High-twist effects in e+A and p+A collisions

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Outline

- Why study high twist effects?

  probe QCD dynamics beyond single scattering / normal PDFs

- High twist effects - observables

  single transverse spin asymmetry

  nuclear modifications: small-x suppression and large-x enhancement

  transverse momentum broadening

- Summary

high-twist/power expansion

❑ Generalized factorization theorem

perturbative expansion →

$$\sigma^h_{phys} = \left[\alpha_s^0 C_2^{(0)} + \alpha_s^1 C_2^{(1)} + \alpha_s^2 C_2^{(2)} + \ldots\right] \otimes T_2(x) \longrightarrow \text{leading twist}$$

$$+ \frac{1}{Q}\left[\alpha_s^0 C_3^{(0)} + \alpha_s^1 C_3^{(1)} + \alpha_s^2 C_3^{(2)} + \ldots\right] \otimes T_3(x) \longrightarrow \text{twist-3}$$

$$+ \frac{1}{Q^2}\left[\alpha_s^0 C_4^{(0)} + \alpha_s^1 C_4^{(1)} + \alpha_s^2 C_4^{(2)} + \ldots\right] \otimes T_4(x) \longrightarrow \text{twist-4}$$

$$+ \ldots$$

power expansion ↓

- High twist effects = power corrections = multiple scattering contributions
- What's the size of the next power corrections?

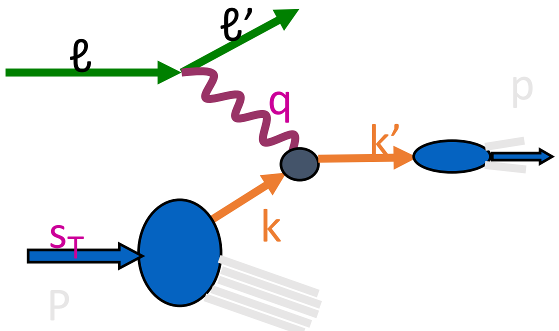
  in general small compare to leading power term
- Observables

  leading power vanishes

  nuclear enhanced power correction $\frac{1}{Q^2} \rightarrow \frac{A^{1/3}}{Q^2}$

Observable: single transverse spin asymmetry

- Spin dependent cross section



Spin averaged xsec $\sigma(\ell) = \frac{1}{2}[\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})]$

Spin dependent xsec $\Delta\sigma(\ell, \vec{s}) = \frac{1}{2}[\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})]$

Single transverse spin asymmetry

$$A(\ell, \vec{s}) \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

SSA vanishes at leading twist $A_N \sim \alpha_s \frac{m_q}{\sqrt{s}} \to 0$

Spin dependent xsec, sensitive to twist-3 multi-parton correlation function

$$\Delta\sigma(Q, s_T) = \frac{1}{Q} H_1(Q/\mu_f, \alpha_s) \otimes T_3(\mu_f) \otimes D(\mu_f)$$ → Sivers effect

$$+ \frac{1}{Q} H_1'(Q/\mu_f, \alpha_s) \otimes T_2(\mu_f) \otimes D_3(\mu_f)$$ → Collins effect (see Kang's talk)

$$+ \ldots$$

Twist-3 matrix element

$$T_{q,F}(x_1, x_2) = \int \frac{dy_1^- dy_2^-}{4\pi} e^{ix_1 P^+ y_1^- + i(x_2 - x_1) P^+ y_2^-} \langle P, s_T | \bar{\psi}_q(0) \gamma^+ \epsilon^{\alpha\beta} S_{\perp\alpha} F^+{}_\beta(y_2^-) \psi_q(y_1^-) | P, s_T \rangle.$$

$$T_{q,F}(x, x) = \int d^2k_\perp \frac{|k_\perp|^2}{M_h} f_{1T}^\perp(x, k_\perp^2)$$ → Sivers function

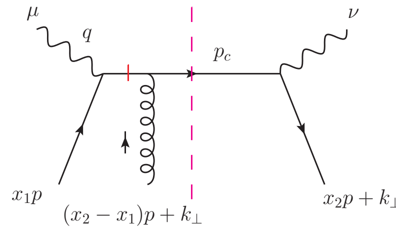
❑ Transverse momentum weighted SSA in SIDIS

Kang, Vitev, HX, PRD, 2013

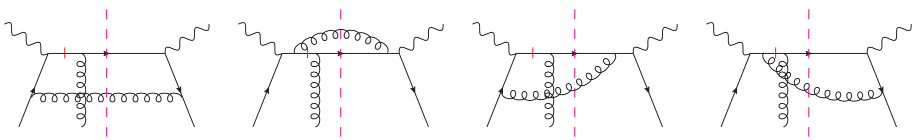
$$\frac{d\langle P_{h\perp}\Delta\sigma(S_\perp)\rangle}{dx_B dy dz_h} \equiv \int d^2P_{h\perp}\epsilon^{\alpha\beta}S_\perp^\alpha P_{h\perp}^\beta \frac{d\Delta\sigma(S_\perp)}{dx_B dy dz_h d^2P_{h\perp}}$$

- Leading order

$$\frac{d\langle P_{h\perp}\Delta\sigma(S_\perp)\rangle}{dx_B dy dz_h} = -\frac{z_h\sigma_0}{2}\sum_q e_q^2 \int \frac{dx}{x}\frac{dz}{z} T_{q,F}(x,x) D_{q\to h}(z)\delta(1-\hat{x})\delta(1-\hat{z})$$

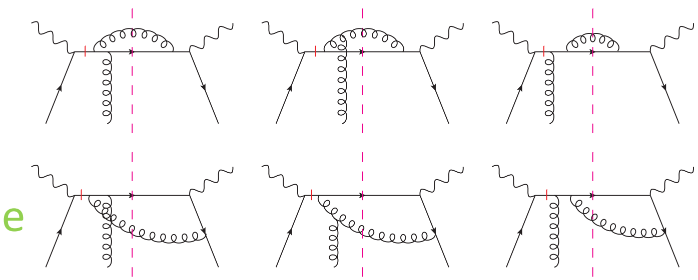


- Next-to-leading order



soft-pole

hard-pole



- QCD evolution of Qiu-Sterman function

$$\frac{\partial}{\partial \ln\mu^2} T_{q,F}(x_B, x_B, \mu^2) = \frac{\alpha_s}{2\pi}\int_{x_B}^1 \frac{dx}{x}\left\{ T_{q,F}(x,x,\mu^2) C_F\left[\frac{1+\hat{x}^2}{(1-\hat{x})_+} + \frac{3}{2}\delta(1-\hat{x})\right] - N_c\delta(1-\hat{x}) T_{q,F}(x,x,\mu^2)\right.$$
$$\left. + \frac{N_c}{2}\left[\frac{1+\hat{x}}{(1-\hat{x})_+} T_{q,F}(x, x\hat{x}, \mu^2) - \frac{1+\hat{x}^2}{(1-\hat{x})_+} T_{q,F}(x,x,\mu^2)\right]\right\}.$$

- Complete next-to-leading order result

LO

$$\frac{d\langle P_{h\perp}\Delta\sigma(S_\perp)\rangle}{dx_B dy dz_h} = -\frac{z_h\sigma_0}{2}\sum_q e_q^2\int_{x_B}^1\frac{dx}{x}\int_{z_h}^1\frac{dz}{z}T_{q,F}(x,x,\mu^2)D_{q\to h}(z,\mu^2)\delta(1-\hat{a}tx)\delta(1-\hat{a}tz)$$

$$-\frac{z_h\sigma_0}{2}\frac{\alpha_s}{2\pi}\sum_q e_q^2\int_{x_B}^1\frac{dx}{x}\int_{z_h}^1\frac{dz}{z}D_{q\to h}(z,\mu^2)\Bigg\{\ln\left(\frac{Q^2}{\mu^2}\right)\left[\delta(1-\hat{x})T_{q,F}(x,x,\mu^2)P_{qq}(\hat{z})\right.$$

$$\left.+\delta(1-\hat{z})P_{qg\to qg}\otimes T_{q,F}(x,x\hat{x},\mu^2)\right]$$

$$+x\frac{d}{dx}T_{q,F}(x,x,\mu^2)\frac{1}{2N_c}\left[\frac{1-\hat{z}}{\hat{z}}+\frac{(1-\hat{x})^2+2\hat{x}\hat{z}}{\hat{z}(1-\hat{z})_+}-\delta(1-\hat{z})\left((1+\hat{x}^2)\ln\frac{\hat{x}}{1-\hat{x}}+2\hat{x}\right)\right]$$

$$+T_{q,F}(x,x,\mu^2)\delta(1-\hat{z})\frac{1}{2N_c}\left[(2\hat{x}^2-\hat{x}-1)\ln\frac{\hat{x}}{1-\hat{x}}-2\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ +\frac{2\hat{x}(2-\hat{x})}{(1-\hat{x})_+}+2\frac{\ln\hat{x}}{1-}\right.$$

$$+T_{q,F}(x,x,\mu^2)\delta(1-\hat{x})C_F\left[-(1+\hat{z})\ln\hat{z}(1-\hat{z})+2\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ -\frac{2\hat{z}}{(1-\hat{z})_+}+2\frac{\ln\hat{z}}{1-\hat{z}}\right]$$

$$+T_{q,F}(x,x,\mu^2)\frac{1}{2N_c\hat{z}}\left[\frac{2\hat{x}^3-3\hat{x}^2-1}{(1-\hat{x})_+(1-\hat{z})_+}+\frac{1+\hat{z}}{(1-\hat{x})_+}-2(1-\hat{x})\right]$$

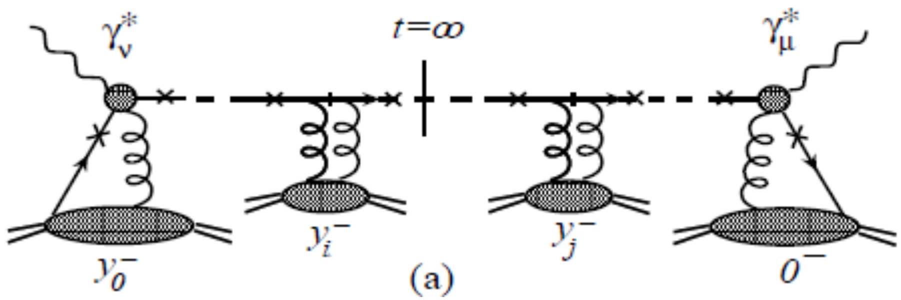
$$+T_{q,F}(x,x\hat{x},\mu^2)\delta(1-\hat{z})\frac{N_c}{2}\left[\ln\frac{\hat{x}}{1-\hat{x}}+2\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ -2\frac{\ln\hat{x}}{1-\hat{x}}-\frac{1+\hat{x}}{(1-\hat{x})_+}\right]$$

$$+T_{q,F}(x,x\hat{x},\mu^2)\frac{1+\hat{x}\hat{z}^2}{(1-\hat{x})_+(1-\hat{z})_+}\left(C_F+\frac{1}{2N_c\hat{z}}\right)-T_{q,F}(x,x,\mu^2)6C_F\delta(1-\hat{x})\delta(1-\hat{z})\Bigg\}$$

NLO

Coherent multiple scattering (small-x)

❑ Nuclear dynamic shadowing - structure function in DIS



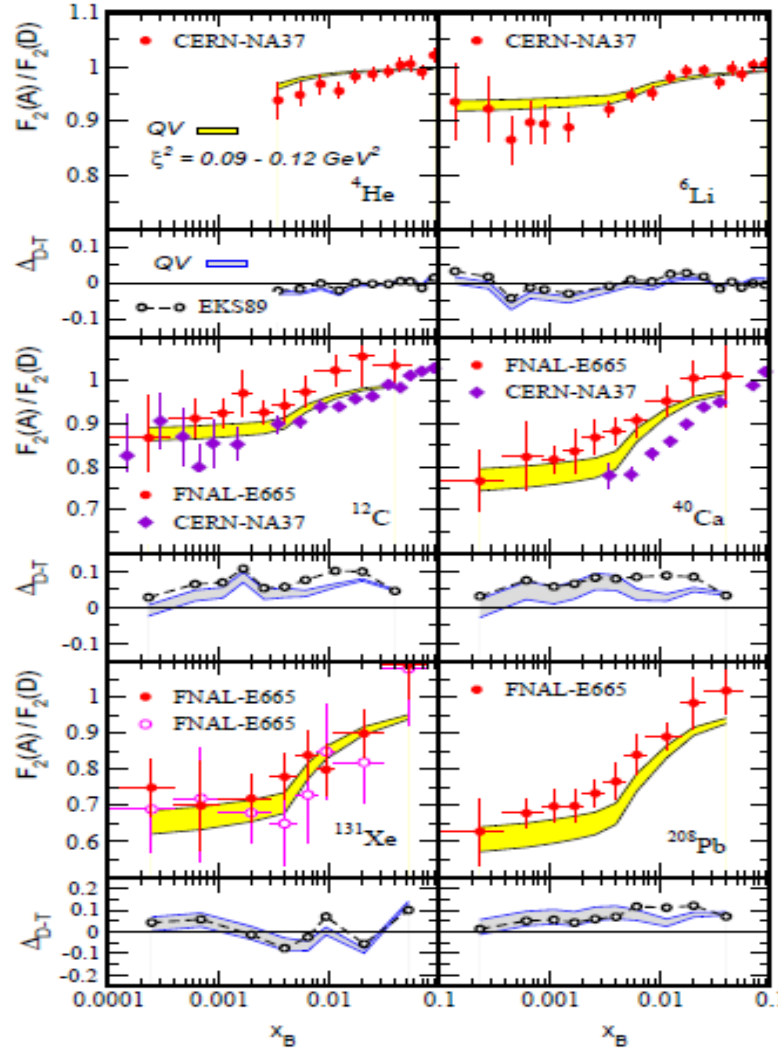
$$F_T^A(x,Q^2) \approx \sum_{n=0}^{N} \frac{A}{n!}\left[\frac{\xi^2(A^{1/3}-1)}{Q^2}\right]^n x^n \frac{d^n F_T^{(\mathrm{LT})}(x,Q^2)}{d^n x}$$

$$\approx A\, F_T^{(\mathrm{LT})}\left(x + \boxed{\frac{x\xi^2(A^{1/3}-1)}{Q^2}}, Q^2\right), \qquad (10)$$

$$\xi^2 = \frac{3\pi\alpha_s(Q^2)}{8\, r_0^2}\langle p|\, \hat{F}^2(\lambda_i)\, |p\rangle$$

$$= 0.09 - 0.12\ \mathrm{GeV}^2$$

Only one free parameter

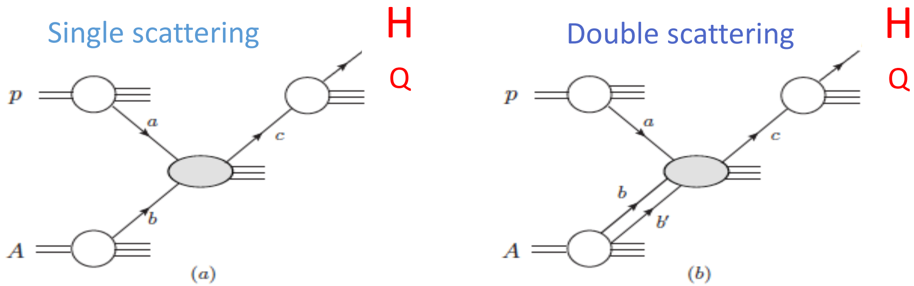


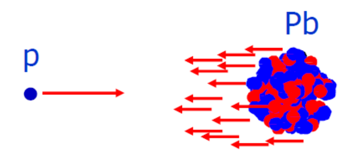
Qiu, Vitev , PRL, 2004

Incoherent multiple scattering – large-x

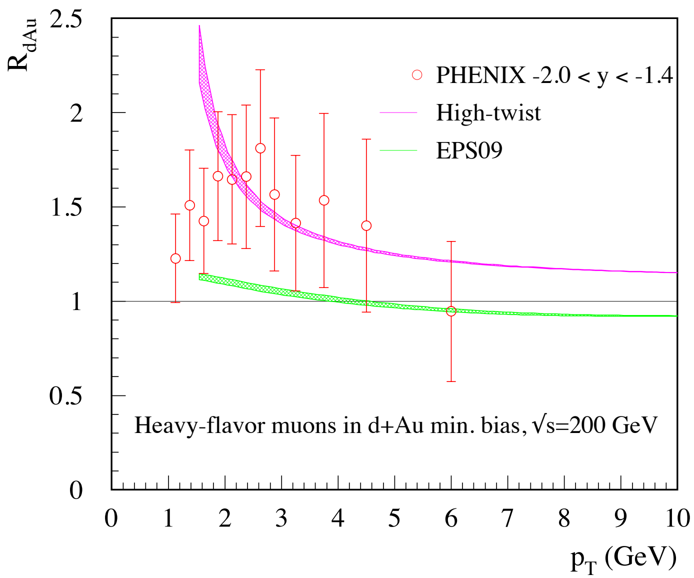
❑ Heavy meson production in pA collisions at backward rapidity

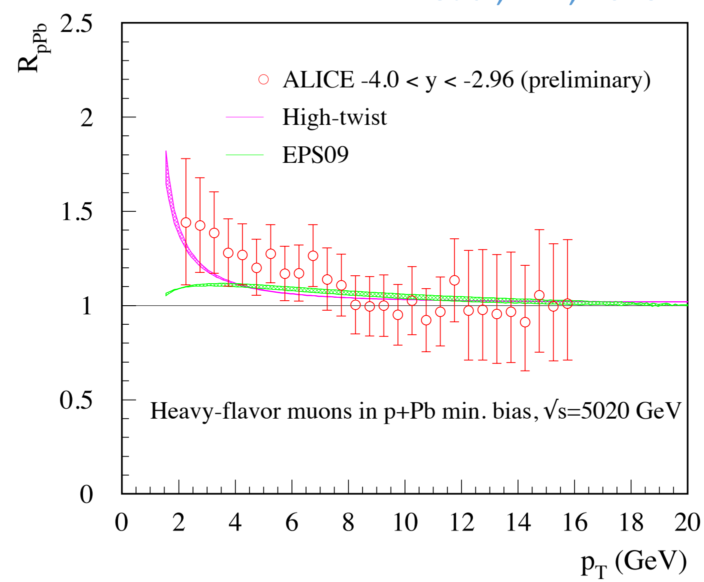
$$d\sigma_{pA\to HX} = d\sigma^{(S)}_{pA\to HX} + d\sigma^{(D)}_{pA\to HX} + \cdots$$





HX et al, PLB, 2015





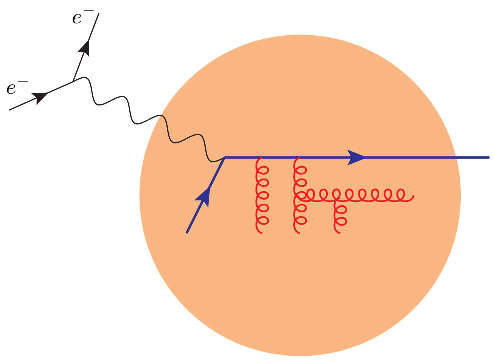
$$\frac{4\pi^2\alpha_s}{N_c} T^{(I)}_{q,g/A}(x) = \frac{4\pi^2\alpha_s}{N_c} T^{(F)}_{q,g/A}(x) = \xi^2 \left(A^{1/3} - 1\right) f_{q,g/A}(x)$$

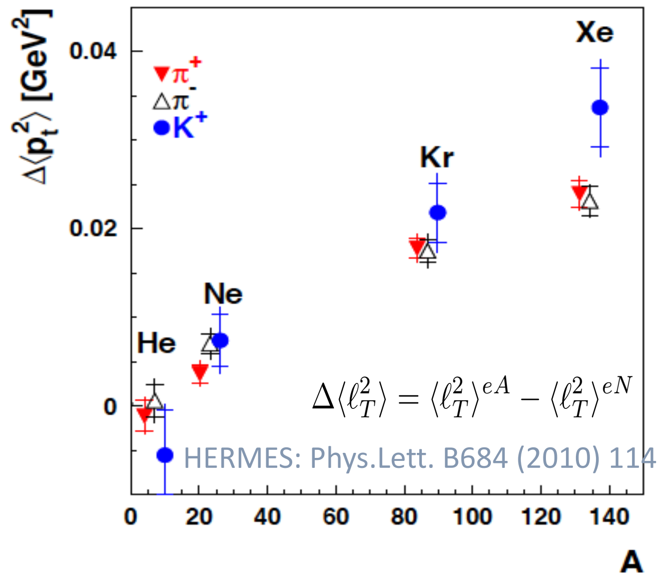
$$\xi^2 = 0.09 - 0.12\ GeV^2$$

Incoherent multiple scattering leads to significant enhancement effect in intermediate p_T region.

Transverse momentum broadening

- Transverse momentum broadening is sensitive to twist-4 (double scattering) effects

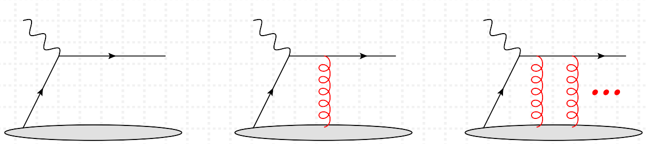




$$
\begin{aligned}
\Delta\langle \ell_T^2\rangle &= \langle \ell_T^2\rangle^{eA} - \langle \ell_T^2\rangle^{eN} \\
&= \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eA}}{dQ^2}} - \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eN}}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eN}}{dQ^2}} \\
&= \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}^S}{dQ^2 d\ell_T^2} + \int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}^D}{dQ^2 d\ell_T^2} + \cdots}{\frac{d\sigma_{eA}}{dQ^2}} - \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eN}^S}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eN}}{dQ^2}} \\
&\approx \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}^D}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eA}}{dQ^2}}
\end{aligned}
$$

Single scattering cancels

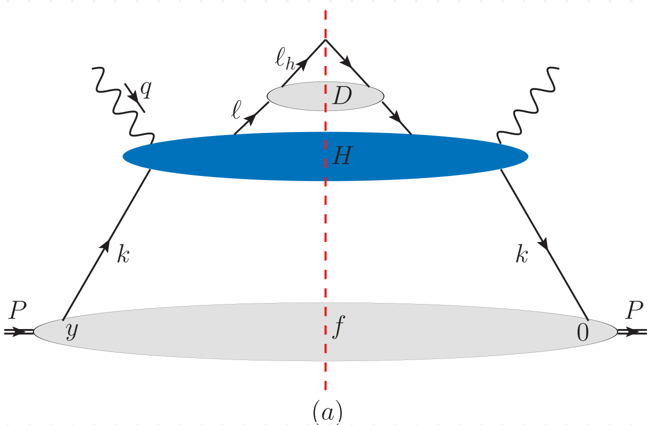
- Double scattering - leading contribution to TMB
- Easy to measure perturbatively calculable



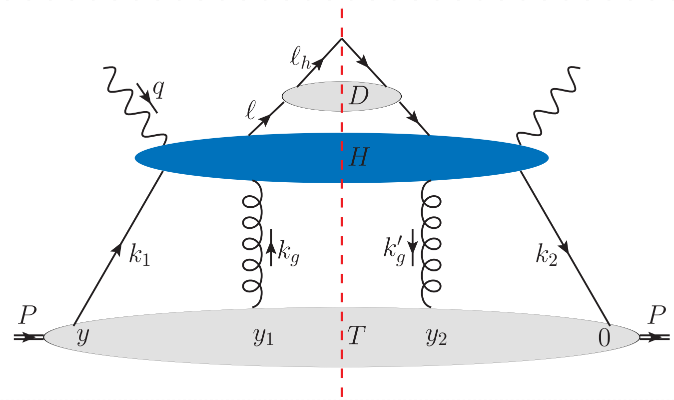
Double scattering in SIDIS

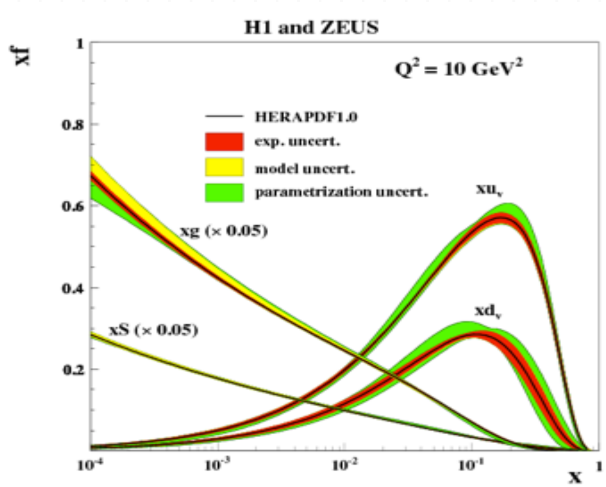
- Final state multiple scattering

Single scattering



Double scattering





Generalized twist-4 factorization formalism

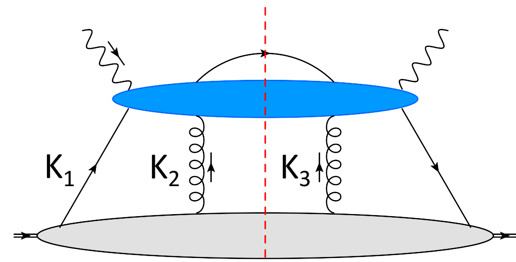
❑ DIS as an example

Qiu, Sterman 1990s

$$E_{L'}\frac{d^3\sigma_{qg}}{dL'^3}=\frac{\alpha_{\rm EM}^2 e_q^2}{2\pi w}\int\frac{d^4l}{(2\pi)^4}\frac{1}{Q^4}L_{\mu\nu}W^{\mu\nu},$$

$$L_{\mu\nu}={\rm Tr}[\not{L}\gamma_\mu\not{L}'\gamma_\nu],$$

$$W^{\mu\nu}=\int\frac{d^4K_1}{(2\pi)^4}\int\frac{d^4K_2}{(2\pi)^4}\int\frac{d^4K_3}{(2\pi)^4}\int d^4z_1\int d^4z_2\int d^4z_3 e^{iK_1z_1}e^{iK_2z_2}e^{iK_3z_3}$$
$$\times{\rm Tr}\{\hat{H}^{\mu\alpha\beta\nu}(K_1,K_2,K_3)\langle A|T[\bar{\psi}(z_1)A_\beta(z_2)A_\alpha(z_3)]\psi(0)|A\rangle\}$$



$$K_1=x_1P^+\bar{n}$$
$$K_2=x_2P^+\bar{n}+\mathbf{K}_{2\perp}$$
$$K_3=x_3P^+\bar{n}+\mathbf{K}_{3\perp}$$

- Pick up the leading contribution to nuclear enhancement

$$W^{\mu\nu}=\int\frac{P^+dx_1}{2\pi}\int\frac{P^+dx_2}{2\pi}\frac{d^2K_{2\perp}}{(2\pi)^2}\int\frac{P^+dx_3}{2\pi}\frac{d^2K_{3\perp}}{(2\pi)^2}\int dz_1^-dz_2^-d^2z_{2\perp}dz_3^-d^2z_{3\perp}$$
$$\times e^{ix_1P^+z_1^-}e^{ix_2P^+z_2^-}e^{-iK_{2\perp}\cdot z_{2\perp}}e^{ix_3P^+z_3^-}e^{-iK_{3\perp}\cdot z_{3\perp}}$$
$$\times{\rm Tr}[\hat{H}^{\mu\alpha\beta\nu}(x_1P^+,K_2,K_3)\langle A|\bar{\psi}(z_1)A_\beta(z_2)A_\alpha(z_3)\psi(0)|A\rangle]$$

- Collinear expansion

gauge link

$$\hat{H}^{\mu\alpha\beta\nu}(x_1P^+,K_2,K_3)=\hat{H}^{\mu\alpha\beta\nu}(x_1P^+,x_2P^+,x_3P^+)+\frac{\partial\hat{H}^{\mu\alpha\beta\nu}(x_1P^+,K_2,x_3P^+)}{\partial K_{2\rho}}\bigg|_{\mathbf{K}_{2\perp}=\mathbf{0}_\perp}(K_2-x_2P^+)_\rho$$
$$+\frac{\partial\hat{H}^{\mu\alpha\beta\nu}(x_1P^+,x_2P^+,K_3)}{\partial K_{3\sigma}}\bigg|_{\mathbf{K}_{3\perp}=\mathbf{0}_\perp}(K_3-x_3P^+)_\sigma$$
$$+\frac{1}{2}\frac{\partial^2\hat{H}^{\mu\alpha\beta\nu}(x_1P^+,K_2,K_3)}{\partial K_{2\rho}\partial K_{3\sigma}}\bigg|_{\mathbf{K}_{2\perp}=\mathbf{K}_{3\perp}=\mathbf{0}_\perp}(K_2-x_2P^+)_\rho(K_3-x_3P^+)_\sigma+\cdots,$$

polarized

unpolarized

- Separate the Dirac trace

$$\mathrm{Tr}\left[\frac{\partial^2 \hat{H}^{\mu\alpha\beta\nu}(x_1 P^+, K_2, K_3)}{\partial K_{2\rho}\partial K_{3\sigma}}\bigg|_{\mathbf{K}_{2\perp}=\mathbf{K}_{3\perp}=0_\perp}\langle A|\bar{\psi}(y_1^-)n\cdot A(z_2)n\cdot A(z_3)\psi(0)|A\rangle\right]$$

$$=\frac{1}{4(P\cdot n)}\frac{\partial^2}{\partial K_{2\rho}\partial K_{3\sigma}}\{\mathrm{Tr}[\hat{H}^{\mu\alpha\beta\nu}(x_1 P^+, K_2, K_3)\not{P}]\}|_{\mathbf{K}_{2\perp}=\mathbf{K}_{3\perp}=0_\perp}\mathrm{Tr}[\langle A|\bar{\psi}(y_1^-)\not{n}n\cdot A(z_2)n\cdot A(z_3)\psi(0)|A\rangle]$$

- Integrate by parts in z_2 and z_3

$$\int d^2z_{2\perp}d^2z_{3\perp}e^{-iK_{2\perp}\cdot z_{2\perp}}e^{-iK_{3\perp}\cdot z_{3\perp}}\,\omega_\rho{}^{\rho'}K_{2\rho'}\omega_\sigma{}^{\sigma'}K_{3\sigma'}\mathrm{Tr}[\langle A|\bar{\psi}(y_1^-)\not{n}n\cdot A(z_2)n\cdot A(z_3)\psi(0)|A\rangle]$$

$$=\frac{1}{(-i)^2}\omega_\rho{}^{\rho'}\omega_\sigma{}^{\sigma'}\int d^2z_{2\perp}d^2z_{3\perp}e^{-iK_{2\perp}\cdot z_{2\perp}}e^{-iK_{3\perp}\cdot z_{3\perp}}\mathrm{Tr}\left[\langle A|\bar{\psi}(y_1^-)\not{n}\frac{\partial[n\cdot A(z_2)]}{\partial z_2^{\rho'}}\frac{\partial[n\cdot A(z_3)]}{\partial z_3^{\sigma'}}\psi(0)|A\rangle\right]$$

- Nuclear enhanced twist-4 contribution

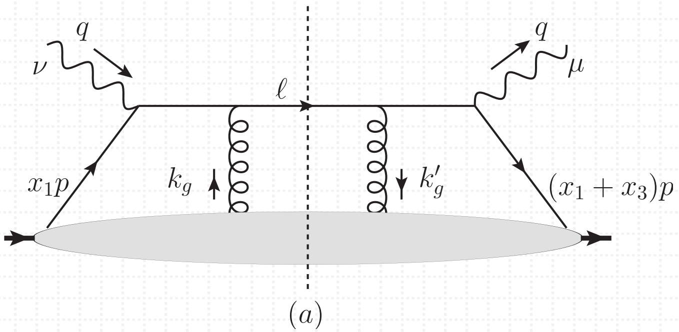
$$W^{\mu\nu}=-\frac{1}{16}\int\frac{dy_1^-}{2\pi}\frac{dy_2^-}{2\pi}\frac{dy_3^-}{2\pi}\mathrm{Tr}[\langle A|\bar{\psi}(y_1^-)\not{n}F_{+\perp}(y_2^-)F_+{}^{\perp}(y_3^-)\psi(0)|A\rangle]$$

$$\times\frac{\partial^2}{\partial K_{2\perp}\partial K_3^{\perp}}\left\{\int dx_1dx_2dx_3e^{ix_1P^+y_1^-}e^{ix_2P^+y_2^-}e^{ix_3P^+y_3^-}\right.$$

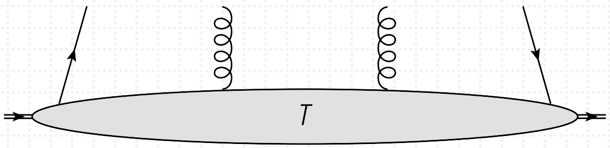
$$\left.\times\mathrm{Tr}[\hat{H}^{\mu\alpha\beta\nu}(x_1P^+,K_2,K_3)\not{P}P_\alpha P_\beta]\right\}\bigg|_{\mathbf{K}_{2\perp}=\mathbf{K}_{3\perp}=0_\perp}$$

Twist-4 at leading order

❑ Leading order contribution



T-4 q-g correlation



- TMB LO   Guo, 1998; Guo, Qiu 2000

$$\Delta\langle \ell_{hT}^2 \rangle = \left(\frac{4\pi^2\alpha_s}{N_c} z_h^2\right) \frac{\sum_q e_q^2 T_{qg}(x_B, 0, 0) D_{h/q}(z_h)}{\sum_q e_q^2 f_{q/A}(x_B) D_{h/q}(z_h)}$$

- T-4 q-g correlation function

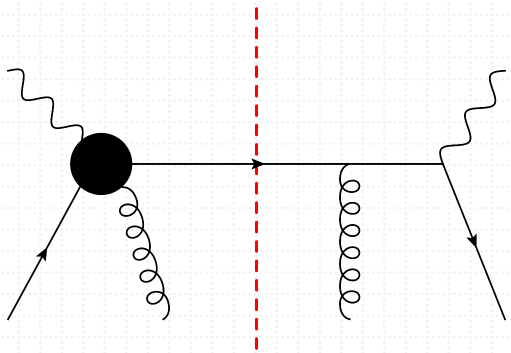
$$T_{qg}(x_1, x_2, x_3) = \int \frac{dy^-}{2\pi} e^{ix_1 p^+ y^-} \int \frac{dy_1^- dy_2^-}{4\pi} e^{ix_2 p^+(y_1^- - y_2^-)} e^{ix_3 p^+ y_2^-}$$
$$\times \langle A | \bar{\psi}_q(0) \gamma^+ F_\sigma^+(y_2^-) F^{\sigma +}(y_1^-) \psi_q(y^-) | A \rangle \theta(y_2^-) \theta(y_1^- - y^-)$$

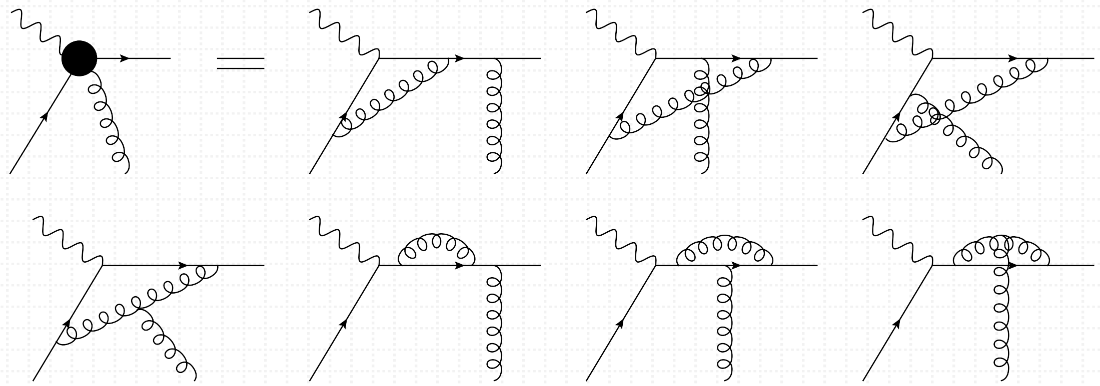
Provide a way to measure the T-4 quark-gluon correlation function.

Double scattering NLO

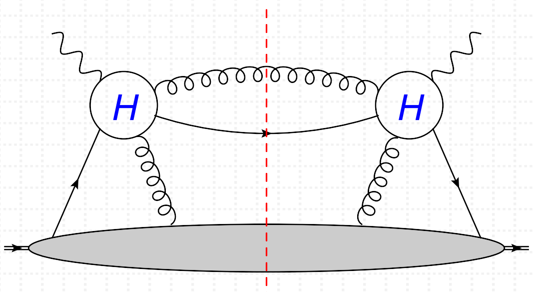
❑ Virtual

Kang, Wang, Wang, HX, PRL 112, 102001 (2014)

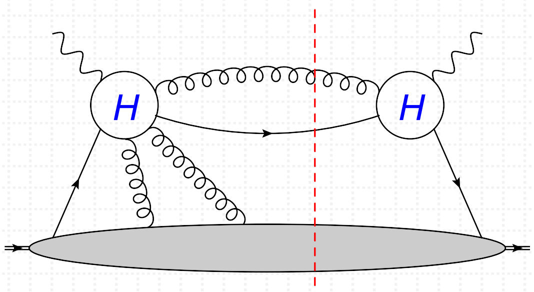




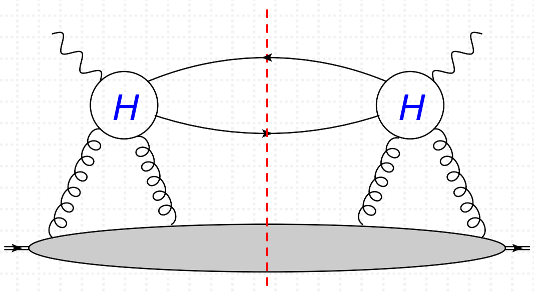
❑ Real



q-g double scattering



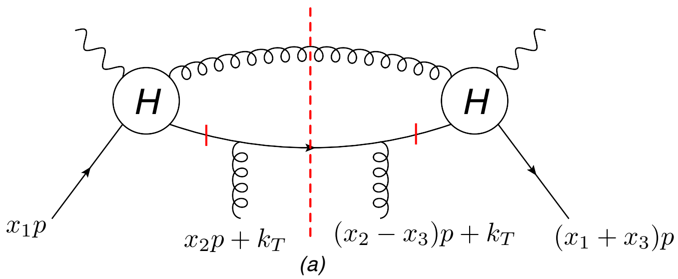
q-g single-triple interference

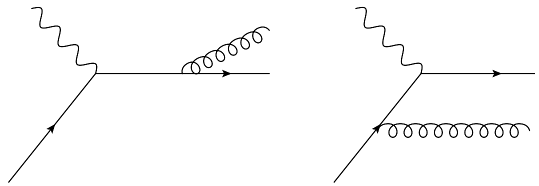


g-g double scattering

❑ Example: soft-soft double scattering in SIDIS

Kang, Wang, HX, 2014





- Soft poles

$$\frac{1}{(\ell - x_2 p - k_T)^2 + i\epsilon} = \frac{x}{\hat{u}} \frac{1}{x_2 - x_D - i\epsilon},$$
$$\frac{1}{\left[\ell - (x_2 - x_3)p - k_T\right]^2 - i\epsilon} = \frac{x}{\hat{u}} \frac{1}{x_2 - x_3 - x_D + i\epsilon}.$$

$$x = \frac{Q^2 + 2q\cdot\ell}{2p\cdot(q-\ell)}, \qquad x_C = x\frac{k_T^2 - 2\ell\cdot k_T}{\hat{t}}, \qquad x_D = x\frac{2\ell\cdot k_T - k_T^2}{\hat{u}}.$$

- Final state phase space

$$dPS^{(C)} = \frac{1}{8\pi}\left(\frac{4\pi}{Q^2}\right)^{\epsilon} \frac{1}{\Gamma(1-\epsilon)} \int dx\, \delta\left(x_1 + x_2 - x - x_C\right) \hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon} \hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon}$$

- Contour integration

$$\int dx_1 dx_2 dx_3 e^{ix_1 p^+ y^-} e^{ix_2 p^+ (y_1^- - y_2^-)} e^{ix_3 p^+ y_2^-} \frac{1}{x_2 - x_D - i\epsilon} \frac{1}{x_2 - x_3 - x_D + i\epsilon} \delta\left(x_1 + x_2 - x - x_C\right)$$
$$= e^{i(x + x_C - x_D)p^+ y^-} e^{ix_D p^+ (y_1^- - y_2^-)} (2\pi)^2 \theta(y_2^-) \theta(y_1^- - y^-)$$

momentum fractions are fixed

$$x_1 = x + x_C - x_D, \qquad x_2 = x_D, \qquad x_3 = 0.$$

- Collinear expansion

$$\frac{\partial^2\left[T(\{x_i\})H_{\mu\nu}(\{x_i\},k_T)\right]}{\partial k_T^\alpha \partial k_T^\beta} = \frac{\partial^2 T}{\partial x_i \partial x_j}\left[\frac{\partial x_i}{\partial k_T^\alpha}\frac{\partial x_j}{\partial k_T^\beta}H_{\mu\nu}\right] + \frac{\partial T}{\partial x_i}\left[\frac{\partial^2 x_i}{\partial k_T^\alpha \partial k_T^\beta}H_{\mu\nu} + \frac{\partial x_i}{\partial k_T^\alpha}\frac{\partial H_{\mu\nu}}{\partial k_T^\beta} + \frac{\partial x_i}{\partial k_T^\beta}\frac{\partial H_{\mu\nu}}{\partial k_T^\alpha}\right]$$
$$+T\frac{\partial^2 H_{\mu\nu}}{\partial k_T^\alpha \partial k_T^\beta},$$

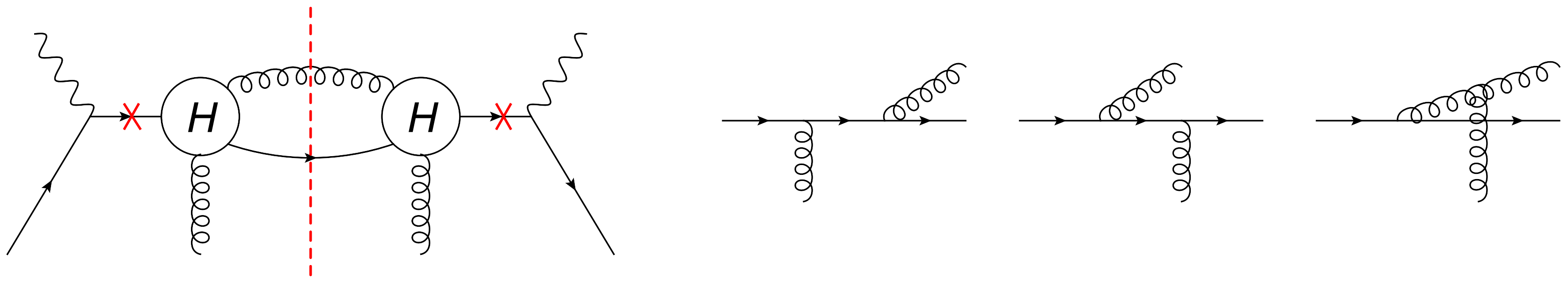
- Weighted cross section

$$\frac{d\langle \ell_{hT}^2 W^D\rangle_C^{ss}}{dz_h} = \frac{2\alpha_s}{N_c} z_h^2 (2\pi)^3(1-\epsilon)\frac{\alpha_s}{2\pi}\int\frac{dx}{x}\int\frac{dz}{z}D_{h/q}(z)\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}\hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon}\hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon}$$
$$\times\left[x^2\frac{d^2}{dx^2}T_{qg}(x,0,0)D_2^{ss} + x\frac{d}{dx}T_{qg}(x,0,0)D_1^{ss} + T_{qg}(x,0,0)D_0^{ss}\right].$$

- Divergent piece

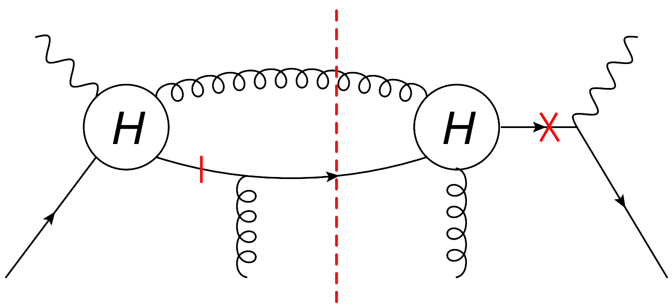
$$\sim C_F\int_{x_B}^1\frac{dx}{x}T_{qg}(x,0,0)\left[\frac{2}{\epsilon^2}\delta(1-\hat{x})\delta(1-\hat{z}) - \frac{1}{\epsilon}\delta(1-\hat{x})\frac{1+\hat{z}^2}{\hat{z}^2(1-\hat{z})_+} - \frac{1}{\epsilon}\delta(1-\hat{z})\frac{1+\hat{x}^2}{(1-\hat{x})_+}\right]$$

❑ hard-hard double scattering in SIDIS

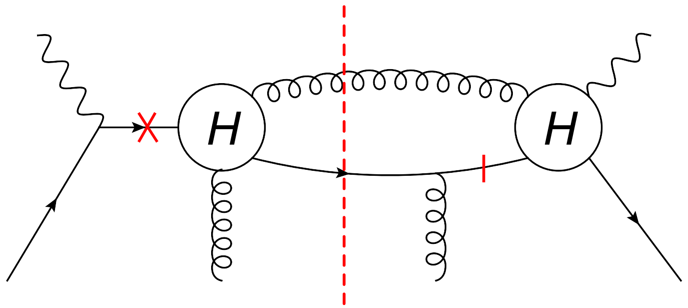


(b)

❑ Interference between soft and hard rescatterings



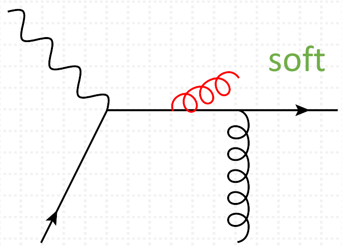
(c)



(d)

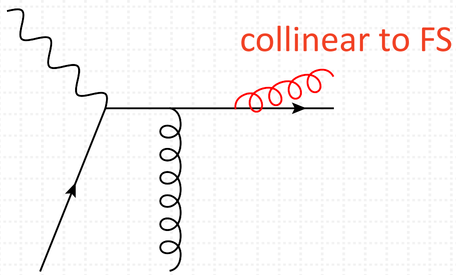
Phase space identification and renormalization

- Soft divergence: p_g --> 0



Real + virtual = 0

- collinear divergence I: p_g // p_q



$$-\frac{1}{\epsilon}\delta(1-\hat{x})T_{qg}(x,0,0)P_{qq}(\hat{z})$$

$\overline{MS}$

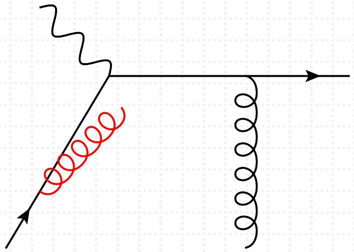
$$D_{h/q}(z_h,\mu_f^2) = D_{h/q}(z_h) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu^2}{\mu_f^2}\right)\int_{z_h}^{1}\frac{dz}{z}P_{qq}(\hat{z})D_{h/q}(z)$$

Redefinition of FF

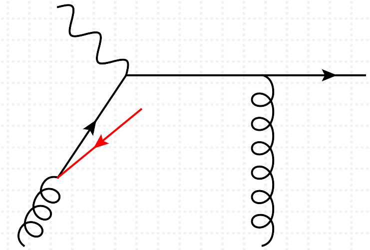
$$\mu^2\frac{\partial D_{h/q}(z_h,\mu^2)}{\partial\mu^2} = \frac{\alpha_s}{2\pi}\int\frac{dz}{z}P_{qq}(\hat{z})D_{h/q}(z,\mu^2)$$

DGLAP of FF

collinear divergence II: p_g // k_q



$$-\frac{1}{\epsilon}\delta(1-\hat{z})\mathcal{P}_{qg\to qg}(\hat{x})\otimes T_{qg}D_{h/q}(z)$$



$$-\frac{1}{\epsilon}\delta(1-\hat{z})P_{qg}(\hat{x})T_{gg}(x,0,0)D_{h/q}(z)$$

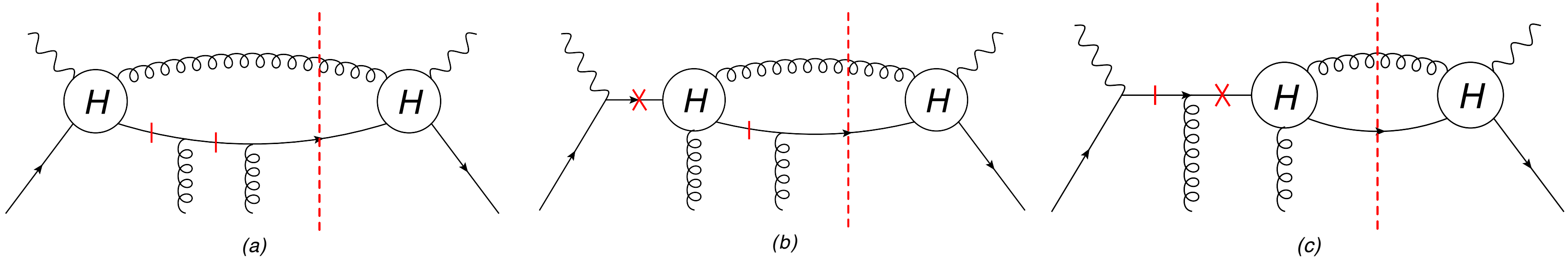
New splitting kernel

$$\mathcal{P}_{qg\to qg}\otimes T_{qg}$$
$$\equiv P_{qq}(\hat{x})T_{qg}(x,0,0)+\frac{C_A}{2}\left\{\frac{4}{(1-\hat{x})_+}T_{qg}(x_B,x-x_B,0)\right.$$
$$\left.-\frac{1+\hat{x}}{(1-\hat{x})_+}\left[T_{qg}(x,0,x_B-x)+T_{qg}(x_B,x-x_B,x-x_B)\right]\right\}$$
$$+2C_A\delta(1-\hat{x})T_{qg}(x,0,0)$$

$$T_{qg}(x_B,0,0,\mu_f^2)=T_{qg}(x_B,0,0)-\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}}+\ln\frac{\mu^2}{\mu_f^2}\right)\int_{x_B}^{1}\frac{dx}{x}\left[\mathcal{P}_{qg\to qg}\otimes T_{qg}+P_{qg}(\hat{x})T_{gg}(x,0,0)\right]$$

Redefinition of T-4 quark-gluon correlation function (MS-bar)

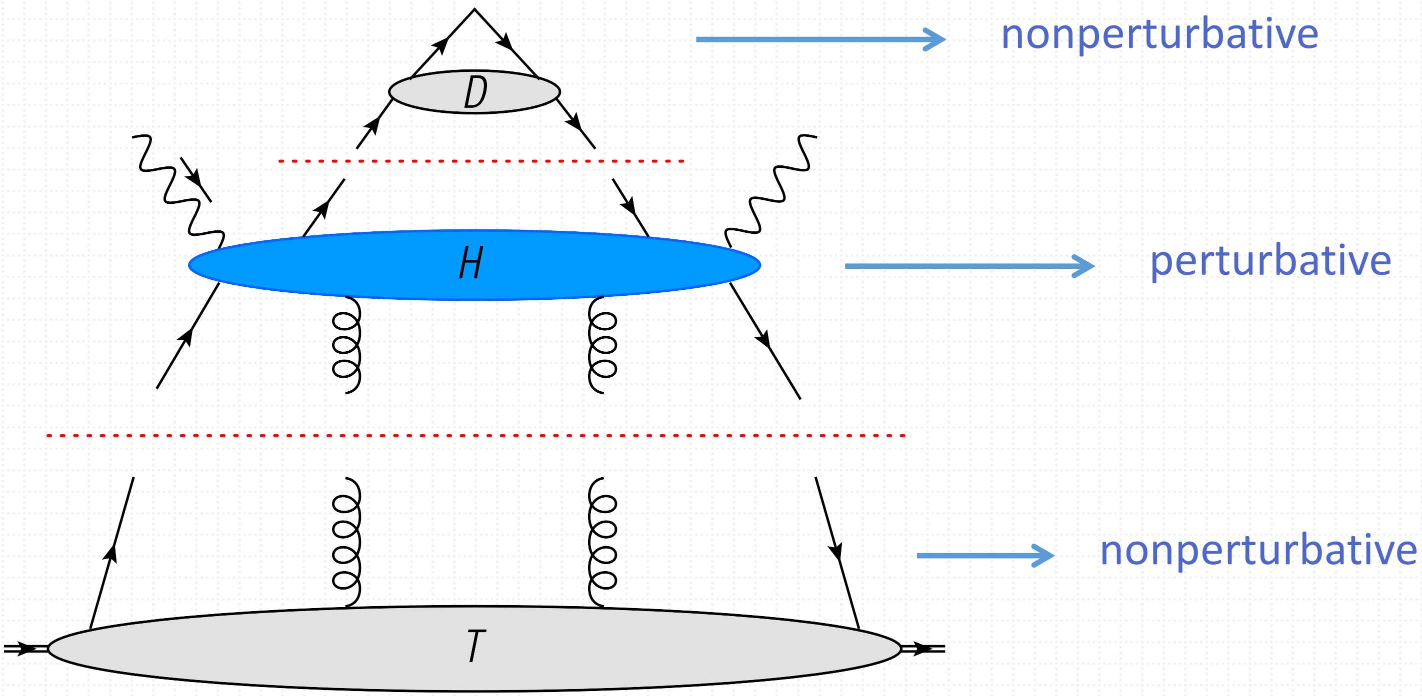
Single-triple interference



Interference between single and triple scatterings is free of any divergences.

Twist-4 NLO in SIDIS

❑ Factorization

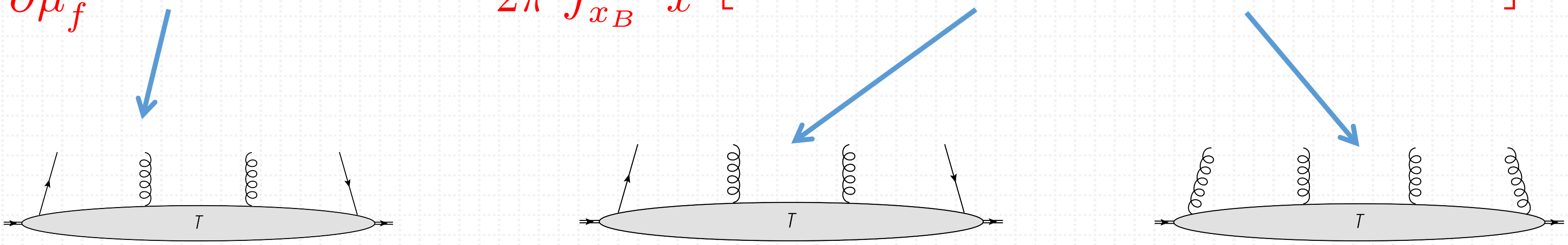


$$\frac{d\langle \ell_{hT}^2 \sigma \rangle}{dz_h} \propto D_{q/h}(z,\mu^2) \otimes H^{LO}(x,z) \otimes T_{qg}(x,0,0,\mu^2)$$
$$+ \frac{\alpha_s}{2\pi} D_{q/h}(z,\mu^2) \otimes H^{NLO}(x,z,\mu^2) \otimes T_{qg(gg)}(x,0,0,\mu^2)$$

Multiple scattering hard part coefficients and twist-4 matrix element (medium properties) can be factorized!!!

❑ Evolution equation for q-g correlation function

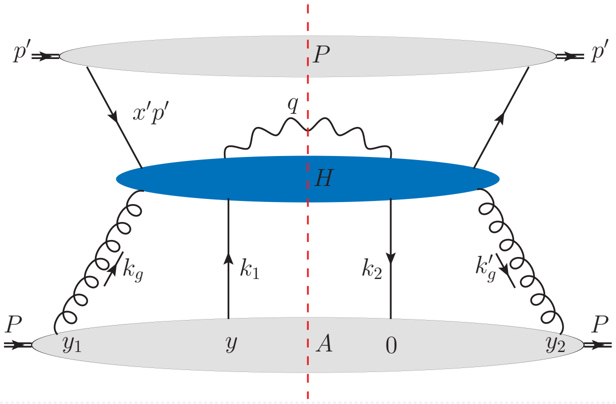
$$\mu_f^2 \frac{\partial}{\partial \mu_f^2} T_{qg}(x_B, 0, 0, \mu_f^2) = \frac{\alpha_s}{2\pi} \int_{x_B}^{1} \frac{dx}{x} \left[\mathcal{P}_{qg \to qg} \otimes T_{qg} + P_{qg}(\hat{x}) T_{gg}(x, 0, 0, \mu_f^2) \right]$$



QCD evolution equation for g-g correlation function is needed to close the equation

❑ SIDIS VS DY: Final state vs initial state

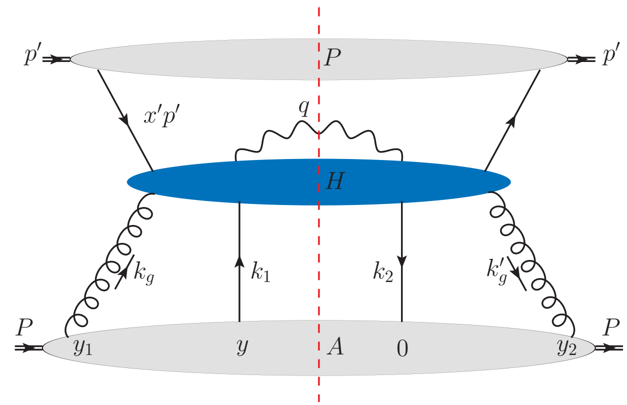
Check factorization (NLO) for processes involving initial state multiple scattering and the universality of twist-4 q-g correlation function



Drell-Yan NLO at twist-4

Kang, Qiu, Wang, HX, PRD, 2016

- Soft divergence cancel (real + virtual)



- collinear divergence

  - Redefinition of beam PDF

$$f_q(x',\mu^2) = f_q^0(x') + \frac{\alpha_s}{2\pi}\int\frac{d\xi}{\xi}\left(-\frac{1}{\hat{\epsilon}} + \ln\frac{\mu^2}{\mu_f^2}\right)P_{qq}(z)f_q(\xi)$$

  - Redefinition of nuclear T-4 gluon-quark correlation function

$$T_{gq}(x_B,0,0,\mu_f^2) = T_{gq}(x_B,0,0) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu^2}{\mu_f^2}\right)\int_{x_B}^1\frac{dx}{x}\left[\mathcal{P}_{qg\to qg}\otimes T_{gq} + P_{qg}(\hat{x})T_{gg}(x,0,0)\right]$$

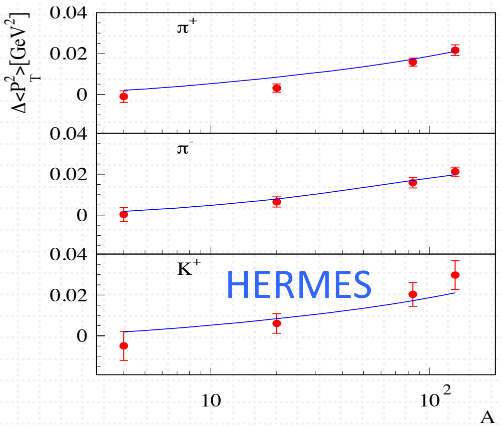
$$T_{qg}^{DIS}(x_1,x_2,x_3) = \int\frac{dy^-}{2\pi}e^{ix_1p^+y^-}\int\frac{dy_1^-dy_2^-}{4\pi}e^{ix_2p^+(y_1^--y_2^-)}e^{ix_3p^+y_2^-}$$
$$\times\langle A|\bar{\psi}_q(0)\gamma^+F_\sigma^+(y_2^-)F^{\sigma+}(y_1^-)\psi_q(y^-)|A\rangle\theta(y_2^-)\theta(y_1^- - y^-)$$

$$T_{qg}^{DY}(x_1,x_2,x_3) = \int\frac{dy^-}{2\pi}e^{ix_1p^+y^-}\int\frac{dy_1^-dy_2^-}{4\pi}e^{ix_2p^+(y_1^--y_2^-)}e^{ix_3p^+y_2^-}$$
$$\times\langle A|\bar{\psi}_q(0)\gamma^+F_\sigma^+(y_2^-)F^{\sigma+}(y_1^-)\psi_q(y^-)|A\rangle\theta(-y_2^-)\theta(y^- - y_1^-)$$

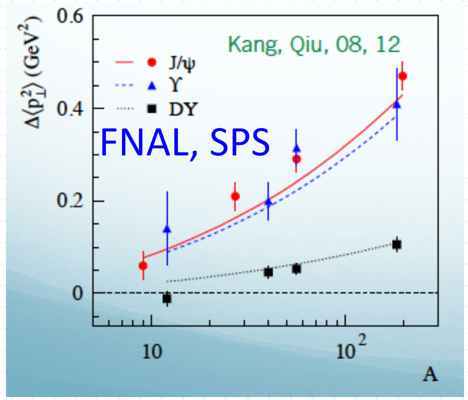
Exactly the same as that in SIDIS
it is universal!

Different channels to probe T-4 parton-parton correlation function

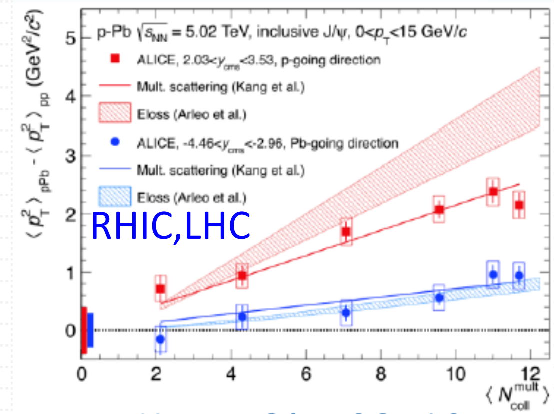
SIDIS



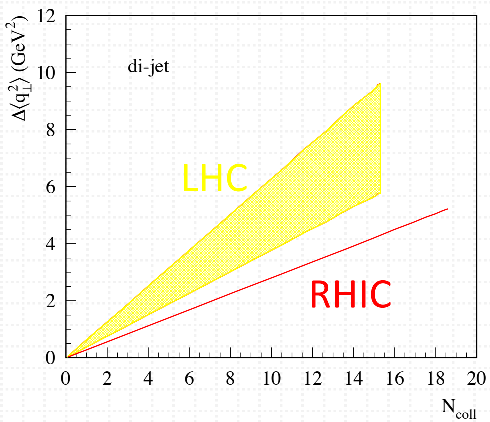
DY

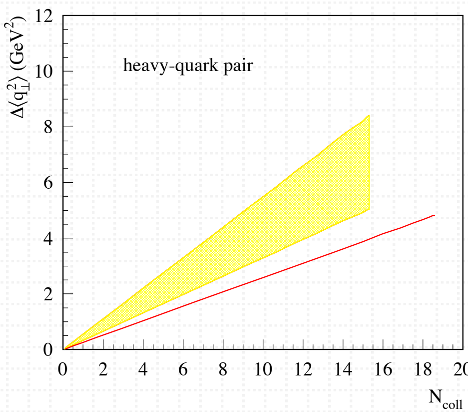


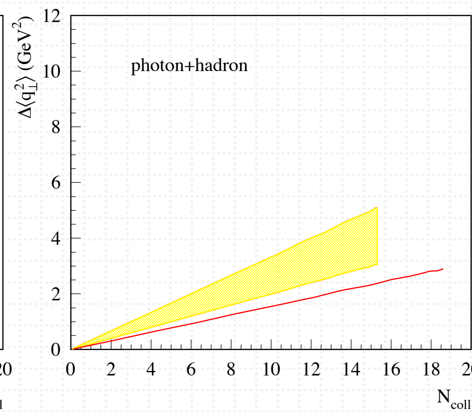
J/Psi

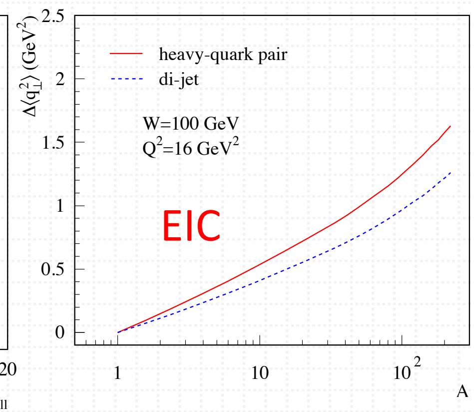


Kang, Qiu 08, 12









Transverse momentum imbalance for back-to-back particle in pA and eA collisions

HX et al, PRD, 2012

Summary

- High-twist effects provide rich information of nonperturbative QCD beyond leading twist PDFs

- Many observables are sensitive to high-twist effects, which provide us good opportunities in e+p, p+p spin programs to study fundamental structures of the nucleon, as well as in e+A and p+A heavy ion programs to understand the nuclear medium properties.

- High-twist effects are in general small, but still might be important for high precision determination of PDFs.

Extracting the jet transport parameter in HI collisions

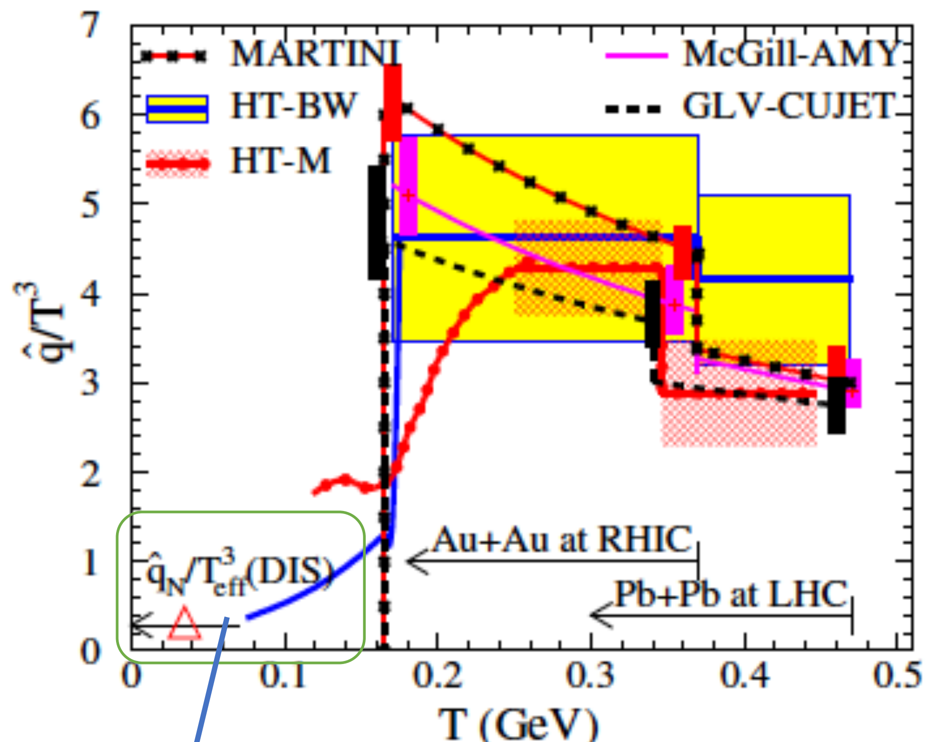
- Multiple scatterings in nuclear medium

- As a first step, only single inclusive hadron production data was included in global fitting, how about jet? dijet? photon+jet …

→ universality of the medium property

- Born level investigation of qhat

→ Any kinematic dependence?
(e.g., energy dependence of qhat)
QCD evolution of qhat



JET Collaboration, 2014

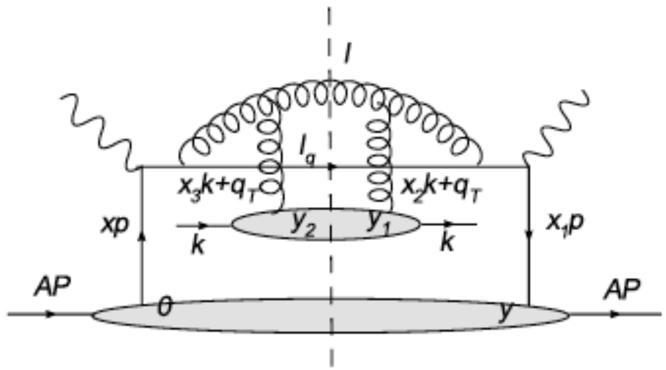
cold nuclear matter

JET Collaboration

Evolution of jet transport parameter

- Related to jet transport parameter

J. Casalderrey-Solana and X.-N. Wang (2008)



$$T_{qg}(x_B, 0, 0, \mu_f^2) \approx \frac{N_c}{4\pi^2\alpha_s} f_{q/A}(x_B, \mu_f^2) \int dy^- \hat{q}(\mu_f^2, y^-)$$

$$\hat{q}(\mu^2, y^-) = \frac{4\pi^2\alpha_s C_R}{N_c^2 - 1} \rho_N^A(y^-) x f_{g/N}(x)$$