The Proton Radius Puzzle

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Introduction: The proton radius puzzle

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)|\sum_q e_q\, \bar{q}\gamma^\mu q|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q^\nu\right] u(p_i)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\boxed{\langle r^2\rangle_E^p = 6\frac{dG_E^p}{dq^2}\bigg|_{q^2=0}}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2\rangle_E^p}$

- The proton *magnetic* radius

$$\langle r^2\rangle_M^p = \frac{6}{G_M^p(0)}\frac{dG_M^p(q^2)}{dq^2}\bigg|_{q^2=0}$$

Charge radius from atomic physics

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
  more recently $r_E^p = 0.84087(39)$ fm [Antognini et al. Science **339**, 417 (2013)]
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.87680(690)$ fm
  more recently $r_E^p = 0.87510(610)$ fm [Mohr et al. RMP **88**, 035009 (2016)]
  extracted mainly from (electronic) hydrogen
- **5σ discrepancy!**

- This is the proton radius puzzle

What could be the reason for the discrepancy?

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1) Problem with the electronic extraction? (Part 1 of this talk)

2) Hadronic Uncertainty? (Part 2 of this talk)

3) New Physics?

Outline

- Introduction: The proton radius puzzle
- Part 1: Proton radii from scattering
- Part 2: Hadronic Uncertainty?
- Backup slides: Connecting $\mu - p$ scattering and muonic hydrogen
- Conclusions and outlook

Part 1: Proton radii from scattering

Problem with the electronic extraction?

- Recent development: use of the z expansion based on known analytic properties of form factors
- **The** method for **meson** form factors [Flavor Lattice Averaging Group, EPJ C **74**, 2890 (2014)]
- Now applied successfully to **baryon** form factors to extract r_E^p, r_M^p, r_M^n, m_A...

PDG 2016: r_E^p

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

p CHARGE RADIUS

This is the rms electric charge radius, $\sqrt{\langle r_E^2 \rangle}$.

VALUE (fm)			*DOCUMENT ID*		*TECN*	*COMMENT*
0.8751	**±0.0061**		MOHR	16	RVUE	2014 CODATA value
0.84087	**±0.00026**	**±0.00029**	ANTOGNINI	13	LASR	μp-atom Lamb shift
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.895	±0.014	±0.014	[1] LEE	15	SPEC	Just 2010 Mainz data
0.916	±0.024		LEE	15	SPEC	World data, no Mainz
0.8775	±0.0051		MOHR	12	RVUE	2010 CODATA, *e p* data
0.875	±0.008	±0.006	ZHAN	11	SPEC	Recoil polarimetry
0.879	±0.005	±0.006	BERNAUER	10	SPEC	$e p \to e p$ form factor
0.912	±0.009	±0.007	BORISYUK	10		reanalyzes old *e p* data
0.871	±0.009	±0.003	HILL	10		z-expansion reanalysis
0.84184	±0.00036	±0.00056	POHL	10	LASR	See ANTOGNINI 13
0.8768	±0.0069		MOHR	08	RVUE	2006 CODATA value
0.844	$^{+0.008}_{-0.004}$		BELUSHKIN	07		Dispersion analysis
0.897	±0.018		BLUNDEN	05		SICK 03 + 2γ correction
0.8750	±0.0068		MOHR	05	RVUE	2002 CODATA value
0.895	±0.010	±0.013	SICK	03		$e p \to e p$ reanalysis

[Hill, GP PRD **82** 113005 (2010)]

[Lee, Arrington, Hill, PRD **92**, 013013 (2015)]

PDG 2016: r_M^p

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

p MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\langle r_M^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*TECN*	*COMMENT*
0.776±0.034±0.017	[1] LEE	15	SPEC	Just 2010 Mainz data
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.914±0.035	LEE	15	SPEC	World data, no Mainz
0.87 ±0.02	EPSTEIN	14		Using $e p$, $e n$, $\pi\pi$ data
0.867±0.009±0.018	ZHAN	11	SPEC	Recoil polarimetry
0.777±0.013±0.010	BERNAUER	10	SPEC	$e p \to e p$ form factor
0.876±0.010±0.016	BORISYUK	10		Reanalyzes old $e p \to e p$ data
0.854±0.005	BELUSHKIN	07		Dispersion analysis

[1] Authors also provide values for a combination of all available data.

[Epstein, GP, Roy PRD **90**, 074027 (2014)]
[Lee, Arrington, Hill, PRD **92**, 013013 (2015)]

PDG 2016: r_M^n

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

n MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\langle r_M^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*COMMENT*
$0.864^{+0.009}_{-0.008}$ OUR AVERAGE			
0.89 ± 0.03	EPSTEIN	14	Using $e\,p$, $e\,n$, $\pi\pi$ data
$0.862^{+0.009}_{-0.008}$	BELUSHKIN	07	Dispersion analysis

[Epstein, GP, Roy PRD **90**, 074027 (2014)]

Part 2: Hadronic Uncertainty?

[Hill, GP PRD **95**, 094017 (2017), arXiv:1611.09917]

The bottom line

- Scattering:
- World $e-p$ data [Lee, Arrington, Hill '15]
  $r_E^p = 0.918 \pm 0.024$ fm
- Mainz $e-p$ data [Lee, Arrington, Hill '15]
  $r_E^p = 0.895 \pm 0.020$ fm
- Proton, neutron and π data [Hill , GP '10]
  $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$

- Muonic hydrogen
- [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
- [Antognini et al. Science **339**, 417 (2013)]
  $r_E^p = 0.84087(39)$ fm

- The bottom line:
  using z expansion scattering disfavors muonic hydrogen

- Is there a problem with muonic hydrogen *theory*?

Muonic hydrogen theory

- Is there a problem with muonic hydrogen *theory*?

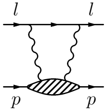
- Potentially yes!
  [Hill, GP PRL **107** 160402 (2011)]

- Muonic hydrogen measures ΔE and translates it to r_E^p

- [Pohl et al. Nature **466**, 213 (2010) Supplementary information]
  $\Delta E = 206.0573(45) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3 \text{ meV}$

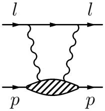
- [Antognini et al. Science **339**, 417 (2013), Ann. of Phy. **331**, 127]
  $\Delta E = 206.0336(15) - 5.2275(10)(r_E^p)^2 + 0.0332(20) \text{ meV}$

- In both cases apart from r_E^p need two-photon exchange



Two photon exchange

- Apart from r_E^p we have two-photon exchange (TPE)



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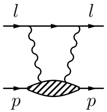

$$W^{\mu\nu} = \frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \boldsymbol{k}, s|\, T\{J^\mu_{\rm e.m.}(x) J^\nu_{\rm e.m.}(0)\}|\boldsymbol{k}, s\rangle$$

$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1 + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right)W_2$$

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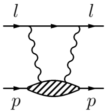

$$
\begin{aligned}
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&= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1+\left(k^\mu-\frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu-\frac{k\cdot q\, q^\nu}{q^2}\right)W_2
\end{aligned}
$$

- Dispersion relations ($\nu = 2k\cdot q,\ Q^2=-q^2$)

$$W_1(\nu, Q^2) = W_1(0, Q^2)+\frac{\nu^2}{\pi}\int_{\nu_{\mathrm{cut}}(Q^2)^2}^{\infty} d\nu'^2\, \frac{\mathrm{Im}\, W_1(\nu', Q^2)}{\nu'^2(\nu'^2-\nu^2)}$$

$$W_2(\nu, Q^2) = \frac{1}{\pi}\int_{\nu_{\mathrm{cut}}(Q^2)^2}^{\infty} d\nu'^2\, \frac{\mathrm{Im}\, W_2(\nu', Q^2)}{\nu'^2-\nu^2}$$

- W_1 requires subtraction...

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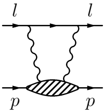

- Imaginary part of TPE related to data:

  form factors, structure functions

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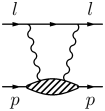

- Imaginary part of TPE related to data:
  form factors, structure functions

- Cannot reproduce it from its imaginary part:
  Dispersion relation requires subtraction

- Need poorly constrained non-perturbative function $W_1(0, Q^2)$

Two Photon exchange: small Q^2 limit

- *Small* Q^2 limit using NRQED [Hill, GP, PRL **107** 160402 (2011)]
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$$W_1(0,Q^2) = 2a_p(2+a_p) + \frac{Q^2}{m_p^2}\left\{\frac{2m_p^3\bar{\beta}}{\alpha} - a_p - \frac{2}{3}\left[(1+a_p)^2 m_p^2 (r_M^p)^2 - m_p^2 (r_E^p)^2\right]\right\} + \mathcal{O}\left(Q^4\right)$$

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- $a_p = 1.793$, $\bar{\beta} = 2.5(4)\times 10^{-4}$ fm^3
- $r_M = 0.776(34)(17)$ fm,
- $r_E^H = 0.8751(61)$ fm or $r_E^{\mu H} = 0.84087(26)(29)$ fm

$$W_1(0,Q^2) = 13.6 + \frac{Q^2}{m_p^2}(-54\pm 7) + \mathcal{O}\left(Q^4\right)$$

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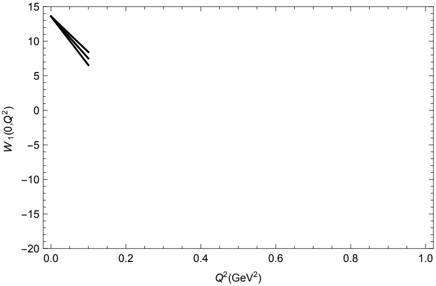
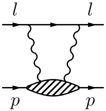


Two Photon Exchange: large Q^2 limit



- Calculable in *large* Q^2 limit using Operator Product Expansion (OPE) [J. C. Collins, NPB **149**, 90 (1979)]
  The photon "sees" the quarks and gluons inside the proton

$$W_1(0, Q^2) = c/Q^2 + \mathcal{O}\left(1/Q^4\right)$$

- Result was used to estimate two photon exchange effects
- c calculated in [J. C. Collins, NPB **149**, 90 (1979)]

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John C. COLLINS *
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540, USA

Received 23 October 1978

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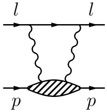

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- Was it?

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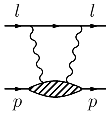

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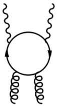
- $W_1(0, Q^2)$ is dimensionless

$$W_1 \sim \frac{\langle \text{Proton}|O|\text{Proton}\rangle}{Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)$$

- O is a dimension 4 operator:

- Quarks: Spin 0: $m_q\bar{q}q$ Spin 2: $\bar{q}\left(iD^\mu\gamma^\nu + iD^\nu\gamma^\mu - \frac{1}{4} i\not{D}\, g^{\mu\nu}\right)q$
- Gluons: must be color singlet: $G_a^{\alpha\beta} G_a^{\rho\sigma}$
- What gluon operators can we have?

Gluon operators



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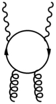
  A product of (E^i, B^i) and (E^j, B^j) has $7 \times 6/2 = 21$ components:

Gluon operators



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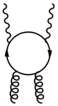

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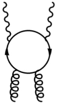

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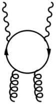
- 9 components of traceless symmetric tensor: $G^{\mu\alpha} G_\alpha^{\ \nu} - \frac{1}{4} G^{\alpha\beta} G_{\alpha\beta} g^{\mu\nu}$
  chromomagnetic stress-energy tensor
- What else?

Gluon operators



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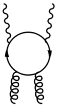

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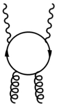
$$O^{\mu\alpha\nu\beta} = -\frac{1}{4}\left(\epsilon^{\mu\alpha\rho\sigma}\epsilon^{\nu\beta\kappa\lambda} + \epsilon^{\mu\beta\rho\sigma}\epsilon^{\nu\alpha\kappa\lambda}\right) G_{\rho\kappa} G_{\sigma\lambda} - \text{all possible traces}$$

For example $O^{0123} = G^{01} G^{23} + G^{03} G^{21} = E^1 B^1 - E^3 B^3$

Gluon operators



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  chromomagnetic stress-energy tensor
- What else? 10 components of

$$O^{\mu\alpha\nu\beta} = -\frac{1}{4}\left(\epsilon^{\mu\alpha\rho\sigma}\epsilon^{\nu\beta\kappa\lambda} + \epsilon^{\mu\beta\rho\sigma}\epsilon^{\nu\alpha\kappa\lambda}\right) G_{\rho\kappa} G_{\sigma\lambda} - \text{all possible traces}$$

  For example $O^{0123} = G^{01} G^{23} + G^{03} G^{21} = E^1 B^1 - E^3 B^3$
- For protons: $\langle \text{Proton} | O^{\mu\alpha\nu\beta} | \text{Proton} \rangle = 0$
  What about $\langle \text{Medium} | O^{\mu\alpha\nu\beta} | \text{Medium} \rangle$?
  Solution looking for a problem...

Summary: Possible operators

- In total we have four operators with non-zero proton matrix elements.

- Quarks:
- Spin 0: $m_q \bar{q} q$
- Spin 2: $\bar{q}\left(iD^\mu\gamma^\nu + iD^\nu\gamma^\mu - \frac{1}{4} i \not{D}\, g^{\mu\nu}\right) q$

- Gluons:
- Spin 0: $G^{\mu\nu} G_{\mu\nu}$
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RENORMALIZATION OF THE COTTINGHAM FORMULA

John C. COLLINS *
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540, USA

Received 23 October 1978

Large Q^2 behavior

- In 1978 Collins calculated EM corrections to the nucleon mass with an emphasis on $m_n - m_p$
  The mass only depends on spin-0 operators (q quark, $G^{\mu\nu}$ gluon)

$$\langle P|m_q\bar{q}q|P\rangle, \qquad \langle P|G^{\mu\nu}G_{\mu\nu}|P\rangle$$

	Quark	Gluon
Spin-0	Collins '78	Collins '78

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Spin-0	Collins '78	Collins '78

- For $W_1(0,Q^2)$ you need also spin-2 operators

$$\langle P|\bar{q}\Big(iD^\mu\gamma^\nu+iD^\nu\gamma^\mu-\frac{1}{4}i\not{D}\,g^{\mu\nu}\Big)q|P\rangle, \qquad \langle P|G^{\mu\alpha}G^{\nu}_{\alpha}-\frac{1}{4}G^{\alpha\beta}G_{\alpha\beta}g^{\mu\nu}|P\rangle$$

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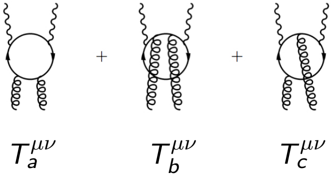
- Need to calculate the spin-2 contribution [Hill, GP arXiv:1611.09917]

	Quark	Gluon
Spin-0	Collins '78	Collins '78
Spin-2	Hill, GP '16	Hill, GP '16

- Collins's result is not enough for muonic hydrogen!

Large Q^2 behavior

- We calculated all the contributions
- Quark: Tree level matching
- Gluon: 1-loop calculation



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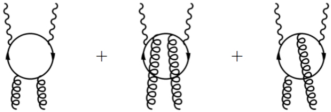
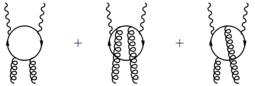


$$T_a^{\mu\nu} \qquad T_b^{\mu\nu} \qquad T_c^{\mu\nu}$$

- Use a background field analysis in Fock-Schwinger gauge
  [Novikov, Shifman, Vainshtein, Zakharov, Fort. Phy. **32**, 585 (1984)]

$$T_c^{\mu\nu} = -\frac{ig^2}{4}\text{Tr}\left[t^a t^b\right] G^a_{\rho\alpha}(0) G^b_{\sigma\beta}(0) \sum_f e_f^2 \int \frac{d^d l}{(2\pi)^d} \frac{\partial}{\partial k_\rho} \frac{\partial}{\partial k'_\sigma}$$
$$\text{Tr}\left[\gamma^\mu \frac{1}{\not l - m_f}\gamma^\alpha \frac{1}{\not l - \not k - m_f}\gamma^\nu \frac{1}{\not l - \not q + \not k' - m_f}\gamma^\beta \frac{1}{\not l - \not q - m_f}\right]_{k=k'=0}$$

Large Q^2 behavior



$T_a^{\mu\nu}$ $T_b^{\mu\nu}$ $T_c^{\mu\nu}$

- Set $m_f = 0$ and use dim. reg. for IR divergences:
  Effective theory loop diagrams vanish
  Read off matching coefficients from full theory finite pieces
  See e.g. [Manohar, PRD **56**, 230 (1997)]

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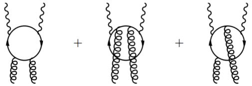

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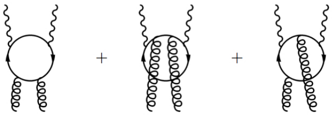
- $T_a^{\mu\nu} = T_b^{\mu\nu}$ IR divergent, $T_c^{\mu\nu}$ IR finite

$$T_c^{\mu\nu} = \sum_f e_f^2 \frac{g^2}{16\pi^2}\frac{1}{q^2} \times$$
$$\times \left[\frac{1}{3}O_g^{(0)}\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right) + 2\frac{q^\mu q_\alpha}{q^2}O_g^{(2)\alpha\nu} + 2\frac{q^\nu q_\alpha}{q^2}O_g^{(2)\alpha\mu} - 4\frac{q_\alpha q_\beta}{q^2}O^{\mu\alpha\nu\beta}\right]$$

$\longleftarrow$ Spin 0 $\longrightarrow$ $\longleftarrow$ Spin 2 $\longrightarrow$ $\langle N|O^{\mu\alpha\nu\beta}|N\rangle = 0$

- Only sum of $T_a^{\mu\nu}$, $T_b^{\mu\nu}$ and $T_c^{\mu\nu}$ obeys current conservation

Large Q^2 behavior



- Performing the complete calculation, we found a mistake in Collins spin-0 calculation from 1978...

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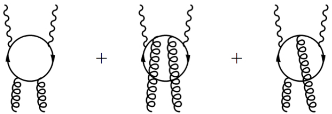

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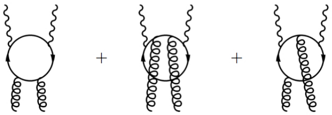

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- For three light quark u, d, s
  Correct result: $\sum_q e_q^2 = (\frac{2}{3})^2 + (\frac{1}{3})^2 + (\frac{1}{3})^2 = \frac{2}{3}$
  Collins: $\sum_q = 3$
  Too large by a factor of 4.5...

- 1978:

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Corrigendum to "Renormalization of the Cottingham formula" [Nucl. Phys. B 149 (1979) 90–100]

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Received 19 December 2016; accepted 20 December 2016

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References

[1] J.C. Collins, Renormalization of the Cottingham formula, Nucl. Phys. B 149 (1979) 90–100, http://dx.doi.org/10.1016/0550-3213(79)90158-5, Nucl. Phys. B 153 (1979) 546 (Erratum).
[2] R.J. Hill, G. Paz, Nucleon spin-averaged forward virtual Compton tensor at large Q^2, arXiv:1611.09917.

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- Even worse, quark spin-0 and gluon spin-0 come with opposite signs
  After correcting the mistake they largely cancel
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- Lesson: It is important to do a full calculation

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  since gluon contribution is the same at lowest order in isospin breaking

- Flip side: You cannot use $m_n - m_p$ to constrain muonic hydrogen

Large Q^2 behavior: Results

- The *correct* spin 0 result

$$\frac{Q^2}{2m_p^2} W_1^{(\mathrm{spin}-0)}(0, Q^2) = -2\sum_q e_q^2 f_q^{(0)} + \left(\sum_q e_q^2\right) \frac{\alpha_s}{12\pi} \tilde{f}_g^{(0)}$$

- $\langle N(k)|O_q^{(0)}|N(k)\rangle \equiv 2m_N^2 f_{q,N}^{(0)}, \quad \langle N(k)|O_g^{(0)}(\mu)|N(k)\rangle \equiv -2m_N^2 \tilde{f}_{g,N}^{(0)}(\mu)$

$$1 = (1-\gamma_m)\sum_q f_q^{(0)} - \frac{\beta}{2g}\tilde{f}_g^{(0)}$$

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$$1 = (1-\gamma_m)\sum_q f_q^{(0)} - \frac{\beta}{2g}\tilde{f}_g^{(0)}$$

- The *new* spin 2 result

$$\frac{Q^2}{2m_p^2} W_1^{(\mathrm{spin}-2)}(0, Q^2) = 2\sum_q e_q^2 f_q^{(2)}(\mu) + \left(\sum_q e_q^2\right) \frac{\alpha_s}{4\pi} \left(-\frac{5}{3} + \frac{4}{3}\log\frac{Q^2}{\mu^2}\right) f_g^{(2)}(\mu)$$

- $\langle N(k)|O_q^{(2)\mu\nu}(\mu)|N(k)\rangle \equiv 2\left(k^\mu k^\nu - \frac{g^{\mu\nu}}{4} m_N^2\right) f_{q,N}^{(2)}(\mu)$

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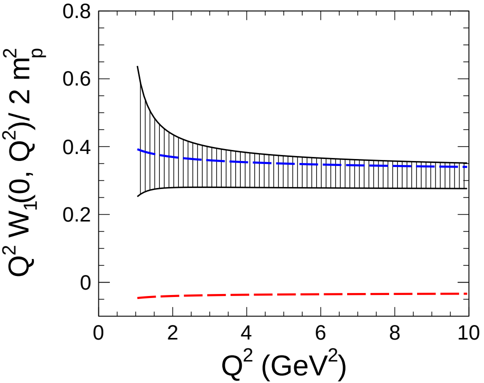
$$\sum_q f_q^{(2)}(\mu) + f_g^{(2)}(\mu) = 1$$

Large Q^2 behavior: Total contribution

- The total contribution



- Dashed red: spin 0
- Dashed blue: spin 2
- Vertical stripes: total contribution with perturbative and hadronic errors added in quadrature

Two Photon exchange: small Q^2 and large Q^2

- Using NRQED we have control over low Q^2

$$W_1(0,Q^2) = 2a_p(2+a_p) + \frac{Q^2}{m_p^2}\left\{\frac{2m_p^3\bar{\beta}}{\alpha} - a_p - \frac{2}{3}\left[(1+a_p)^2 m_p^2 (r_M^p)^2 - m_p^2 (r_E^p)^2\right]\right\} + \mathcal{O}\left(Q^4\right)$$

- Using OPE we *now* have control over the high Q^2

$$\frac{Q^2}{2m_p^2} W_1^{(\mathrm{spin}-0)}(0,Q^2) = -2\sum_q e_q^2 f_q^{(0)} + \left(\sum_q e_q^2\right)\frac{\alpha_s}{12\pi}\tilde{f}_g^{(0)}$$

$$\frac{Q^2}{2m_p^2} W_1^{(\mathrm{spin}-2)}(0,Q^2) = 2\sum_q e_q^2 f_q^{(2)}(\mu) + \left(\sum_q e_q^2\right)\frac{\alpha_s}{4\pi}\left(-\frac{5}{3} + \frac{4}{3}\log\frac{Q^2}{\mu^2}\right) f_g^{(2)}(\mu)$$

- The problem, like the joke, is how to make a whole fish from a head and a tail...

- Before this work we had only the low Q^2
  knowing the large Q^2 allows to connect the dots

Two Photon Exchange: Modeling

- "Aggressive" modeling: use OPE for $Q^2 \geq 1$ GeV^2

- Model unknown Q^4: add $\Delta_L(Q^2) = \pm Q^2/\Lambda_L^2$ with $\Lambda_L \approx 500$ MeV
- Model unknown $1/Q^4$: add $\Delta_H(Q^2) = \pm \Lambda_H^2/Q^2$ with $\Lambda_H \approx 500$ MeV

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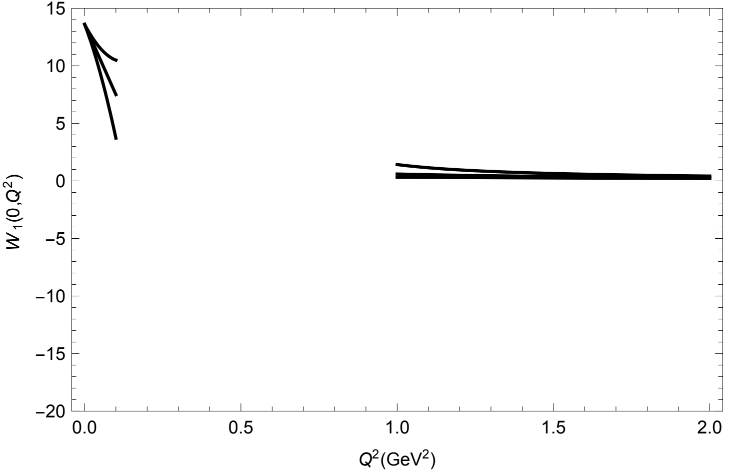
- How to connect the curves?



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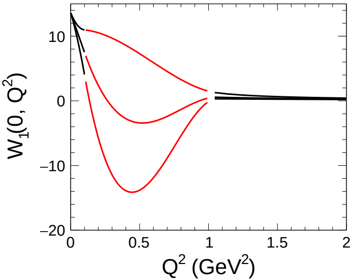

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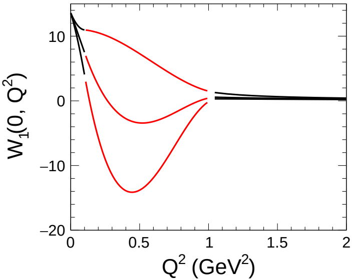

- Energy contribution: $\delta E(2\text{S})^{W_1(0,Q^2)} \in [-0.046\,\text{meV},\ -0.021\,\text{meV}]$
  To explain the puzzle need this to be ~ -0.3 meV
- Caveats: OPE might be only valid for larger Q^2
  $W_1(0, Q^2)$ might be different than the interpolated lines

Experimental test

- How to test?
- New experiment: $\mu - p$ scattering
  MUSE (MUon proton Scattering Experiment) at PSI
  [R. Gilman et al. (MUSE Collaboration), arXiv:1303.2160]

- Need to connect muon-proton scattering and muonic hydrogen can use a new effective field theory: QED-NRQED
  [Hill, Lee, GP, Mikhail P. Solon, PRD **87** 053017 (2013)]
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]

Conclusions

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- Proton radius puzzle: $> 5\sigma$ discrepancy between
- r_E^p from muonic hydrogen
- r_E^p from hydrogen and $e - p$ scattering

- Recent muonic deuterium results find similar discrepancies
  [Pohl et al. Science **353**, 669 (2016)]

- After more than 6 years the origin is still not clear

1) Is it a problem with the electronic extraction?
2) Is it a hadronic uncertainty?
3) is it new physics?

- Motivates a reevaluation of our understanding of the proton

Conclusions

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3) Direct connection between muon-proton scattering and muonic hydrogen using a new effective field theory: QED-NRQED

- Much more work to do!

- Thank you

Backup: Connecting muon-proton scattering and muonic hydrogen

MUSE

- Muonic hydrogen:

  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV

  Both proton and muon non-relativistic

- MUSE:

  Muon momentum $\sim m_\mu \sim 100$ MeV

  Muon is relativistic, proton is still non-relativistic

- QED-NRQED effective theory:

- Use QED for muon
- Use NRQED for proton

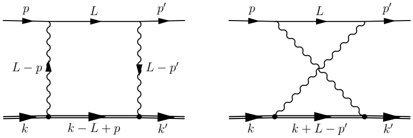
  $m_\mu/m_p \sim 0.1$ as expansion parameter

- A *new* effective field theory suggested in

  [Hill, Lee, GP, Mikhail P. Solon, PRD **87** 053017 (2013)]

QED-NRQED Effective Theory

- Example: TPE at the lowest order in $1/m_p$
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]
- Consider muon-proton scattering $\mu(p) + p(k) \to \mu(p') + p(k')$
- At lowest order in $1/m_p$: $p^0 = p'^0 \Rightarrow \delta(p^0 - p'^0)$
- At the proton rest frame $k = (m_p, \vec{0}) \Rightarrow k^0 = 0$ in NRQED
- NRQED propagator: $\dfrac{1}{l^0 - \vec{l}^2/2M + i\epsilon}$

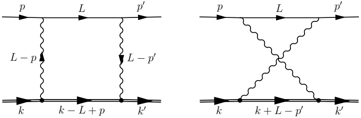


$$\frac{1}{p^0 - L^0 + i\epsilon} + \frac{1}{L^0 - p^0 + i\epsilon} \Rightarrow \delta(L^0 - p^0)$$

- In total

$$\delta(p^0 - p'^0)\,\delta(L^0 - p^0) = \delta(L^0 - p^0)\,\delta(L^0 - p'^0)$$

QED-NRQED Effective Theory



- The amplitude

$$i\mathcal{M}(2\pi)^4\delta^4(k+p-k'-p') = Z^2e^4\int\frac{d^4L}{(2\pi)^4}\frac{1}{(L-p)^2L-p')^2}$$

$$\times \quad \bar{u}(p')\gamma^0\frac{i}{\not{L}-m}\gamma^0u(p)\chi^\dagger\chi(2\pi)\delta(L^0-p^0)\,(2\pi)\delta(L^0-p'^0)$$

$$\times \quad (2\pi)^3\delta^4(\vec{p}-\vec{p}'-\vec{k}')$$

- The cross section

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2 4E^2\left(1-v^2\sin^2\frac{\theta}{2}\right)}{\vec{q}^{\,4}}\left[1+\frac{Z\alpha\pi v\sin\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)}{1-v^2\sin^2\theta}\right]$$

$Z=1$, $E=$ muon energy, $v=|\vec{p}|/E$, $q=p'-p,\theta$ scattering angle

QED-NRQED Effective Theory

- QED-NRQED result

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2 4E^2\left(1-v^2\sin^2\frac{\theta}{2}\right)}{\vec{q}^{\,4}}\left[1+\frac{Z\alpha\pi v\sin\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)}{1-v^2\sin^2\theta}\right]$$

- *Same result* as scattering relativistic lepton off static $1/r$ potential [Dalitz, Proc. Roy. Soc. Lond. **206**, 509 (1951)] reproduced in [Itzykson, Zuber, "Quantum Field Theory"]

- *Same result* as $m_p \to \infty$ of "point particle proton" QED scattering (For $m_p \to \infty$ only proton charge is relevant)

QED-NRQED Effective Theory beyond $m_p \to \infty$ limit

- QED-NRQED allows to calculate $1/m_p$ corrections
- Example: one photon exchange $\mu + p \to \mu + p$:
  QED-NRQED = $1/m_p$ expansion of form factors
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]

Connecting muon-proton scattering to muonic hydrogen

- Matching

QED, QCD	$G_{E,M}$, Structure func., $W_1(0, Q^2)$
Scale: $m_p \sim 1$ GeV	$\Downarrow$
QED-NRQED: *MUSE*	r_E^p, $\bar{\mu}\gamma^0\mu\psi_p^\dagger\psi_p$
Scale: $m_\mu \sim 0.1$ GeV	$\Downarrow$
NRQED-NRQED: *muonic H*	r_E^p, $\psi_\mu^\dagger\psi_\mu\psi_p^\dagger\psi_p$

- Need to match QED-NRQED contact interaction, e.g. $\bar{\mu}\gamma^0\mu\psi_p^\dagger\psi_p$ to NRQED-NRQED contact interaction, e.g. $\psi_\mu^\dagger\psi_\mu\psi_p^\dagger\psi_p$
[Dye, Gonderinger, GP *in progress*]

Connecting muon-proton scattering to muonic hydrogen

- To do list:

1) Relate QED-NRQED contact interactions
   to NRQED contact interactions and $W_1(0, Q^2)$

2) Calculate $d\sigma(\mu + p \to \mu + p)$ and asymmetry in terms of r_E^p and d_2

3) *Direct* relation between μ–p scattering and muonic H