OpenLoops 2

A new method to generate and reduce one-loop amplitudes

Federico Buccioni

in collaboration with

S. Pozzorini M. Zoller

Universität Zürich[UZH]

FNSNF
FONDS NATIONAL SUISSE
SCHWEIZERISCHER NATIONALFONDS
FONDO NAZIONALE SVIZZERO
SWISS NATIONAL SCIENCE FOUNDATION

LoopFest, Argonne National Laboratory 06/01/2017

Contents

OpenLoops

- Fully automated numerical algorithm for tree and one-loop amplitudes **[Cascioli, Lindert, Maierhöfer, Pozzorini]**
- hybrid tree-loop recursion ⇒ very high speed
- NLO QCD and NLO EW corrections are fully implemented
- Publicly available at openloops.hepforge.org

Third party tools for the tensor integral reduction to MIs and evaluation thereof:

- `Collier` **[Denner, Dittmaier, Hofer '16]**, `Cuttools` **[Ossola, Papadopoulos, Pittau '08]** `OneLOop` **[van Hameren '10]**

Successful applications:

- used for NLO calculations for processes with $\mathcal{O}(10^5)$ loop diagrams/channel
- used in several NNLO calculations e.g. $(p\,p \to V_1\,V_2,\ H\,H)$, $V_i = \gamma, Z, W$
- interfaced to Sherpa, Powheg, Herwig, Whizard, Geneva, Munich, Matrix

Long term goal of $2 \to 3$ NNLO automation and challenges

$\Rightarrow$ construction/reduction of 2-loops amplitudes

$\Rightarrow$ speed and stability of $2 \to 4$ at NLO are crucial

OpenLoops 2 [F.B., Lindert, Maierhöfer, Pozzorini, Zoller '17]

New one-loop approach that merges amplitude construction and reduction

OpenLoops 1	OpenLoops 2
• relies on external reduction libraries	• integrand reduction merged with OpenLoops approach
• high complexity at high tensor rank	• tensor rank ≤ 2 at any stage of the calculation
• stability in the IR region for $2 \to 4$ processes is challenging	• stability issue can be addressed in the new reduction approach

OpenLoops 1

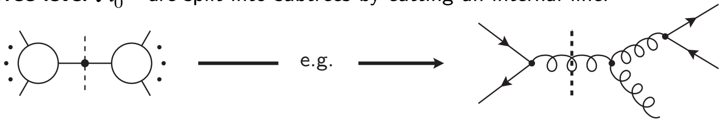
Tree level and one-loop amplitudes as sums of individual Feynman diagrams

$$\mathcal{M}_0 = \sum_d \mathcal{M}_0^{(d)}\,, \qquad \mathcal{M}_1 = \sum_d \mathcal{M}_1^{(d)}$$

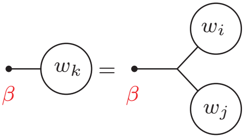
and each diagram is factorized into a color factor and a color stripped amplitude:

$$\mathcal{M}_l^{(d)} = \mathcal{C}_l^{(d)}\,\mathcal{A}_l^{(d)}\,.$$

Tree level $\mathcal{A}_0^{(d)}$ are split into subtrees by cutting an internal line:

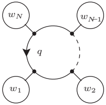


Subtrees computed numerically via recursive merging: universal kernels ⇒ automation



$$w_k^\beta = \frac{X^\beta_{\gamma\delta}}{p_i^2 - m_i^2 + \mathrm{i}\epsilon}\, w_i^\gamma w_j^\delta$$

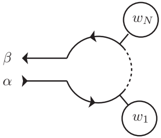
Tensor representation of 1-loop diagrams $\qquad D_i = (q+p_i)^2 - m_i^2$

$\mathcal{A}_1^d =$ 

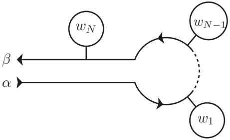
$$= \int \mathrm{d}^D q \frac{\mathcal{N}(\mathcal{I}_N, q)}{D_0 D_1 \cdots D_{N-1}} = \sum_{r=0}^{N} \mathcal{N}_{\mu_1 \cdots \mu_r} \underbrace{\int \mathrm{d}^D q \frac{q^{\mu_1} \cdots q^{\mu_r}}{D_0 D_1 \cdots D_{N-1}}}_{\text{fed to a tensor integral reduction library}}$$

One-loop diagrams $\mathcal{A}_1^d$ are cut open and generated with hybrid tree-loop recursion

$$\mathcal{N}_N^{\alpha\beta}(q) \equiv \sum_{r=0}^{N} \mathcal{N}^{\alpha\beta}_{\mu_1 \cdots \mu_r} q^{\mu} \cdots q^{\mu_r} =$$



$=$



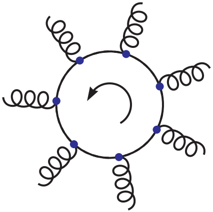
Recursive construction of tensor coefficients $\Rightarrow$ very high speed

$$\mathcal{N}_N^{\alpha\beta}(q) = X^{\beta}_{N;\gamma\delta}(q) \cdot w_N^{\delta} \cdot \mathcal{N}^{\alpha\gamma}_{N-1}(q)$$

$$\text{"segments"} \equiv \mathcal{X}^{\beta}_{N;\gamma}(q) = \left(Y^{\beta}_{N;\gamma\delta} + q^{\nu} Z^{\beta}_{N;\gamma\delta\nu} \right) w_N^{\delta}$$

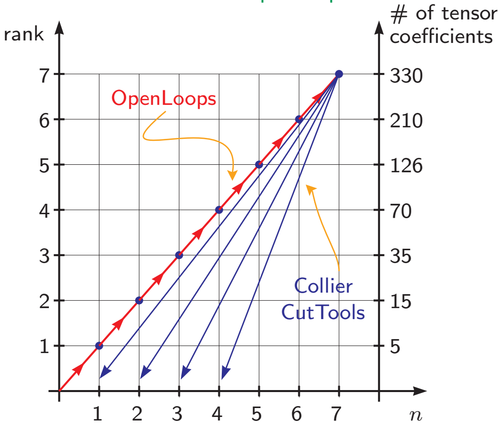
Factorized expression $\Rightarrow \mathcal{N}_N(q) \simeq \mathcal{X}_N(q) \cdots \mathcal{X}_{k+1}(q) \mathcal{N}_k(q) = \prod_{k=1}^{N} \mathcal{X}_k(q)$

Interplay between amplitude construction and reduction in OpenLoops 1



Example of a 7g diagram

complexity grows exponentially with tensor rank



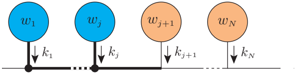
n: # of attached external legs

Numerical tensor integral reduction to scalar integrals

OpenLoops 2

- exploits factorization properties of OpenLoops recursion
- performs an on-the-fly integrand reduction during amplitude construction
- keeps the rank ≤ 2 at any stage of the calculation

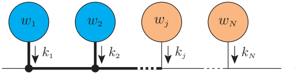
Factorized structure of N-point OpenLoops integrand.



$$\Rightarrow \mathcal{N}_N(q) \simeq \overbrace{\mathcal{X}_1(q)\cdots\mathcal{X}_j(q)}^{\text{first j steps}}\underbrace{\mathcal{X}_{j+1}(q)\cdots\mathcal{X}_N(q)}_{\text{future N-j steps}}$$

The factorized representation in $\mathcal{X}_j(q)$ allows for an *on-the-fly Reduction (OFR)*

$$\frac{\mathcal{N}_N(q)}{D_0\cdots D_N} = \frac{\mathcal{X}_1(q)\mathcal{X}_2(q)\cdots\mathcal{X}_j(q)\cdots\mathcal{X}_N(q)}{D_0 D_1 D_2 D_3\cdots D_N},$$



→ integrand reduction applicable after any $j \geq 2$ step (independently of future steps!)

Integrand reduction of $\geq$ 3-point integrals with rank ≥ 2 **[del Aguila, Pittau '05]**

$$q^\mu q^\nu = A^{\mu\nu} + B^{\mu\nu}_\lambda q^\lambda$$

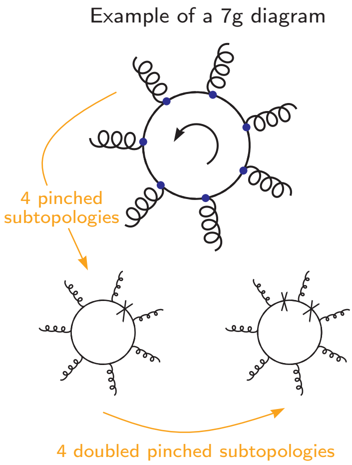
rank-2 monomials are reduced to rank-1 on-the-fly, i.e. at any OL construction step.

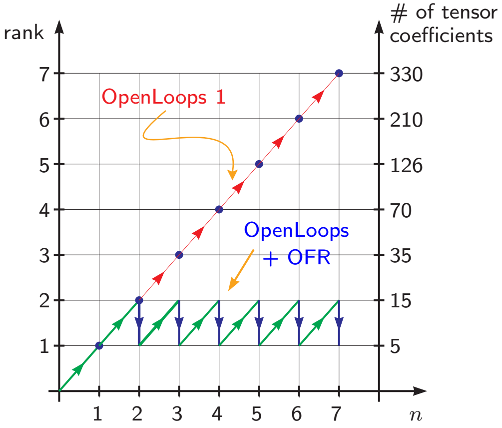
unpinched contributions

$$\begin{cases} A^{\mu\nu} &= A^{\mu\nu}_{up} + A^{\mu\nu}_0 D_0 \\ B^{\mu\nu}_\lambda &= B^{\mu\nu}_{up,\lambda} + \sum_{i=0}^{3} B^{\mu\nu}_{i,\lambda} D_i \end{cases}$$

reconstructed denominators

Interplay between amplitude construction and reduction in OpenLoops 2

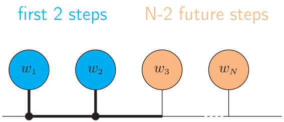


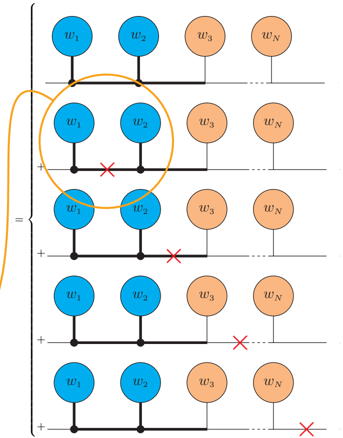


complexity associated with tensor rank remains small

Problem: huge computational due to proliferation of subtopologies from reduction

$$\underbrace{\mathcal{N}_k(q)}_{\text{rank}=2} = \underbrace{\mathcal{N}_{up}(q)}_{\text{rank}=1} + \sum_{i=0}^{3} \underbrace{\mathcal{N}_i(q)}_{\text{rank}=1} D_i + \underbrace{\mathcal{N}_{R_1}\tilde{q}^2}_{\text{rational terms}}$$





naive recursion + reduction
$\Rightarrow$ potentially highly inefficient

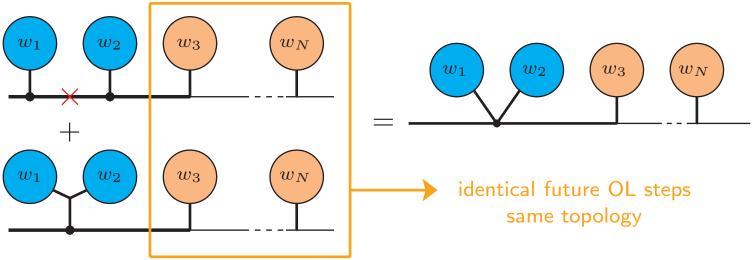
e.g. D_1-pinched subtopology

future steps need to be performed for all (1+4)

subtopologies $\sim$ factor 5 higher in complexity at each step

Solution: Diagrams Merging

Pinched subtopologies from the on-the-fly reduction of N-point open loops can be merged with unpinched $(N-1)$-point open loops.



- The condition is two open loops sharing identical future steps and same topology.
- Recursive merging works for any tensor rank and can be iterated after any OpenLoops+OFR step

OpenLoops+OFR opens the doors to a fast one-loop algorithm with low tensor rank!

CPU performance of OpenLoops 1 + Collier/Cuttools vs OpenLoops 2 with OFR

Runtimes ($10^{-3}s$) per phase-space point. Helicity and color sums included. The last column shows the timing ratios between the fastest OL1 interface and OL2.

	OL1 (Collier)	OL1 (Cuttools)	OL2	OL1/OL2
$u\bar{u} \to t\,\bar{t}$	0.2355	0.4034	0.2385	0.99
$u\bar{u} \to t\,\bar{t}\,g$	4.259	7.066	3.828	1.1
$u\bar{u} \to t\,\bar{t}\,g\,g$	$1.154 \cdot 10^2$	$1.612 \cdot 10^2$	$0.7884 \cdot 10^2$	1.5
$gg \to t\,\bar{t}$	1.408	2.486	1.019	1.4
$gg \to t\,\bar{t}\,g$	35.03	50.23	23.32	1.5
$gg \to t\,\bar{t}\,g\,g$	$1.330 \cdot 10^3$	$1.519 \cdot 10^3$	$0.600 \cdot 10^3$	2.2
$u\bar{d} \to W^+\,g$	0.2972	0.6274	0.3349	0.89
$u\bar{d} \to W^+\,g\,g$	5.690	11.3	5.664	1.0
$u\bar{d} \to W^+\,g\,g\,g$	$1.787 \cdot 10^2$	$2.380 \cdot 10^2$	$1.142 \cdot 10^2$	1.6
$u\bar{u} \to W^+\,W^-$	0.2622	0.4140	0.1756	1.5
$u\bar{u} \to W^+\,W^-\,g$	8.528	12.04	7.145	1.2
$u\bar{u} \to W^+\,W^-\,g\,g$	$2.441 \cdot 10^2$	$2.817 \cdot 10^2$	$1.340 \cdot 10^2$	1.8

Factor ~ 2 speedup wrt OpenLoops 1 for nontrivial processes!

Numerical Stability

Integrand reduction formula for ≥ 3-point functions of rank≥ 2 **[del Aguila, Pittau '05].**

The main idea is to decompose q^μ into a basis of lightlike l_i^μ:

$$l_1^\mu = p_1^\mu - \alpha_1 p_2^\mu, \qquad l_3^\mu = \bar{v}(l_1)\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) u(l_2), \qquad l_{1,2}\cdot l_{3,4} = 0$$

$$l_2^\mu = p_2^\mu - \alpha_2 p_1^\mu, \qquad l_4^\mu = \bar{v}(l_2)\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) u(l_1), \qquad l_1\cdot l_2 = -\frac{l_3\cdot l_4}{4}$$

$$q^\mu = \frac{1}{\gamma} D^\mu - \frac{1}{2\gamma} Q^\mu$$

$$D^\mu = 2(q\cdot l_2) l_1^\mu + 2(q\cdot l_1) l_2^\mu$$
$$Q^\mu = (q\cdot l_4) l_3^\mu + (q\cdot l_3) l_4^\mu$$

$$\gamma = \frac{4\Delta}{p_1\cdot p_2 \pm \Delta}$$

Δ - Gram Determinant

$$(q\cdot l_j) \to D_i$$

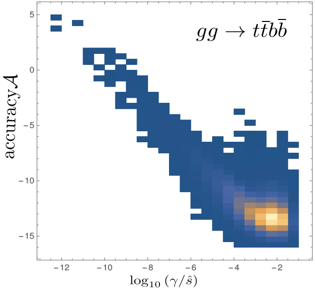
$$q^\mu q^\nu = A^{\mu\nu} + B_\lambda^{\mu\nu} q^\lambda$$

factors $\sim \Delta^{-2}$
$\Rightarrow$ potential instabilities

$A^{\mu\nu}, B_\lambda^{\mu\nu}$ are functions of the p_j, $j = 1, 2(3)$, Δ and D_i

$A^{\mu\nu}, B_\lambda^{\mu\nu}$ suffer from severe γ^{-2} instabilities

Clear and neat correlation of instabilities with Gram-determinants Δ from individual parameters of the reduction.



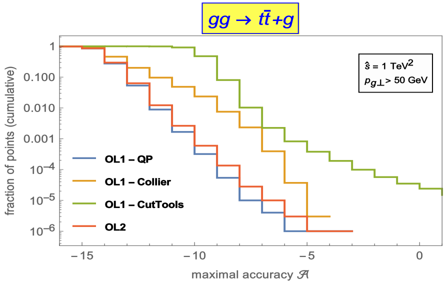
$$\gamma = \frac{4\Delta}{p_1 \cdot p_2 \pm \Delta}$$

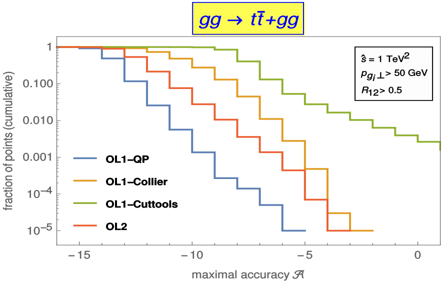
→ correlation between the smallest γ for all Feynman diagrams and the number of accurate digits (wrt to a QP benchmark) for 10^5 $gg \to t\bar{t}b\bar{b}$ events

Simple and powerful solution to numerical instability problems

- for $N \geq 4$ integrals, there are 3 external momenta available in $D_0 \cdots D_3$: we choose the pair (p_1, p_2) with the largest $\gamma \Rightarrow$ no $\Delta-$expansions needed.
- for $N = 3$, $\Delta-$expansions are performed only in few special cases: analytic $\sqrt{\Delta}$ expansions up to $\mathcal{O}(\Delta)$ of full reduction to scalars.

Probability of relative accuracy $\mathcal{A}$ or less (assessed wrt OL+Cuttools QP benchmarks with uniform random points)





Stability of OpenLoops 2 in double precision (DP)

- behaviour in the tails crucial for real-life applications
- orders of magnitude improvement wrt Cuttols in DP
- very significant improvement also wrt Collier in DP

Excellent stability thanks to on-the fly reduction with rank ≤ 2 and minimal Δ-expansions

Conclusions

We have presented the new **OpenLoops 2** algorithm for the automated generation of one-loop matrix elements in the Standard Model

1. the generation and reduction of loop integrands are merged in a single recursion that keeps the tensor rank ≤ 2 throughout
2. merging pinched and unpinched open loops avoids the proliferation of subtopologies and permits to reach very high CPU performance
3. the main sources of numerical instabilities have been understood and addressed with simple and general solutions that yield very high stability

Backup Slides

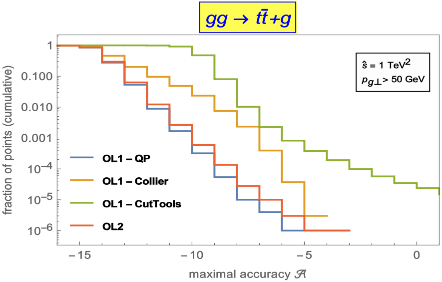
- Benchmarks computed in quadruple precision (OL.1 + `Cuttools`)
- The accuracy $\mathcal{A}$ of a result r wrt to a benchmark b is defined as

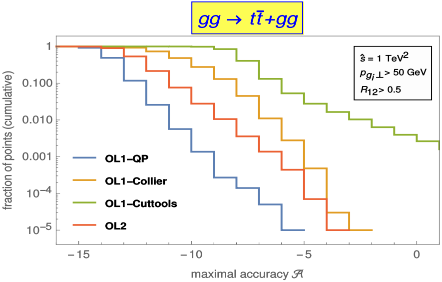
$$\mathcal{A} = \log_{10}\left[\mathrm{Max}\left(\left|\frac{b-r}{r}\right|, \left|\frac{b-r}{b}\right|\right)\right]$$

- all other results are evaluated in DP.

Stability plots:

Probability of accuracy of $\mathcal{A}$ or less for two selected processes $gg \to t\bar{t}g$ and $gg \to t\bar{t}gg$ in samples of 10^5 and 10^6 uniformly distributed phase-space points respectively.





Stability Studies:

The stability of the QP benchmarks is assessed through a rescaling test: at a given phase space point one-loop matrix elements are recomputed by rescaling all dimensionful input parameters by a factor $\xi \Rightarrow$

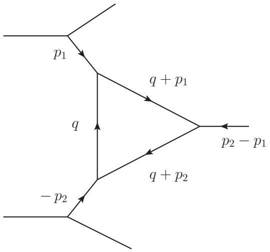
$$\mathcal{M}_R \equiv \mathcal{M}(\xi p_i, \xi m_i) = \xi^d \mathcal{M}(p_i, m_i) \;\Rightarrow\; |\mathcal{M}_R|^2 = \xi^{2d} |\mathcal{M}|^2$$

The intrinsic accuracy of the QP benchmark $\mathcal{A}_{QP}$ is then:

$$\mathcal{A}_{QP} = \log_{10}\left[\text{Max}\left(1 - \xi^{2d}\frac{|\mathcal{M}|^2}{|\mathcal{M}_R|^2}, 1 - \xi^{-2d}\frac{|\mathcal{M}_R|^2}{|\mathcal{M}|^2}\right)\right]$$

The accuracy of the DP results is assessed comparing the latter with the QP benchmarks.

Targeted expansions in the Gram determinant Δ. A special case



- $p_1^2 = -p^2, \quad p_2^2 = -p^2\,(1+\delta)$
- $(p_2 - p_1)^2 = 0$
- $\sqrt{\Delta} = \frac{p^2}{2}\delta \;\Rightarrow\; \gamma = -p^2\delta^2$

In the reduction one has γ^{-2} factors at each step.

When $\delta \ll 1$, severe $\frac{1}{\delta^4}$ instabilities show up.