

Analytic resummation for TMD observables

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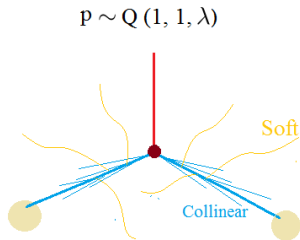
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Outline

- 1 Factorization in SCET for Gauge boson transverse spectra
- 2 Resummation schemes
- 3 Scale Choice in momentum space
- 4 Analytical expression
- 5 Numerical results
- 6 Summary

Factorization in SCET

- $P+P \rightarrow H+X$, $P+P \rightarrow l^+ + l^- + X$.



$$p_c \sim Q(1, \lambda^2, \lambda),$$

$$p_{\bar{c}} \sim Q(\lambda^2, 1, \lambda),$$

$$p_s \sim Q(\lambda, \lambda, \lambda),$$

$$\lambda = q_T/Q$$

Motivation

- Resum large logs of q_T/Q by setting renormalization scales in momentum space
- Obtain, for the first time, an analytic expression for resummed cross section.

Factorization in SCET

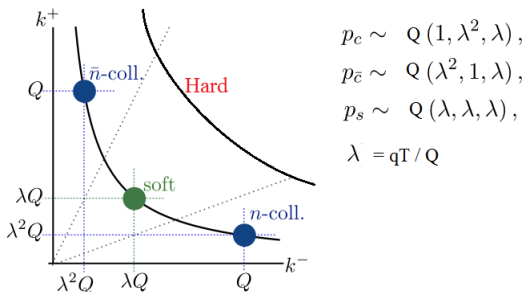


Figure: IR modes have the same virtuality.

- Need a regulator ν that breaks boost invariance to factorize the Soft from the collinear sector

Transverse momentum cross section

$$\frac{d\sigma}{dq_T^2 dy} \propto H\left(\frac{\mu}{Q}\right) \times \int d^2\vec{q}_{T_s} d^2\vec{q}_{T_1} d^2\vec{q}_{T_2} S(\vec{q}_{T_s}, \mu, \nu) \times f_1^\perp(x_1, \vec{q}_{T_1}, \mu, \nu, Q) f_2^\perp(x_2, \vec{q}_{T_2}, \mu, \nu, Q) \delta^2(\vec{q}_T - \vec{q}_{T_s} - \vec{q}_{T_1} - \vec{q}_{T_2})$$

- The function f_i^\perp along with the soft function S forms the **TMDPDF**.
- RG equations in two scales, μ, ν .
- RG equations in momentum space are convolutions of distribution functions and hard to solve directly.

$$\nu \frac{d}{d\nu} G_i(\vec{q}_{T_i}, \nu) = \gamma_\nu^i(\vec{q}_{T_i}) \otimes G_i(\vec{q}_{T_i}, \nu)$$

b space formulation

$$\frac{d\sigma}{dq_T^2 dy} \propto H\left(\frac{\mu}{Q}\right) \int b db J_0(bq_T) S(b, \mu, \nu) f_1^\perp(x_1, b, \mu, \nu, Q) f_2^\perp(x_2, b, \mu, \nu, Q)$$

RG equations in b space are simple

$$\mu \frac{d}{d\mu} F_i(\mu, \nu, b) = \gamma_\mu^i F_i(\mu, \nu, b), \quad F_i \in (H, S, f_i^\perp)$$

$$\nu \frac{d}{d\nu} G_i(\mu, \nu, b) = \gamma_\nu^i G_i(\mu, \nu, b), \quad G_i \in (S, f_i^\perp)$$

$$\sum_{F_i} \gamma_\mu^i = \sum_{G_i} \gamma_\nu^i = 0$$

Resummation schemes

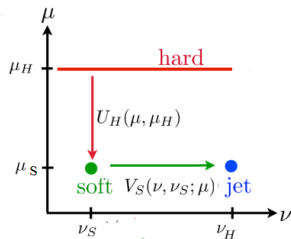


Figure: Choice of resummation path.

- b space resummation: Default choice of $\mu = \nu = 1/b$, 1007.2351 De Florian et.al., 1503:00005 V. Vaidya et. al
- Momentum space resummation: Both μ, ν in momentum space, distributional scale setting, 1611.08610 Tackmann et.al., 1604.02191 P. Monni et. al.
- Hybrid: μ in momentum space(1007.4005, 1109.6027 Becher et.al.)

Scale choice in momentum space

- Can we choose scales in momentum space for μ and ν ?
- Naively, we expect, $\mu_H, \nu_H \sim Q, \mu_L, \nu_L \sim qT$.
- Assume a power counting
 $\alpha_s \log(Q/\mu_L), \log(\mu_L b_0), \alpha_s \log(Q/\nu_L), \log(\nu_L b_0) \sim 1$

Attempt at NLL \rightarrow running Soft function in ν^a

$$^a b_0 = b e^{-\gamma_E}/2$$

$$\begin{aligned} \frac{d\sigma}{dq_t^2} &\propto U_H^{NLL}(H, \mu_L) \int db b J_0(bq_t) U_S(\nu_H, \nu_L, \mu_L) \\ &= U_H^{NLL}(H, \mu_L) \int db b J_0(bq_t) e^{\int_{\nu_L}^{\nu_H} d \ln \nu (\gamma_\nu^{(0)S})} \\ &= U_H^{NLL}(H, \mu_L) \int db b J_0(bq_t) e^{-\Gamma_0 \frac{\alpha_s}{\pi} \log\left(\frac{\nu_H}{\nu_L}\right) \log(\mu_L b_0)} \end{aligned}$$

- Cross section is singular due to divergence at small b .

Scale choice in momentum space

- Divergence caused due to single log structure ($\log(\mu_L b)$) in the Soft exponent
- Contribution from highly energetic soft contributions.
- Need damping at low b to stabilize b space exponent.

Scale choice in momentum space

- Resum all logarithms of the form $\alpha_s \log^2(\mu_L b_0)$

A choice for ν in b space \rightarrow include sub-leading terms

$$\nu_L = \frac{\mu_L^n}{b_0^{1-n}}, \quad n = \frac{1}{2} \left(1 - \alpha(\mu_L) \frac{\beta_0}{2\pi} \log\left(\frac{\nu_H}{\mu_L}\right) \right)$$

Soft exponent at NLL \rightarrow Quadratic in $\log(\mu_L b_0)$

$$\log(U_S^{NLL}(\nu_H, \nu_L, \mu_L)) = -2\Gamma_0 \frac{\alpha(\mu_L)}{2\pi} \times$$
$$\left(\log\left(\frac{\nu_H}{\mu_L}\right) \log(\mu_L b_0) + \frac{1}{2} \log^2(\mu_L b_0) + \alpha(\mu_L) \frac{\beta_0}{4\pi} \log^2(\mu_L b_0) \log\left(\frac{\nu_H}{\mu_L}\right) \right)$$

Scale choice in momentum space

A choice for μ_L in momentum space

- A choice that justifies the power counting $\log(\mu_L b_0) \sim 1$
- A choice that will minimize contributions from residual fixed order logs $\log^n(\mu_L b_0)$.
- Scale shifted away from q_T due to the scale Q in b space exponent.

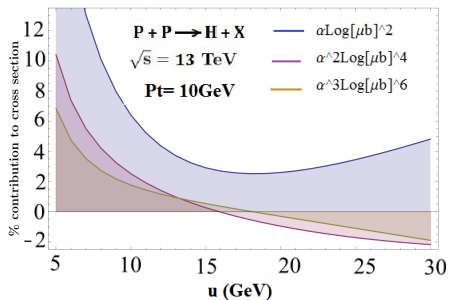
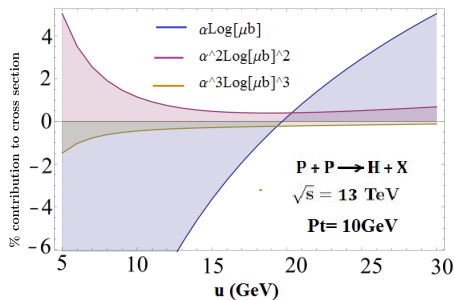
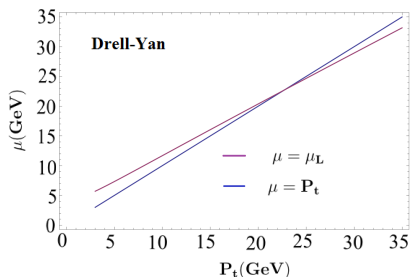
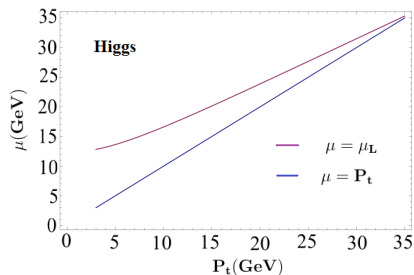


Figure: Percentage contribution of the fixed order logs as a function of the scale  

Scale choice in momentum space



- $\mu_L \sim 1/b^*$, b^* is the value at which b space integrand peaks.

$$\frac{d}{db} (b J_0(b q_T) U_{\text{soft}}(b)) \Big|_{(b=1/\mu_L)} = 0$$

Analytical expression for cross section

Mellin-Barnes representation of Bessel function

- Polynomial integral representation for Bessel function is needed

$$J_0(z) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dt \frac{\Gamma[-t]}{\Gamma[1+t]} \left(\frac{1}{2}z\right)^{2t}$$

b space integral

$$\begin{aligned} U_S &= C_1 \text{Exp}[-A \log^2(Ub)] \\ I_b &= \int_0^\infty db b J_0(bq_T) U_S \quad \text{No Landau pole} \\ &= C_1 \int_{-i\infty}^{i\infty} dt \frac{\Gamma[-t]}{\Gamma[1+t]} \int_0^\infty db b \left(\frac{bq_T}{2}\right)^{2t} \text{Exp}[-A \log^2(Ub)] \end{aligned}$$

Analytical expression for cross section

$$I = \frac{2C_1}{iq_T^2} \frac{1}{\sqrt{4\pi A}} \int_{c-i\infty}^{c+i\infty} dt \frac{\Gamma[-t]}{\Gamma[1+t]} \text{Exp}\left[\frac{1}{A}(t-t_0)^2\right]$$
$$t_0 = -1 + A \log(2U/q_T) \rightarrow \text{saddle point}$$

- Path of steepest descent is parallel to the imaginary axis
- Suppression controlled by $1/A \sim \frac{4\pi}{\alpha_s} 1/\Gamma_{cusp}^{(0)}$
- $t = c + ix$

$$I = \frac{2C_1}{q_T^2} \frac{1}{\sqrt{4\pi A}} \int_{-\infty}^{\infty} dx \frac{\Gamma[-c-ix]}{\Gamma[1+c+ix]} \text{Exp}\left[-\frac{1}{A}(x-i(c-t_0))^2\right]$$

Analytical expression

Motivated by two reasons

- Reproduce resummed cross section to sub percent accuracy.
- Reproduce fixed order to two loop (α_s^2) accuracy.

We choose $c = -1$ as the default contour .

Expand out the function $\Gamma[-c - ix]/\Gamma[1 + c + ix]$ in a Gaussian Polynomial basis

Analytical expression for cross section

An expansion for $F(x) = \Gamma[-1 - ix]/\Gamma[ix]$

A general basis $x^n e^{\alpha x^2 + \beta x}$ for expansion

$$\hat{F}_R(x) = g_1(\text{Exp}[-g_2 x^2] - \cos[g_3 x])$$

$$\hat{F}_I(x) = f_1 \sin[f_2 x] + f_3 \sinh(f_4 x)$$

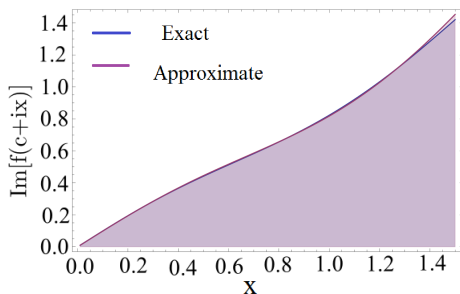
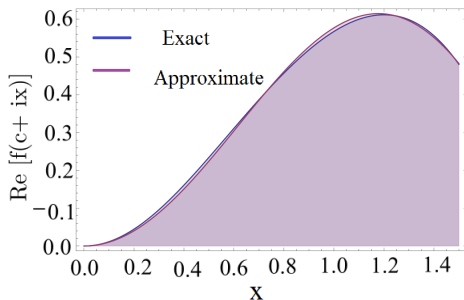


Figure: Expansion for real and imaginary parts of $f(t)$, c is chosen to be ≈ 1

Analytical expression for cross section

Expression for resummed Soft function

$$I_b(A, L) = \frac{2C_1}{q_T^2} e^{-AL^2} \left[-g_1 e^{-\frac{Ag_3^2}{4}} \cosh[ALg_3] + \frac{g_1 e^{\frac{A^2 L^2 g_2}{1+Ag_2}}}{\sqrt{1+Ag_2}} \right. \\ \left. + f_1 e^{-\frac{Af_2^2}{4}} \sinh[ALf_2] + f_3 e^{\frac{Af_4^2}{4}} \sin[ALf_4] \right]$$

Parameters at NLL

$$A = -2\Gamma_{cusp}^{(0)} \frac{\alpha(\mu_L)}{4\pi} \left(1 + \frac{\alpha(\mu_L)\beta_0}{2\pi} \log\left(\frac{\nu_H}{\mu_L}\right) \right) \\ C_1 = \text{Exp}[A \log^2(\eta)], \quad U = \mu_L \eta e^{-\gamma_E}/2 \\ \eta = \text{Exp}\left[\frac{\log(\nu_H/\mu_L)}{1 + \frac{\alpha(\mu_L)\beta_0}{2\pi} \log(\frac{\nu_H}{\mu_L})} \right], \quad L = \log\left(\frac{2U}{q_T}\right)$$

Fixed order terms

- I_b acts as a generating function for residual fixed order logs

$$I_{even} = C_1 \int bdb J_0(bq_T) \log^{2n}(Ub) \text{Exp}[-A \log^2(Ub)]$$

$$= (-1)^n \frac{d^n}{dA^n} I_b(A, L)$$

$$I_{odd} = C_1 \int bdb J_0(bq_T) \log^{2n+1}(Ub) \text{Exp}[-A \log^2(Ub)]$$

$$= (-1)^n \frac{d^n}{dA^n} \frac{(-1)}{2A} \frac{d}{dL} I_b(A, L)$$

Numerical results

- Easily extended to NNLL, b space exponent kept quadratic in $\log(\mu b)$

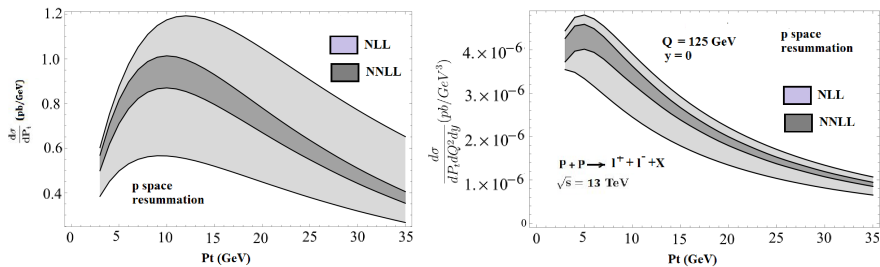


Figure: Resummation in momentum space.

- Excellent convergence for both the Higgs and Drell-Yan spectrum
- No arbitrary b space cut-off while estimating perturbative errors.

Comparison with b space resummation

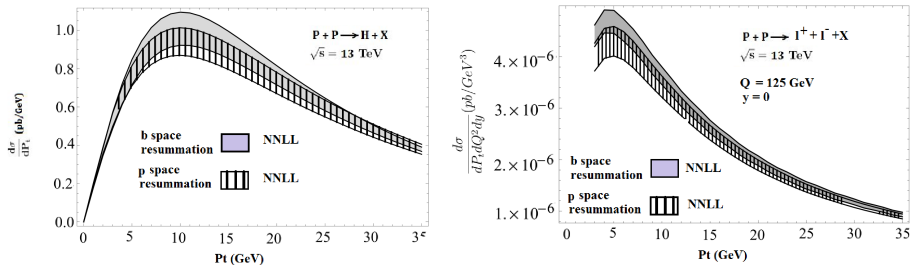


Figure: comparison of nnll cross section in two schemes

- Difference of the order of sub-leading terms.
- More reliable perturbative error estimation in the absence of Landau pole.

Matching to fixed order

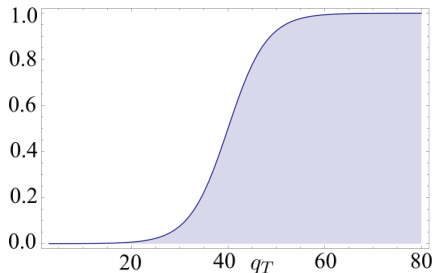
- Implement profiles in μ and ν to turn off resummation

$$S = S_L^{(1-z(q_T))} Q^{z(q_T)} \quad S \in \mu, \nu$$

- Soft exponent scales as $(1 - z(q_T))$

$$U_S = \text{Exp}[(1 - z)\gamma_S^\nu \log\left(\frac{Q}{\nu_L}\right)]$$

- This is equivalent to $A \rightarrow A(1-z)$ in $I_b(A, L)$



$$z(q_T) = \frac{1}{2} \left(1 + \tanh \left[r \left(\frac{q_T}{t} - 1 \right) \right] \right)$$

Summary

- Implementation **momentum space resummation** for transverse spectra of gauge bosons
- Rapidity choice in impact parameter space
- Virtuality choice in momentum space.
- **Analytical expression for cross section across the entire range of q_T** obtained for the first time.
- Numerical accuracy controlled by the accuracy of the expansion for **process independent function** $\frac{\Gamma[-t]}{\Gamma[1+t]}$
- Outlook
 - Promising approach for other observables with similar factorization structure.
 - Non-perturbative effects need to be included for low Q as well as the low q_T regime.

Backup

Analytical expression for cross section

- What choice do we make for c ? Obvious choice $c = t_0$? c depends on A and hence on the details of the process.
- For percent level accuracy, we need info about $F(x) = \frac{\Gamma[-c-ix]}{\Gamma[1+c+ix]}$ out to $x_I \sim \sqrt{2A \log(10)}$
- Worst case scenario $A \sim 0.5 \implies x_I \sim 1.5$
- A Taylor series expansion around the saddle point is not enough.
- Choose $c = -1$, the saddle point in the limit $A \rightarrow 0$ for all observables and use a more suitable basis for expanding $F(x)$

Guidelines for choosing a basis for expansion

- Fixed order cross section

$$I_{exact}^{O(\alpha_s)} = -2\Gamma_{cusp}^{(0)} \frac{\alpha(\mu_L)}{4\pi} \frac{2}{q_T^2} \left(F'[0] \log \left(\frac{\mu_L e^{-\gamma_E}}{q_T} \right) + \frac{F''[0]}{4} \right)$$

- To correctly reproduce the fixed order cross section upto α_s^n , we need $2n^{th}$ derivative of the expansion to match the exact function $F(x)$
- We need the expansion in a basis to be accurate upto $x \sim 1.5$
- The basis functions for the expansion should be chosen so as to yield a rapidly converging and analytical result.

(A more accurate) Analytical expression for cross section

An expansion for $F(x) = \Gamma[-1 - ix] / \Gamma[ix]$

A general basis $x^n e^{\alpha x^2 + \beta x}$ for expansion

$$\hat{F}_R(x) = g_1(\text{Exp}[-g_2 x^2] - \cos[g_3 x]) + g_4 x^2 \text{Exp}[-g_5 x^2]$$

$$\hat{F}_I(x) = f_1 \sin[f_2 x^2] + f_3 \sinh(f_4 x)$$

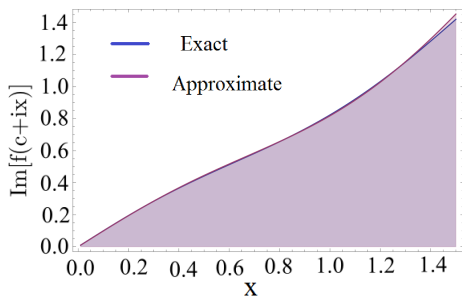
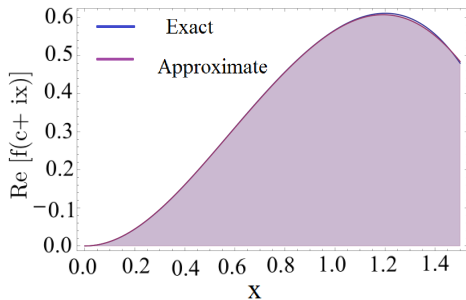


Figure: (Expansion for real and imaginary parts of $f(t)$, c is chosen to be -1)