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Power corrections in the N-jettiness subtraction scheme

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LoopFest XVI @ Argonne National Laboratory, 2017

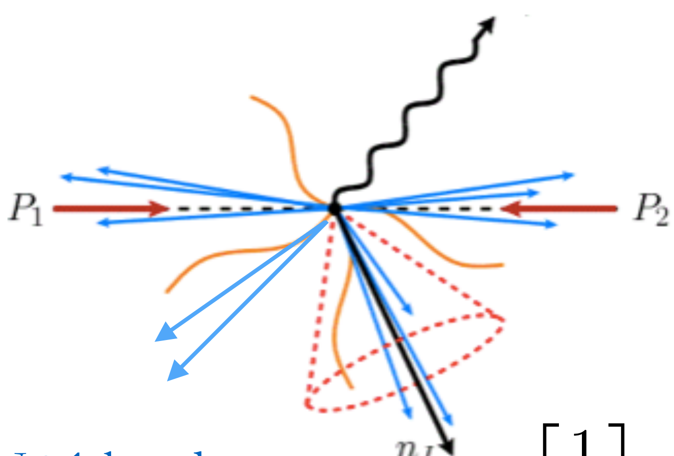
Outline

- Current status
- Power corrections
  - NLO
  - NNLO
- Conclusion

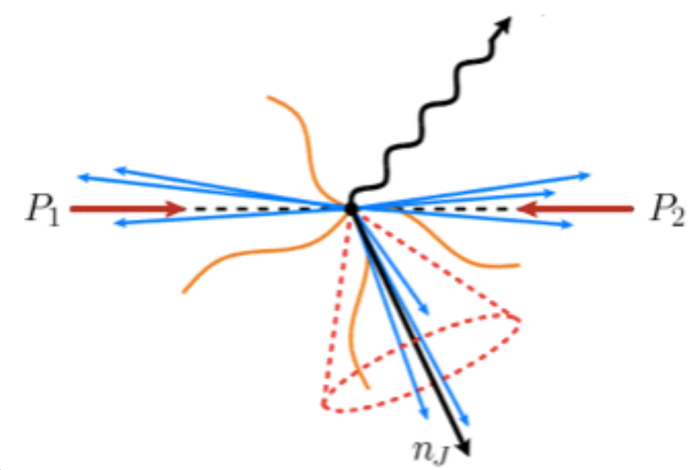
Current Status of the N-Jettiness Scheme

- Non-local schemes

Catani, Grazzini; Gao, Li and Zhu; Boughezal, Focke, XL, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh



$$\tau_{cut}$$



At least N+1 hard radiations

$$\left[\frac{1}{\tau}\right]_+ = \frac{1}{\tau}\theta(\tau - \tau_{cut}) + \delta(\tau)\log(\tau_{cut}) + \ldots$$

$$\tau_{cut} \to 0$$

- a physical observable to set the boundary between NLO and NNLO
- ignorant of the NLO details
- NNLO using EFT based on LP F.T.
- universal building blocks
- conceptually appealing to implement

$$\mathrm{Tr}[H \cdot S_N] \otimes B_a \otimes B_b \otimes J_i + \ldots$$

Stewart,Tackmann,Waalewijn

Current Status of the N-Jettiness Scheme

- 0-jet
    - Color singlet production at NNLO in MCFM

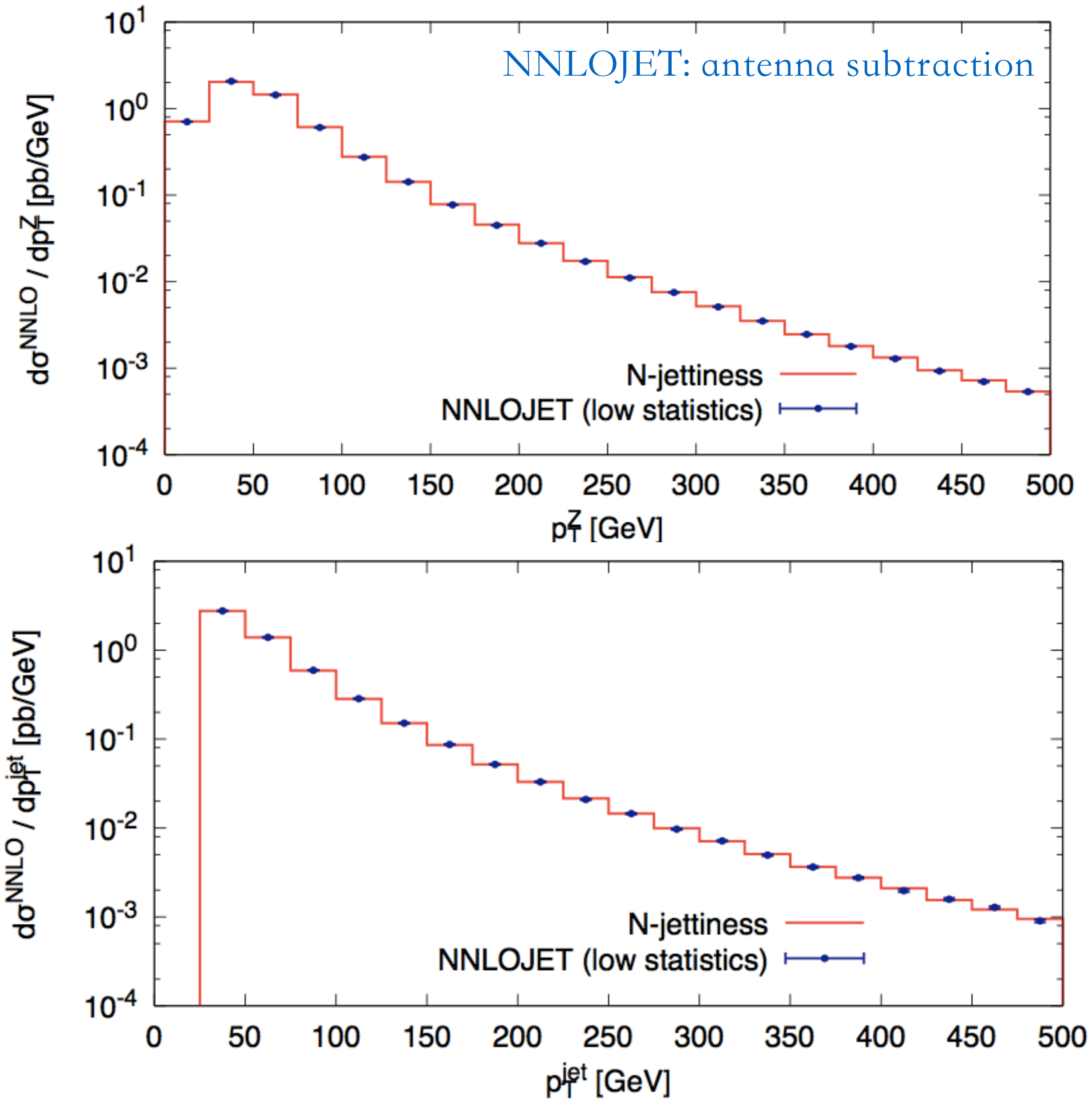
      Boughezal, Campbell, Ellis, Focke, Giele, XL, Petriello, Williams
- 1-jet
    - H/V + 1-jet

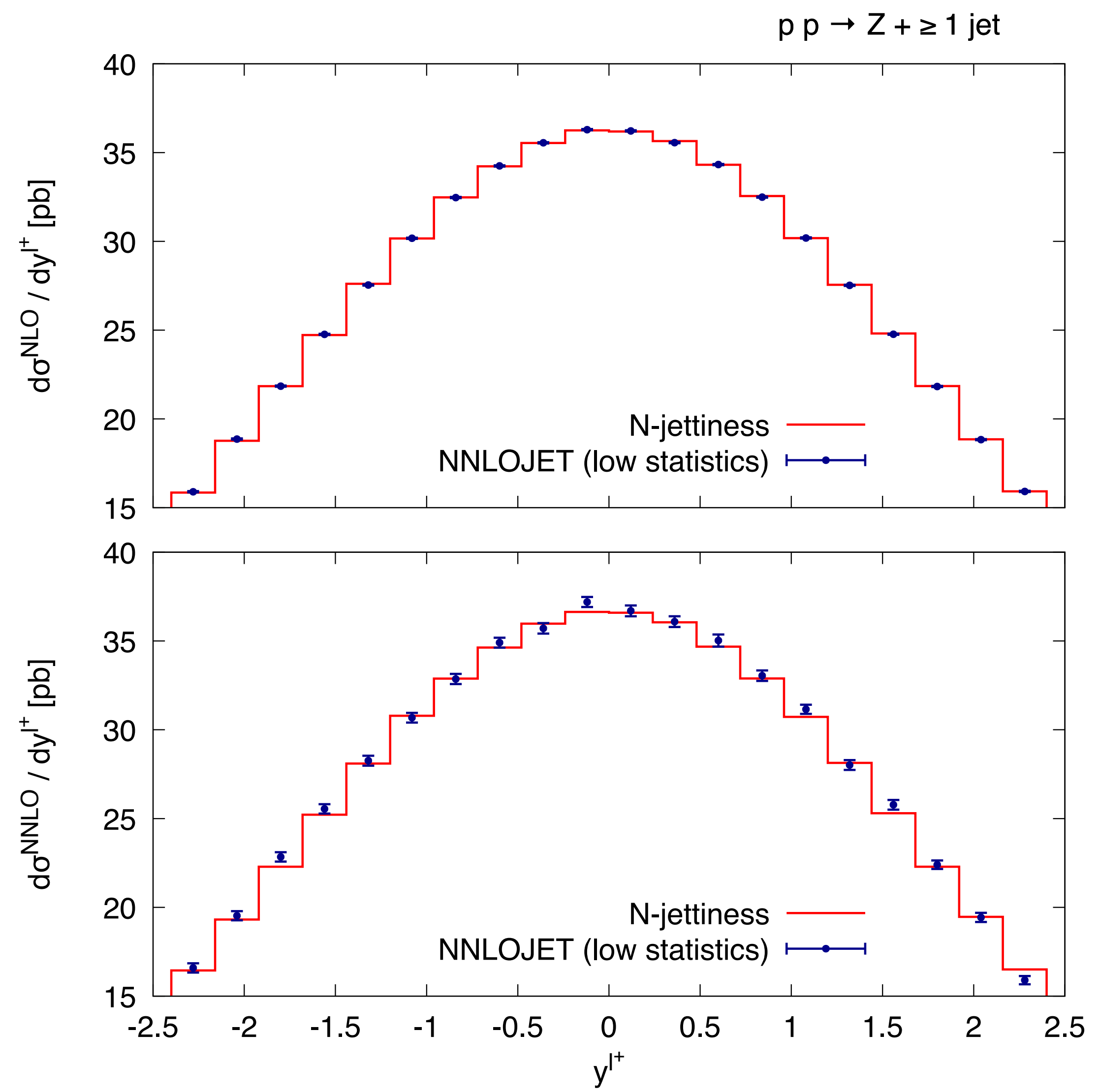
      Boughezal, Focke, XL, Petriello;
      Boughezal, Focke, Giele, XL, Petriello;
      Boughezal, Campbell, Ellis, Focke, Giele, XL, Petriello
      Campbell, Ellis, Williams
    - DIS + 1-jet

      Ablof, Boughezal, XL, Petriello

Current Status of the N-Jettiness Scheme

- Good agreement between independent calculations



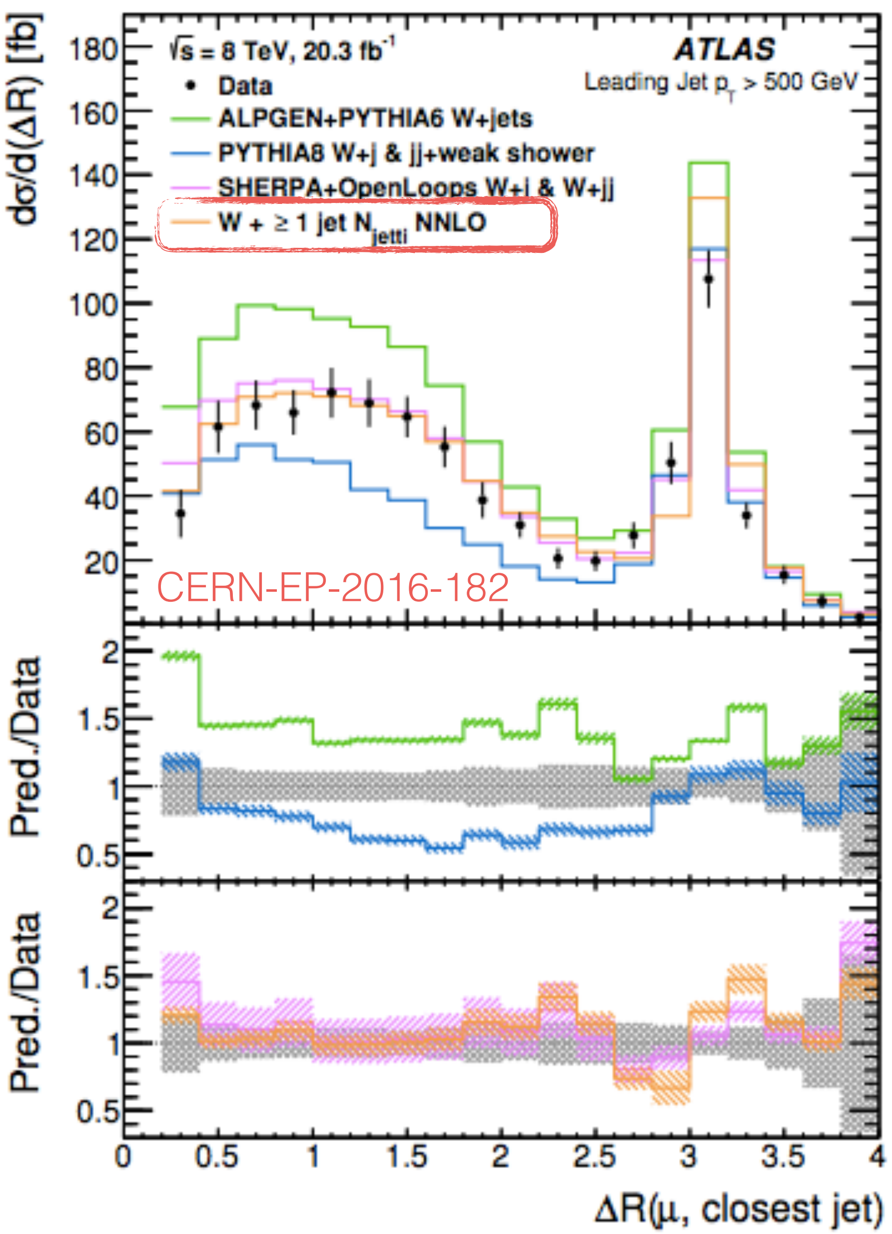


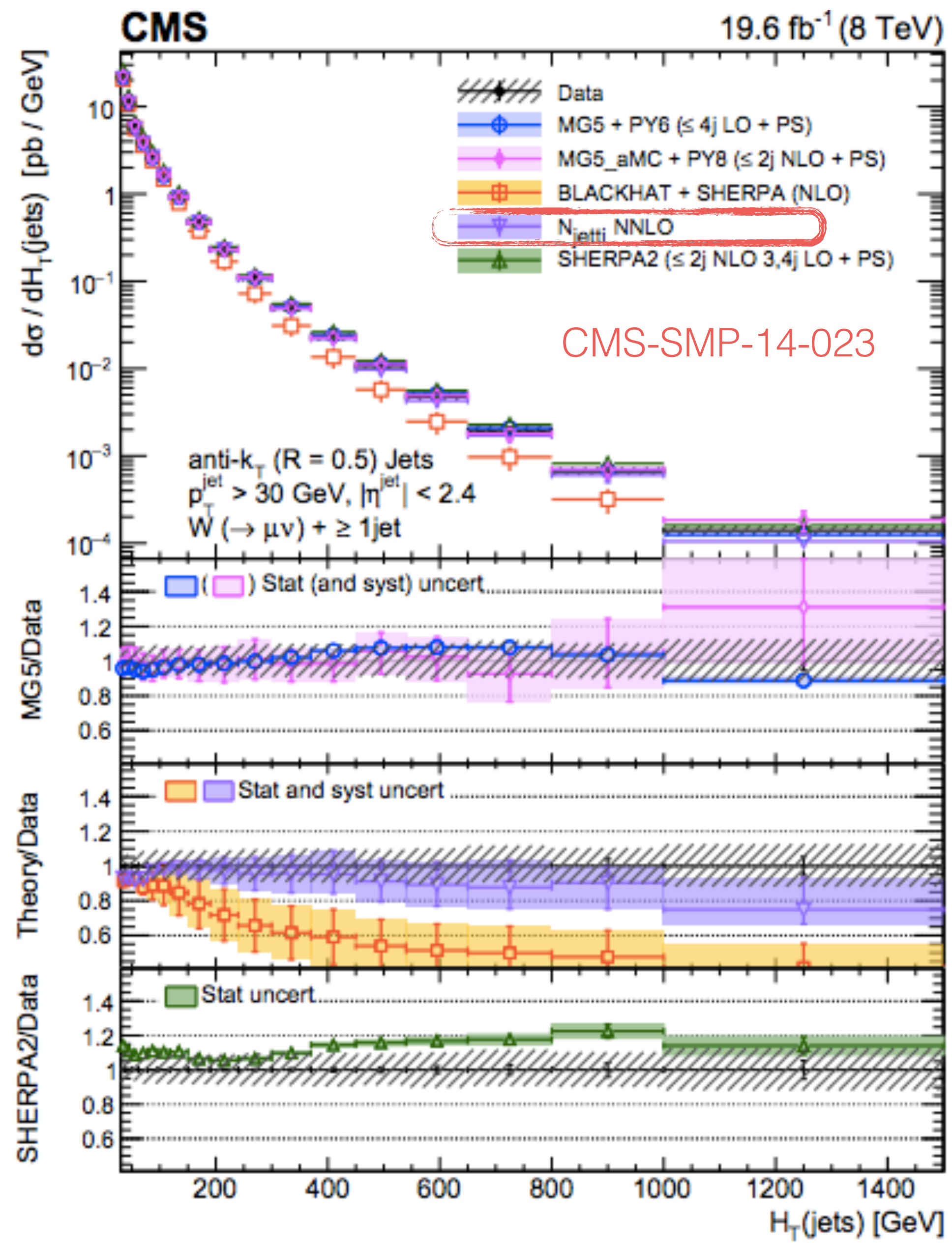
Boughezal, Campbell, Ellis, Focke, Giele, XL, Petriello

Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan

Current Status of the N-Jettiness Scheme

- Good agreement with data



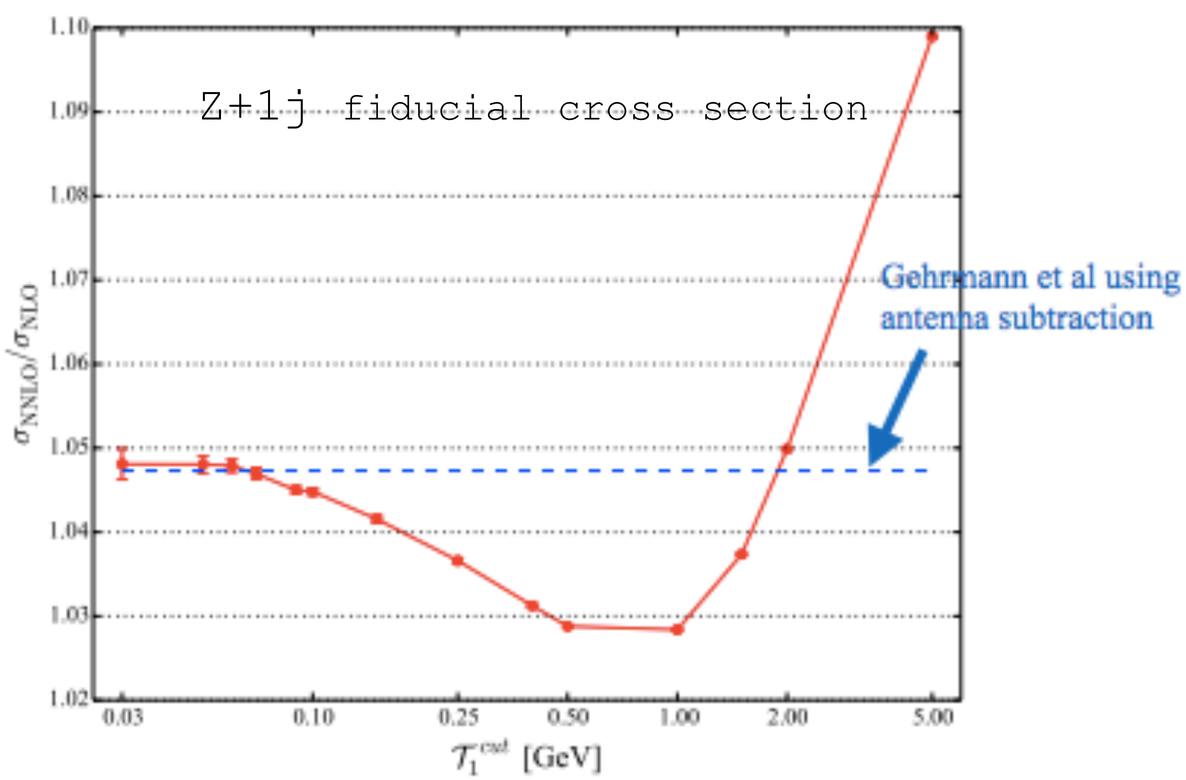


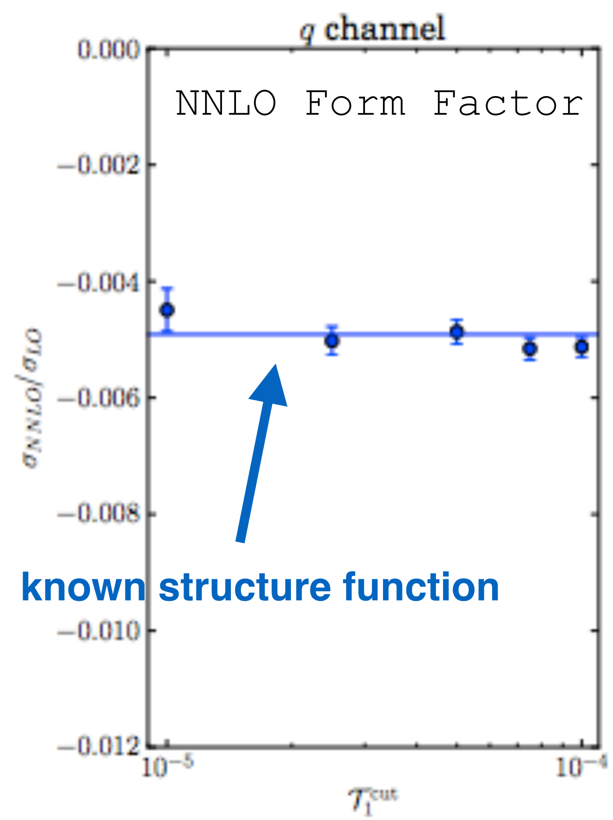
- Improved agreements with all measured distributions

Bougezhal, XL, Petriello

Current Status of the N-Jettiness Scheme

- Require tiny tau-cut to suppress power corrections





Ablof, Boughezal, XL, Petriello

- numerically challenging for NNLO

ways to improve:

- analytic matrix elements
- more efficient PS generators
- optimize the N-jettiness definition
- include power corrections

$$\mathrm{Tr}[H \cdot S_N] \otimes B_a \otimes B_b \otimes J_i + \ldots$$

Power Corrections

Bougezhal, XL, Petriello; see also: Moult, Rothen, Stewart, Tackmann, Zhu

- Logarithmic nature

$$\alpha_s^2\,\tau_{cut}\ \left[C_{23}\log^3(\tau_{cut}) + C_{22}\log^2(\tau_{cut})\ldots\right]$$

  - arise in the soft/collinear limits
  - amenable to a direct FO calculation ← this talk
  - EFT approach Ian Moult’s talk
  - higher order can be predicted via lower order calculation

Power Corrections

- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

$$\tau_N = \sum_k \min\left[\frac{p_k \cdot n_a}{Q_a}, \frac{p_k \cdot n_b}{Q_b}, \frac{p_k \cdot n_1}{Q_1}, \ldots, \frac{p_k \cdot n_N}{Q_N}\right]$$

Stewart,Tackmann,Waalewijn

ggH/Drell-Yan:

$$d\tau \, \left[\delta(k^+ - \tau)\Theta(k_- - k_+) + \delta(k^- - \tau)\Theta(k_+ - k_-)\right]$$

at NLO, for power corrections, we only need to consider real emission. The entire virtual information has been included in the LP F.T.

$$\mathrm{Tr}[H \cdot S_N] \otimes B_a \otimes B_b \otimes J_i + \ldots$$

Power Corrections

- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

$$\int \mathrm{d}\,\Phi\,|\mathcal{M}|^2_{ij}\, f_i(x_a) f_j(x_b)$$

$$\frac{1}{4}\frac{\Omega_\perp}{(2\pi)^{d-2}}\frac{m_H^2}{s}\frac{e^{Y'}}{m_H}\frac{\mathrm{d}z}{z^2}\,\mathrm{d}Y'\,\mathrm{d}\tau\,\mathrm{d}k_-\,\left(\tau\, k_-\right)^{-\epsilon}\,\delta\left(\frac{m_H e^{Y'}(1-z)}{z} - \frac{1}{z}e^{2Y'}\tau - k^-\right)\Theta(k^- - \tau)$$

$$x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}\,, \qquad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

expand everything to $\mathcal{O}(\tau)$

$$|\mathcal{M}|^2 = \frac{1}{\hat{t}\,\hat{u}} \times M(\hat{s},\hat{t},\hat{u};\epsilon) = \frac{z}{m^2}\frac{1}{\tau\, k^-} \times M(\hat{s},\hat{t},\hat{u};\epsilon)$$

$$\sim \frac{1}{\tau\,(1-z)} H_0(0,z;\epsilon) + \frac{1}{(1-z)^2}\frac{e^{Y'}}{m} H_0(0,z;\epsilon) + \frac{1}{1-z}\partial_\tau H_0(0,z;\epsilon) + \mathcal{O}(\tau)$$

power divergence means the ambiguity in defining the leading power matrix element

Power Corrections

- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

$$\int \mathrm{d}\,\Phi\,|\mathcal{M}|^2_{ij}\, f_i(x_a) f_j(x_b)$$

$$\frac{1}{4}\frac{\Omega_\perp}{(2\pi)^{d-2}}\frac{m_H^2}{s}\frac{e^{Y'}}{m_H}\frac{\mathrm{d}z}{z^2}\,\mathrm{d}Y'\,\mathrm{d}\tau\,\mathrm{d}k_-\;(\tau\, k_-)^{-\epsilon}\,\delta\left(\frac{m_H e^{Y'}(1-z)}{z}-\frac{1}{z}e^{2Y'}\tau - k^-\right)\Theta(k^- - \tau)$$

$$x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}\,, \qquad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

expand everything to $\mathcal{O}(\tau)$

$$|\mathcal{M}|^2 = \frac{1}{\hat{t}\,\hat{u}} \times M(\hat{s},\hat{t},\hat{u};\epsilon) = \frac{z}{m^2}\frac{1}{\tau\, k^-} \times M(\hat{s},\hat{t},\hat{u};\epsilon)$$

$$\sim \frac{1}{\tau\,(1-z')} H_0(0,1;\epsilon) + \frac{1}{1-z'}\partial_\tau H_0(0,1;\epsilon) + \mathcal{O}(\tau)$$

It has to be soft to generate the LLs in the power corrections

rescale z to avoid power divergence

$$z \to z' = \frac{z}{1-\frac{\tau}{m}e^{'Y}}$$

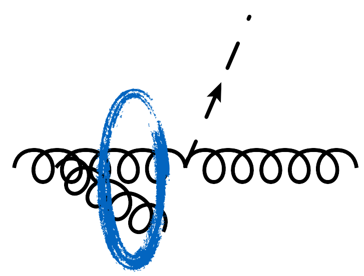
$$\int^{1-a\tau} \mathrm{d}z' \,\frac{\mathcal{N}(1-z')}{1-z'} = \int^{1-a\tau} \mathrm{d}z' \left[\frac{\mathcal{N}_0}{1-z'} + \mathcal{N}_1 + \dots\right]$$

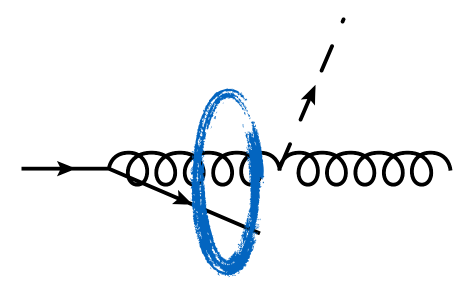
Power Corrections

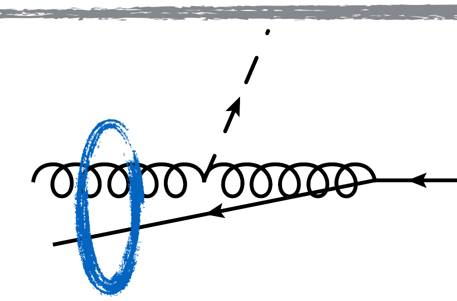
- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

e.g. 1-real emission in ggH







$|\mathcal{M}|^2$:

$$\frac{16\pi\alpha_s C_i}{\tau\,(1-z')} H_0 \quad + \frac{1}{1-z'} \times 0 \qquad\qquad \frac{8\pi\alpha_s C_i}{\tau} H_0 \qquad\qquad \frac{8\pi\alpha_s C_i}{1-z'} \frac{e^{Y'}}{m} H_0$$

$\mathrm{d}\Phi$: LP/SLP — LP

$\mathcal{L}_{ij}$: $\mathcal{L}|_{z'=1}\,,\ \tau \times x_1 \partial_{x_1} \mathcal{L}|_{z'=1}$ — $\mathcal{L}|_{z'=1}$

$$\frac{\alpha_s C_A}{2\pi} L\,[2x_1\partial_{x_1}]\mathcal{L}_{g_1 g_2} + \{x_1 \leftrightarrow x_2\,, Y' \leftrightarrow -Y'\}$$

$$L = \frac{e^{Y'}}{m} \log\left(\frac{\tau\, m\, e^{Y'}}{\tau^2}\right)$$

$$\frac{\alpha_s C_F}{2\pi} L\,\mathcal{L}_{g_1 q_2} + \{x_1 \leftrightarrow x_2\,, Y' \leftrightarrow -Y'\}$$

$$x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}\,, \qquad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

Power Corrections

- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

The results are free of divergence.

$|\mathcal{M}|^2$: $\dfrac{16\pi\alpha_s C_i}{\tau\,(1-z')} H_0 \quad + \dfrac{1}{1-z'} \times 0$ $\qquad \dfrac{8\pi\alpha_s C_i}{\tau} H_0$ $\qquad \dfrac{8\pi\alpha_s C_i}{1-z'} \dfrac{e^{Y'}}{m} H_0$

$d\Phi$: LP/SLP $\qquad$ LP

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Power Corrections

- ggH/Drell-Yan @ NLO

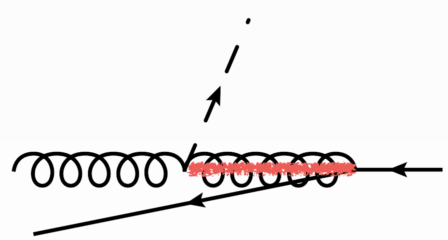
$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

The MEs contribute to the leading log can be deduced from the leading power collinear kernel, if we relate (1-z ')~0 to the collinear limit (k- ~ 0)

$|\mathcal{M}|^2$: $\frac{16\pi\alpha_s C_i}{\tau\,(1-z')} H_0 \quad + \frac{1}{1-z'} \times 0$ $\frac{8\pi\alpha_s C_i}{\tau} H_0$ $\frac{8\pi\alpha_s C_i}{1-z'} \frac{e^{Y'}}{m} H_0$

dΦ : LP/SL LP

$\mathcal{L}_{ij}$: $\mathcal{L}|_{z'=1}\,,\ \tau\times$ $\mathcal{L}|_{z'=1}$



$$\propto \frac{1}{k^-} P_{qg}(x)$$

offshellness (1-z ') ~ k- ~ 0

1-x ~ k^+ ~ tau

$\frac{\alpha_s C_A}{2\pi} L\, [2x_1\partial_{x_1}]\mathcal{L}_{g_1 g_2} + \{x$ $\frac{C_F}{2\pi} L\, \mathcal{L}_{g_1 q_2} + \{x_1 \leftrightarrow x_2\,, Y' \leftrightarrow -Y'\}$

$$L = \frac{e^{Y'}}{m} \log\left(\frac{\tau\, m\, e^{Y'}}{\tau^2}\right) \qquad x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}\,, \qquad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

Power Corrections

- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

The one which does not contribute to the leading log in the leading power does not contribute to the leading log in the sub-leading power

$$|\mathcal{M}|^2 \ : \quad \frac{16\pi\alpha_s C_i}{\tau\,(1-z')} H_0 \quad + \frac{1}{1-z'} \times 0 \qquad \frac{8\pi\alpha_s C_i}{\tau} H_0 \qquad \frac{8\pi\alpha_s C_i}{1-z'} \frac{e^{Y'}}{m} H_0$$

$$d\Phi \ : \quad \text{LP/SLP} \qquad \text{LP}$$

$$\mathcal{L}_{ij} \ : \quad \mathcal{L}|_{z'=1}\,,\ \tau \times x_1 \partial_{x_1} \mathcal{L}|_{z'=1} \qquad \mathcal{L}|_{z'=1}$$

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$$x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}\,, \qquad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

Power Corrections

- ggH/Drell-Yan @ NNLO

$$\alpha_s^2\, \tau_{cut}\, C_{23} \log^3(\tau_{cut})$$

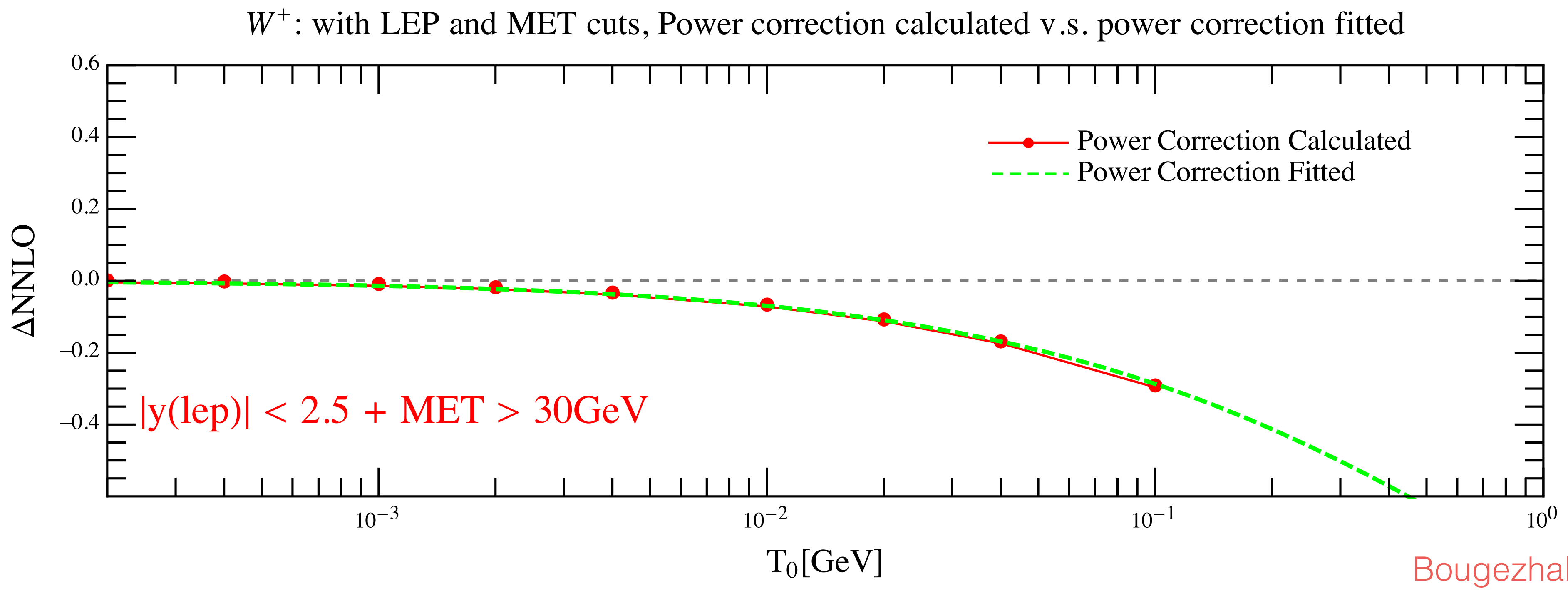
$$\mathrm{d}\sigma_{q\bar{q}\to V} = \frac{1}{2}\mathrm{d}\hat{\sigma}_0 \left(\frac{\alpha_s C_F}{2\pi}\right)^2 \left[4\log^2\left(\frac{\tau}{\mu}\right)\right] \times \left[\frac{1}{m}\log\frac{\tau}{m}\left[2x_1\partial_{x_1}\right]\mathcal{L}_{q\bar{q}} + x_1 \leftrightarrow x_2\right]$$

$$\mathrm{d}\sigma_{gq+qg\to V} = \mathrm{d}\hat{\sigma}_0 \left(\frac{\alpha_s}{2\pi}\right)^2 (C_F + C_A)T_R \left[\frac{1}{m}\mathcal{L}_{q_i g}\log^3\left(\frac{\tau}{m}\right) + x_1 \leftrightarrow x_2\right]$$

- free of poles: poles from RV have to cancel against RR
- soft limit leads to the leading logs
- MEs deducible from collinear limits for q final state in RV and qg final state in RR
- MEs takes the strongly-order limit, for instance, in the case of the qg final state, Eg<<Eq

Power Corrections

- Numerical consequence

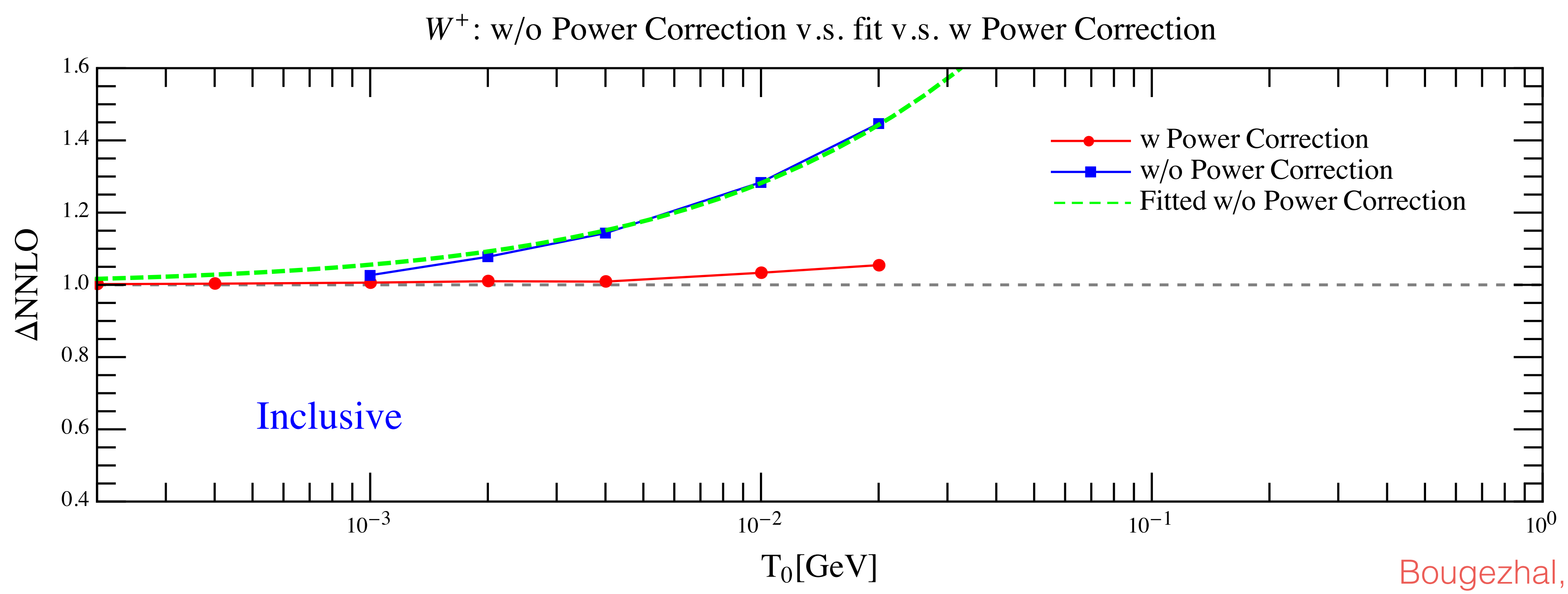


Bougezhal, XL, Petriello

- in ggH/Drell-Yan, the form of the power correction is simple can be fitted
- Analytic calculations agree well with the fitting

Power Corrections

- Numerical consequence

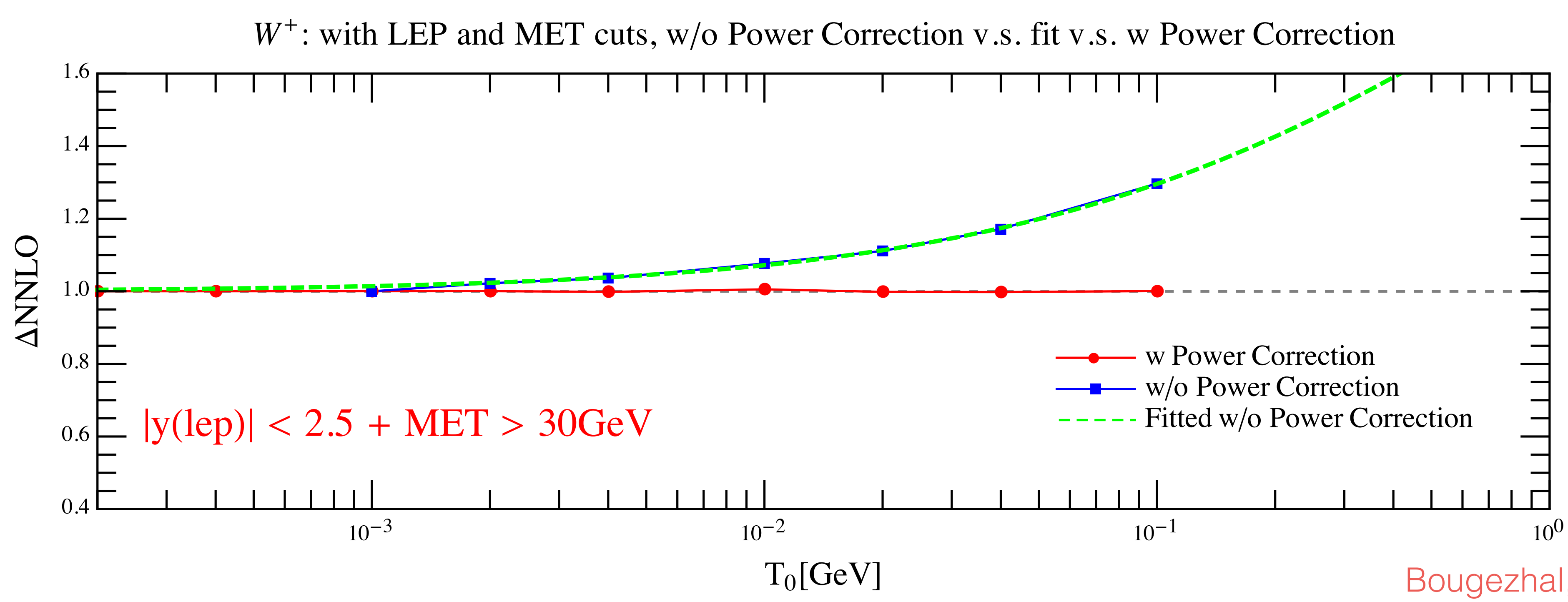


Bougezhal, XL, Petriello

- Including the power corrections helps improving the convergence of the N-jettiness subtraction scheme.
- allows us to relax the cut-off dramatically.
- improves the numerical stability

Power Corrections

- Numerical consequence



Bougezhal, XL, Petriello

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Power Corrections

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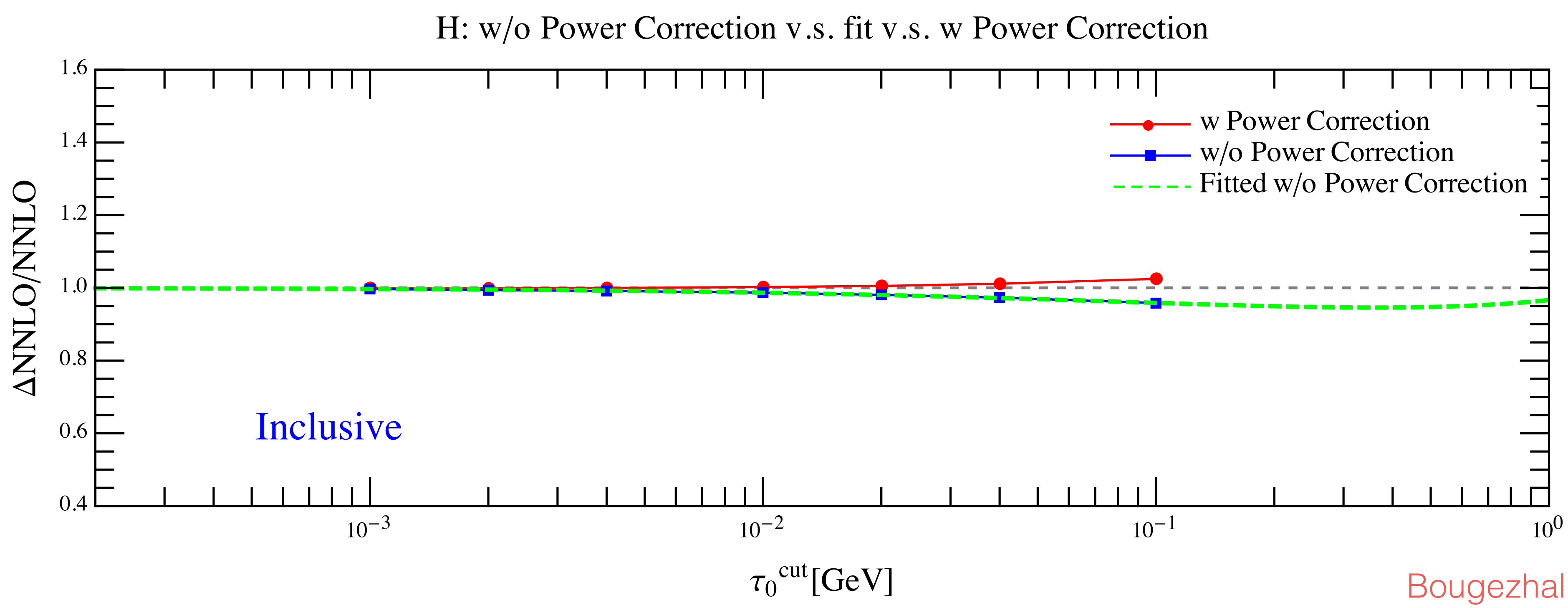

Bougezhal, XL, Petriello

- Including the power corrections helps improving the convergence of the N-jettiness subtraction scheme.
- allows us to relax the cut-off dramatically.
- improves the numerical stability.

Conclusions

- A first step toward understanding the power corrections in the jettiness subtraction
- Calculate the leading logs in the power corrections
- Can increase tau-cut substantially and improve numerical stabilities

Outlook

- Go beyond Drell-Yan/ggH

thanks