Three-loop quark form factor at high energy: the leading mass corrections

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Form factor

$$\mathcal{F} = \bar{q}(p_2)\left(\gamma_\mu F_1 + \frac{i\sigma_{\mu\nu}Q^\nu}{2m_q}F_2\right)q(p_1)$$

three-loop massless [Baikov,Chetyrkin,Smirnov^2,Steinhauser 2009]
[Gehrmann,Glover,Huber,Ikizlerli,Studerus 2010]

four-loop massless [Henn,Lee,Smirnov^2,Steinhauser 2016; Manteuffel,Schabinger 2016]
[Lee,Smirnov^2,Steinhauser 2017]

two-loop massive [Mastrolia,Remiddi 2003; Bernreuther et al 2005]

three-loop massive [Henn,Smirnov^2,Steinhauser 2016]

High energy limit

$$p_1^2 = p_2^2 = m_q^2 \ll Q^2, \quad \rho = m_q^2/Q^2$$

$$F_1 = \exp\left[-\frac{\alpha_s}{2\pi}\frac{C_F \ln\rho(1+\mathcal{O}(\rho^2))}{\varepsilon}\right]\sum_{n=0}^{\infty}\rho^n F_1^{(n)}$$

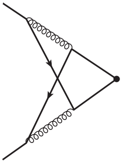
Leading power:

Sudakov logs [Sudakov 1956; Frenkel,Taylor 1976]
subleading logs also exponentiate [Muller 1979; Collins 1980; Sen 1981; ...]

$\mathcal{O}(\rho)$ corrections:

expansion by regions [Beneke, Smirnov 1998]
double logs come from soft quark exchange [Penin 2015; Penin,Zerf 2016]

Two loop



- Sudakov parameterization: $l_i = u_i p_1 + v_i p_2 + l_{i\perp}$
- eikonal glue: $\frac{1}{(p_1+l_i)^2} \approx \frac{1}{Q^2 v_i}$
- soft quark: $\frac{m_q}{(l_i^2 - m_q^2)} \approx -i\pi m_q \delta(Q^2 u_i v_i + l_{i\perp}^2 - m_q^2)$
- $\eta_i = \ln v_i / \ln \rho$, $\xi_i = \ln u_i / \ln \rho$

Result:

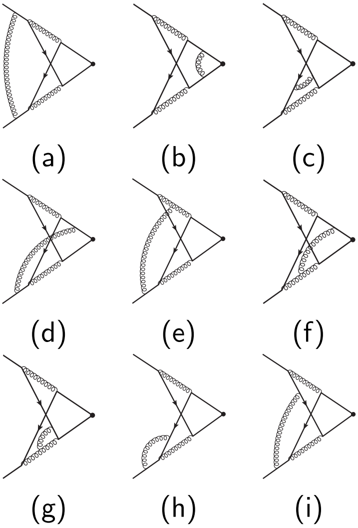
$$x = \frac{\alpha_s}{4\pi} \ln^2 \rho$$

$$K = \theta(1-\eta_1-\xi_1)\theta(1-\eta_2-\xi_2)\theta(\eta_2-\eta_1)\theta(\xi_1-\xi_2)$$

$$F_1^{(1,2l)} = 2\,(C_A - 2C_F)\,x^2 \times \int K(\eta_1,\eta_2,\xi_1,\xi_2)\mathrm{d}\eta_1\mathrm{d}\eta_2\mathrm{d}\xi_1\mathrm{d}\xi_2$$

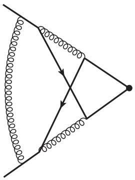
Diagrams

Dressing the two-loop non-planar digram with a soft glue:



Others either have no proper region or vanshing color factor.

Dia(a)



- region: $v_3 \ll v_2$, $u_3 \ll u_1$
- eikonal quark: $\frac{1}{(p_1+l_3)^2-m_q^2} \approx \frac{1}{Q^2(v_3+2\rho u_3)}$
- soft glue: $\frac{1}{l_3^2} \approx -i\pi\delta(Q^2 u_3 v_3 + l_{3\perp}^2)$

$$\propto \int_{\rho u_3}^{v_2} \frac{\mathrm{d}v_3}{v_3} \int_{\rho v_3}^{u_1} \frac{\mathrm{d}u_3}{u_3}$$

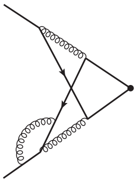
After subtractions one get infrared finite integrals:

$$\propto -\left(\int_{v_2}^{1} \frac{\mathrm{d}v_3}{v_3} \int_{\rho v_3}^{u_1} \frac{\mathrm{d}u_3}{u_3} + \int_{\rho u_3}^{v_2} \frac{\mathrm{d}v_3}{v_3} \int_{u_1}^{1} \frac{\mathrm{d}u_3}{u_3} + \int_{v_2}^{1} \frac{\mathrm{d}v_3}{v_3} \int_{u_1}^{1} \frac{\mathrm{d}u_3}{u_3}\right)$$

subtraction reproduces the factorized singular term.

Dia(h)

Sudakov parameterization: $l_3 = u_3 l_1 + v_3 p_2 + l_{3\perp}$



l_3 flow down at the vertex:

$$\propto \int_{\rho u_3/u_1}^{1} \frac{\mathrm{d}v_3}{v_3} \int_{\rho v_3/u_1}^{1} \frac{\mathrm{d}u_3}{u_3}$$

l_3 flow up at the vertex:

$$\propto - \int_{\rho u_3/u_1}^{v_1} \frac{\mathrm{d}v_3}{v_3} \int_{\rho v_3/u_1}^{1} \frac{\mathrm{d}u_3}{u_3}$$

The above integrals are divergent separately and the sum of them is finite.

Results

$$F_1^{(1,3l)} = \frac{C_F\,(C_A - 2C_F)}{2} \sum_\lambda c_\lambda d_\lambda\, x^3$$

$$d_\lambda = 4 \int w_\lambda(\eta,\xi) K(\eta_1,\eta_2,\xi_1,\xi_2) \mathrm{d}\eta_1 \mathrm{d}\eta_2 \mathrm{d}\xi_1 \mathrm{d}\xi_2$$

λ	w_λ	d_λ	c_λ
a	$-((\eta_2+2)\eta_2 + (\xi_1 - 2\eta_2 + 2)\xi_1 - 1)$	$-\frac{17}{45}$	$-C_F$
b	$2\xi_2\eta_1$	$\frac{1}{45}$	$-C_F$
c	$2(\xi_1-\xi_2)(\eta_2-\eta_1)$	$\frac{1}{15}$	$C_A - C_F$
d	$-\eta_1(\eta_1 - 2\xi_1 + 2)$	$-\frac{1}{10}$	$C_A - C_F$
e	$(\eta_2-\eta_1)(2 - 2\xi_1 + \eta_1 + \eta_2)$	$\frac{8}{45}$	$-\frac{C_A}{2}$
f	$2\eta_1(\xi_1-\xi_2)$	$\frac{1}{30}$	$-\frac{C_A}{2}$
g	$2\eta_2(\xi_1-\xi_2)$	$\frac{1}{10}$	$-\frac{C_A}{2}$
h	$\eta_1(\eta_1 - 2\xi_1 + 2)$	$\frac{1}{10}$	$\frac{C_A}{2} - C_F$
i	$\eta_2(\eta_2 - 2\xi_1 + 2)$	$\frac{5}{18}$	$\frac{C_A}{2} - C_F$

Results

$$F_1^{(1)} = \frac{C_F\left(C_A - 2C_F\right)}{6} x^2 \left[1 - \frac{C_A + 4C_F}{5} x + \mathcal{O}(x^2)\right]$$

$C_A - 2C_F = 1/N_c$.

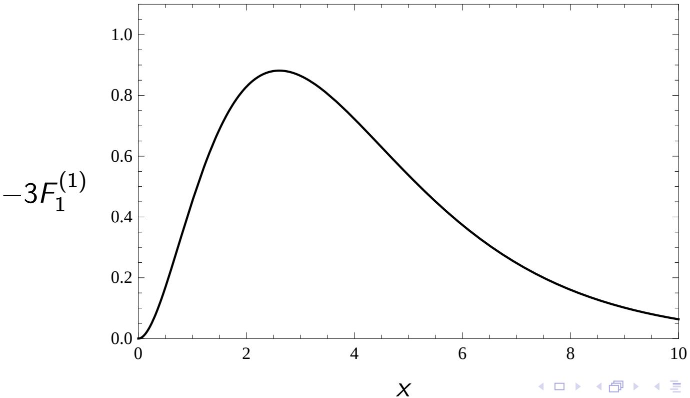
Our result agrees with the analysis of leading-color approximation.

[Henn,Smirnov,Smirnov,Steinhauser 2016]

QED resummation

$$F_1^{(0)} = \exp\left[-x\right]$$

$$F_1^{(1)} = -4x^2 \int \exp\Big[-x\,(1 - 2\eta_1\xi_1 + 4\eta_1\xi_2 - 2\eta_2\xi_2)\Big] K \mathrm{d}\eta_1 \mathrm{d}\eta_2 \mathrm{d}\xi_1 \mathrm{d}\xi_2$$



Conclusion

- Our result could be used to cross check future calculations
- To be done: 1. all order resummation for nonabelian case
  2. double logs for other physical processes

Thanks for your attention!