## NLO merging with MiNLO and NNLOPS: VV and VH

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LoopFest XVI
Argonne National Laboratory, 31 May 2017

## plan of the talk

1. vector boson pair production


- access to anomalous gauge couplings
- background for several searches, for instance $H \rightarrow W W$
- NLO+PS merging of $p p \rightarrow W W$ and $p p \rightarrow W W j$ using MiNLO

2. associated Higgs production

- new-Physics in $V V H$ vertex
- $H \rightarrow b \bar{b}$ decay (in boosted regime)
- NNLO+PS matching for $p p \rightarrow H W$



## 1. vector boson pair production

[Hamilton, Melia, Monni, ER, Zanderighi '16]

## MiNLO

Multiscale Improved NLO

- original goal: method to a-priori choose scales in multijet NLO computation
- non-trivial task: hierarchy among scales can spoil accuracy (large logs can appear, without being resummed)
- how: correct weights of different NLO terms with CKKW-inspired approach (without spoiling formal NLO accuracy)


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## (1)중 Sudakov FF included on $V+j$

 Born kinematics- MiNLO-improved VJ yields finite results also when 1 st jet is unresolved ( $q_{T} \rightarrow 0$ )
- $\bar{B}_{\text {MiNLO }}$ allows extending the validity of VJ-POWHEG [called "VJ-MinLo" hereafter]
- formal accuracy of VJ-MiNLO for inclusive observables carefully investigated
[Hamilton et al., 1212.4504]
- possible to improve $\mathrm{VJ}-\mathrm{MiNLO}$ such that inclusive NLO is recovered $\left(\mathrm{NLO}^{(0)}\right)$, without spoiling NLO accuracy of $V+j\left(\mathrm{NLO}^{(1)}\right)$ :
$\underline{M i N L O}{ }^{\prime}$ : NLO+PS merging, without merging scale
- accurate control of subleading small- $p_{T}$ logarithms is needed:
- include $B_{2}$ (NNLL) coefficient in MiNLO-Sudakov
- set scales in $R, V$ and subtraction terms equal to $q_{T}$ (boson transverse momentum)
- without the above requirements, spurious $\alpha_{\mathrm{S}}^{3 / 2}$ terms show up in $\sigma_{\mathrm{NLO}}^{(0)}$ upon integration over $q_{T}$


## MiNLO'

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## MiNLO' : from Drell-Yan to $W W$

1606.07062: MiNLO' generator that merges WW and WW + 1 jet at NLO+PS:

- POWHEG WWJ generator obtained ex-novo using interfaces to Madgraph and Gosam 2.0
[Campbell et al. 1202.547; Luisoni et al. 1306.2542; Cullen et al. 1404.7096]
- starting from the Drell-Yan case, we extracted the $B_{2}^{(\mathrm{WW})}$ term from the virtual ( $V^{(\mathrm{WW})}$ ) and Born ( $B^{(\mathrm{WW})}$ ) contributions of $p p \rightarrow W W$
- for Drell-Yan, $V^{(V)}$ and $B^{(V)}$ are proportional, hence $B_{2}^{(V)}$ is just a number
- in $p p \rightarrow W W$, this is no longer true: $B_{2}^{(\mathrm{WW})}=B_{2}^{(\mathrm{WW})}\left(\Phi_{W W}\right)$
- for $q \bar{q}$-initiated color singlet production, $B_{2}$ has the form

$$
B_{2}=-2 \gamma^{(2)}+\beta_{0} C_{F} \zeta_{2}+2\left(2 C_{F}\right)^{2} \zeta_{3}+\beta_{0} H_{1}(\Phi)
$$

- from which

$$
B_{2}^{(\mathrm{WW})}=B_{2}^{(\mathrm{V})}-\beta_{0} H_{1}^{(\mathrm{V})}+\beta_{0} H_{1}^{(\mathrm{WW})}\left(\Phi_{W W}\right)
$$

- process-dependent part of $B_{2}$ extracted on an event-by-event basis:
projection of $\Phi_{W W J}$ onto $\Phi_{W W}$, used FKS ISR mapping (smooth collinear limit)


## WWJ-MiNLO' : technical details and choices

- All off-shell and single-resonant diagrams included. Full matrix-element with leptonic decays.

- worked in the 4 F scheme: no interference with $W t$ and $t \bar{t}$
- for same-family leptons, " $Z(\rightarrow \ell \bar{\ell}) Z\left(\rightarrow \nu_{\ell} \bar{\nu}_{\ell}\right)$ " not included:
- will be part of $Z Z$ generator
- interference between WW and ZZ shown to be extremely small
[Melia et al. 1107.5051]

- option to include/exclude fermionic loop corrections (at most 1-2\% difference in tails, x2 difference in speed)


## WWJ-MiNLO': results I

WW generator vs. WWJ-MiNLO generator


- total cross-section agrees at the level of $4 \%$ (although MiNLO uncertainty bands are wider than the WW ones)
- part of the shape difference in $y_{W W}$ is correlated with the differences in the $p_{T, W W}$ spectrum


## WWJ-MiNLO': results II

WW generator vs. WWJ-MiNLO generator


- NLO corrections sizeable in the spectrum
- small $p_{T}$ region: different terms in the two approaches.
. at small $p_{T}$, there's also a non-zero contribution from the two-emissions matrix element (missing in the WW case)
- underestimated WW uncertainty band


## color-singlet $p_{T}$ spectrum



[Monni,ER,Torrielli '16]

- to implement MiNLO', possible also to use ( $p_{T}^{j_{1}}$ ) as resolution variable (using Sudakov from "jet-veto" resummation)
- ongoing studies suggest that this would improve the shape at small $p_{T}$ (at least qualitatively)
- probably related to scaling properties at small $p_{T}$
- perhaps some of the aforementioned differences are related to this...


## WWJ-MiNLO' : results III

WW generator vs. WWJ-MiNLO generator
here explicit jet requirement: $25 \mathrm{GeV}, \mathrm{R}=0.4$


- below 25 GeV , at least 2 QCD emissions needed: only PS for WW, LO matrix elements for WWJ-MiNLO
- NLO K-factor visible throughout
- notice WWJ uncertainty bands: WWJ is NLO accurate for $p_{T, W W}>25 \mathrm{GeV}$, whereas WW is only LO.


## WWJ-MiNLO' : results IV

WW generator vs. WWJ-MiNLO generator (WW+j at NLO, partonic) here explicit jet requirement: $25 \mathrm{GeV}, \mathrm{R}=0.4$


- right plot shows that MinLO mantains the formal NLO accuracy in the "1-jet" region
- small differences: Sudakov effects, different scale choices ( $\mu_{\text {NLO }}=2 m_{W}$ vs. $\mu_{\text {MiNLO }}$ )


## 2. VH @ NNLO+PS

[Astill, Bizon, ER, Zanderighi '16]

## NNLO+PS for color-singlet production

- starting from a MinLO' generator, it's possible to match a PS simulation to NNLO
- XJ-MiNLO' (+POWHEG) generator gives X-XJ @ NLOPS

|  | X (inclusive) | $X+j$ (inclusive) | $X+2 \mathrm{j}$ (inclusive) |
| :---: | :---: | :---: | :---: |
| $\checkmark$ X-XJ @ NLOPS | NLO | NLO | LO |
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- reweighting (differential on $\Phi_{B}$ ) of "MiNLO-generated" events:

$$
W\left(\Phi_{B}\right)=\frac{\left(\frac{d \sigma}{d \Phi_{B}}\right)_{\mathrm{NNLO}}}{\left(\frac{d \sigma}{d \Phi_{B}}\right)_{\mathrm{XJ}-\mathrm{MiNLO}^{\prime}}}
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- by construction NNLO accuracy on fully inclusive observables
- to reach NNLOPS accuracy, need to be sure that the reweighting doesn't spoil the NLO accuracy of XJ-MiNLO in 1-jet region


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- by construction NNLO accuracy on fully inclusive observables
- to reach NNLOPS accuracy, need to be sure that the reweighting doesn't spoil the NLO accuracy of XJ-MiNLO in 1-jet region
- notice: formally works because no spurious $\mathcal{O}\left(\alpha_{\mathrm{S}}^{1.5}\right)$ terms in X-XJ @ NLOPS (relative to $\sigma_{X}$ )


## NNLO+PS for color-singlet production

- Variants for reweighting $W\left(\Phi_{B}\right)$ are also possible:

$$
\begin{gathered}
W\left(\Phi_{B}, p_{T}\right)=h\left(p_{T}\right) \frac{\int d \sigma_{A}^{\mathrm{NNLO}} \delta\left(\Phi_{B}-\Phi_{B}(\mathbf{\Phi})\right)}{\int d \sigma_{A}^{\mathrm{MiNLO}} \delta\left(\Phi_{B}-\Phi_{B}(\boldsymbol{\Phi})\right)}+\left(1-h\left(p_{T}\right)\right) \\
d \sigma_{A}=d \sigma h\left(p_{T}\right), \quad d \sigma_{B}=d \sigma\left(1-h\left(p_{T}\right)\right), \quad h\left(p_{T}\right)=\frac{(\beta M)^{2}}{(\beta M)^{2}+p_{T}^{2}}
\end{gathered}
$$

- freedom to distribute "NNLO/NLO K-factor" only over medium-small $p_{T}$ region
- $h\left(p_{T}\right)$ controls where the NNLO/NLO K-factor is distributed (in the high- $p_{T}$ region, there is no improvement in including it)
- $\beta$ cannot be too small, otherwise resummation spoiled: for Higgs, chosen $\beta=1 / 2$; for DY and HW, $\beta=1$
- in practice, we used

$$
W\left(\Phi_{B}, p_{T}\right)=h\left(p_{T}\right) \frac{\int d \sigma^{\operatorname{NNLO}} \delta\left(\Phi_{B}-\Phi_{B}(\boldsymbol{\Phi})\right)-\int d \sigma_{B}^{\mathrm{MiNLO}} \delta\left(\Phi_{B}-\Phi_{B}(\boldsymbol{\Phi})\right)}{\int d \sigma_{A}^{\mathrm{MiNLO}} \delta\left(\Phi_{B}-\Phi_{B}(\boldsymbol{\Phi})\right)}+\left(1-h\left(p_{T}\right)\right)
$$

- one gets exactly $\left(d \sigma / d \Phi_{B}\right)_{\mathrm{NNLOPS}}=\left(d \sigma / d \Phi_{B}\right)_{\mathrm{NNLO}}$
- chosen $h\left(p_{T}^{j_{1}}\right)$


## WH @ NNLOPS: technical details and choices

- started from existing HWJ-MiNLO' generator
- NNLO input from HVNNLO by [Ferrera et al. '11]
- to compute the $W\left(\Phi_{B}\right)$ function, $\left(d \sigma_{\mathrm{NNLO}} / d \Phi_{B}\right)$ needed!
$\Rightarrow$ albeit conceptually simple, in practice it gets quickly complicated
- Higgs and Drell-Yan production: extracted ( $d \sigma_{\mathrm{NNLO}} / d \Phi_{B}$ ) numerically as a (multi-dimensional) histogram: 25 bins/dimension
- not possible for $W(\rightarrow \ell \nu) H$ [at least using brute-force approach]
- used properties of final state: $\left(y_{\mathrm{HW}}, \Delta y_{\mathrm{HW}}, p_{t, \mathrm{H}}\right)+$ Collins-Soper angles

$$
\begin{aligned}
\frac{d \sigma}{d \Phi_{B}} & =\frac{d \sigma}{d y_{\mathrm{HW}} d \Delta y_{\mathrm{HW}} d p_{t, \mathrm{H}} d \cos \theta^{*} d \phi^{*}} \\
& =\frac{3}{16 \pi}\left(\frac{d \sigma}{d \Phi_{\mathrm{HW}^{*}}}\left(1+\cos ^{2} \theta^{*}\right)+\sum_{i=0}^{7} A_{i}\left(\Phi_{\mathrm{HW}^{*}}\right) f_{i}\left(\theta^{*}, \phi^{*}\right)\right)
\end{aligned}
$$

- moreover verified that reweighting factor is independent from lepton pair invariant mass
- $(25)^{5}$ bins $\rightarrow 9 \times 3$-d histograms (still tough, but manageable)


- left: CS coefficient $A_{4}$ as a function of $y_{\mathrm{Hw}}$. NNLOPS agrees with NNLO
- right: $\theta^{*}$ distribution in a $y_{\mathrm{HW}}$ window.
- very good agreement between NNLOPS, the NNLO result, and the differential NNLO cross section reconstructed from the CS parametrization.

- left: inclusive observable $\rightarrow$ NNLO+PS is NNLO accurate, and displays non-flat K-factor w.r.t. NLO+PS
- right: $p_{t}$ of color-singlet: standard observable to visualize Sudakov resummation.
- $p_{t}$ dependence of NNLO reweighting visible
- NNLO + PS approach NLO + PS at large $p_{t}$


## conclusions

- presented a recent non-trivial application of the "improved" MinLo method:
- aside from applying it to processes of the same class $(p p \rightarrow V V)$, one obvious avenue to be explored is NNLOPS simulations for $2 \rightarrow 2$ processes.
. in principle, for color singlet (as $V V$ production), all ingredients are there.
- including the $g g$-initiated channels at NLO + PS can also be studied results for $g g \rightarrow Z Z$ : [Alioli et al. 1609.09719]
- presented NNLO+PS results for $p p \rightarrow H W$ :
- the $H Z$ case is straightforward
- boosted regime will become increasingly important: possible to include also $H \rightarrow b \bar{b}$ at NLO (techniques developed for resonance-aware NLO+PS)
- variants of the MiNLO method might allow better resummation properties for the final results


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## Extra slides

## "Improved" MiNLO \& NLOPS merging: details

- Resummation formula can be written as

$$
\begin{gathered}
\frac{d \sigma}{d q_{T}^{2} d y}=\sigma_{0} \frac{d}{d q_{T}^{2}}\left\{\left[C_{g a} \otimes f_{a}\right]\left(x_{A}, q_{T}\right) \times\left[C_{g b} \otimes f_{b}\right]\left(x_{B}, q_{T}\right) \times \exp S\left(q_{T}, Q\right)\right\}+R_{f} \\
S\left(q_{T}, Q\right)=-2 \int_{q_{T}^{2}}^{Q^{2}} \frac{d q^{2}}{q^{2}} \frac{\alpha_{\mathrm{S}}\left(q^{2}\right)}{2 \pi}\left[A_{f} \log \frac{Q^{2}}{q^{2}}+B_{f}\right]
\end{gathered}
$$

- If $C_{i j}^{(1)}$ included and $R_{f}$ is $\mathrm{LO}^{(1)}$, then upon integration we get $\mathrm{NLO}^{(0)}$
- Minlo formula is not written as a total derivative: "expand" the above expression, then compare with MinLO :

$$
\sim \sigma_{0} \frac{1}{q_{T}^{2}}\left[\alpha_{\mathrm{S}}, \alpha_{\mathrm{S}}^{2}, \alpha_{\mathrm{S}}^{3}, \alpha_{\mathrm{S}}^{4}, \alpha_{\mathrm{S}} L, \alpha_{\mathrm{S}}^{2} L, \alpha_{\mathrm{S}}^{3} L, \alpha_{\mathrm{S}}^{4} L\right] \exp S\left(q_{T}, Q\right)+R_{f} \quad L=\log \left(Q^{2} / q_{T}^{2}\right)
$$

- highlighted terms are needed to reach $\mathrm{NLO}^{(0)}$ :

$$
\int^{Q^{2}} \frac{d q_{T}^{2}}{q_{T}^{2}} L^{m} \alpha_{\mathrm{S}}{ }^{n}\left(q_{T}\right) \exp S \sim\left(\alpha_{\mathrm{S}}\left(Q^{2}\right)\right)^{n-(m+1) / 2}
$$

(scaling in low $-p_{T}$ region is $\alpha_{S} L^{2} \sim 1$ !)

- if I don't include $B_{2}$ in MinLO $\Delta_{g}$, I miss a term $\left(1 / q_{T}^{2}\right) \boxed{\alpha_{\mathrm{S}}^{2}} B_{2} \exp S$
- upon integration, violate $\mathrm{NLO}^{(0)}$ by a term of relative $\mathcal{O}\left(\alpha_{\mathrm{S}}^{3 / 2}\right)$

