

# Soft Gluon Resummation in Higgs plus jets Production at LHC

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# Outline

- Soft gluon resummation (  $K_T$  resummation )
- soft gluon resummation in Higgs plus jets production at LHC
- Summary

# soft gluon resummation ( $K_T$ resummation)

- Consider the production process  $pp \rightarrow H(Z)+X$

$$\frac{d\sigma}{dQ_T^2} \sim \frac{1}{Q_T^2} \left\{ \begin{aligned} &\alpha_S(L+1) + \alpha_S^2(L^3 + L^2) + \alpha_S^3(L^5 + L^4) + \alpha_S^4(L^7 + L^6) + \dots \\ &+ \alpha_S^2(L+1) + \alpha_S^3(L^3 + L^2) + \alpha_S^4(L^5 + L^4) + \dots \\ &+ \alpha_S^3(L+1) + \alpha_S^4(L^3 + L^2) + \dots \end{aligned} \right\}$$

Where  $Q_T$  is the transverse momentum, and  $Q$  is the mass of  $H(Z)$ , and  $L = \text{Log}[Q^2 / Q_T^2]$ .

- We have to resum these large logs to make reliable predictions

Differential cross section can be written as :

$$\frac{d^3\sigma(M^2, q_\perp, y)}{d^2q_\perp dy} = \sigma_0 \int \frac{d^2\vec{b}}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} W(x_1, x_2, b, M^2)$$

W term can be factorized into :

$$W(x_1, x_2, b, M^2) = x_1 f(x_1, b, \xi_1, \mu) x_2 f(x_2, b, \xi_2, \mu) H(M^2, \mu) S(b, \mu)$$

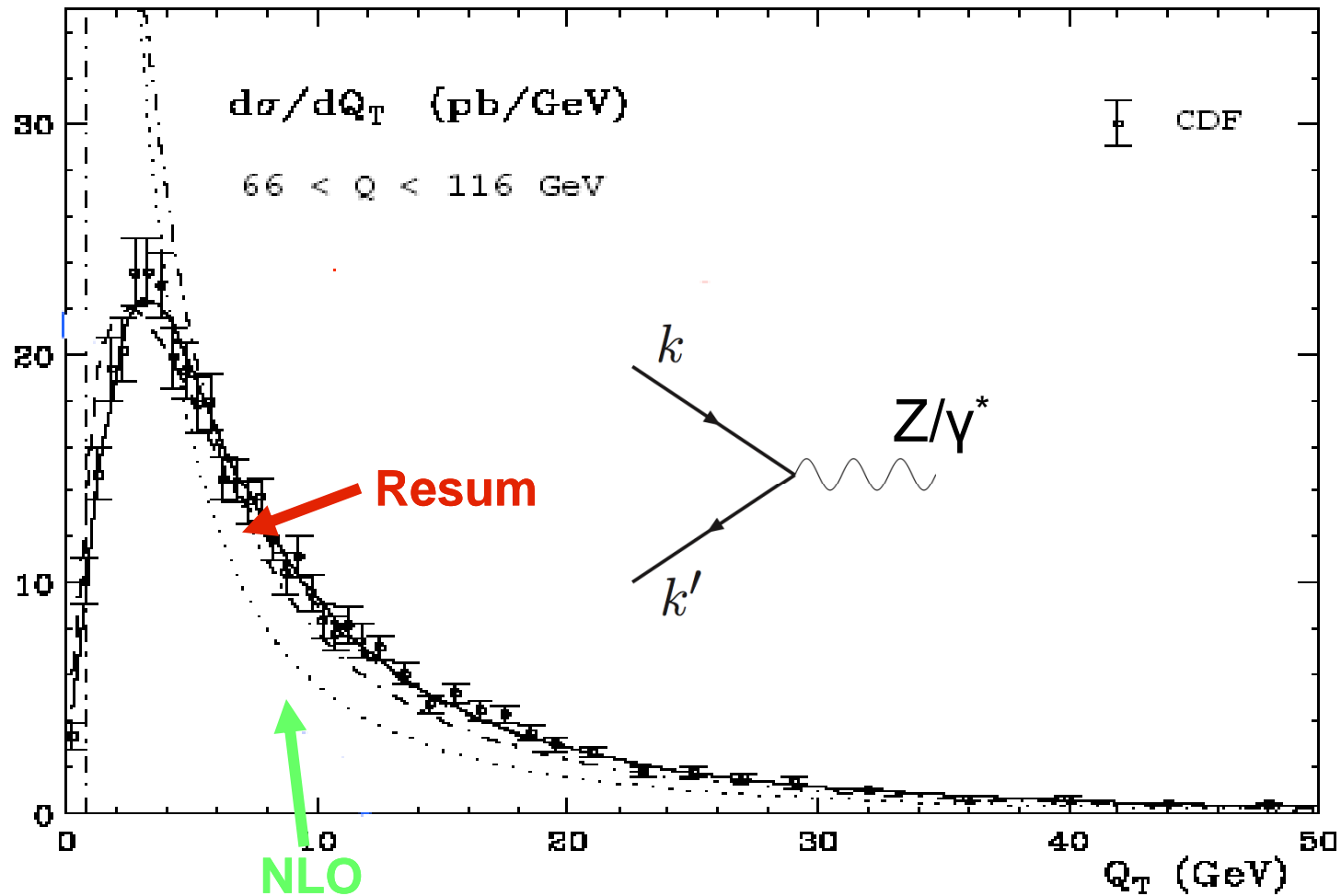
After evolving all the scales in W term to the same point

$$W(x_1, x_2, b, M^2) = x_1 f(x_1, b, b, b) x_2 f(x_2, b, b, b) H(M, \mu) \exp \left\{ - \int_{b_0^2/b^2}^{\mu^2} \left( A \ln \frac{M^2}{\mu^2} + B \right) \frac{d\mu^2}{\mu^2} \right\}$$

$$\text{For } g \rightarrow H + X \quad A^{(1)} = C_A a_s / \pi \quad B^{(1)} = -2C_A \beta_0 a_s / \pi$$

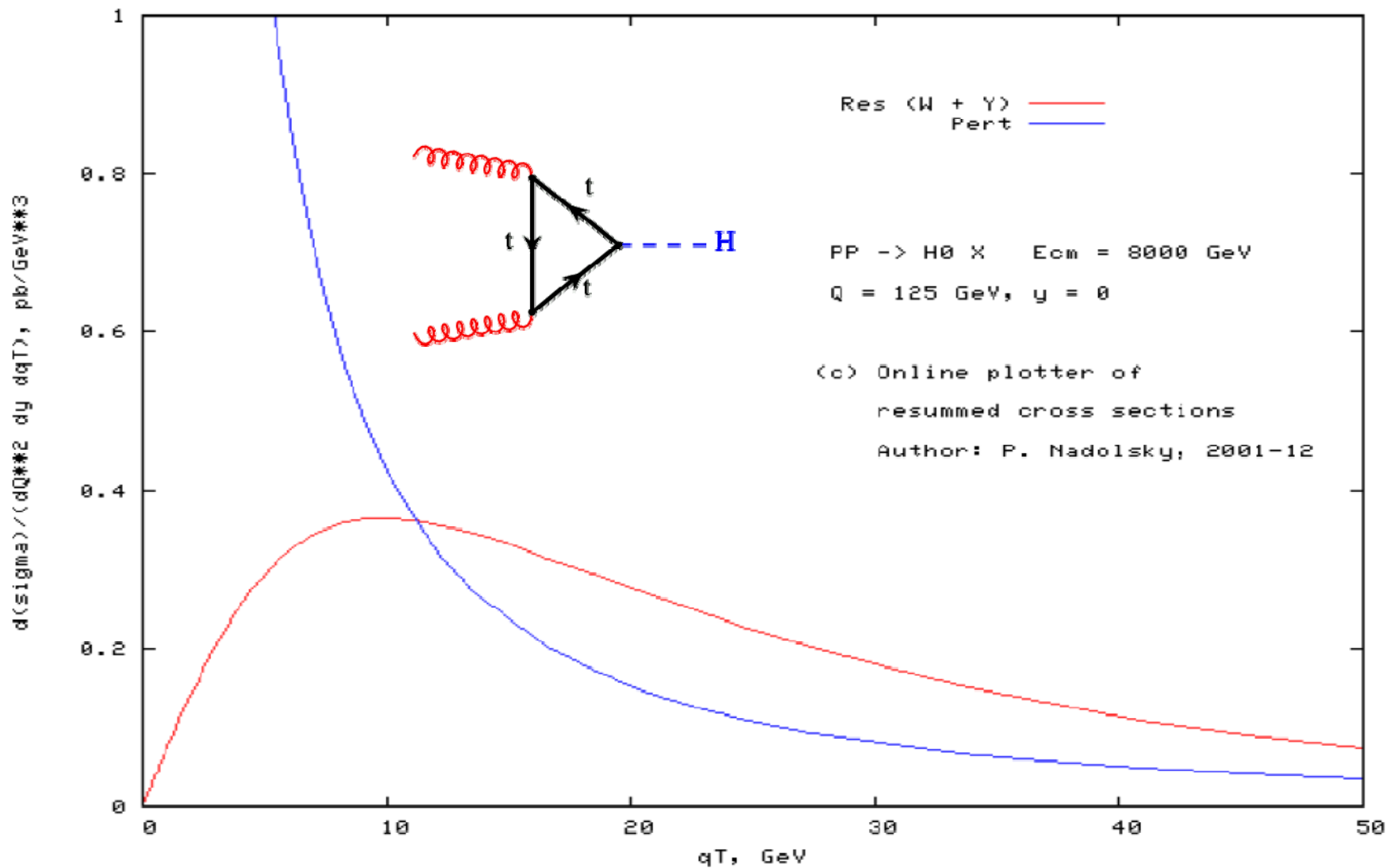
$$\text{For } q \rightarrow Z + X \quad A^{(1)} = C_F a_s / \pi \quad B^{(1)} = -2C_F / 3 a_s / \pi$$


# Z production in pp collision



Kulesza, Sterman, Vogelsang, 02

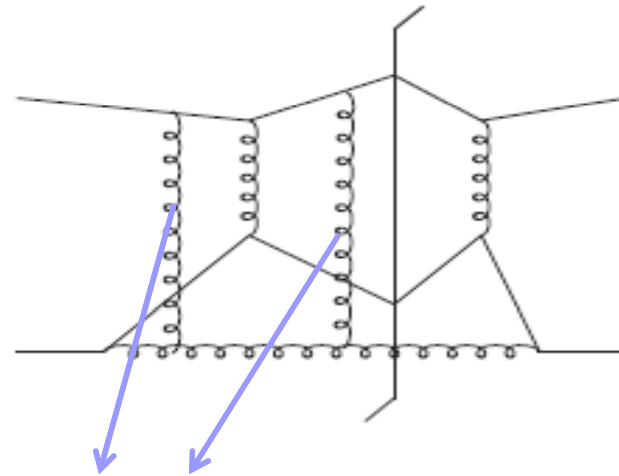
# Higgs production in pp collision



- 
- The  $K_T$  resummation works very well in Drell-Yan or Higgs inclusive production processes
  - We can apply it into the processes including three, four or more color particals

# Factorization breaking

factorization  
breaking effects  
Collins-Qiu, 2007;  
Vogelsang-Yuan,  
2007; Rogers-  
Mulders 2010



Glauber gluon ( $l^+ l^- \ll l_t^2$  &  $l_t \sim \lambda$ )  
It breaks ward identities

- 1) It shows at NNLO for unpolarized pp collision, unless you integrate out  $K_T$
- 2) We assume the breaking effect is about  $M_p/K_T$ , when  $M_p \ll K_T \ll Q$ , the breaking effect is not large comparing with pQCD contribution



# Heavy quark pair production at the hadron colliders

C. S. Li et al Phys.Rev.  
D88 (2013) 074004

The cross section can be factorized into

$$W_{kl}(x_i, b) = x_1 f_l(x_1, b, \xi_1^2, \mu^2, \rho) x_2 f_k(x_2, b, \xi_2^2, \mu^2, \rho) \text{Tr} [\mathbf{H}(Q^2, \mu^2, \rho) \mathbf{S}(b, \mu^2, \rho)]$$

The Hard and soft part have to be expanded by a group of color basis

For the channel  $q(i) + \bar{q}(j) \rightarrow t(k) + \bar{t}(l)$ , we adopt the bases

$$C_1(i, j, k, l) = \delta_{ij} \delta_{kl}, \quad C_2(i, j, k, l) = T_{ij}^d T_{kl}^d.$$

While for  $g(a) + g(b) \rightarrow t(k) + \bar{t}(l)$ , we have the bases

$$C_1(a, b, k, l) = \delta^{ab} \delta_{kl}, \quad C_2(a, b, k, l) = i f^{abd} T_{kl}^d, \quad C_3(a, b, k, l) = d^{abd} T_{kl}^d$$

# The soft factor's definition

We can define the soft factor as:

$$\mathcal{L}_n^\dagger(\zeta^-, \zeta_\perp) = P \exp \left\{ -ig \int_{-\infty}^0 d\lambda n \cdot A^a(\lambda n + \zeta) T_a \right\}$$

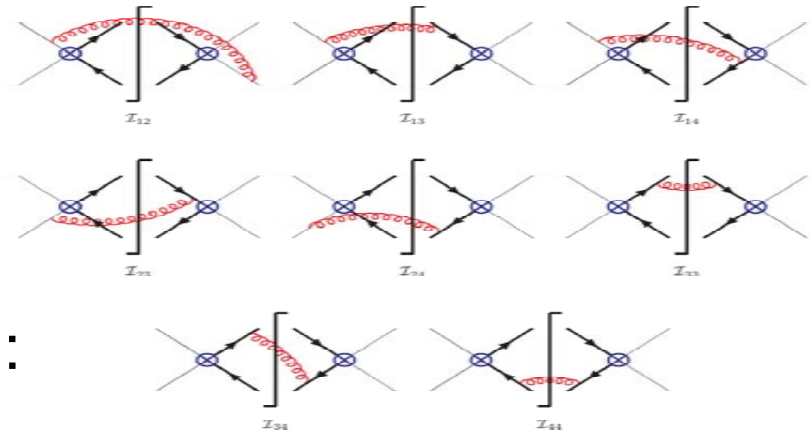
$$S_{IJ} = \int_0^\pi \frac{(\sin \phi)^{-2\epsilon}}{\frac{\sqrt{\pi} \Gamma(\frac{1}{2} - \epsilon)}{\Gamma(1 - \epsilon)}} d\phi C_{Iii'}^{bb'} C_{Jll'}^{aa'} \langle 0 | \mathcal{L}_{vcb'}^\dagger \mathcal{L}_{vbc} \mathcal{L}_{\bar{v}ca'}^\dagger \mathcal{L}_{\bar{v}ac} \mathcal{L}_{nji}^\dagger \mathcal{L}_{ni'k} \mathcal{L}_{nkl}^\dagger \mathcal{L}_{nl'j} | 0 \rangle$$

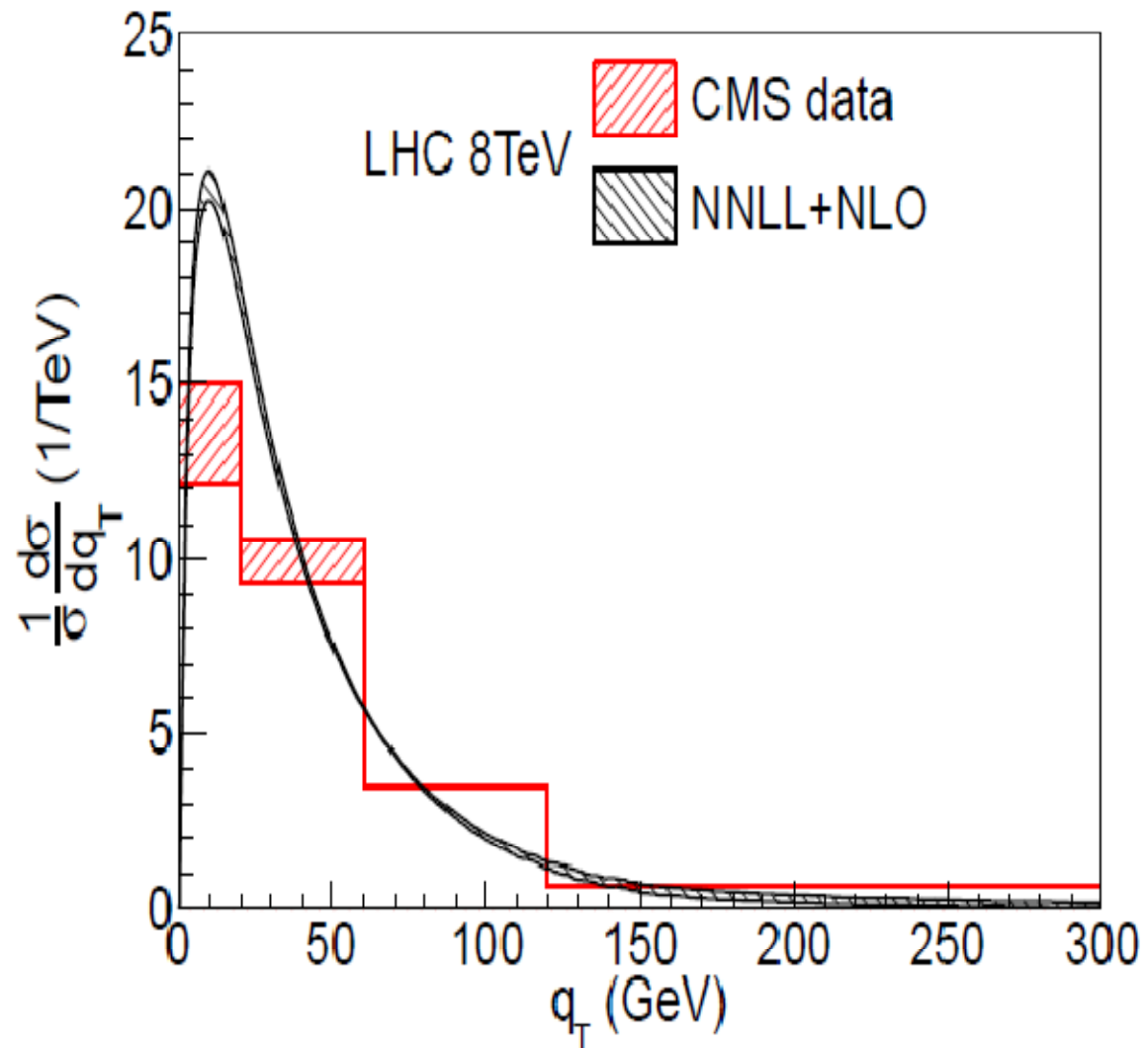
the evolution equation:

$$\frac{d}{d \ln \mu} S_{i\bar{i}}(\mu) = -\gamma_{i\bar{i}}^{s\dagger} S_{i\bar{i}}(\mu) - S_{i\bar{i}}(\mu) \gamma_{i\bar{i}}^s$$


Then you can get the W function:

$$W_{kl} \left( x_i, b_\perp, \frac{C_1^2}{C_2^2 b_\perp^2} \right) = f_k(x_A, C_1^2 / (C_2^2 / b_\perp^2)) f_l(x_B, C_1^2 / (C_2^2 / b_\perp^2)) \\ \times Tr \left[ \mathbf{H}(M_{c\bar{c}}^2, M_{c\bar{c}}^2) \text{EXP} \left\{ - \int_{C_1^2 / b_\perp^2}^{M_{c\bar{c}}^2} \frac{d\mu}{\mu} \gamma_{i\bar{i}}^{s\dagger} \right\} \mathbf{S}(b, \frac{C_1^2}{C_2^2 b_\perp^2}) \text{EXP} \left\{ - \int_{C_1^2 / b_\perp^2}^{M_{c\bar{c}}^2} \frac{d\mu}{\mu} \gamma_{i\bar{i}}^s \right\} \right]$$





**C. S. Li et al Phys.Rev. D88 (2013) 074004**



# Soft Gluon Resummations in Dijet Azimuthal Angular Correlations at the Collider

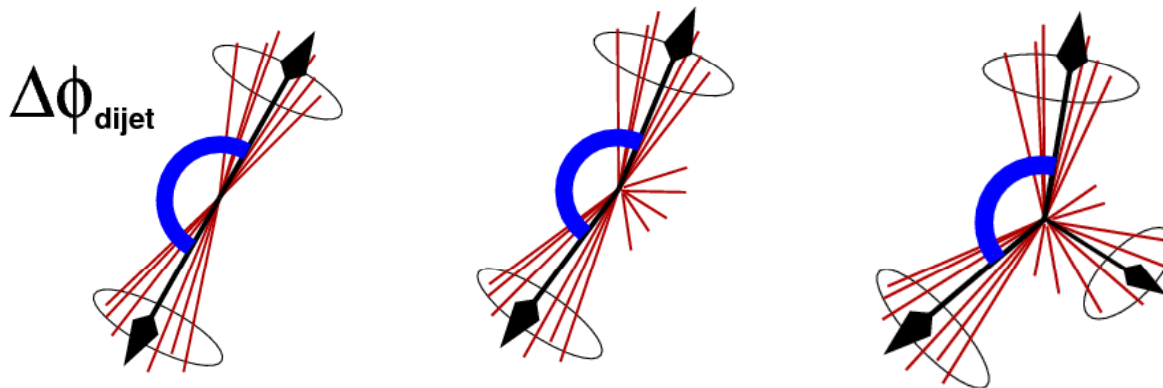
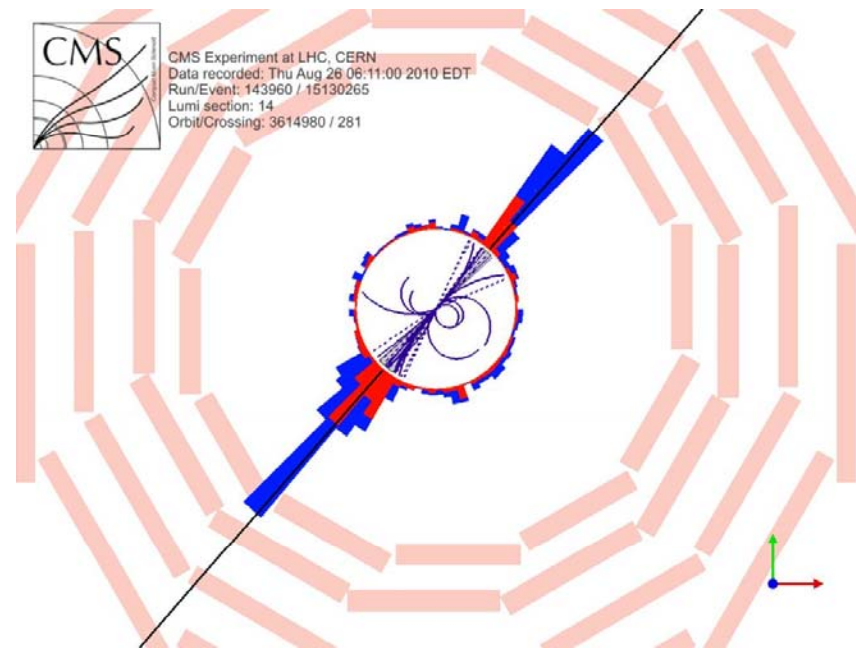
Peng Sun,<sup>1</sup> C.-P. Yuan,<sup>2</sup> and Feng Yuan<sup>1</sup>

## Abstract

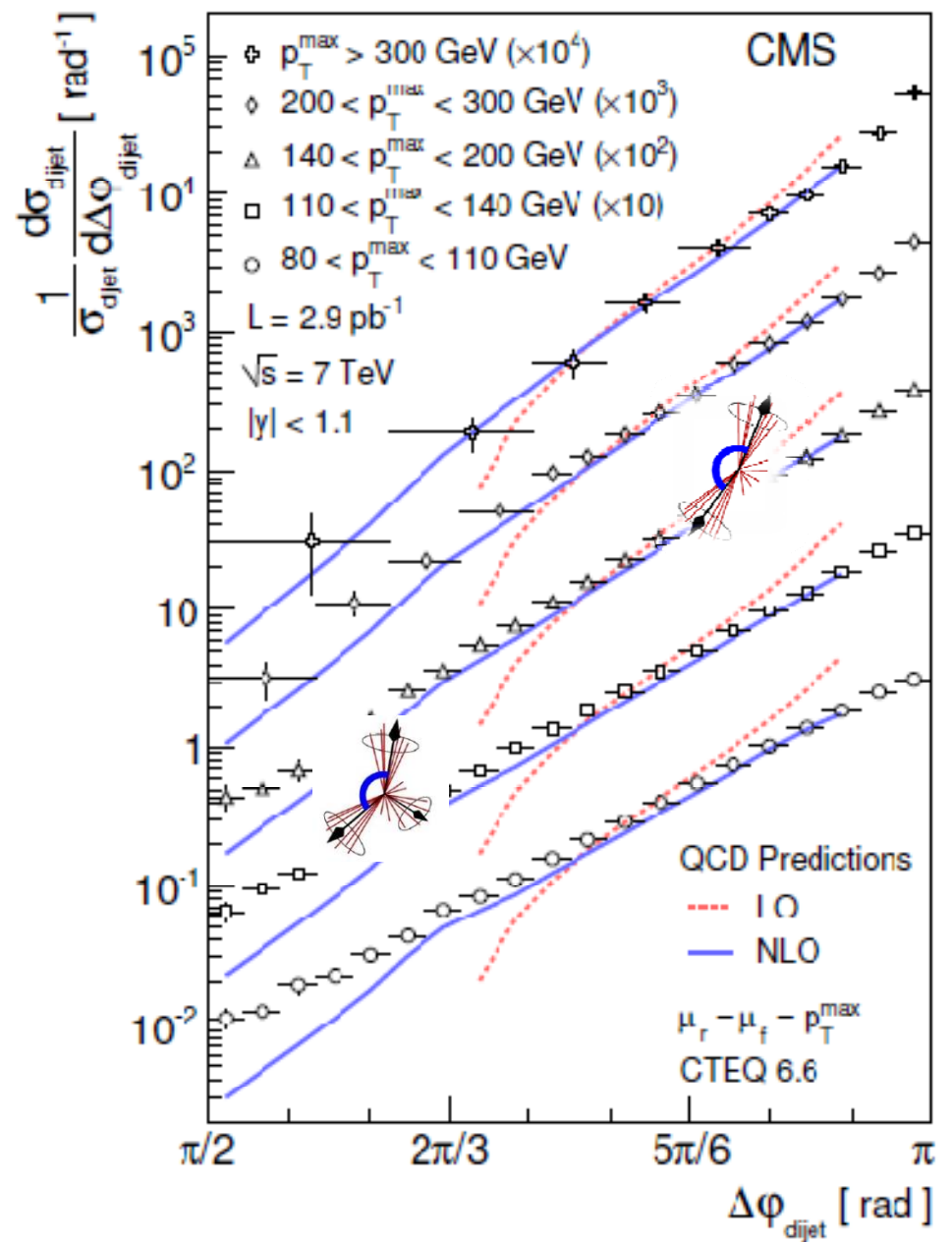
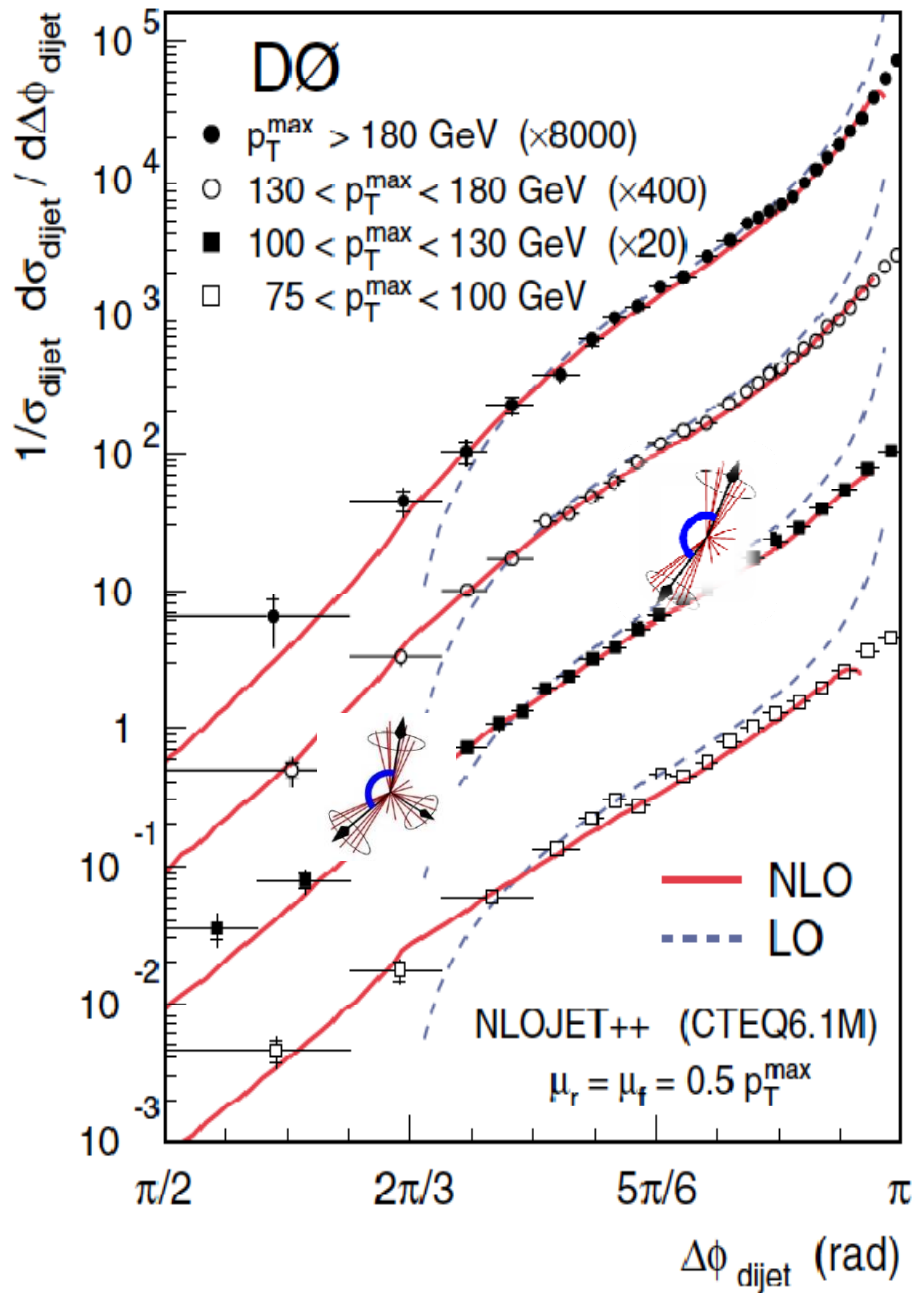
We derive the all order soft gluon resummation in dijet azimuthal angular correlation in  $pp$  collisions at the next-to-leading logarithmic level. The relevant coefficients for the resummation Sudakov factor, and the soft and hard factors are calculated. The theory predictions agree well with the experimental data from D0 collaboration at the Tevatron.

# Dijet production at the hadron colliders

- Most abundant events
- Almost back-to-back
- De-correlation comes
  - Hard gluon jet
  - Soft gluon radiation



2016/10/27

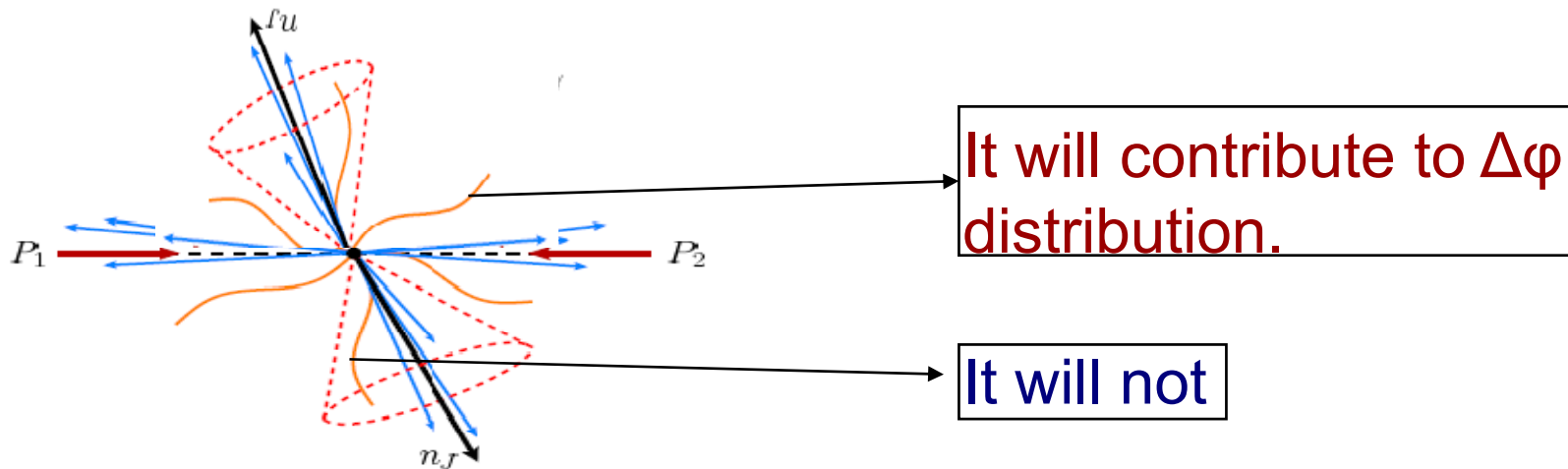


There three kinds of large logarithms in the processes:  
 $(\text{Log}(q_{\perp}/P_J))^2$  ,  $\text{Log}(q_{\perp}/P_J)$  and  $\text{Log}(R)\text{Log}(q_{\perp}/P_J)$

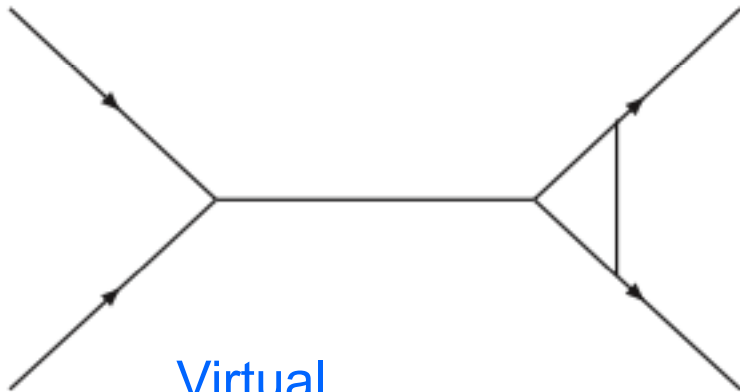
$$\frac{d^4\sigma}{dy_1 dy_2 dP_J^2 d^2q_{\perp}} = \sum_{ab} \sigma_0 \left[ \int \frac{d^2\vec{b}_{\perp}}{(2\pi)^2} e^{-iq_{\perp} \cdot b_{\perp}} W_{ab \rightarrow cd}(x_1, x_2, b_{\perp}) + Y_{ab \rightarrow cd} \right]$$

where

$$W_{ab \rightarrow cd}(x_i, b) = x_1 f_a(x_1, b, \zeta^2, \mu^2, \rho) x_2 f_b(x_2, b, \bar{\zeta}^2, \mu^2, \rho) \text{Tr} [\mathbf{H}_{ab \rightarrow cd}(Q^2, \mu^2, \rho) \mathbf{S}_{ab \rightarrow cd}(b, \mu^2, \rho)]$$

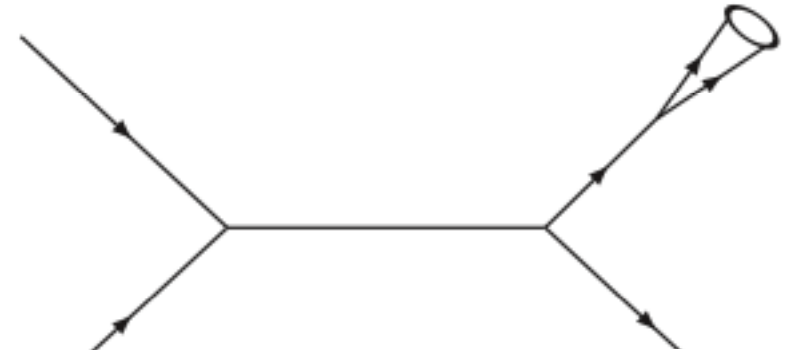


# Soft and collinear gluon at one-loop



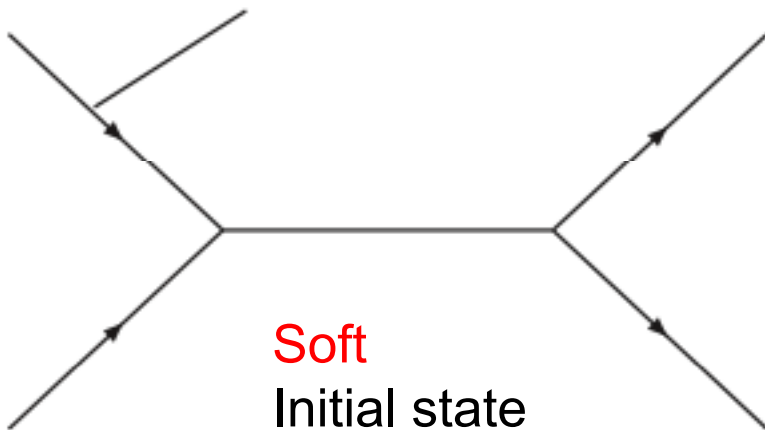
Virtual

Ellis-Sexton 86



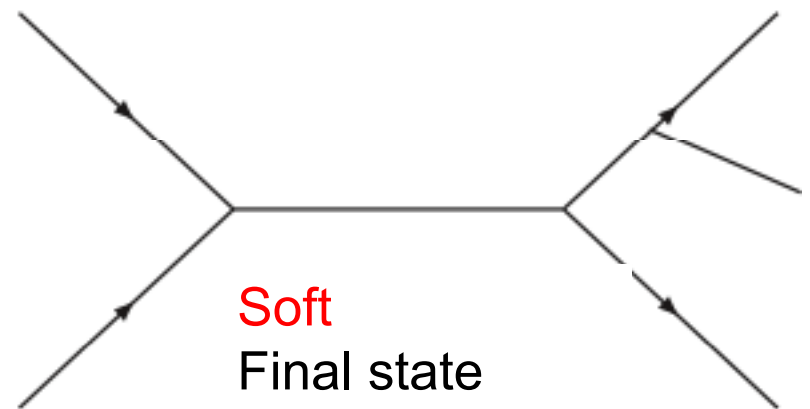
Jet (Narrow Jet Approx.)

Jager-Stratmann-Vogelsang  
2004



Soft

Initial state



Soft

Final state  
(out of jet cone)



$$S_{IJ} = \int_0^\pi \frac{d\phi_0}{\pi} C_{Ii i'}^{bb'} C_{Jl l'}^{aa'} \langle 0 | \mathcal{L}_{vcb'}^\dagger(b_\perp) \mathcal{L}_{vbc}(b_\perp) \mathcal{L}_{\bar{v}ca'}^\dagger(0) \mathcal{L}_{\bar{v}ac}(0) \mathcal{L}_{nji}^\dagger(b_\perp) \mathcal{L}_{\bar{n}i'k}(b_\perp) \mathcal{L}_{\bar{n}kl}^\dagger(0) \mathcal{L}_{nl'j}(0) | 0 \rangle$$

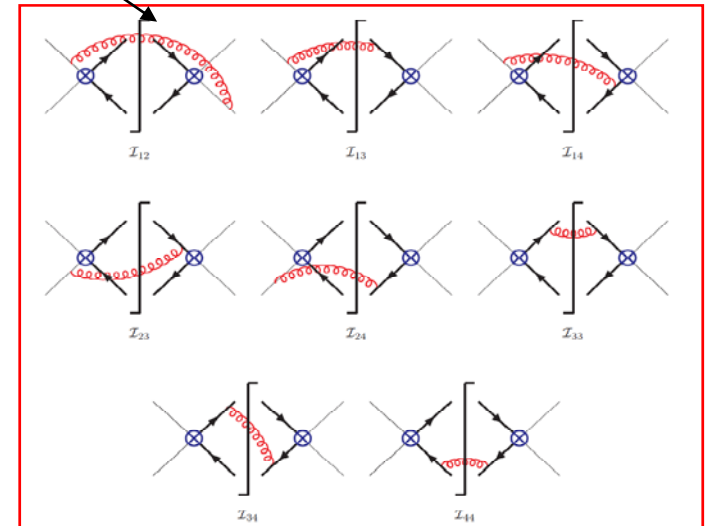
The soft factor satisfies

$$\frac{d}{d \ln \mu} S_{IJ}(\mu) = -\Gamma_{IJ'}^{s\dagger} S_{J'J}(\mu) - S_{IJ'}(\mu) \Gamma_{J'J}^s$$

$$c_1 = f^{a_1 a_2 c_1} f_{a_3 a_4 c_1}, \quad c_2 = f^{a_1 a_3 c_1} f_{a_2 a_4 c_1} + f^{a_1 a_4 c_1} f_{a_2 a_3 c_1}, \quad c_3 = d^{a_1 a_2 c_1} f_{a_3 a_4 c_1},$$

$$c_4 = f^{a_1 a_2 c_1} d_{a_3 a_4 c_1}, \quad c_5 = d^{a_1 a_4 c_1} f_{a_2 a_3 c_1}, \quad c_6 = \delta^{a_1 a_2} \delta^{a_3 a_4}, \quad c_7 = \delta^{a_1 a_3} \delta^{a_2 a_4}, \quad c_8 = \delta^{a_1 a_4} \delta^{a_2 a_3}$$

$$c_1 = \delta^{a_1 a_2} \delta_{a_3 a_4}, \quad c_2 = i f^{a_1 a_2 c} t_{a_3 a_4}^c, \quad c_3 = d^{a_1 a_2 c} t_{a_3 a_4}^c$$



# Cross checks

- Divergences cancelled out between virtual, jet, soft contributions (dimension regulation applied)
- Final results : **double logs**, **single logs**, ..

$$\begin{aligned}
 W^{(1)}(b_{\perp})|_{logs.} = & \frac{\alpha_s}{2\pi} \left\{ h_{q_i q_j \rightarrow q_i q_j}^{(0)} \left[ -\ln \left( \frac{\mu^2 b_{\perp}^2}{b_0^2} \right) (\mathcal{P}_{qq}(\xi)\delta(1-\xi') + \mathcal{P}_{qq}(\xi')\delta(1-\xi)) - \delta(1-\xi) \right. \right. \\
 & \times \delta(1-\xi') \left( \underline{C_F \ln^2 \left( \frac{Q^2 b_{\perp}^2}{b_0^2} \right)} + \ln \left( \frac{Q^2 b_{\perp}^2}{b_0^2} \right) \left( \underline{-3C_F + C_F \ln \frac{1}{R_1^2} + C_F \ln \frac{1}{R_2^2}} \right) \right) \left. \right] \\
 & \left. - \delta(1-\xi)\delta(1-\xi') \ln \left( \frac{Q^2 b_{\perp}^2}{b_0^2} \right) \Gamma_{sn}^{(qq')} \right\} , \tag{71}
 \end{aligned}$$

Quark channel:  $q_i q_j \rightarrow q_i q_j$

After solving the evolution equations

$$W_{ab \rightarrow cd}(x_1, x_2, b) = x_1 f_a(x_1, \mu = c_0/b_\perp) x_2 f_b(x_2, \mu = c_0/b_\perp) e^{-S_{Sud}(Q^2, b_\perp)} \\ \times \text{Tr} \left[ \mathbf{H}_{ab \rightarrow cd} \exp\left[-\int_{c_0/b_\perp}^Q \frac{d\mu}{\mu} \gamma^{s\dagger}\right] \mathbf{S}_{ab \rightarrow cd} \exp\left[-\int_{c_0/b_\perp}^Q \frac{d\mu}{\mu} \gamma^s\right] \right]$$

where

$$S_{Sud}(Q^2, b_\perp, C_1, C_2) = \int_{C_1^2/b_\perp^2}^{C_2^2 Q^2} \frac{d\mu^2}{\mu^2} \left[ \ln\left(\frac{Q^2}{\mu^2}\right) A + B + D_1 \ln \frac{2}{R_1} + D_2 \ln \frac{2}{R_2} \right]$$

For  $g g \rightarrow jj$   $A_{gg} = C_A a_s/\pi$        $B_{gg} = -2C_A \beta_0 a_s/\pi$

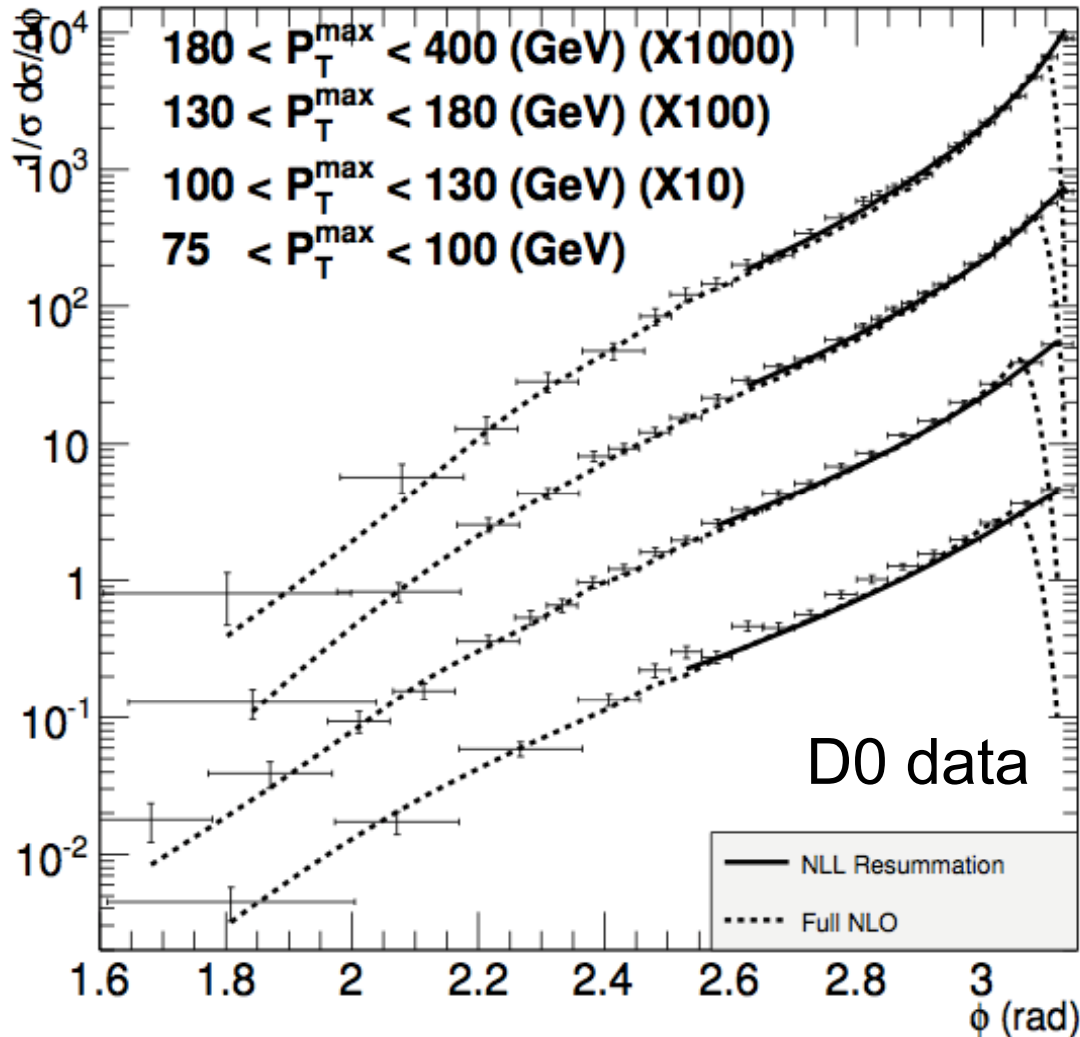
For  $q q \rightarrow jj$   $A_{qq} = C_F a_s/\pi$        $B_{qq} = -2C_F/3 a_s/\pi$

For  $q g \rightarrow jj$   $A_{qg} = (A_{gg} + A_{qq})/2$        $B_{qg} = (B_{gg} + B_{qq})/2$

for quark jet  $D_i = C_F a_s/\pi$

for gluon jet  $D_i = C_A a_s/\pi$

# Compared to the data



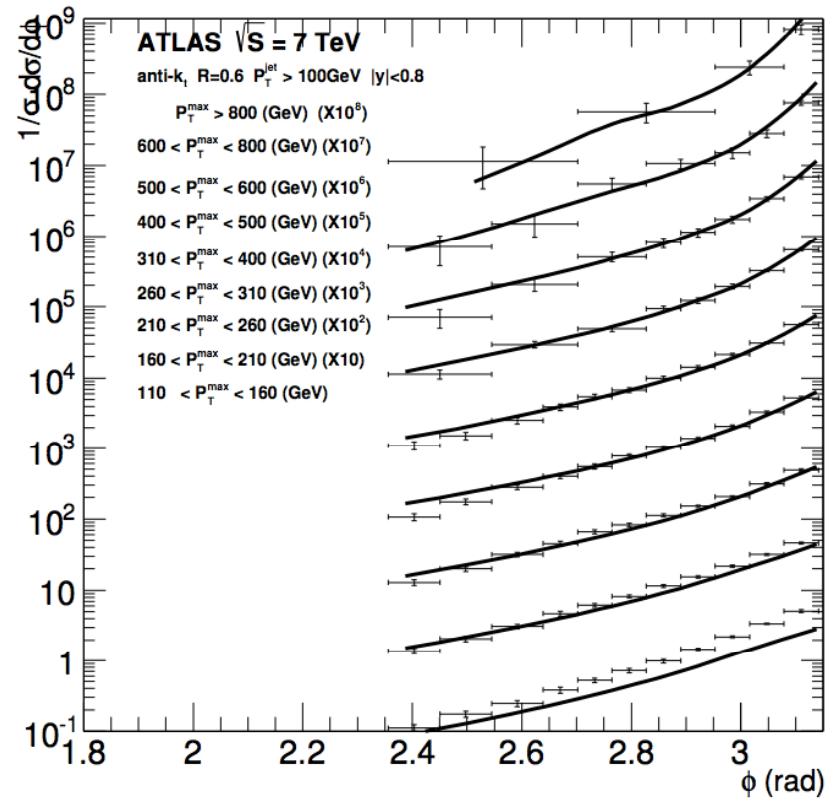
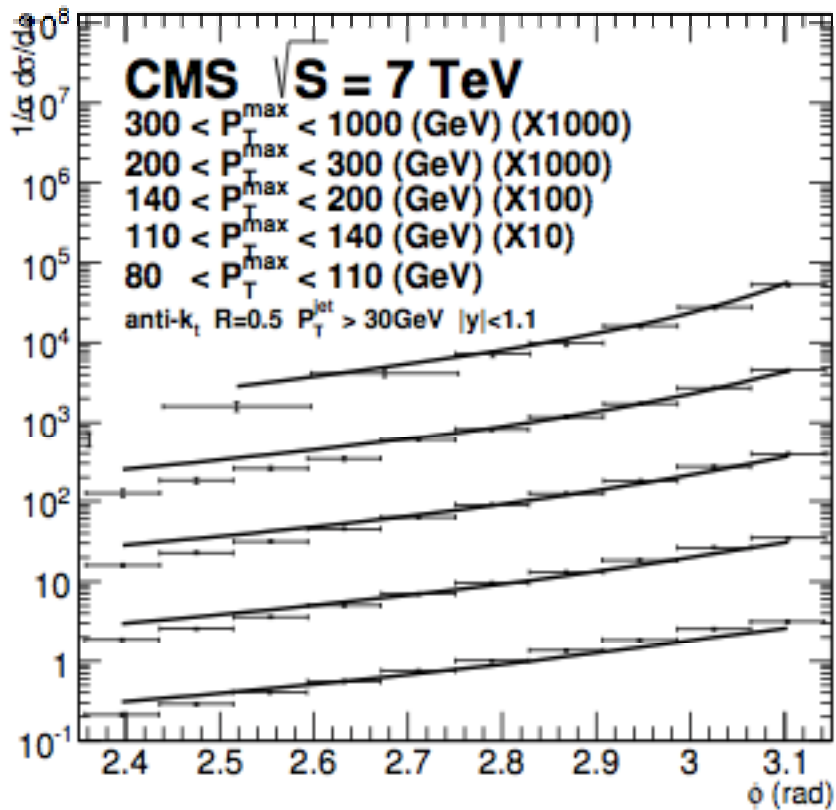
NLL Resummation:  
Sun, C.P. Yuan, F. Yuan, PRL2014

$$x_1 f_a(x_1, \mu = b_0/b_\perp) x_2 f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\text{Sud}}(Q^2, b_\perp)}$$

$$\text{Tr} \left[ \mathbf{H}_{ab \rightarrow cd} \exp \left[ - \int_{b_0/b_\perp}^Q \frac{d\mu}{\mu} \gamma^{s\dagger} \right] \mathbf{S}_{ab \rightarrow cd} \exp \left[ - \int_{b_0/b_\perp}^Q \frac{d\mu}{\mu} \gamma^s \right] \right]$$

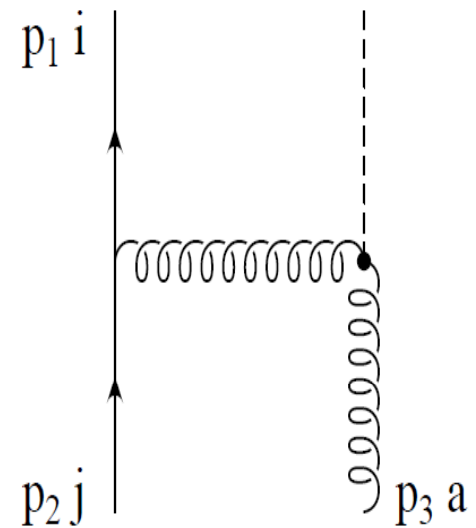
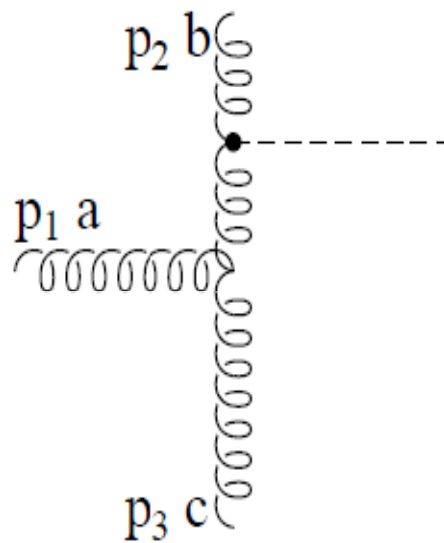
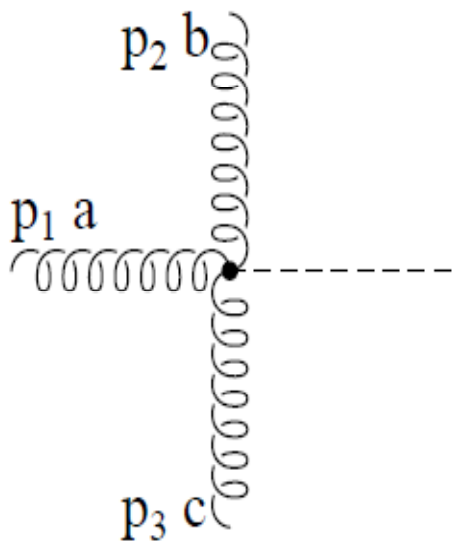
Full NLO: Nagy 2002, NLOJET++

# At the LHC

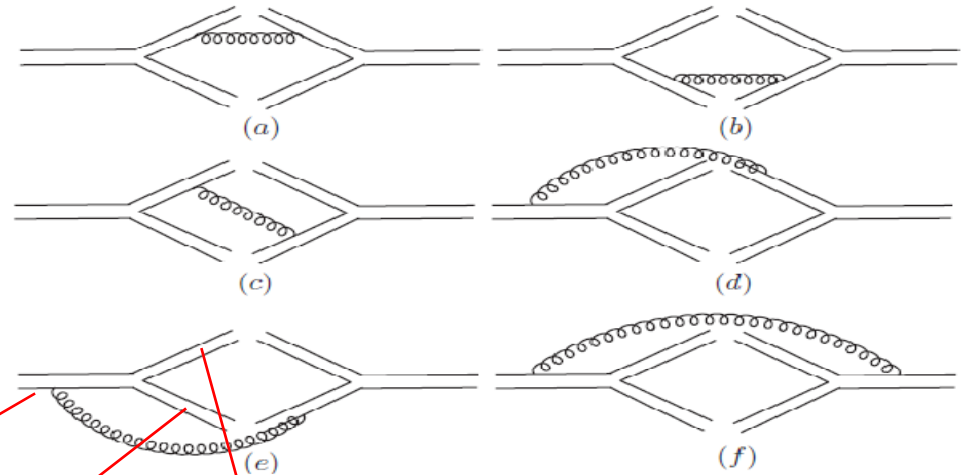


# Higgs plus one jet productions in pp collision

- the leading order feynman diagrams



# The soft factor



The definition of soft factor:

$$\bar{S}(b_{\perp}, \mu, \rho) = \frac{\int_0^{\pi} \frac{(\sin \phi)^{-2\epsilon}}{a_1} d\phi \langle 0 | \mathcal{L}_{\bar{v}ca'}^{\dagger}(b_{\perp}) \text{Tr} \left[ \mathcal{L}_{n_c}^{\dagger}(b_{\perp}) T^{a'} \mathcal{L}_{n_{\bar{c}}}^{\dagger}(b_{\perp}) \mathcal{L}_{n_{\bar{c}}}(0) T^a \mathcal{L}_{n_c}(0) \right] \mathcal{L}_{\bar{v}ac}(0) | 0 \rangle}{\text{Tr}[T^d T^d]}$$

At the one loop order

$$\bar{S}_{\text{JMY}}^{(1)}(b_{\perp}, \mu, \rho) = \frac{\alpha_s}{2\pi} \left\{ C_A \ln \frac{c_0^2}{b_{\perp}^2 \mu^2} (B_{\text{final}} + \ln \rho^2 + \ln \frac{\tilde{Q}^2}{\zeta^2} - 1) + C_{\text{final}} \right\}$$

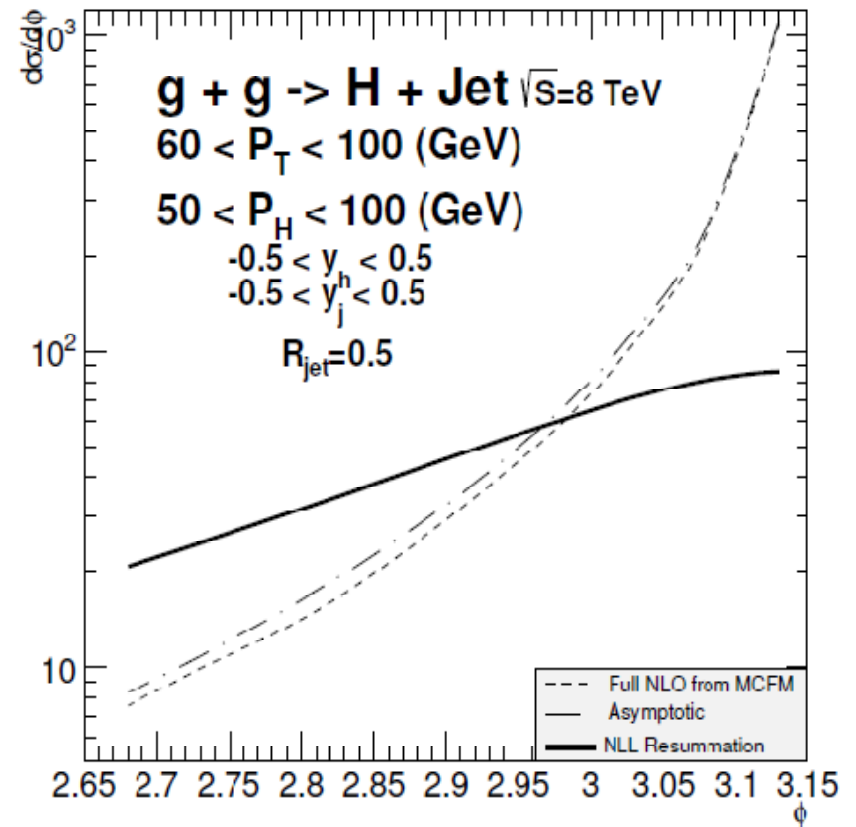
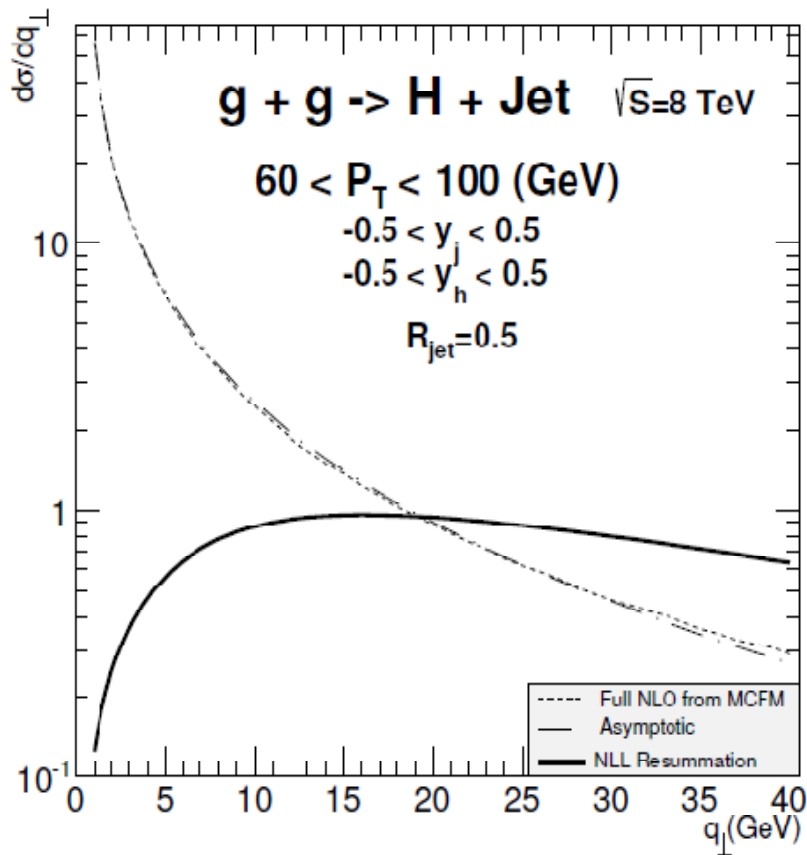
Sudakov form factor:

$$S_{\text{sud}} = - \int_{\tilde{Q}_0}^{\tilde{Q}} \frac{d\mu}{\mu} \left( \ln \frac{\tilde{Q}}{\mu} \gamma_K(\mu) - \gamma_S(\mu, 1) + \frac{\alpha_s C_A}{\pi} (1 - 2\beta_0 - \ln \frac{\tilde{Q}_0^2 b_{\perp}^2}{c_0^2}) \right)$$

$$\gamma_K(\mu) = \frac{2\alpha_s(\mu) C_A}{\pi},$$

$$\gamma_S(\mu, \rho) = - \frac{\alpha_s(\mu) C_A}{\pi} (B_{\text{final}} + \ln \rho - 1)$$

# Higgs + jet production in pp collision



□ Higgs+Jet, Sun, C.-P. Yuan, F. Yuan,  
Phys.Rev.Lett. 114 (2015) 202001



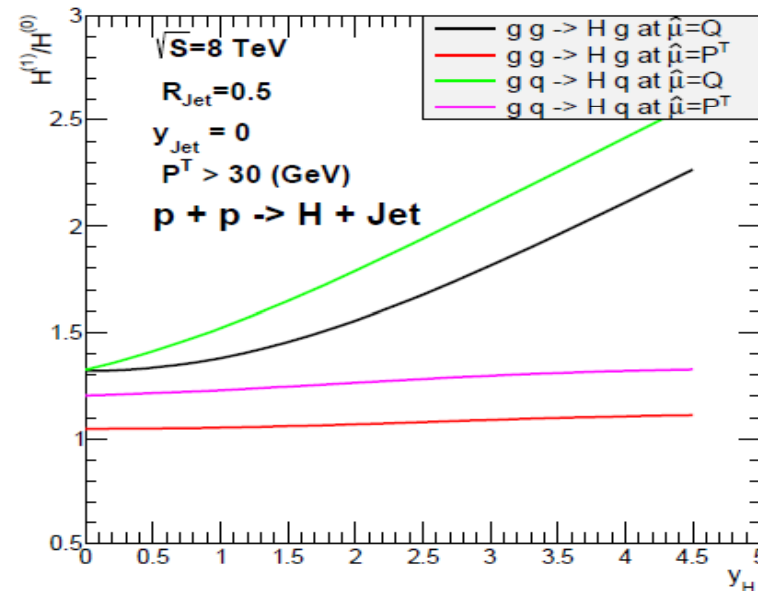
# Resummation scale dependence

$$W_{gg \rightarrow Hg}(x_1, x_2, b) = H_{gg \rightarrow Hg}(s, \hat{\mu}) x_1 f_g(x_1, \mu = b_0/b_\perp) x_2 f_g(x_2, \mu = b_0/b_\perp) e^{-S_{\text{Sud}}(s, \hat{\mu}^2, b_\perp)}$$

$$S_{\text{Sud}}(b) = \int_{b_0^2/b^2}^{\hat{\mu}^2} \frac{d\mu^2}{\mu^2} \left[ \ln \left( \frac{s}{\mu^2} \right) A + B + D \ln \frac{1}{R^2} \right]$$

- The resummation scale dependence between hard part and Sudakov factor will be cancelled to each other order by order
- However, we found

$$s \sim P_t \sqrt{P_t^2 + M_h^2} \text{EXP}(|y_J - y_h|)$$



# Hard factor at NLO

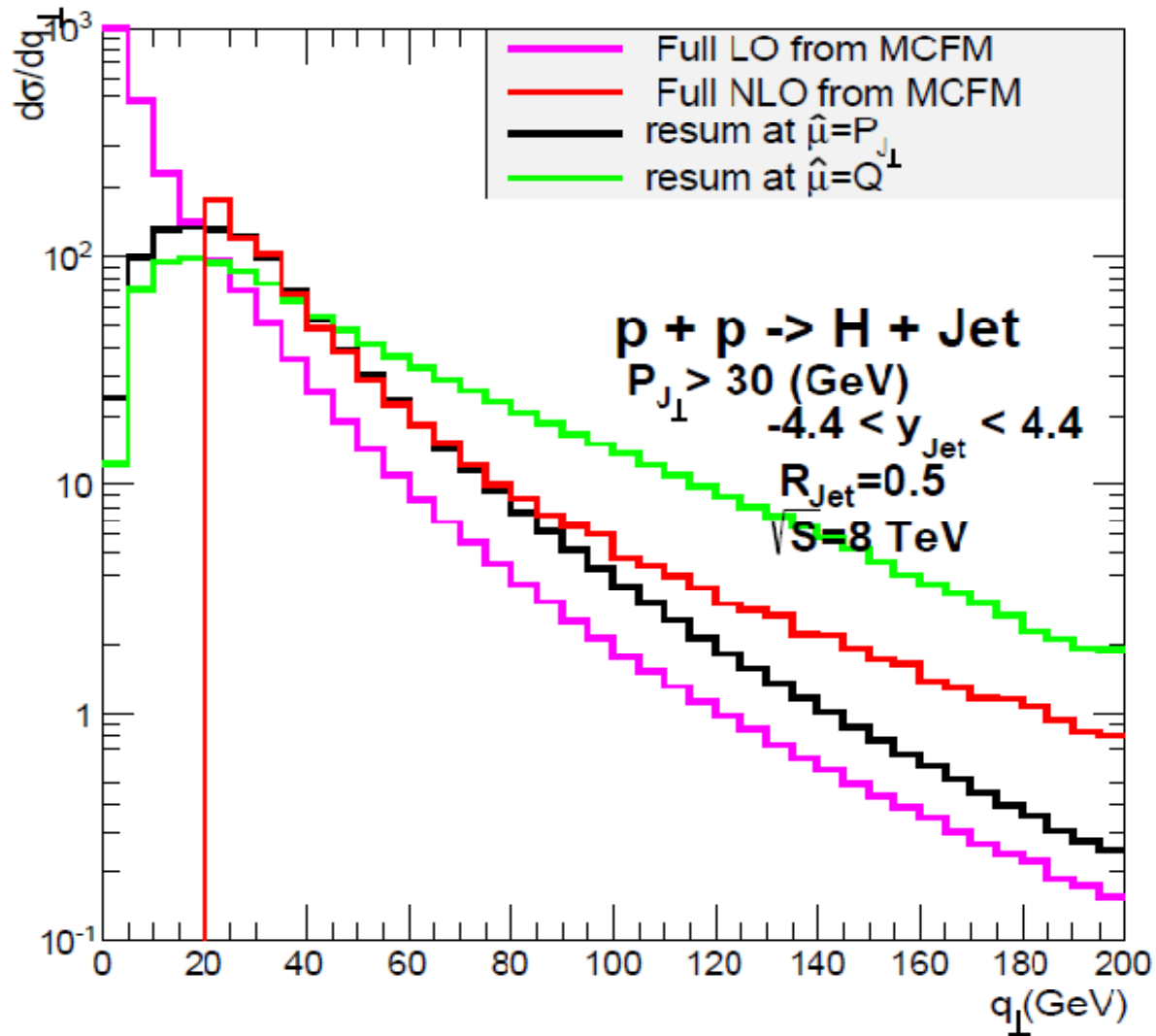
for  $G G \rightarrow G H$

$$\begin{aligned}
 H_{gg \rightarrow Hg}^{(1)} = & H_{gg}^{(0)} \frac{\alpha_s C_A}{2\pi} \left[ \ln^2 \left( \frac{\hat{\mu}^2}{P_{J\perp}^2} \right) + 2\beta_0 \ln \frac{\hat{\mu}^2}{P_{J\perp}^2 R^2} + \ln \frac{1}{R^2} \ln \frac{\hat{\mu}^2}{P_{J\perp}^2} - 6\beta_0 \ln \frac{\hat{\mu}^2}{\tilde{\mu}^2} - 2 \ln \left( \frac{P_{J\perp}^2}{\hat{\mu}^2} \right) \ln \left( \frac{s}{\hat{\mu}^2} \right) \right. \\
 & - 2 \ln \frac{s}{-t} \ln \frac{s}{-u} + \ln^2 \left( \frac{\tilde{t}}{m_h^2} \right) - \ln^2 \left( \frac{\tilde{t}}{-t} \right) + \ln^2 \left( \frac{\tilde{u}}{m_h^2} \right) - \ln^2 \left( \frac{\tilde{u}}{-u} \right) \\
 & \left. + 2\text{Li}_2 \left( 1 - \frac{m_h^2}{s} \right) + 2\text{Li}_2 \left( \frac{t}{m_h^2} \right) + 2\text{Li}_2 \left( \frac{u}{m_h^2} \right) + \frac{67}{9} + \frac{\pi^2}{2} - \frac{23}{54} N_f \right] + \delta H^{(1)}, \quad (6)
 \end{aligned}$$

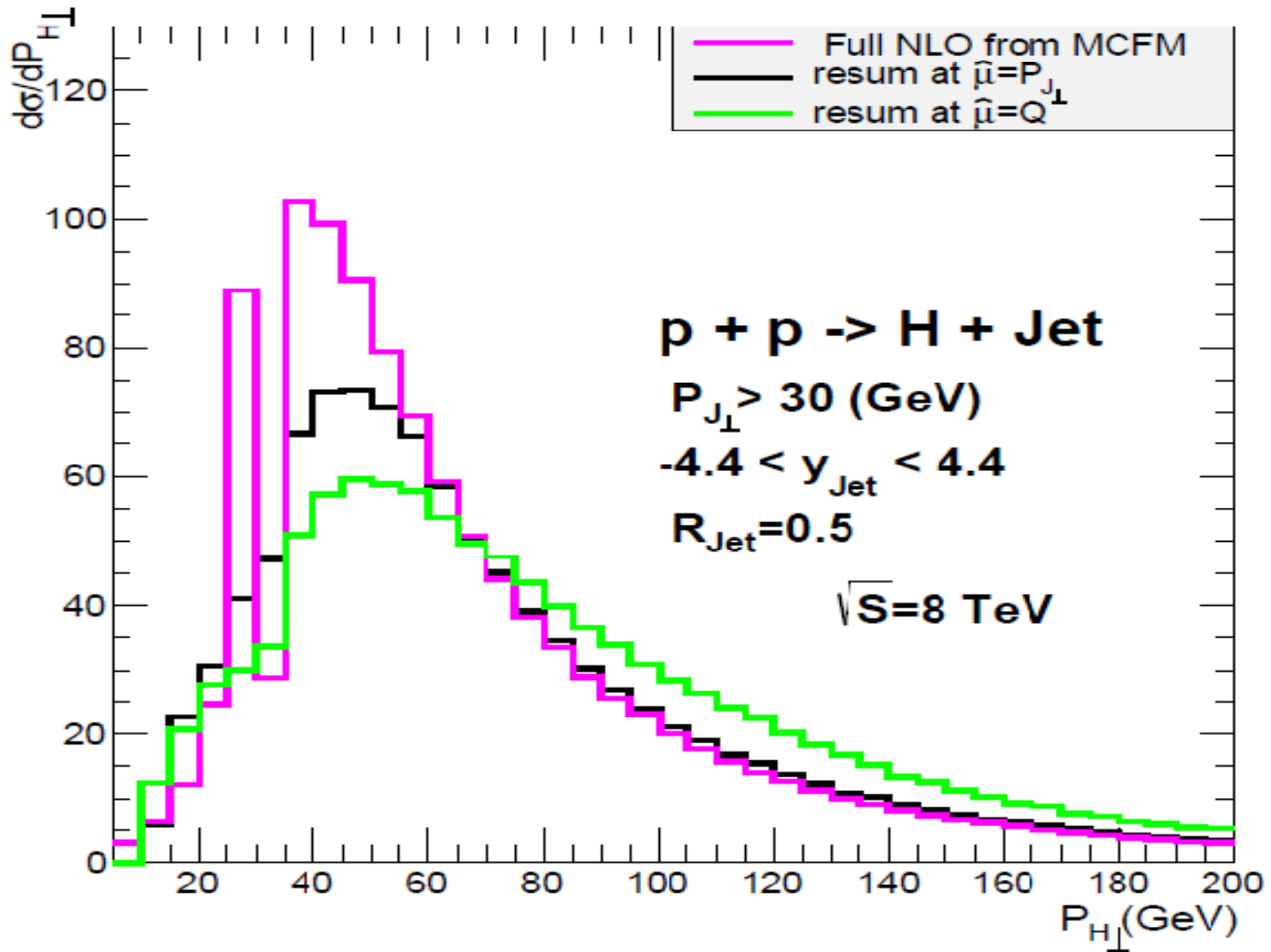
for  $G Q \rightarrow Q H$

$$\begin{aligned}
 H_{gq \rightarrow Hq}^{(1)} = & H^{(0)} \frac{\alpha_s}{2\pi} \left\{ C_A \left[ \frac{1}{2} \ln^2 \left( \frac{\hat{\mu}^2}{P_{J\perp}^2} \right) + \ln \left( \frac{P_{J\perp}^2}{\hat{\mu}^2} \right) \ln \left( \frac{u}{t} \right) + \ln \left( \frac{P_{J\perp}^2}{\hat{\mu}^2} \right) \ln \left( \frac{s}{\hat{\mu}^2} \right) - 2 \ln \frac{-t}{\hat{\mu}^2} \ln \frac{-u}{\hat{\mu}^2} \right. \right. \\
 & \left. \left. - 4\beta_0 \ln \frac{-u}{\hat{\mu}^2} - 6\beta_0 \ln \frac{\hat{\mu}^2}{\tilde{\mu}^2} + 2\text{Li}_2 \left( \frac{u}{m_h^2} \right) - \ln^2 \frac{\tilde{u}}{-u} + \ln^2 \frac{\tilde{u}}{m_h^2} + \frac{7}{3} + \frac{4\pi^2}{3} \right] \right. \\
 & + C_F \left[ \frac{1}{2} \ln^2 \left( \frac{\hat{\mu}^2}{P_{J\perp}^2} \right) + \frac{3}{2} \ln \frac{\hat{\mu}^2}{P_{J\perp}^2 R^2} + \ln \frac{1}{R^2} \ln \frac{\hat{\mu}^2}{P_{J\perp}^2} - \ln \frac{P_{J\perp}^2}{\hat{\mu}^2} \ln \frac{u}{t} - \ln \frac{P_{J\perp}^2}{\hat{\mu}^2} \ln \frac{s}{\hat{\mu}^2} + 3 \ln \frac{-u}{\hat{\mu}^2} \right. \\
 & \left. \left. + 2\text{Li}_2 \left( 1 - \frac{m_h^2}{s} \right) + 2\text{Li}_2 \left( \frac{t}{m_h^2} \right) - \ln^2 \left( \frac{\tilde{t}}{-t} \right) + \ln^2 \left( \frac{\tilde{t}}{m_h^2} \right) - \frac{3}{2} - \frac{5\pi^2}{6} \right] + 20\beta_0 \right\} + \delta H^{(1)}.
 \end{aligned}$$

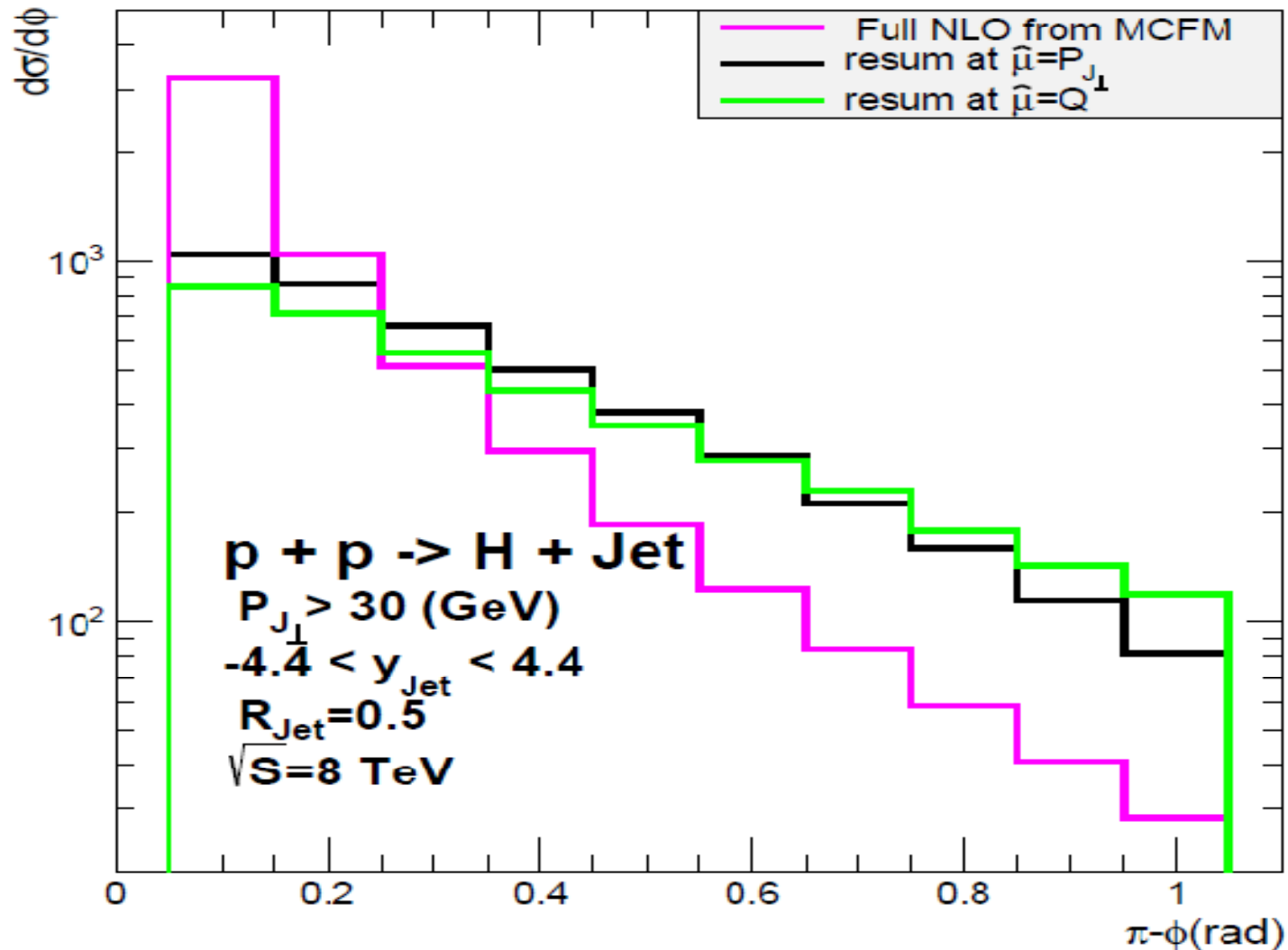
# $q_{\perp}$ distribution of Higgs plus leading jet system



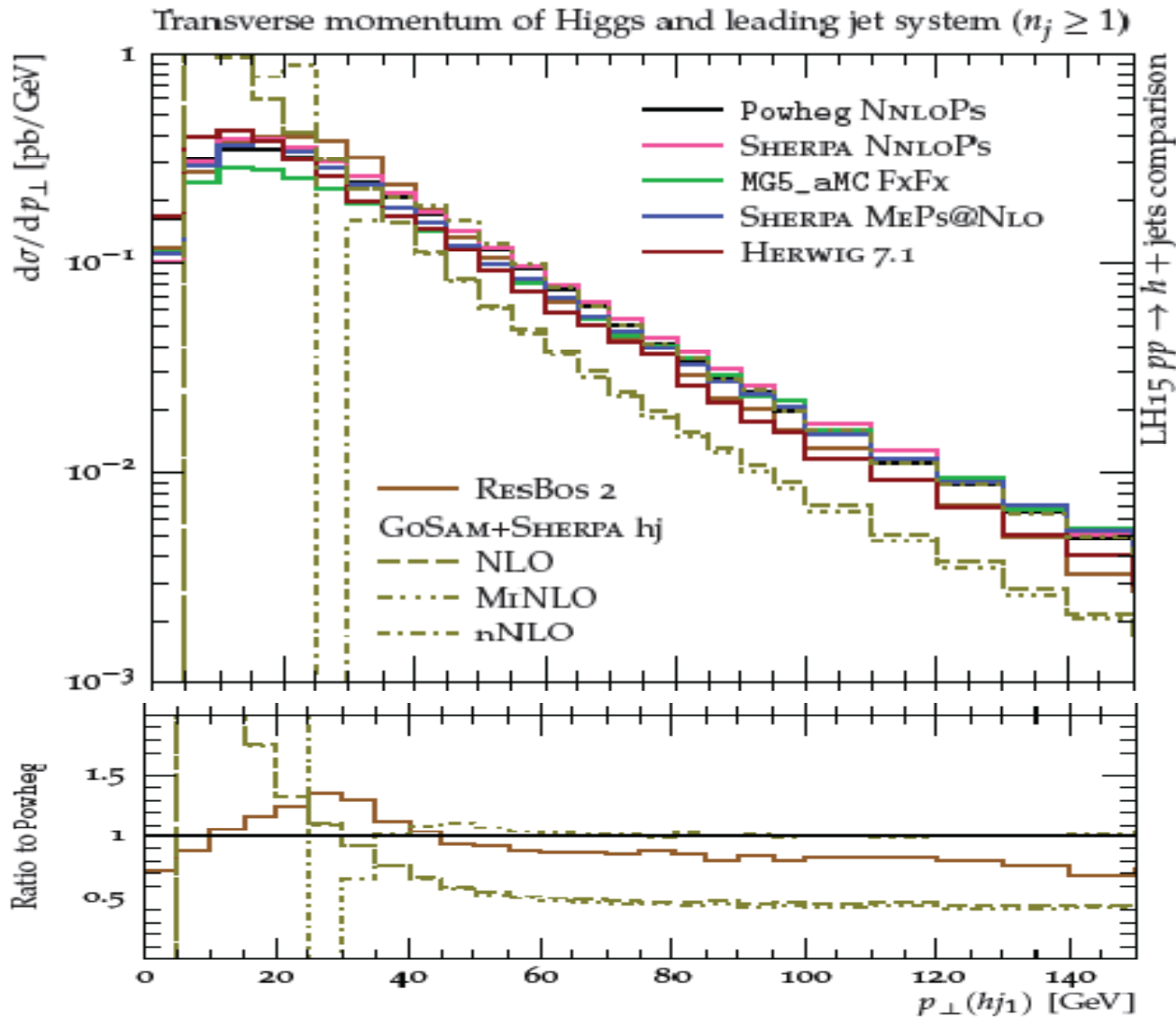
# Higgs $P_{\perp}$ distribution



# distribution of the azimuthal angle between Higgs and leading jet

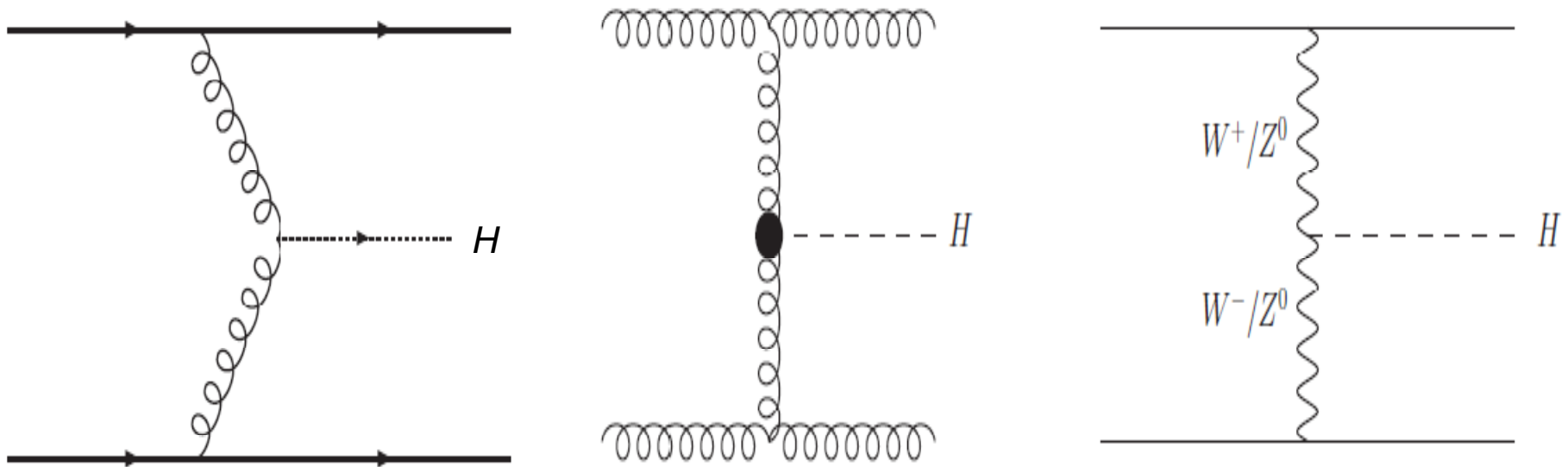


# Comparison to MC generators and Fixed Order



# Higgs plus two jets production in pp collisions at large $\Delta y_{jj}$ region

- The dominant contributions at tree level



# Sudakov factors

$$S_a(\hat{\mu}, b_\perp) = \int_{\mu_b^2}^{\hat{\mu}^2} \frac{d\mu^2}{\mu^2} \left[ \ln \left( \frac{s}{\mu^2} \right) A_a + B_a + D_a \ln \frac{1}{R^2} + \gamma_a'^s \right]$$

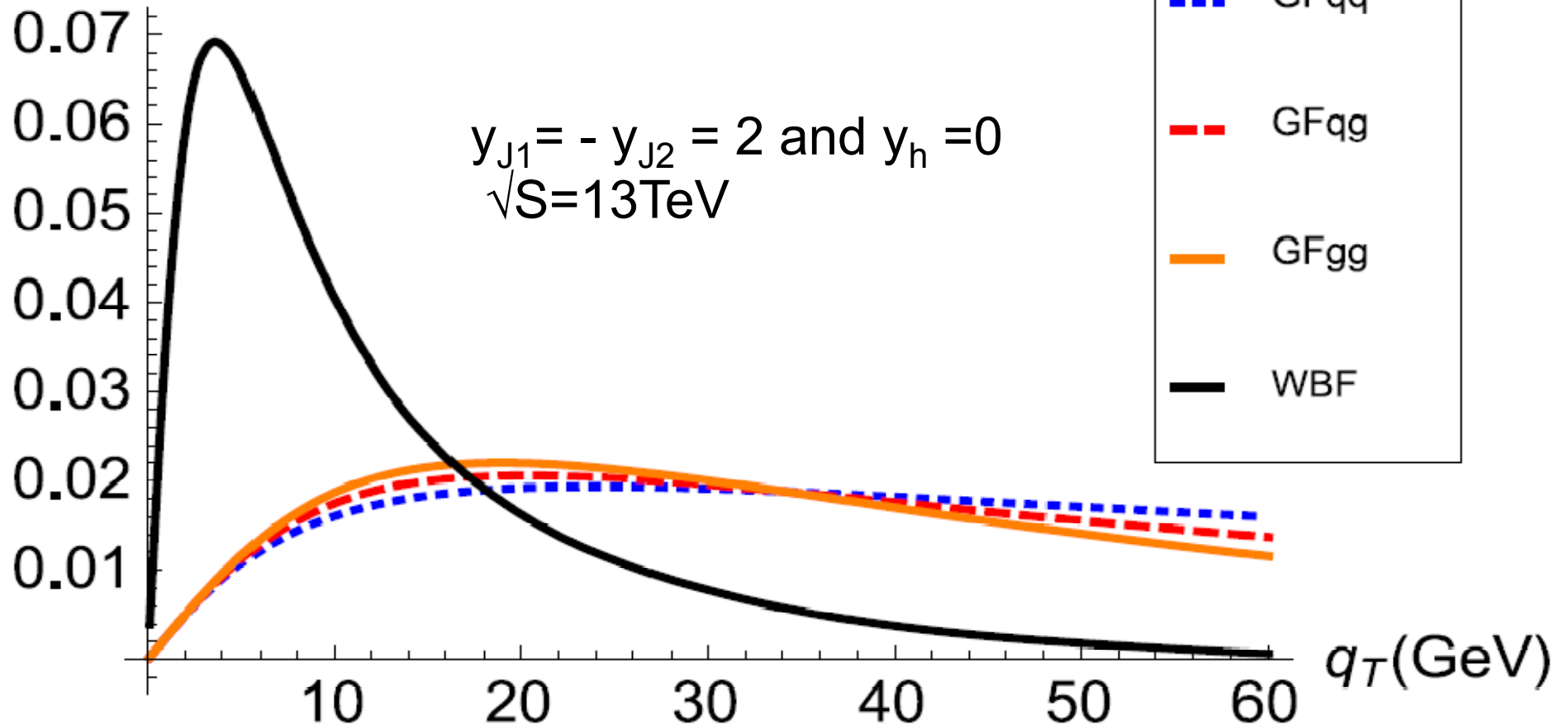
- Where A and B coefficients are the same as Drell-Yan or Higgs plus 0 jet production.
- The coefficient D is decided by color structure of jet.

$$\gamma_{qWBF}'^s = -C_F \ln \frac{u_1}{t_1}, \quad \gamma_{qGF}'^s = (C_A - C_F) \ln \frac{u_1}{t_1}, \quad \gamma_{gGF}'^s = 0$$

- In the large  $\Delta y_{jj}$  region,  $|u_1| \gg |t_1|$



$d\sigma/dq_T/\sigma$



- $q_T$  is the total transverse momentum of Higgs plus two leading jets



# Summary

- The soft gluon resummation can help us to make a precise prediction of the SM
- We also can use the resummation effect to suppress some background event



Thank you very much!