Associated Production of a Top Pair and a Higgs Boson to NNLL accuracy at the LHC

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Outline

- Top quarks and Higgs bosons at the LHC
- Factorization of the partonic cross section in the partonic threshold limit and resummation
- Results at NLO+NNLL accuracy for the total cross section and some differential distributions

In collaboration with:

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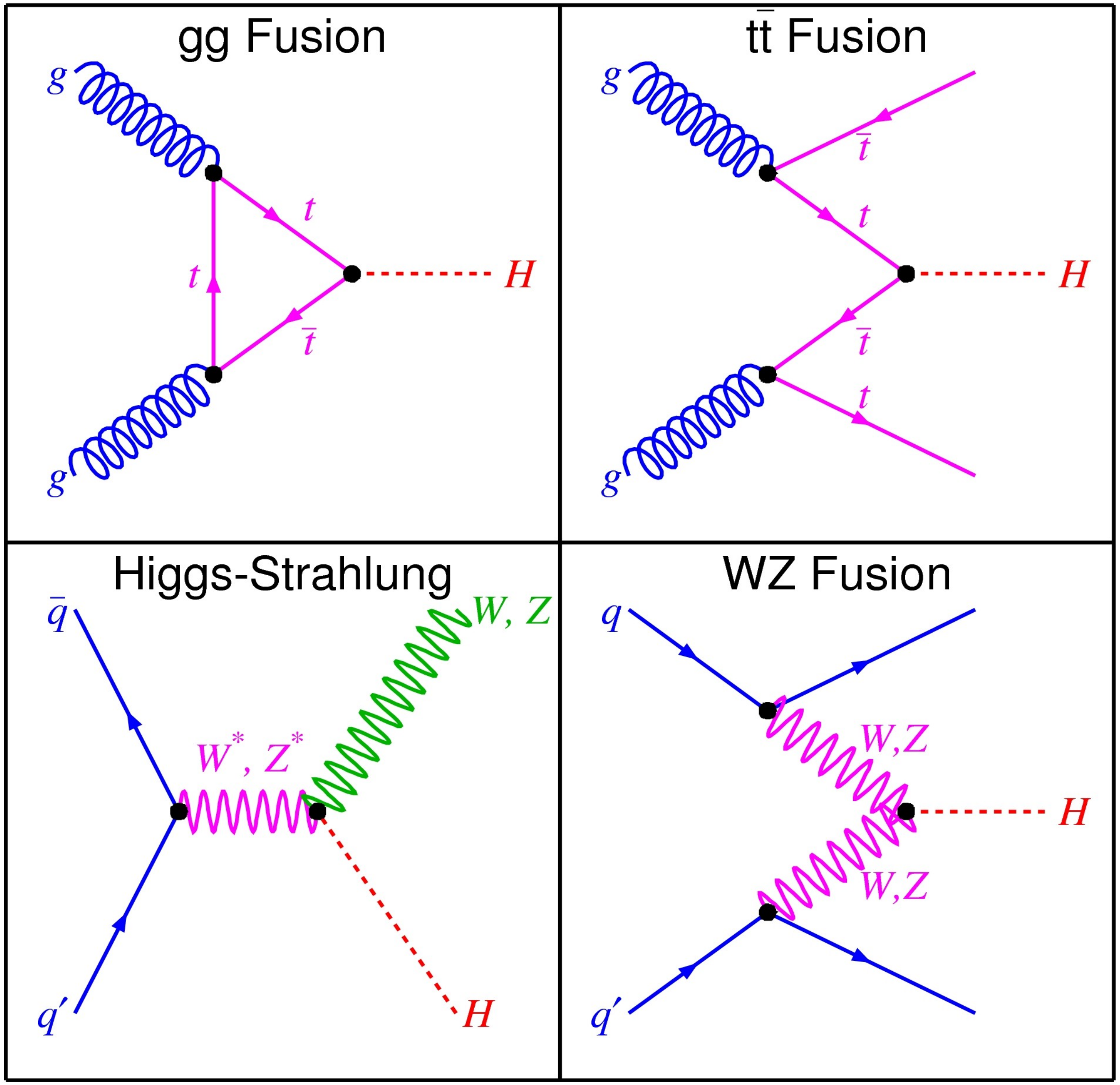
JHEP 1603 (2016) 124 [arXiv:1510.01914]

JHEP 1702 (2017) 127 [arXiv:1611.00049]

Higgs boson production channels

$\sigma \sim 49\,\mathrm{pb}$

$\sigma \sim 0.6\,\mathrm{pb}$



$\sigma_W \sim 1.5\,\mathrm{pb}$

$\sigma_Z \sim 0.97\,\mathrm{pb}$

$\sigma \sim 4.2\,\mathrm{pb}$

LHC @ 14 TeV

Higgs boson production channels

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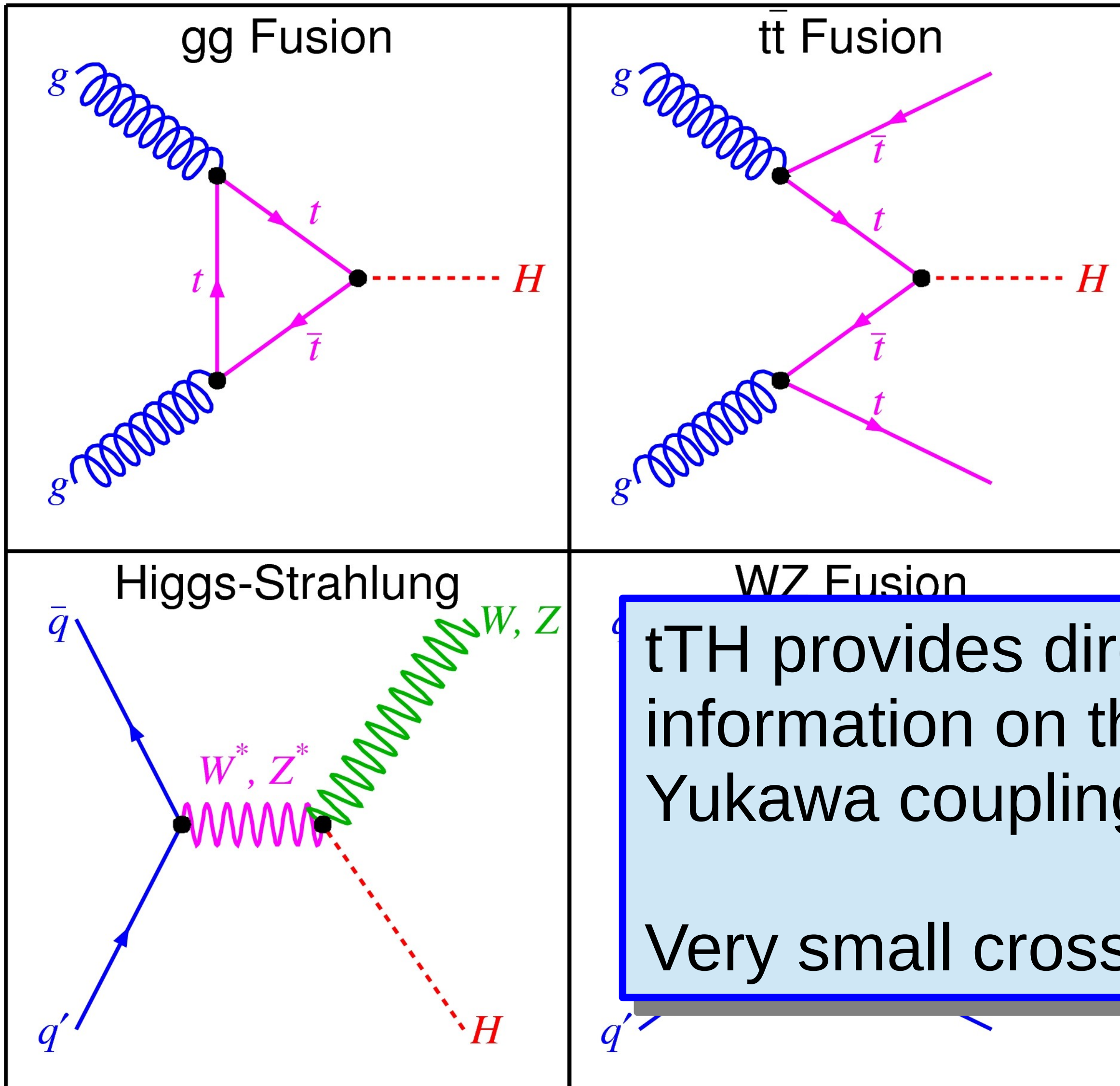

tTH provides direct information on the top-quark Yukawa coupling

Very small cross section

LHC @ 14 TeV

A brief (incomplete) history of top pair + Higgs calculations

- Cross section and some distributions evaluated to NLO QCD

Beenakker, Dittmaier, Kraemer, Pluember, Spira, Zerwas ('01-'02)

Dawson, Reina, Wackeroth, Orr,Jackson ('01,'03)

- Top pair + Higgs production was one of the first processes to be used to test automated tools

  Frixione et al, Hirshi et al., Garzelli et al., Bevilacqua et al.('11)

- EW (QED) corrections

  Frixione at al. ('14-'15),Zhang, Ma, Chen, Guo ('14)

- NLO QCD corrections interfaced with SHERPA and POWHEG BOX

  Gleisberg et al. ('09) Hartanto, Jaeger, Reina, Wackeroth ('15)

- NLO QCD and EW corrections with off shell top quarks

  Denner and Feger ('15) Denner, Lang, Pellen, Uccirati('16)

- Soft gluon emission resummation in the production and partonic threshold limit

  Kulesza, Motyka, Stebel, Theeuwes ('15,'16,’17)

Goal

We want to analyze the factorization properties of

$$p + p \longrightarrow t + \bar{t} + H + X$$

in the soft emission limit (partonic threshold limit) in order to

i. Obtain NNLL resummation formulas for these processes
ii. Evaluate the total cross section and differential distributions depending on the 4-momenta of the final state particles

Large logarithmic corrections

- The partonic cross section for top pair +Higgs (or W,Z) production receives potentially large corrections from soft gluon emission diagrams
- Schematically, the partonic cross section depends on logarithms of the ratio of two different scales:

$$L \equiv \ln\left(\frac{\text{“hard” scale}}{\text{“soft” scale}}\right)$$

- It can be that $\alpha_s L \sim 1$
- One needs to reorganize the perturbative series: Resummation
- The resummation of soft emission corrections can be carried out by means of effective field theory methods

Large logarithmic corrections

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Renormalization group improved perturbation theory schematically:

➔ Separation of scales ↔ factorization

➔ Evaluate each (single-scale) factor in fixed order perturbation theory at a scale for which it is free of large logs

➔ Use Renormalization Group Equations to evolve the factors to a common scale

- So
log

- It c
- On tion
- Th d
out by means of effective field theory methods

“Pair” Invariant Mass kinematics

- For top pair + Higgs production, we have two tree-level partonic processes

$$q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$
$$g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$

- Define the invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4 + p_5)^2$$

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Partonic center of mass energy (squared)

Invariant mass of the heavy particles in the final state

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$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4 + p_5)^2$$

If real radiation in the final state is present, $\hat{s} \neq M^2$

$$z = \frac{M^2}{\hat{s}}$$

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- Define the invariants

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Partonic threshold limit

$$z \longrightarrow 1$$

If real radiation in the final state is

$$z = \frac{M^2}{\hat{s}}$$

Factorization in a nutshell

Differential cross section:

$$\frac{d^2\sigma}{dM^2} = f\!f(z) \otimes C(z)$$

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Partonic luminosity

$$ff = \int_y^1 \frac{dx}{x} f_{i/N_1}(x) f_{j/N_2}\left(\frac{y}{x}\right)$$

Factorization in a nutshell

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Partonic luminosity

Hard scattering kernel
(partonic cross section)

Factorization in a nutshell

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In the soft emission limit a clear scale hierarchy emerges:

$$\hat{s}, M^2, m_t^2, m_H^2 \gg \hat{s}(1-z)^2 \gg \Lambda^2_{\mathrm{QCD}}$$

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In this limit, the partonic cross section factors into two parts:

$$C_{ij} = \mathrm{Tr}\left[\underbrace{\mathbf{H}_{ij}\left(M, \{p_i\}, \mu\right) \mathbf{S}_{ij}}_{\substack{\text{Hard function} \\ \text{(virtual corrections)}}} \underbrace{\left(\sqrt{\hat{s}}(1-z), \{p_i\}, \mu\right)}_{\substack{\text{Soft function} \\ \text{(real soft emission)}}}\right]$$

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Hard function
(virtual corrections)

both are matrices
(in color space)

Soft function
(real soft emission)

Renormalization group equations

- The hard and soft functions are free from large logarithms and can be evaluated in fixed order perturbation theory
- The hard and soft functions satisfy RGEs regulated by anomalous dimensions which can also be calculated up to a given order in the strong coupling constant α_s
- By solving the RGEs one can resum large corrections depending on the ratio of hard and soft scales
- In practice, it is more convenient to solve the RGEs in Laplace space or Mellin space, where convolutions become regular products

$$\frac{d\sigma}{dM^2} \propto \int_\tau^1 \frac{dz}{z} ff\left(\frac{\tau}{z}\right) \mathrm{Tr}\left[\mathbf{H}\mathbf{S}(z)\right] \to \frac{d\tilde{\sigma}}{dM^2} \propto \tilde{ff}\,\mathrm{Tr}\left[\mathbf{H}\tilde{\mathbf{s}}\right]$$

$$\left[\tau = M^2/s\,, \quad s = (\text{collider energy})^2\right]$$

Mellin space

- The resummation can also be carried out in Mellin space (by taking the Mellin transform of the factorized cross section), similar to “direct QCD” resummation

$$\tilde{c}(N,\mu) = \int_0^1 dz\, z^{N-1} \int d\mathrm{PS}_{t\bar{t}H} \mathrm{Tr}\left[\mathbf{H}\left(\{p\},\mu\right)\mathbf{S}\left(\sqrt{\hat{s}}(1-z),\{p\},\mu\right)\right]$$

- The total cross section can be then recovered with an inverse Mellin transform

$$\sigma = \frac{1}{2s}\int_{\tau_{\min}}^1 \frac{d\tau}{\tau}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\,\tau^{-N}\widetilde{ff}(N,\mu)\int d\mathrm{PS}_{t\bar{t}H}\,\widetilde{c}\,(N,\mu)$$

Resummed kernels in Mellin space

- RG evolution is used to obtain $\widetilde{c}$ at the scale μ_f

$$\widetilde{c}(\mu_f) = \mathrm{Tr}\left[\widetilde{\mathbf{U}}(\mu_f, \mu_h, \mu_s)\,\mathbf{H}(\mu_h)\,\widetilde{\mathbf{U}}^{\dagger}(\mu_f, \mu_h, \mu_s)\widetilde{\mathbf{s}}\,(\mu_s)\right]$$

- By rewriting $\alpha_s(\mu_f)$ and $\alpha_s(\mu_s)$ as a function of $\alpha_s(\mu_h)$

$$\widetilde{\mathbf{U}} = \exp\left\{\frac{4\pi}{\alpha_s(\mu_h)} g_1\left(\lambda, \lambda_f\right) + g_2\left(\lambda, \lambda_f\right) + \frac{\alpha_s(\mu_h)}{4\pi} g_3\left(\lambda, \lambda_f\right) + \cdots\right\}$$
$$\times\, \mathbf{u}(\{p\}, \mu_h, \mu_s)$$

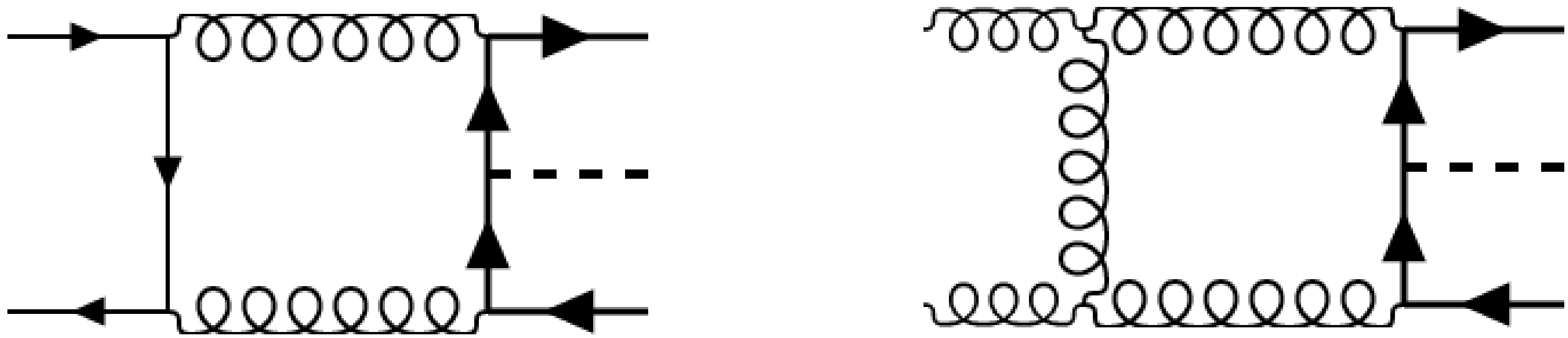
$$\lambda = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0 \ln\frac{\mu_h}{\mu_s}\,, \qquad \lambda_f = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0 \ln\frac{\mu_h}{\mu_f}$$

NNLL resummation requires:

1) g_1, g_2, g_3, u 2) Soft function to NLO 3) Hard function to NLO

Hard function at NLO

In order to evaluate the NLO hard function one needs to calculate one-loop QCD amplitudes. In doing this one need to separate the various components of the amplitude in color space. Ex.



It is convenient to take advantage of the automated tools available on the market. To date, all require some level of customization. We used **Openloops (+ Collier)**, and we cross checked our implementation of the hard function using **GoSam**

Cascioli, Maierhofer, Pozzorini ('12) Denner, Dittmaier, Hofer (`16)

Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola,et al. ('12-'14)

Complete NLO calculations

The resummation formula we have can be evaluated to NNLL

We want to match NNLL and NLO calculations

We want to make sure that soft gluon emission corrections are a sizable part of the radiative corrections (dynamical threshold enhancement) $\Rightarrow$ Compare full NLO with the NLO expanded resummation formula

We need a complete NLO calculation:

MadGraph5_aMC@NLO

Alwall, Frederix, Frixione, Hirshi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (`14)

Implementation of the differential cross section calculation in the code

We employ an in-house parton level MC code to evaluate

$$\sigma = \frac{1}{2s}\int_{\tau_{\min}}^{1}\frac{d\tau}{\tau}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\,\tau^{-N}\widetilde{ff}(N,\mu)\int d\mathrm{PS}_{t\bar{t}H}\,\widetilde{c}\,(N,\mu)$$

$$\tau_{\min} = \frac{(2m_t+m_H)^2}{s}$$

- We want to set up things in such a way that one can easily calculate differential distributions depending on the momenta of the final state particles (invariant mass dist, pT of top-quark and Higgs boson, rapidities, etc…)

- OpenLoops/GoSam performs a numerical calculation in each phase-space point: one needs a fast evaluation of the NLO hard function. Best performance OpenLoops+Collier

Scales

- In fixed order calculations, one needs to choose a default value for the factorization scale $\mu_{f,0}$
- In resummed calculations we also have to pick a value for the default hard and soft scales $\mu_{h,0}$, $\mu_{s,0}$
- In order to keep the hard and soft functions free from large logarithms in Mellin space, one chooses

$$\mu_{h,0} = M \qquad \mu_{s,0} = \frac{M}{\bar{N}}$$

- The choice of the default value for the factorization scale is more delicate (see following slides)

Scale uncertainty

- In fixed order results, the scale uncertainty is evaluated by varying $\mu_f \in [\mu_{f,0}/2, 2\mu_{f,0}]$

- For resummed results, we vary all scales (hard, soft and factorization) independently in the range $\mu_i \in [\mu_{i,0}/2, 2\mu_{i,0}]$

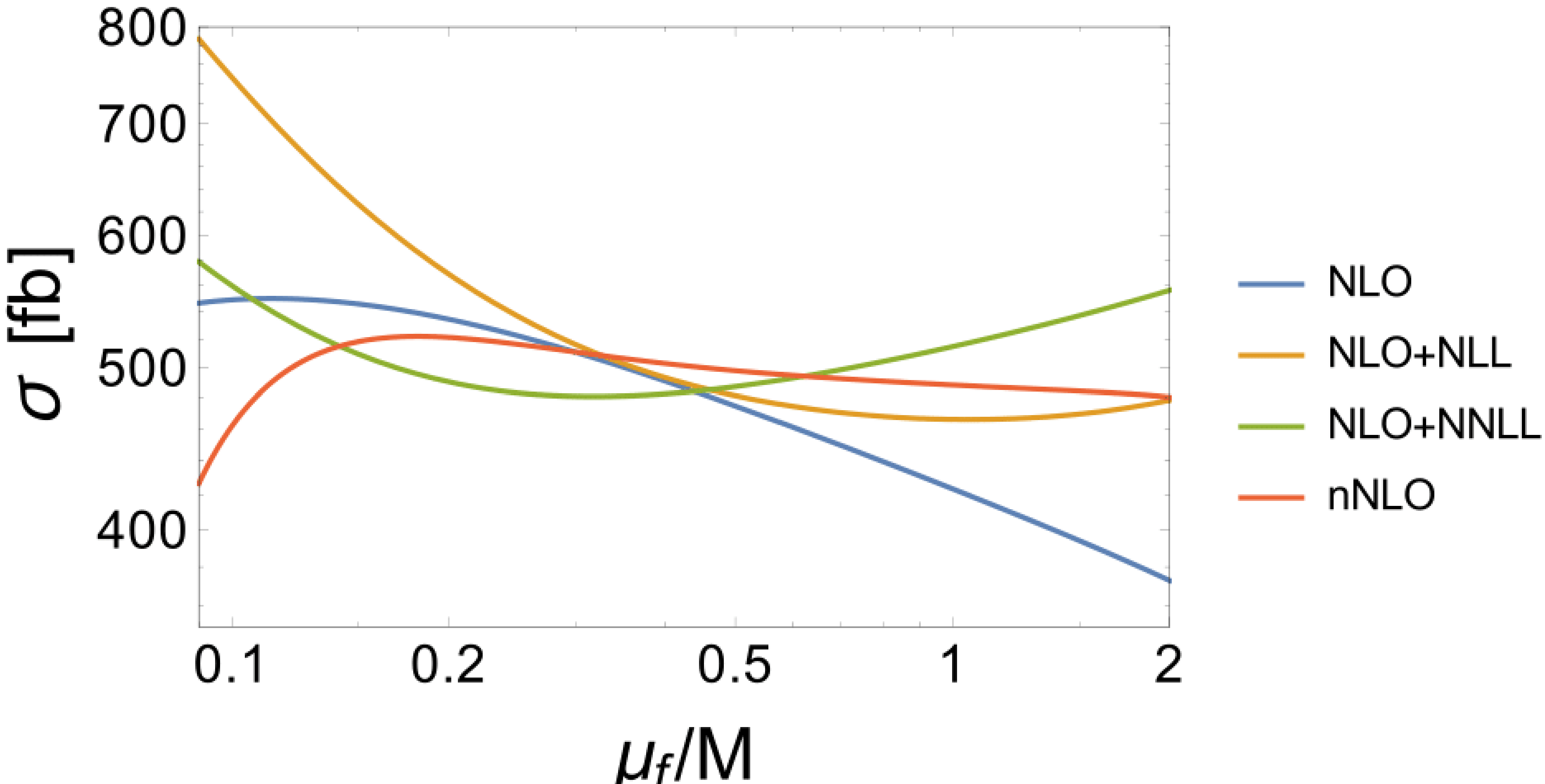
- For an observable O (the total cross section, or the value of a differential cross section in a given bin) one evaluates (for $i = s, f, h \text{ and } k_i = \mu_{i,0}/M$)

$$\Delta O_i^+ = \max\{O(\kappa_i = 1/2), O(\kappa_i = 1), O(\kappa_i = 1)\} - O(\kappa_i = 1)$$

$$\Delta O_i^- = \min\{O(\kappa_i = 1/2), O(\kappa_i = 1), O(\kappa_i = 1)\} - O(\kappa_i = 1)$$

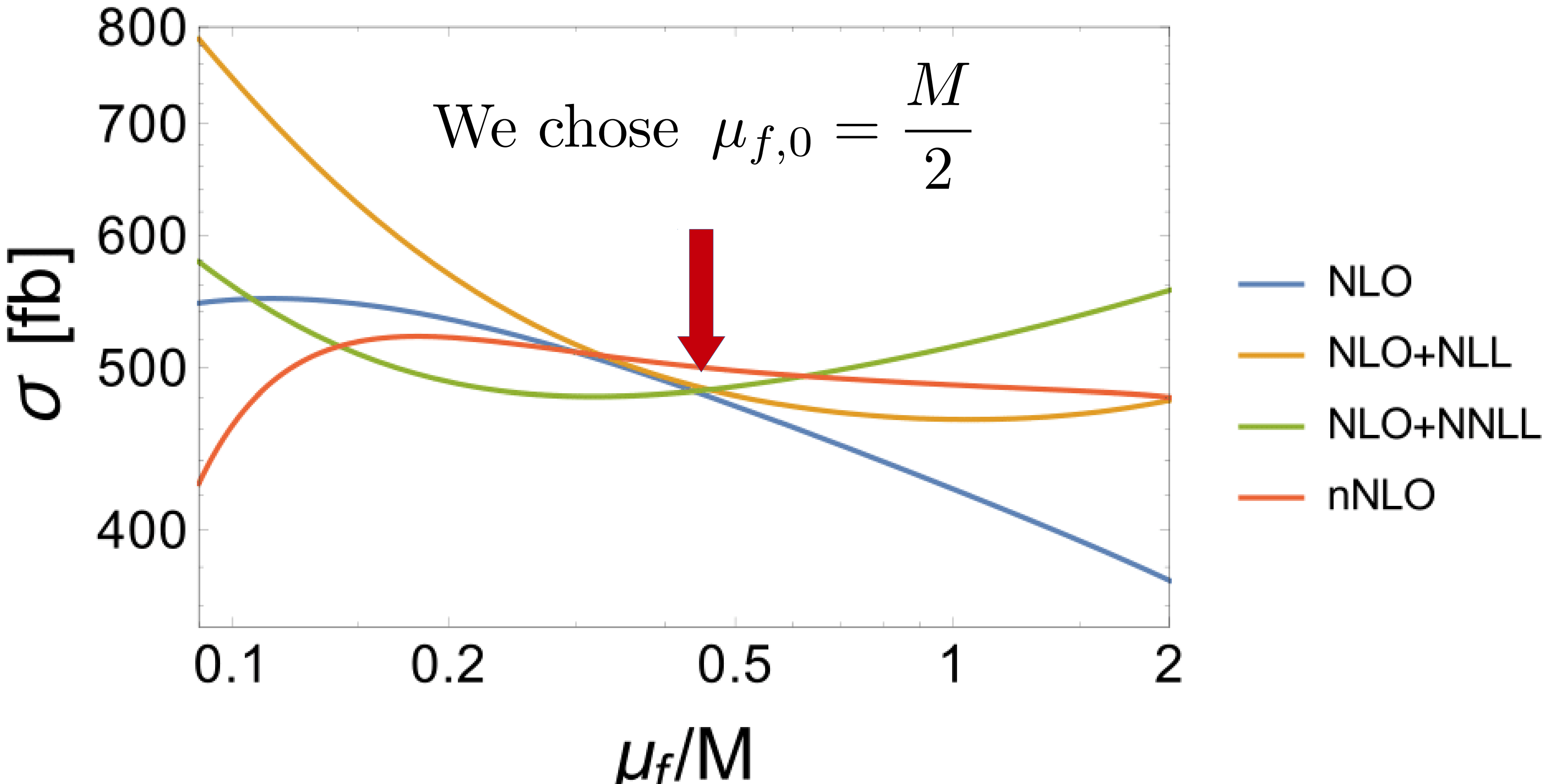
- The quantities $\Delta O_i^+ (\Delta O_i^-)$ are then combined in quadrature in order to obtain the scale uncertainty above (below) the central value

tTH production scale dependence



The factorization scale should be chosen such in such a way that logarithms of the ratio µ_f /M are not large. Since we are working in the partonic threshold limit it is natural to choose a dynamical value for the factorization scale which is correlated with M

tTH production scale dependence



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Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
app. NLO	NLO	MC	$473.3^{+0.0}_{-28.6}$
NLO no qg	NLO	`MG5_aMC`	$482.1^{+10.9}_{-35.1}$
NLO	NLO	`MG5_aMC`	$474.8^{+47.2}_{-51.9}$

$t\bar{t}H$ boson production $\mu_f = M/2$

MMHT 2014 PDFs

App. NLO results include only the leading-power contributions from the gluon fusion and quark-annihilation channels in the soft limit

App NLO vs NLO no qg gives a measure of the power corrections away from the soft limit

Large contribution of the qg channel to the scale uncertainty.

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MMHT 2014 PDFs

The fact that the leading terms in the soft limit make up the bulk of the NLO correction provides a strong motivation to resum these leading terms to all orders.

No information is lost by doing this, as both sources of power corrections are taken into account by matching with NLO as discussed above

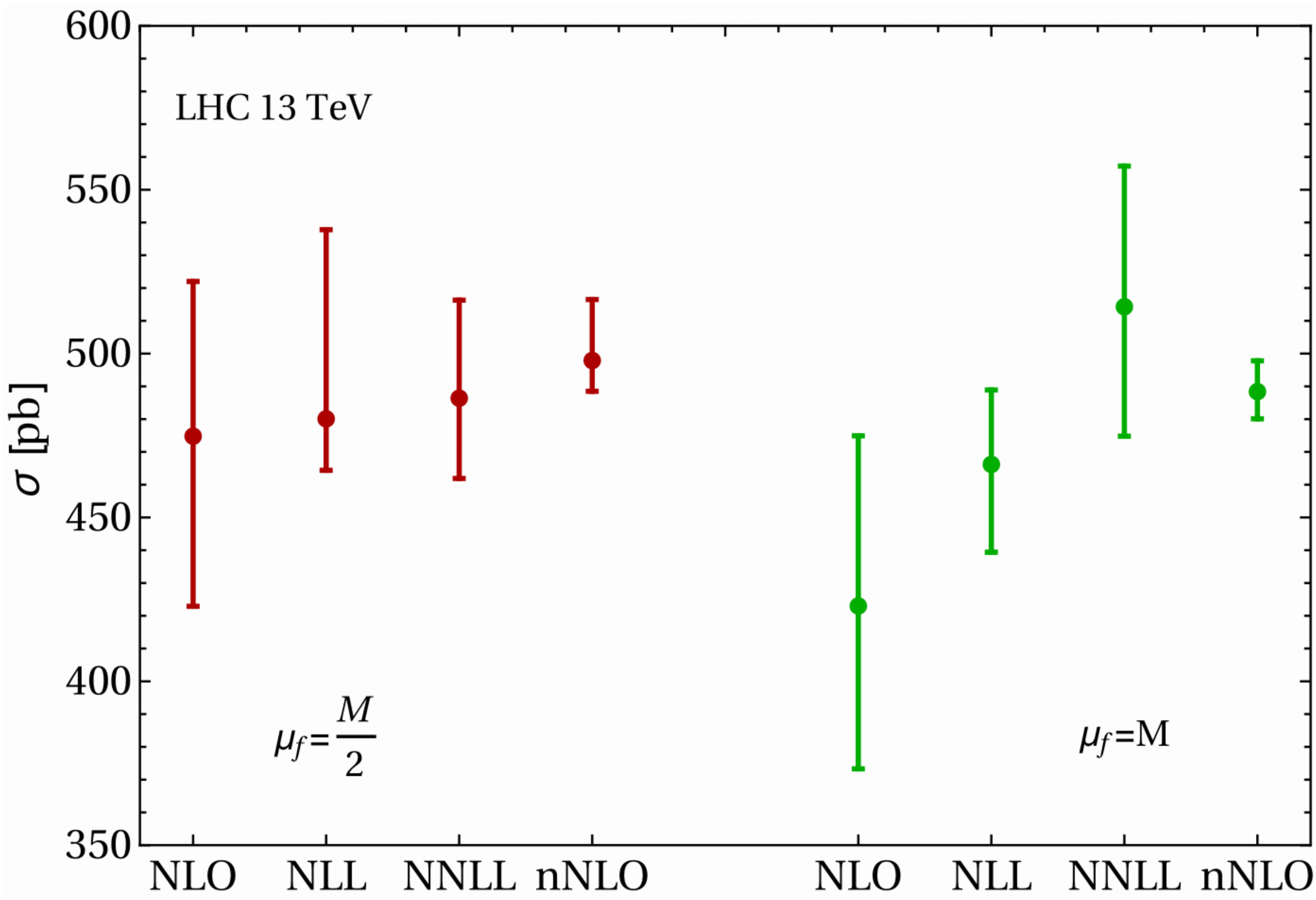
Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
LO	LO	`MG5_aMC`	$378.7^{+120.5}_{-85.2}$
NLO	NLO	`MG5_aMC`	$474.8^{+47.2}_{-51.9}$
NLO+NLL	NLO	MC +`MG5_aMC`	$480.1^{+57.7}_{-15.7}$
NLO+NNLL	NNLO	MC +`MG5_aMC`	$486.4^{+29.9}_{-24.5}$

$t\bar{t}H$ boson production $\mu_f = M/2$

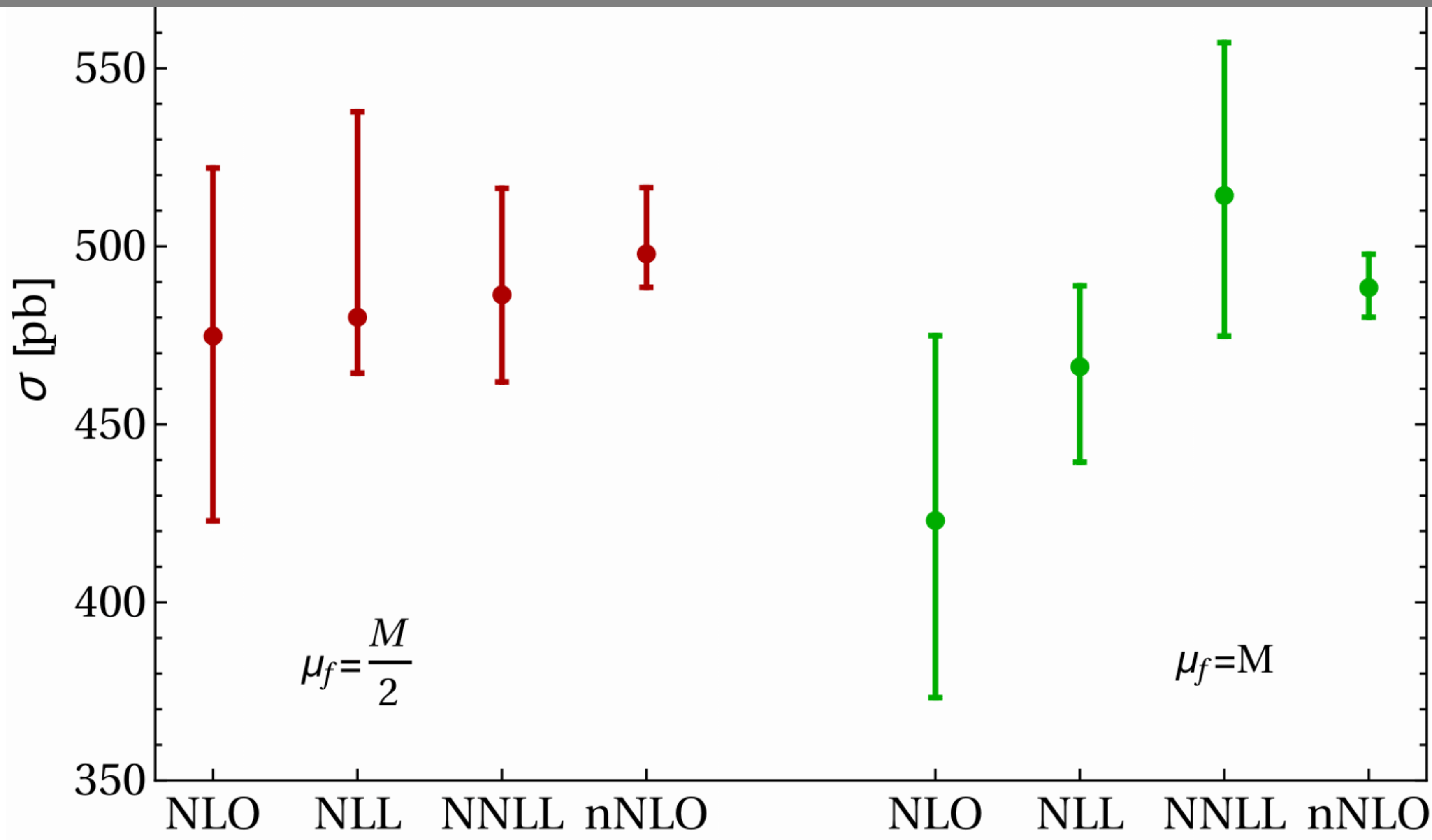
The scale uncertainties get progressively smaller when moving from NLO to NLO+NLL to NLO+NNLL, and the higher-order results are roughly within the range predicted by the uncertainty bands of the lower-order ones.

Comparison among predictions at different factorization scales

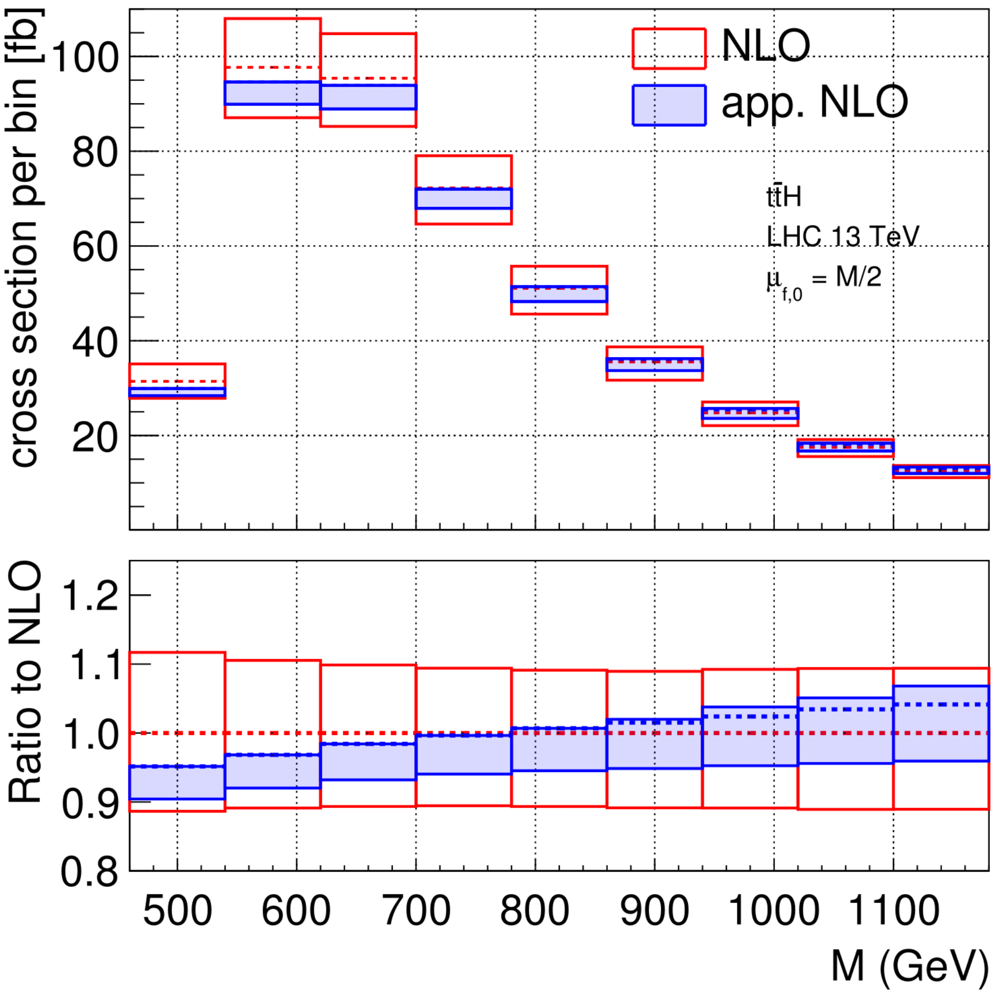


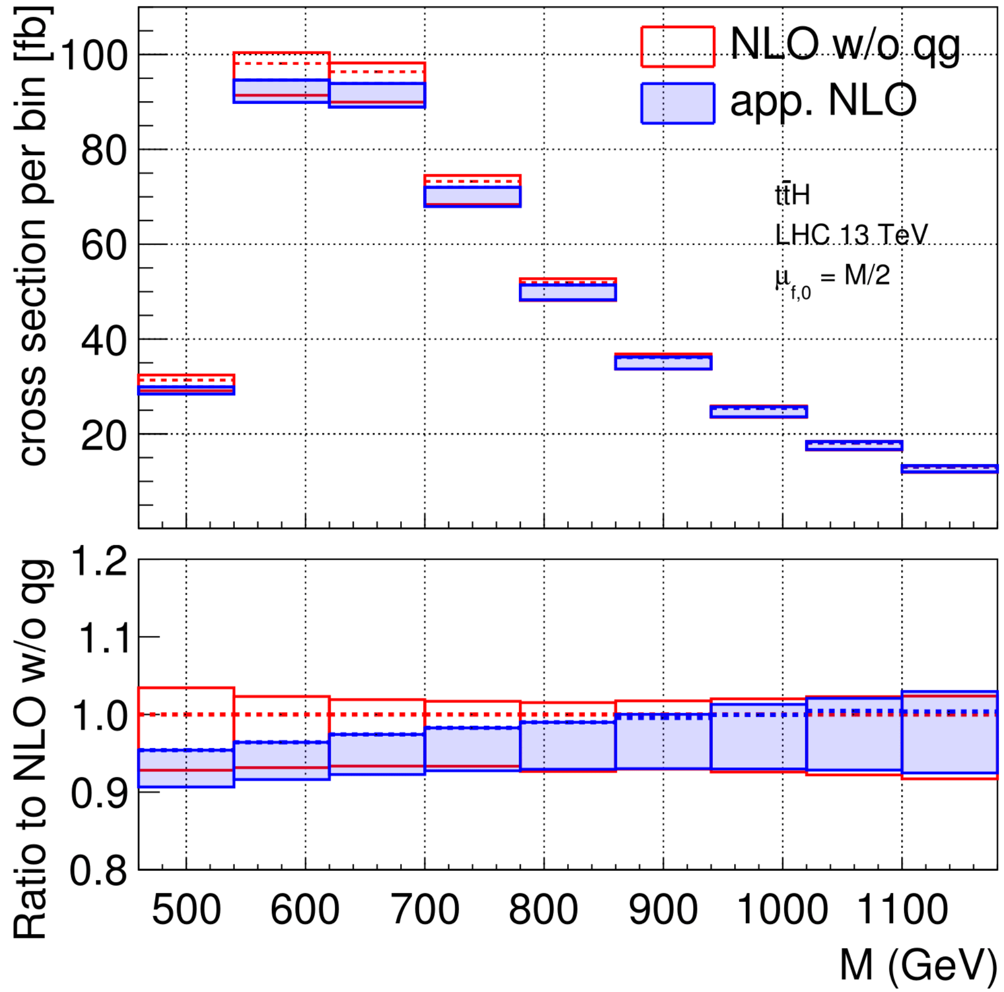
Comparison among predictions at different factorization scales

Good agreement between NLO+NNLL calculations at different μ_f.
(The scale uncertainty of the nNLO results is very small and probably unreliable)



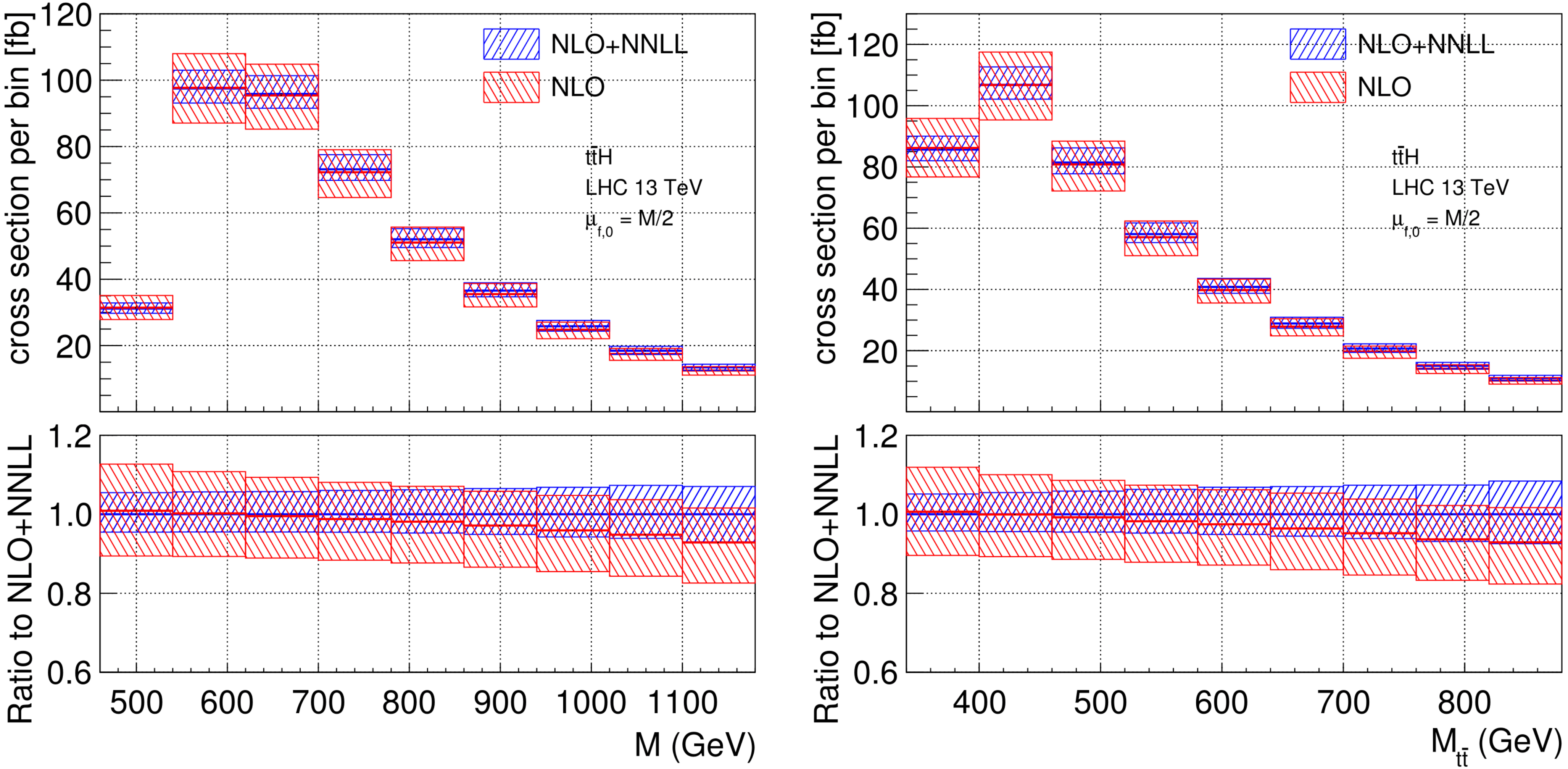
tTH distributions dynamical threshold enhancement





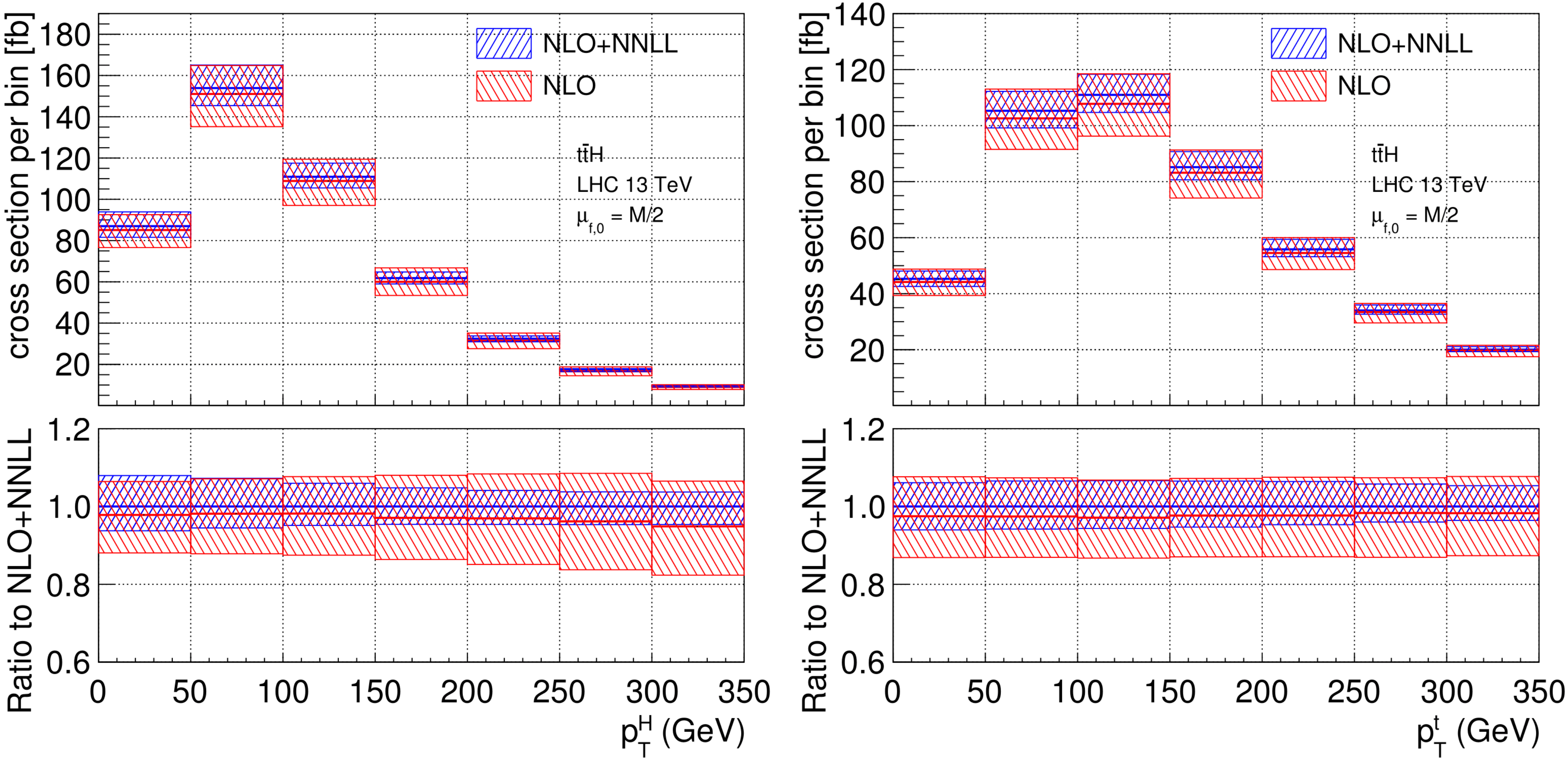
Approximate NLO and NLO distributions without the qg channel agree quite well and the size of the respective uncertainty bands is very similar

tTH distributions at NLO+NNLL



NLO+NNLL distributions overlap with the upper part of the NLO bands.
The NLO+NNLL bands are narrower than the NLO bands

tTH distributions at NLO+NNLL



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Conclusions

- We implemented a method to study partonic threshold corrections to top pair + H boson production
- The resummation formula was evaluated to NNLL by means of in-house parton level Monte Carlo code
- NLO+NNLL results are available for top pair + H production (total cross section + diff. distributions)
- NLO+NNLL results obatained with the same methods are available for top pair + Z/W production (total cross section + diff. distributions)

Backup Slides

Resummation

From a lecture by E. Laenen

("Direct QCD" approach)

Resummation = (re-)arrangement of large logarithms in perturbative expansion

$$\hat{O} = \overbrace{\overbrace{1 + \alpha_s(L^2 + L + 1)}^{\text{NLO}} + \alpha_s^2(L^4 + L^3 + L^2 + L + 1)}^{\text{NNLO}} + \mathcal{O}(\alpha_s^3)$$

$$= \exp\Big(\underbrace{\underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + g_2(\alpha_s L)}_{\text{NLL}} + \alpha_s g_3(\alpha_s L) + \cdots\Big)\ \underbrace{C(\alpha_s)}_{\text{constants}}$$

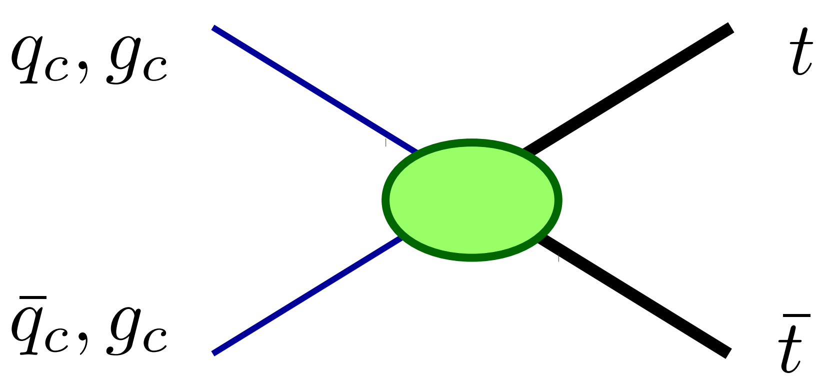
$$+ \text{ suppressed terms}$$

$$Lg_1 \longrightarrow \alpha_s^n L^{n+1}\,, \quad g_2 \longrightarrow \alpha_s^n L^n\,, \quad \alpha_s g_3 \longrightarrow \alpha_s^{n+1} L^n$$

Resummation reduces the theoretical uncertainty on a given observable

Operators in the effective theory

- The effective Lagrangian includes a set of operators describing the partonic process



$$\mathcal{H}_{\mathrm{eff}} = \sum_{I,m,\mathrm{ch}} \int dt_1 dt_1 e^{i m_t (v_3+v_4)\cdot x} C_{Im}^{\mathrm{ch}}(t_1, t_2) O_{Im}^{\mathrm{ch}}(x, t_1, t_2)$$

Wilson coefficients
hard virtual fluctuations

Effective theory operators
depending on collinear and heavy quark fields

Operators in the effective theory

- The interactions between collinear fields and soft gluons, as well as the interactions between collinear quarks and soft gluons are of the eikonal type and they can be absorbed in soft Wilson lines

$$\mathbf{S}_i = \mathbf{P} \exp \left[i g_s \int_0^\infty dt \, v_i \cdot A_s^a(x + t v_i) \mathbf{T}_i^a \right]$$

- Through a decoupling transformation, the operators factorize into a product of collinear, heavy-quark and soft-gluon operators: the different sectors no longer interact with each other

- After interfering squaring the amplitude:
  - "Squared" Wilson coeff. $\longrightarrow$ Hard functions
  - Products of Wilson lines $\longrightarrow$ Soft functions

Dynamical threshold enhancement

- Soft limit = good approximation of the partonic cross section
- However, we are colliding protons: Does the soft limit provide reasonable results for hadronic observables? (ex. invariant mass distribution)

$$\frac{d\sigma}{dM} \propto \int_{\tau}^{1} \frac{dz}{z} \sum_{\text{channels}} ff\left(\frac{\tau}{z}, \mu\right) \underbrace{C(z, \cdots, \mu)}_{\substack{\text{Hard scattering kernel} \\ \text{Partonic cross section}}}$$

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The soft limit (z → 1) after convolutions with the partonic luminosity provides a good approximation to the observable if:

- $\tau \sim 1$, but this situation is not interesting phenomenologically
- $ff \to 0 \quad z \to \tau$: Dynamical threshold enhancement

Dynamical threshold enhancement: tests

- Does dynamical threshold enhancement occur? We need to check if the approximate NLO predictions are reasonably close to the full NLO calculations
- In top pair production dynamical threshold enhancement does take place; we will see that the same is true for top pair + Higgs and top pair + W/Z
- Warning: approximate NLO formulas obtained from the soft limit ignore the contribution of the quark-gluon channel

Top pair + Higgs: Soft function

- The calculation of the soft function at NLO is very similar to the top pair production case: same integrals, but we can't use

$$\hat{s} + \hat{t} + \hat{u} = 2m_t^2$$

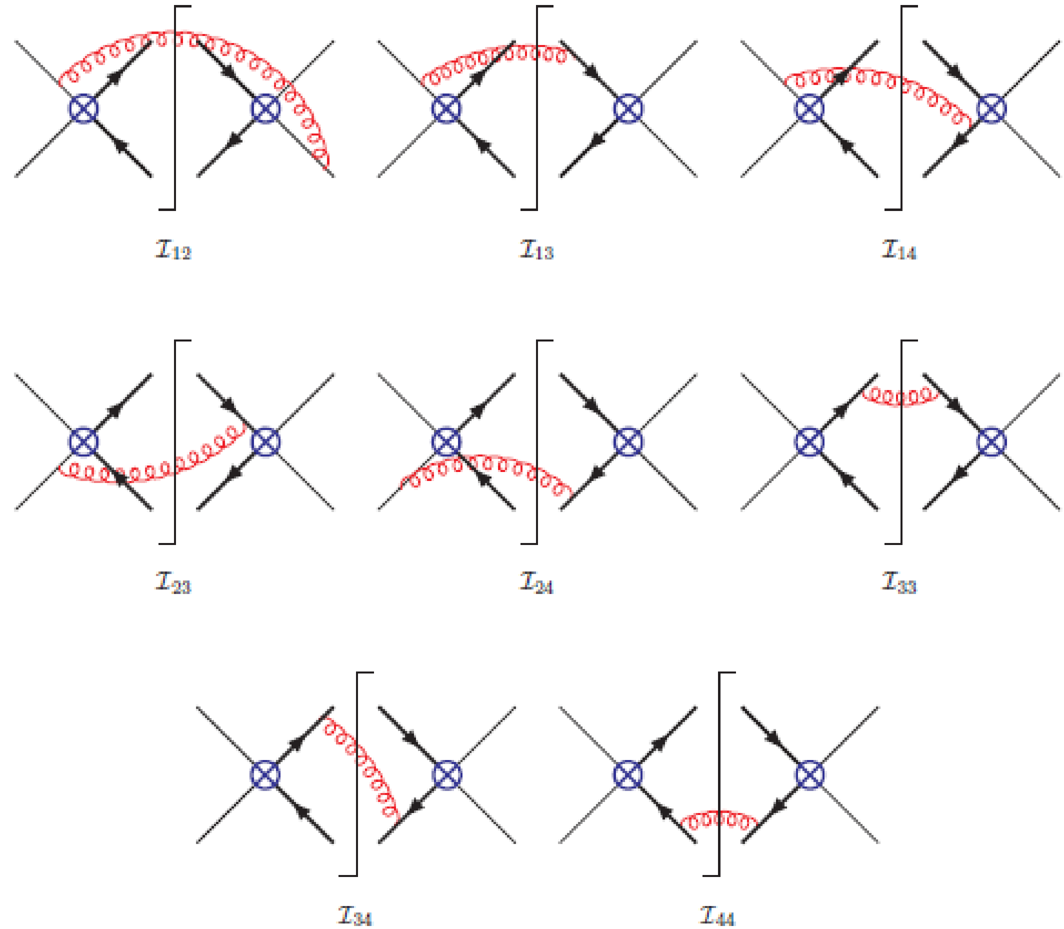
- The subtraction of the IR poles is carried out in the same way as in the top pair production case
- The resulting soft function is exactly the same needed in the calculation of the top pair + W cross section

Li, Li, and Li ('14)

Soft function at NLO

The soft functions are given by the vacuum expectation values of soft Wilson loop operators.

NLO corrections can be calculated directly in position space. Transformation to the Laplace/Mellin space can be easily carried out



$$\mathbf{W}^{(1)}_{\mathrm{bare}} = \sum_{ij} \mathbf{w}_{ij} \mathcal{I}_{ij}$$

Color matrices

$$\mathcal{I}_{ij}(\epsilon, x_0, \mu) = -\frac{(4\pi\mu^2)^\epsilon}{\pi^{2-\epsilon}} v_i \cdot v_j \int d^d k \, \frac{e^{-ik^0 x_0}}{v_i \cdot k \, v_j \cdot k} (2\pi) \, \delta(k^2) \, \theta(k^0)$$

Hard function at NLO

- Perturbative expansion of the hard function

$$\mathbf{H}_{ij} = \alpha_s^2 \frac{1}{d_R} \left(\mathbf{H}_{ij}^{(0)} + \frac{\alpha_s}{4\pi} \mathbf{H}_{ij}^{(1)} + \cdots \right)$$

- The matrix elements can be written in terms of UV finite, IR subtracted QCD amplitudes, projected on a channel dependent color basis

$$\mathbf{H}_{IJ}^{(0)} = \frac{1}{4} \frac{1}{\langle c_I | c_I \rangle \langle c_J | c_J \rangle} \left\langle c_I | \mathcal{M}_{\text{ren}}^{(0)} \right\rangle \left\langle \mathcal{M}_{\text{ren}}^{(0)} | c_J \right\rangle$$

$$\mathbf{H}_{IJ}^{(1)} = \frac{1}{4} \frac{1}{\langle c_I | c_I \rangle \langle c_J | c_J \rangle} \left[\left\langle c_I | \mathcal{M}_{\text{ren}}^{(0)} \right\rangle \left\langle \mathcal{M}_{\text{ren}}^{(1)} | c_J \right\rangle + \left\langle c_I | \mathcal{M}_{\text{ren}}^{(1)} \right\rangle \left\langle \mathcal{M}_{\text{ren}}^{(0)} | c_J \right\rangle \right]$$

- For top-quark pair production, the NLO matrix elements can still be calculated analytically $\longrightarrow$ Things are more complicated for top-pair+Higgs

Final state phase space

- The final state phase space is written as the convolution of two two-particle phase spaces:

$$\int d\Phi_{t\bar{t}H} = \int \frac{ds_{t\bar{t}}}{2\pi} \frac{1}{2M^2} \frac{d\Omega}{16\pi^2} K\left(M^2, s_{t\bar{t}}, m_H^2\right) \frac{1}{2s_{t\bar{t}}} \frac{d\Omega^*}{16\pi^2} K\left(s_{t\bar{t}}, m_t^2, m_t^2\right)$$

$$K(x,y,z) = \sqrt{x^2+y^2+z^2}$$

- Five integrations left in the final state phase space
- Three integrations for the initial state (τ, N, and the luminosity variable x)
- One needs to build a Monte Carlo integration over 8 variables
- The 8 integration variables determine the top, antitop, Higgs and incoming parton momenta: one can bin events and plot distributions

Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
LO	LO	`MG5_aMC`	$378.7^{+120.5}_{-85.2}$
app. NLO	NLO	MC	$473.3^{+0.0}_{-28.6}$
NLO no qg	NLO	`MG5_aMC`	$482.1^{+10.9}_{-35.1}$
NLO	NLO	`MG5_aMC`	$474.8^{+47.2}_{-51.9}$
NLO+NLL	NLO	MC +`MG5_aMC`	$480.1^{+57.7}_{-15.7}$
NLO+NNLL	NNLO	MC +`MG5_aMC`	$486.4^{+29.9}_{-24.5}$
nNLO (Mellin)	NNLO	MC +`MG5_aMC`	$497.9^{+18.5}_{-9.4}$
$(\mathrm{NLO+NNLL})_{\mathrm{exp.}}$	NNLO	MC +`MG5_aMC`	$482.7^{+10.7}_{-21.1}$

$t\bar{t}H$ boson production $\mu_f = M/2$

Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
app. NLO	NLO	MC	$444.7^{+28.6}_{-39.2}$
NLO no qg	NLO	`MG5_aMC`	$447.0^{+35.1}_{-40.4}$
NLO	NLO	`MG5_aMC`	$423.0^{+51.9}_{-49.7}$

$t\bar{t}H$ boson production $\mu_f = M$

Large contribution of the qg channel to the scale uncertainty.

Good agreement approx. NLO (only soft emission) vs NLO without qg

Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
LO	LO	MG5_aMC	$293.5^{+85.2}_{-61.7}$
NLO	NLO	MG5_aMC	$423.0^{+51.9}_{-49.7}$
NLO+NLL	NLO	MC +MG5_aMC	$466.2^{+22.9}_{-26.8}$
NLO+NNLL	NNLO	MC +MG5_aMC	$514.3^{+42.9}_{-39.5}$

$t\bar{t}H$ boson production $\mu_f = M$

Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
LO	LO	MG5_aMC	$293.5^{+85.2}_{-61.7}$
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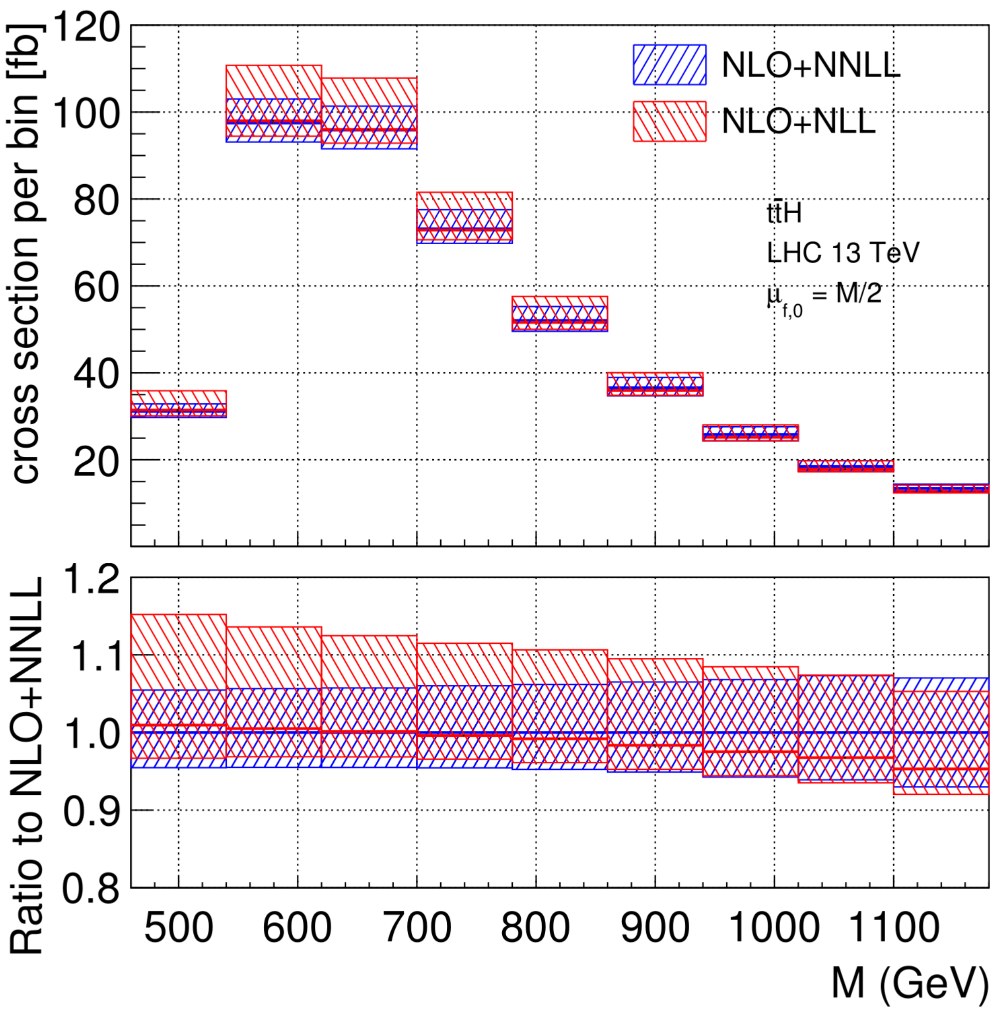
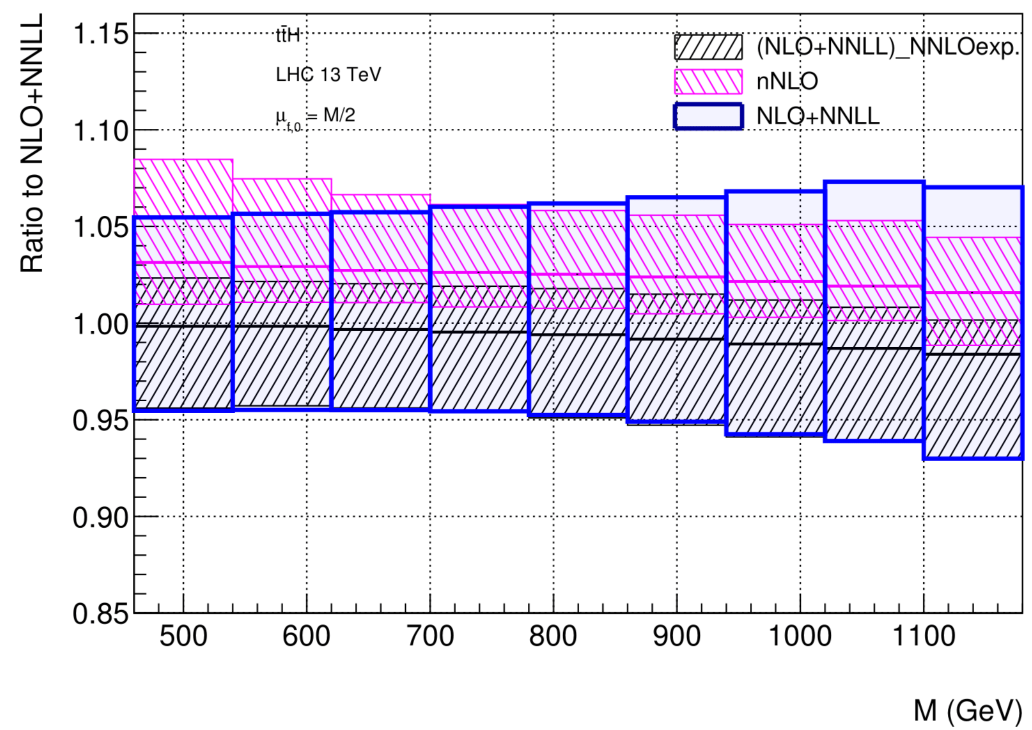
NLO+NLL	NLO	MC +MG5_aMC	$466.2^{+22.9}_{-26.8}$
NLO+NNLL	NNLO	MC +MG5_aMC	$514.3^{+42.9}_{-39.5}$
nNLO (Mellin)	NNLO	MC +MG5_aMC	$488.4^{+9.4}_{-8.3}$
(NLO+NNLL)$_{\mathrm{exp.}}$	NNLO	MC +MG5_aMC	$485.7^{+6.8}_{-15.0}$

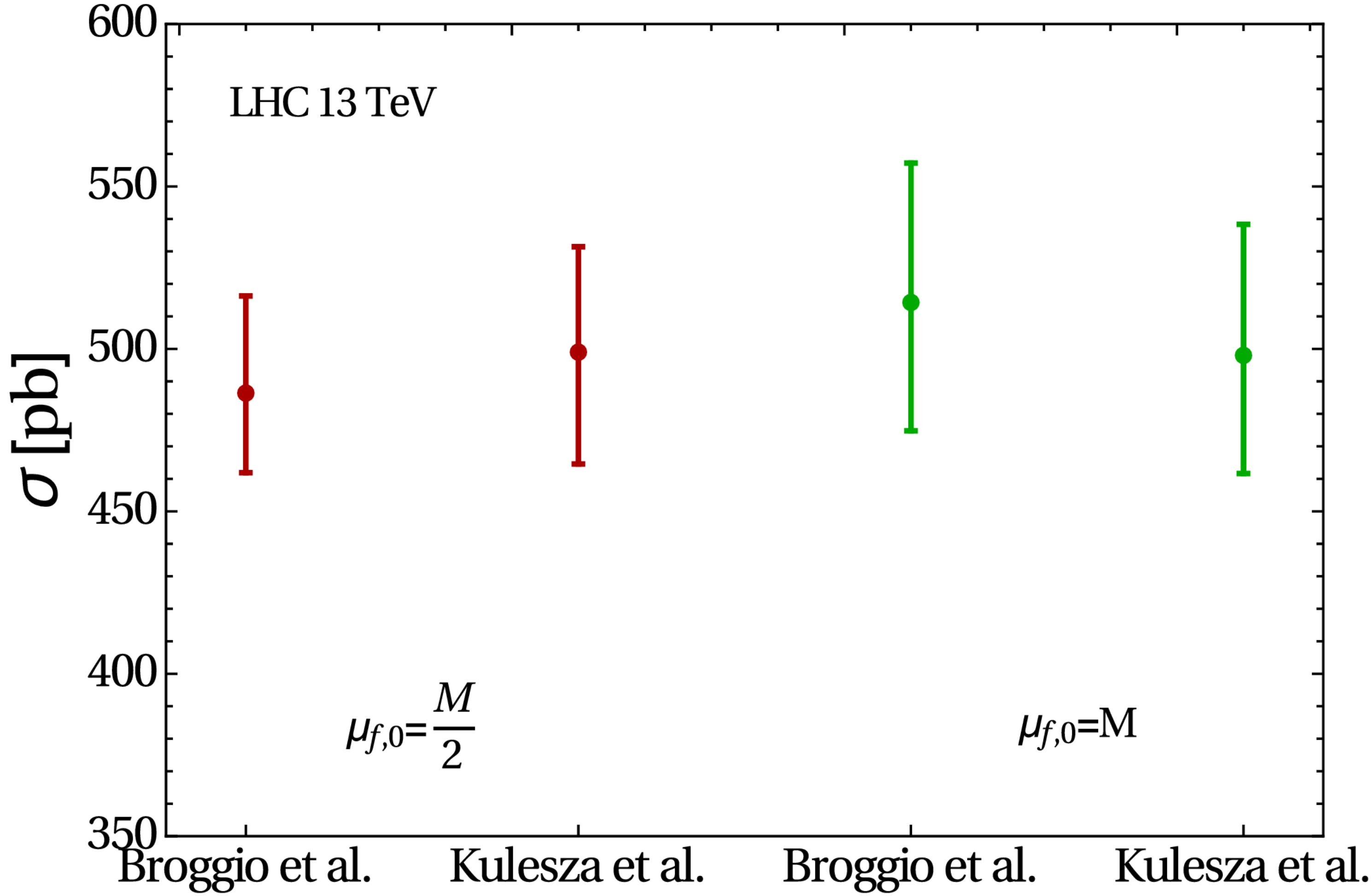
$t\bar{t}H$ boson production $\mu_f = M$

NLO+NNLL vs NLO+NLL and nNLO





Comparison with 1704.03363



Broggio et al MMHT 14 PDFs $m_t = 173.2\ \mathrm{GeV}$
Kulesza et al PDF4LHC15_100 $m_t = 173.0\ \mathrm{GeV}$

Approximate, color averaged hard & soft functions in Kulesza et al.