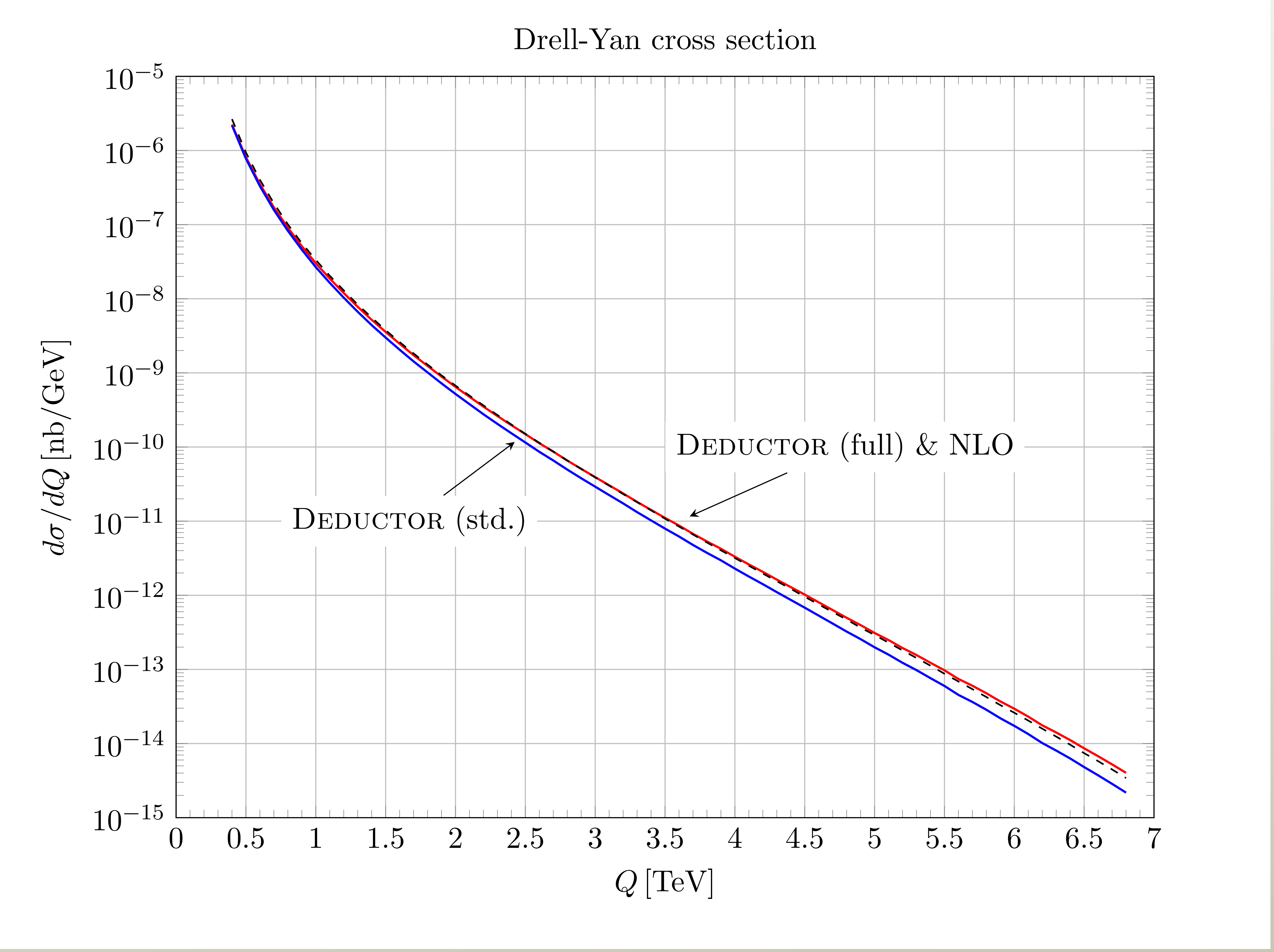
Numerical Results

Drell-Yan process



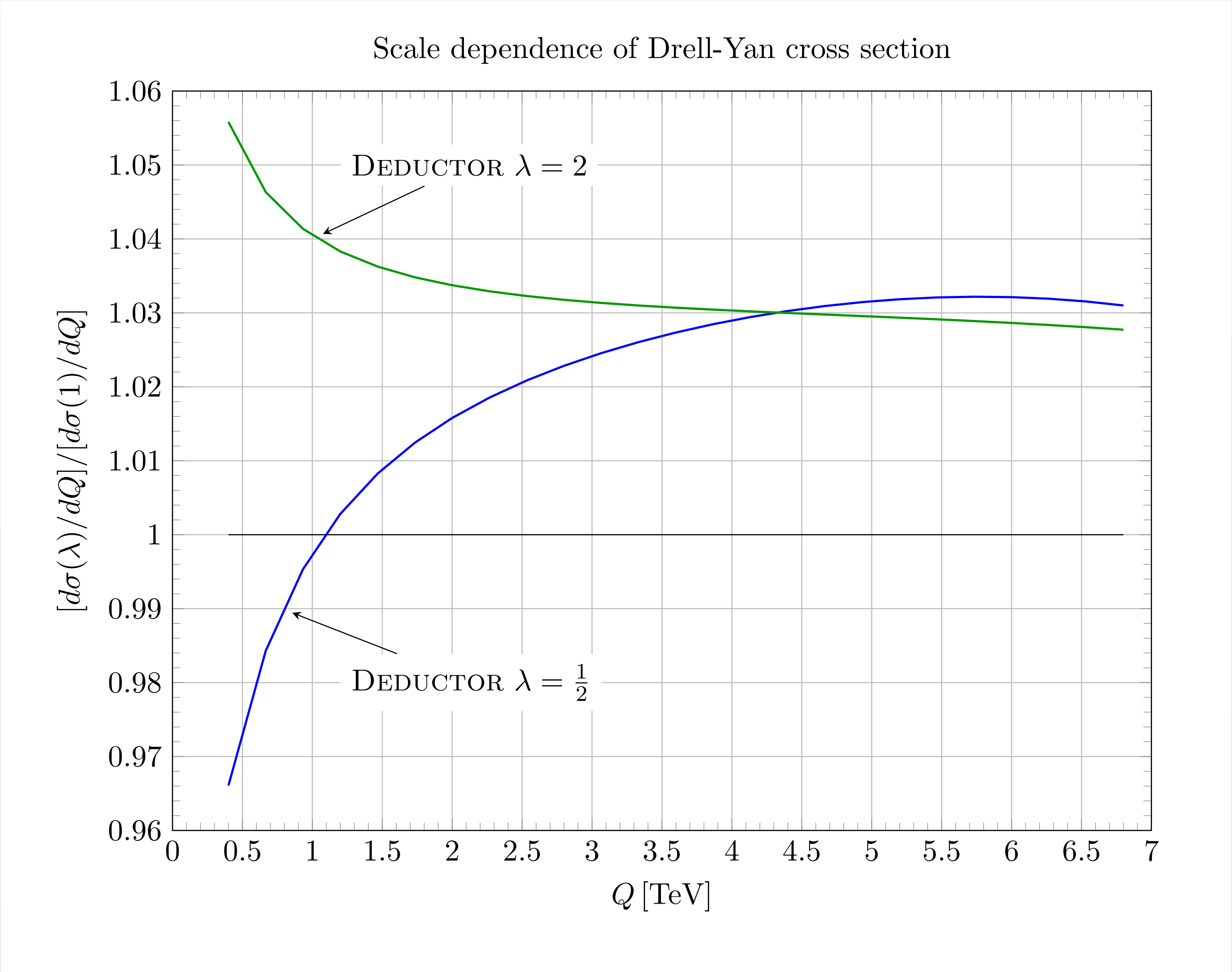
- $\mathrm{p} + \mathrm{p} \to \mu^+ + \mu^- + X$ with $Q^2 = [p(\mu^+) + p(\mu^-)]^2$.
- No threshold effects: DEDUCTOR (std.).
- Full threshold effects: DEDUCTOR (full).
- Next-to-leading order perturbative calculation (with MCFM): NLO.

Cross section $d\sigma/dQ$



Scale dependence

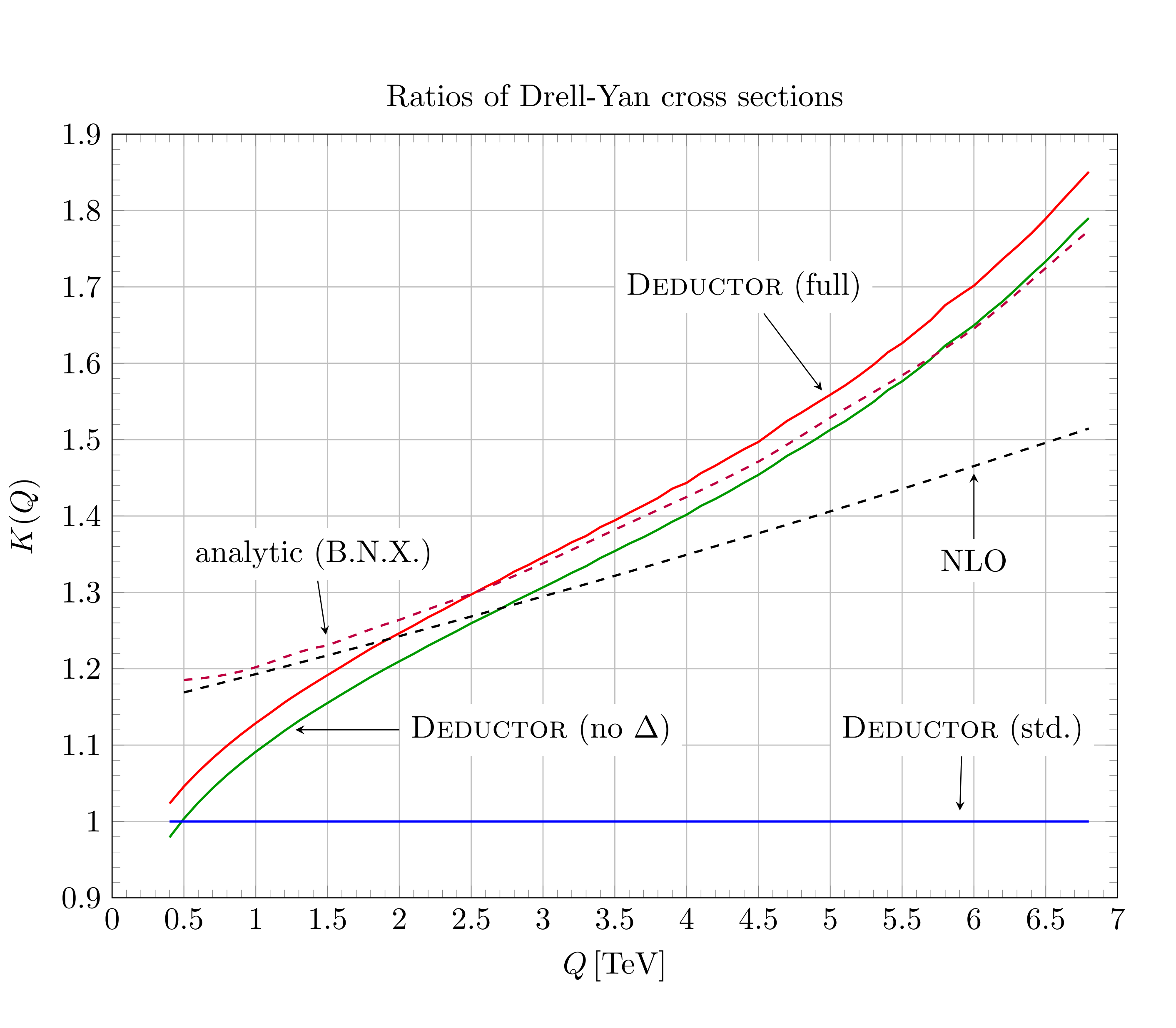
$$\mu_{\mathrm{F}} = \mu_{\mathrm{R}} = \lambda Q$$



Ratios to Deductor (std.)

$$K = \frac{d\sigma/dQ}{d\sigma(\text{std.})/dQ}$$

- (full): all terms in threshold factor
- (no Δ): omit Δ term in threshold factor
- NLO: perturbative calculation at NLO
- (B.N.X.): analytic sum of threshold logs by Becher, Neubert, Xu



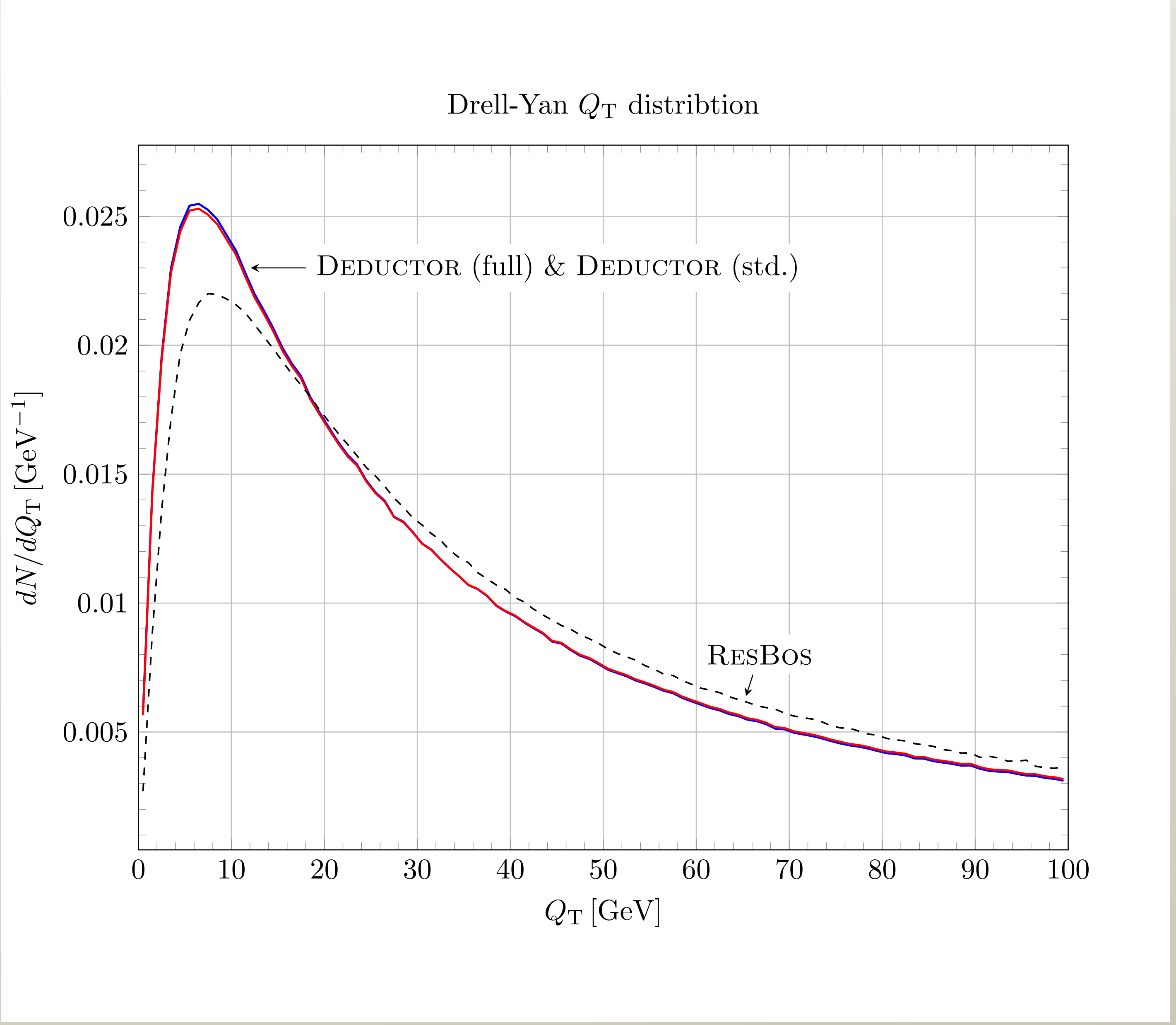
Transverse momentum distribution

- $2.0 \text{ TeV} < Q < 2.1 \text{ TeV}$.
- For $Q_{\rm T}^2 \ll Q^2$, we sum $Q_{\rm T}$ and threshold logs.

$$\int_0^{100\ \text{GeV}} \frac{dN}{dQ_{\rm T}} = 1$$

- Deductor (full) and Deductor (std.) are almost identical.

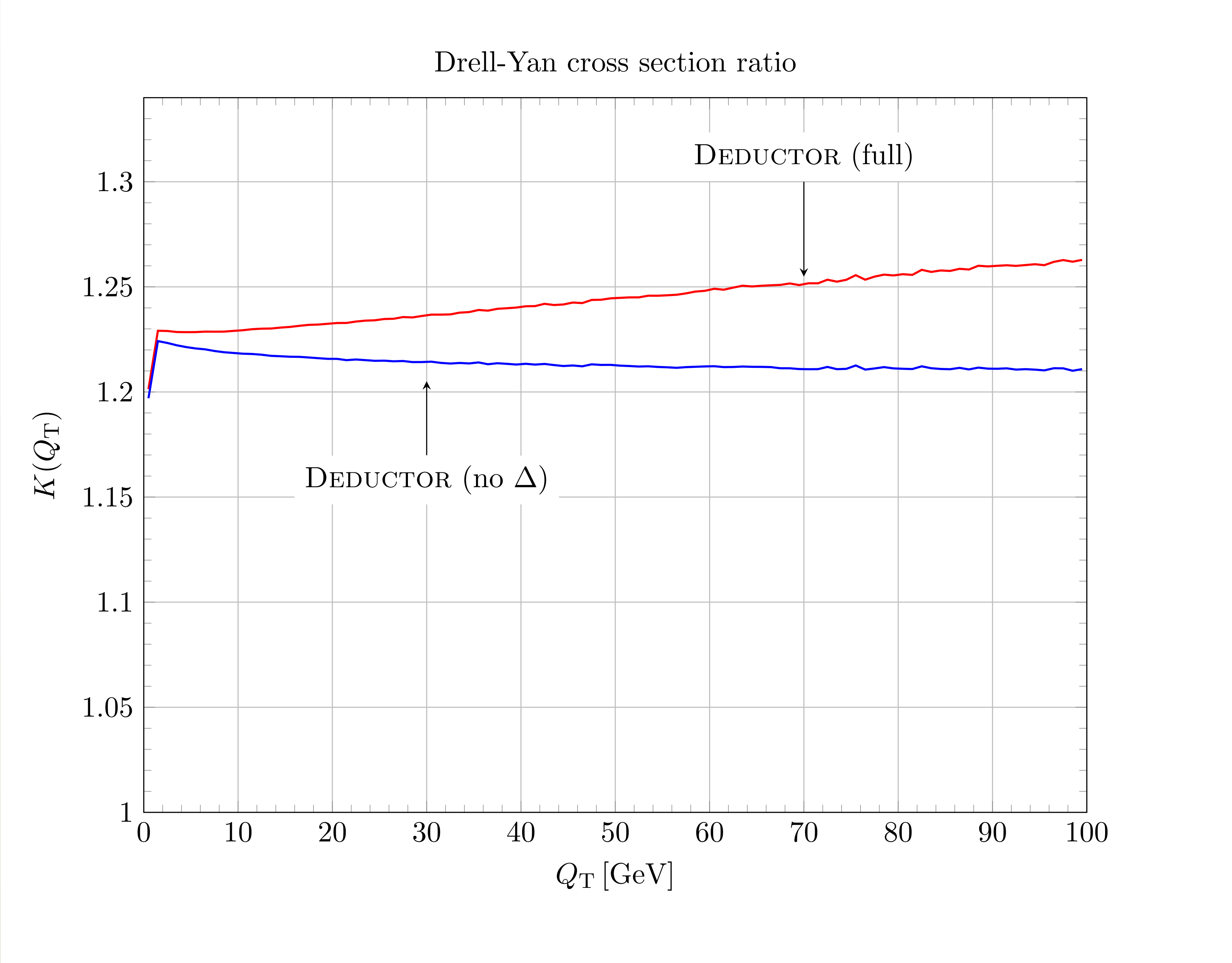
- Analytic ResBos summation should be somewhat broader and is.



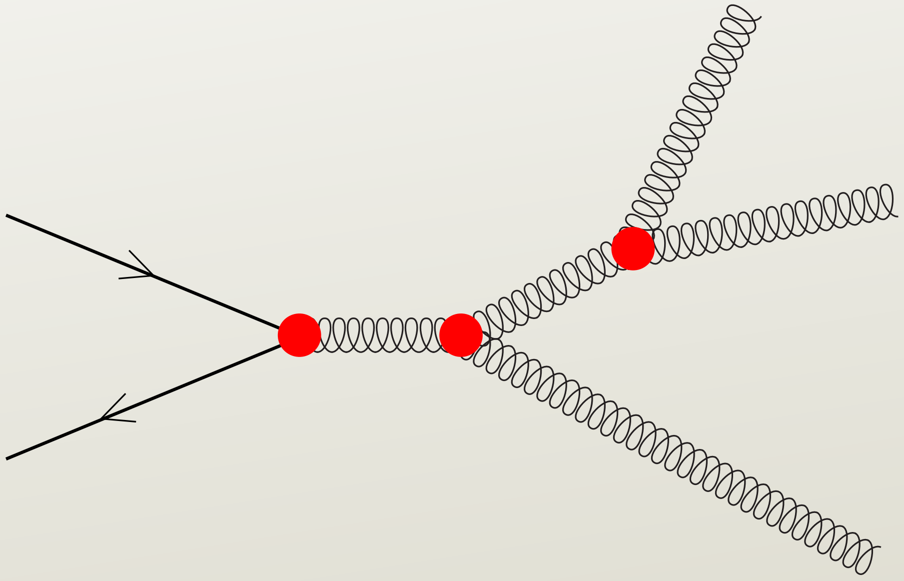
Ratio of cross section to Deductor (std.)

$$K = \frac{d\sigma/dQ_{\rm T}}{d\sigma({\rm std.})/dQ_{\rm T}}$$

- $K(\text{no }\Delta) > 1$ and $K(\text{full}) > 1$.
- Both are pretty flat.

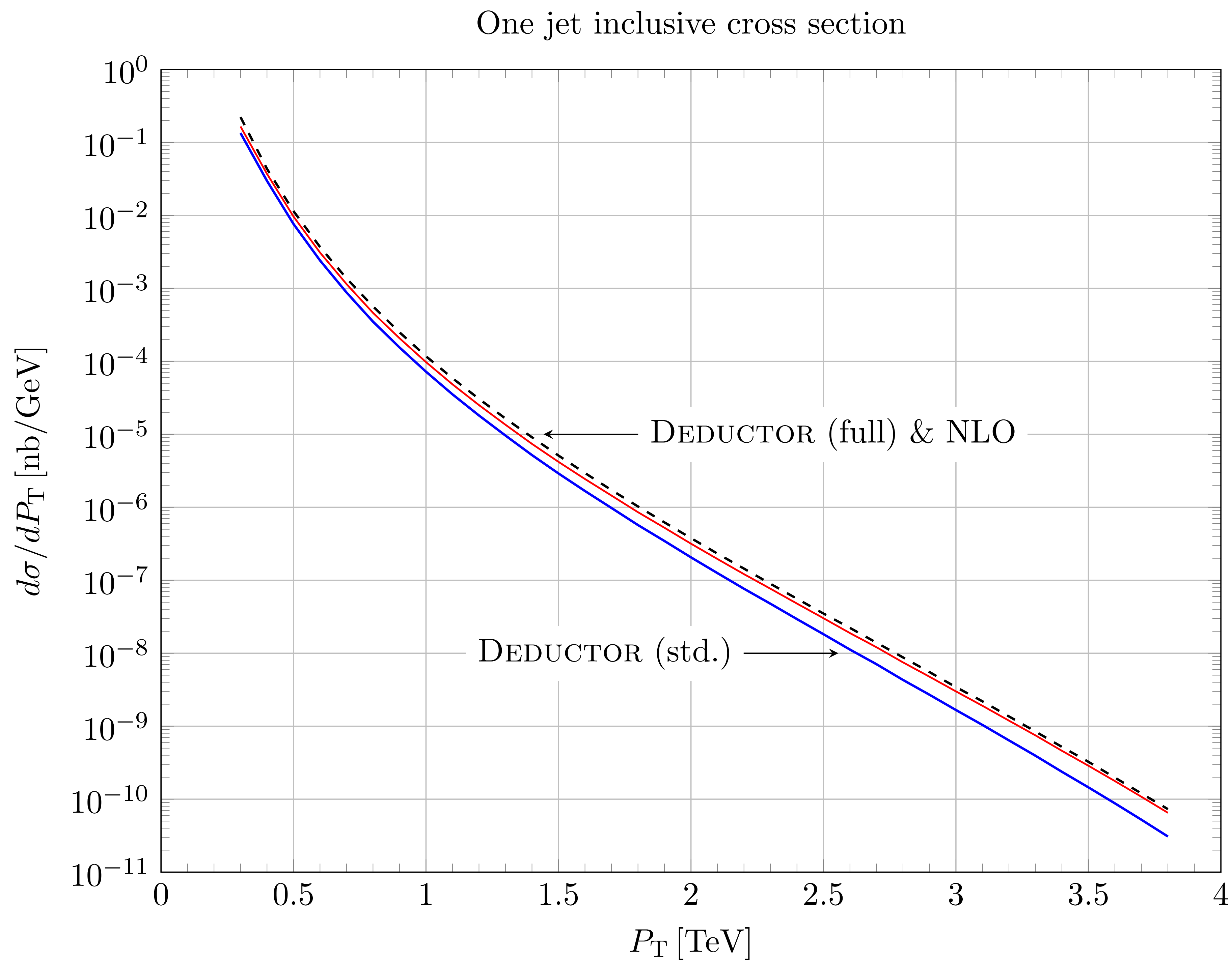


Jet production



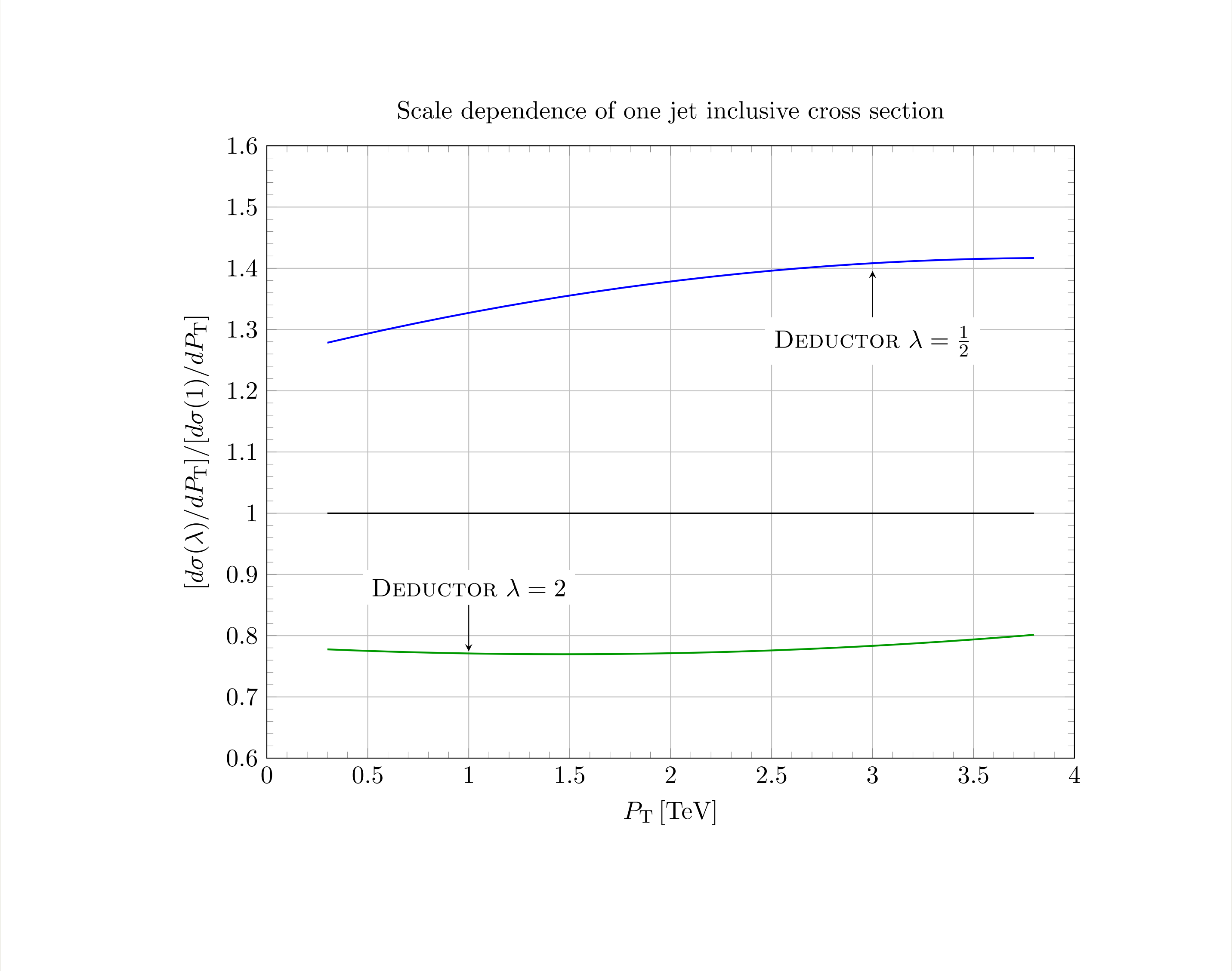
- $\mathrm{p} + \mathrm{p} \to \mathrm{jet} + X$ vs. P_{T} of jet.
- Use anti-k_{T} algorithm with $R = 0.4$.
- Include threshold logs.
- Include probability for daughter partons to make a jet.

Cross section $d\sigma/dP_{\mathrm{T}}$



Scale dependence

$$\mu_F = \mu_R = \lambda P_T$$



Ratios to DEDUCTOR (LO)

$$K = \frac{d\sigma/dP_\mathrm{T}}{d\sigma(\mathrm{LO})/dP_\mathrm{T}}$$

$d\sigma(\mathrm{std.})/dP_\mathrm{T} < d\sigma(\mathrm{LO})/dP_\mathrm{T}$

$d\sigma(\mathrm{full})/dP_\mathrm{T} > d\sigma(\mathrm{std.})/dP_\mathrm{T}$

$d\sigma(\mathrm{NLO})/dP_\mathrm{T}$ is larger.

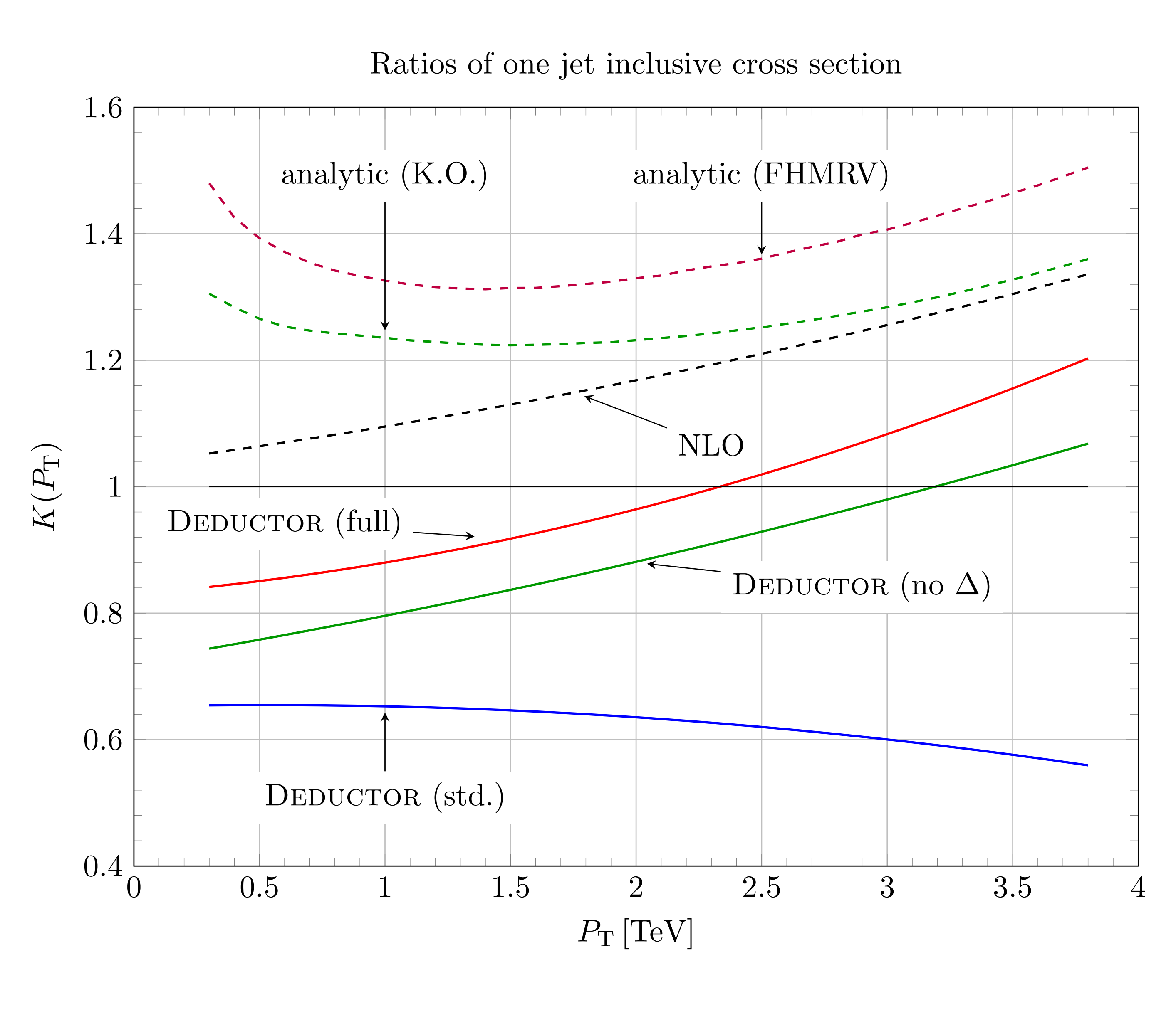
NLO matching needed.

Two analytic summations.

K.O. = Kidonakis-Owens

F.H.M.R.V. =

de Florian, Hinderer, Mukherjee, Ringer, Vogelsang.
(more sophisticated)



General conclusion

- Parton shower event generators can sum logarithms.
- They are leading order, so not as precise as SCET.
- But they are useful because they are more general.
- Summing threshold logs with a parton shower is possible.