



# NLO corrections to massive vector color octet pair production

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Based on arXiv:1705.xxxx  
with A. Freitas



Fig.1 A multi-loop correction to a tree

# The Why, the What and the How

## ○ the Why

- Colored Scalars and Spinors done at NLO for SUSY searches
- Vector Octets still missing → technology not straightforward!

## ○ the What

- Massive Vector Color Octets prominent feature (e.g. **UED**)
- Appropriate low-energy theory (**Coloron**<sup>1</sup>) renormalizable without knowing UV

## ○ the How

- Massless particles lead to **Infrared/Collinear Divergencies**
- Use **Two Cutoff Phase Space Slicing**
- FeynArts/FeynCalc had (until now) no Coloron model/QCD@NLO toolbox

<sup>1</sup>) E.Simmons, A. Atre, R. Chivukula et al hep-ph/1304.0255

# QCD@NLO: Divergencies

$$|\langle in|out\rangle|^2 = \left| \sum_n \alpha_s^n \mathcal{M}_n \right|^2 = \alpha_s^2 |\mathcal{M}_{LO}|^2 + 2\alpha_s^3 \text{Re}|\mathcal{M}_{LO}\mathcal{M}_{NLO}| + \dots$$

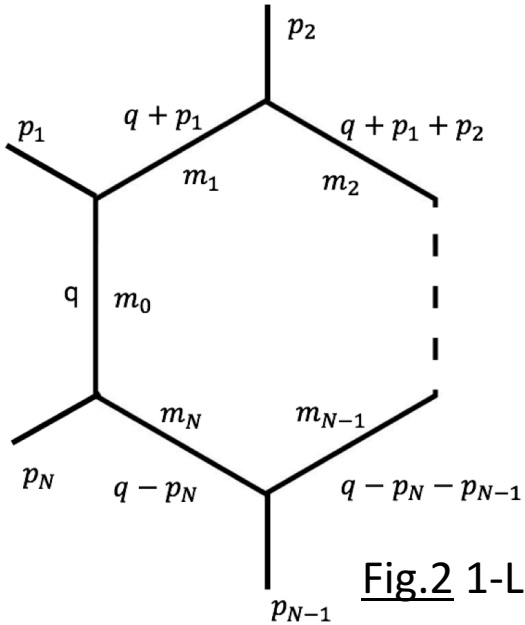


Fig.2 1-Loop conventions

$$T_N^{\mu_1 \mu_2 \dots \mu_k} = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \frac{q^{\mu_1} q^{\mu_2} \dots q^{\mu_k}}{(q^2 - m_0^2) ((q+p_1)^2 - m_1^2) \dots ((q-p_N)^2 - m_N^2)}$$

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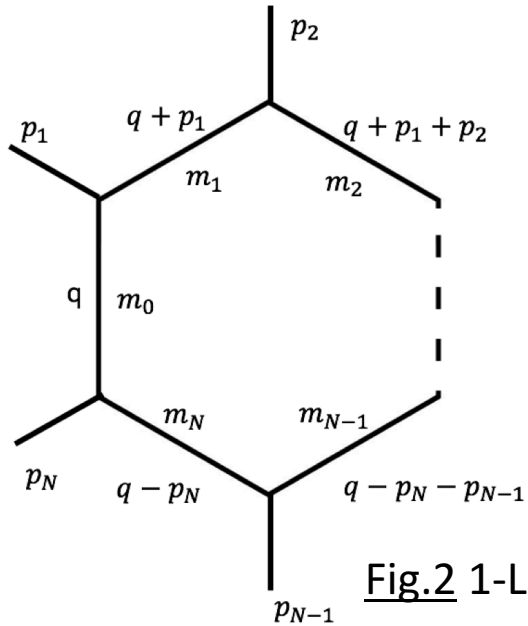


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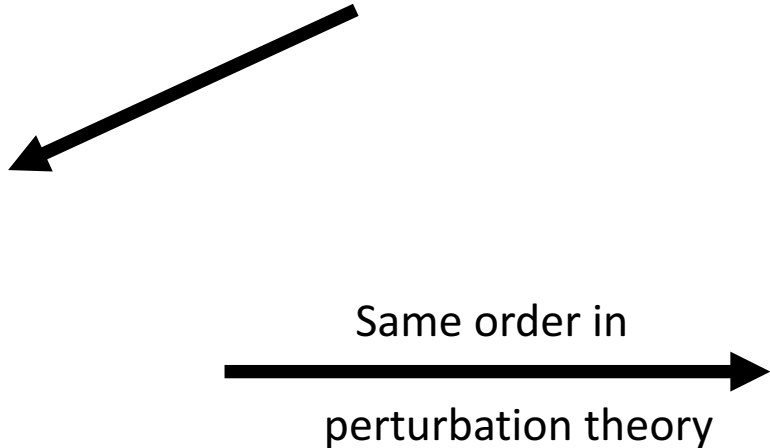
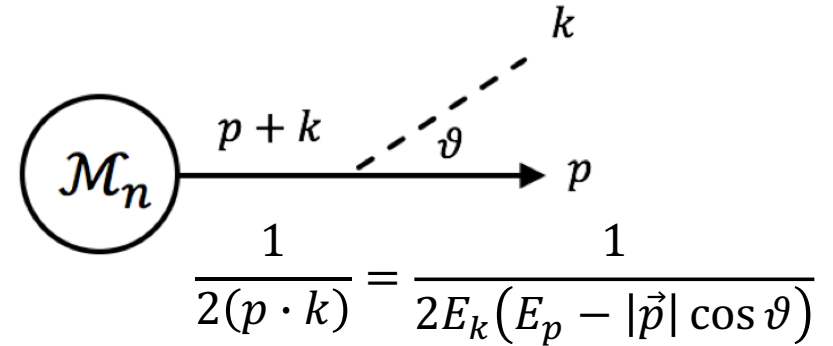


Fig.3 Final state real emission



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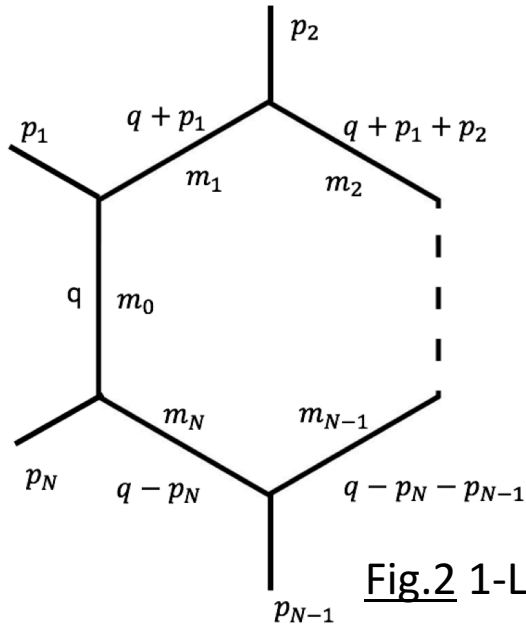


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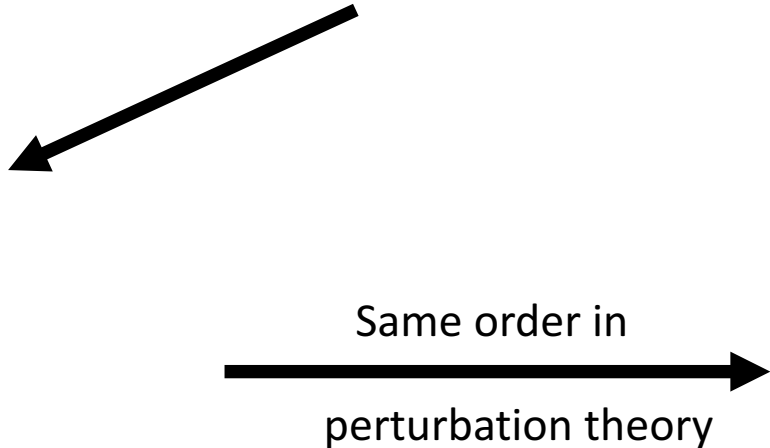
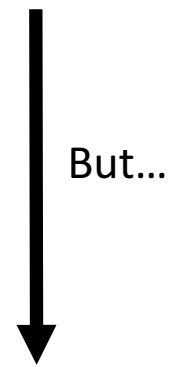
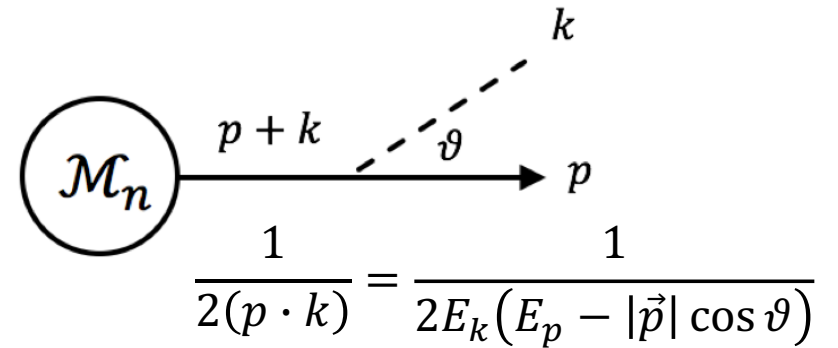


Fig.3 Final state real emission



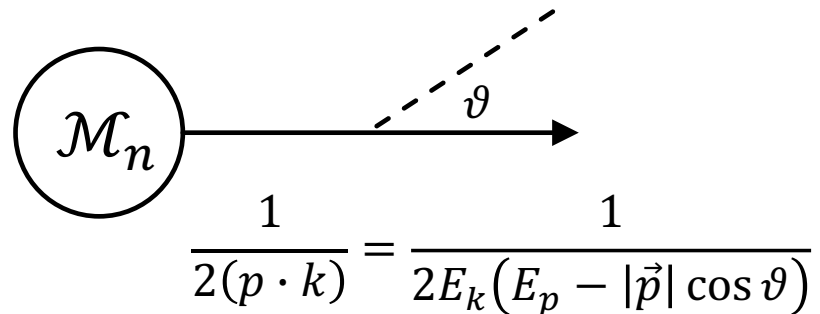
Kinoshita-Lee-Nauenberg: The NLO observables are IR-finite order by order in perturbation theory

# Two Cutoff Phase Space Slicing

soft-collinear region

To extract the soft/collinear poles divide the integration region:

$$\sigma_{2 \rightarrow 3} = \int \sum |M_{2 \rightarrow 3}|^2 d\Gamma_3 = \int \sum_{Soft} |M_{2 \rightarrow 3}|^2 d\Gamma_3 + \int \sum_{Hard} |M_{2 \rightarrow 3}|^2 d\Gamma_3 \quad 0 \leq E_g \leq \delta_s \frac{\sqrt{s}}{2}$$



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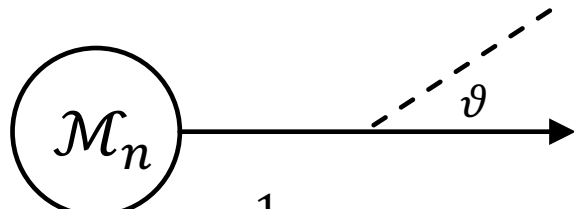
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## hard-collinear region

$$\int \sum_{Hard} |M_{2 \rightarrow 3}|^2 dLIPS_3 = \int \sum_{HC} |M_{2 \rightarrow 3}|^2 dLIPS_3 + \int \sum_{HNC} |M_{2 \rightarrow 3}|^2 dLIPS_3 \quad \begin{aligned} 0 \leq -(p_i - p_g)^2 \\ = \sqrt{s} E_g (1 - \cos \vartheta) \leq \delta_c s \end{aligned}$$



$$\frac{1}{2(p \cdot k)} = \frac{1}{2E_k(E_p - |\vec{p}| \cos \vartheta)}$$

The hard-non collinear region does not contain any poles anymore and can be integrated numerically

# The Eikonal Limit

Soft/Collinear Region

$$0 \leq E_g \leq \delta_s \frac{\sqrt{s}}{2}$$

$$d\sigma_{soft} = \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left(\frac{4\pi\mu^2}{s}\right)^\varepsilon d\sigma_{LO} \\ \times \sum_{f,f'} \int dS \underbrace{\frac{-p_f \cdot p_{f'}}{(p_f \cdot k)(p_{f'} \cdot k)}}_{I_D^{(k,l)}}$$

→ Factorization for **color subamplitudes**

→ Regularized angular integrals  
scattered through literature

<sup>1)</sup> B. Harris, J. Owens hep-ph/0102128



# The Eikonal Limit

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## Hard/Collinear Region

$$(1 - \cos \vartheta) \leq \delta_c \sqrt{s}/E_g$$

$$d\sigma_{coll} = \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left(\frac{4\pi\mu^2}{s}\right)^\varepsilon \left(-\frac{1}{\varepsilon}\right) \delta_c^{-\varepsilon} \times \left\{ \int_0^{1-\delta_s} \frac{dz}{z} \left[ \frac{1-z}{z} \frac{s}{\mu_F^2} \delta_c \right]^{-\varepsilon} P_{ii'}(z) d\sigma_{LO}^{ji'} + (i \leftrightarrow j) \right\}$$

Altarelli-Parisi Kernel

→ Redefine **Parton Density Functions** to absorb the Pole

$$\tilde{\mathcal{F}}(x, \mu_F) = \frac{\alpha_s}{2\pi} \sum_k \int_x^{1-\delta_s} \frac{dz}{z} \mathcal{F}_k\left(\frac{x}{z}\right) \left[ P_{ik}(z) \log\left(\frac{s}{\mu_F^2} \frac{1-z}{z} \delta_c\right) - P'_{ik}(z) \right]$$

Master formula  
for any 2→2  
process in  
QCD@NLO

$$\sigma_{NLO} = \sum_{i,j} \int dx_1 dx_2 \mathcal{F}_i(x_1, \mu_f) \mathcal{F}_j(x_2, \mu_f) \left[ \sigma_{LO}^{ij}(x_1, x_2, \mu) + \sigma_{Virt}^{ij}(x_1, x_2, \mu) + \sigma_{soft}^{ij}(x_1, x_2, \mu) \right]$$

$$+ \frac{\alpha_s}{2\pi} \sum_{i,j} \int dx_1 dx_2 \int_{x_1}^{1-\delta_s} \frac{dz}{z} \left[ \mathcal{F}_i\left(\frac{x_1}{z}, \mu_f\right) \mathcal{F}_j(x_2, \mu_f) + \mathcal{F}_i(x_2, \mu_f) \mathcal{F}_j\left(\frac{x_1}{z}, \mu_f\right) \right]$$

$$\times \sigma_{LO}^{ij} \left[ P_{ik}(z) \log\left(\frac{s(1-z)\delta_c}{z\mu_F^2}\right) - P'_{ik}(z) \right] + (i \leftrightarrow j) + \int \sum_{HNC} |M_{2 \rightarrow 3}|^2 d\Gamma_3$$

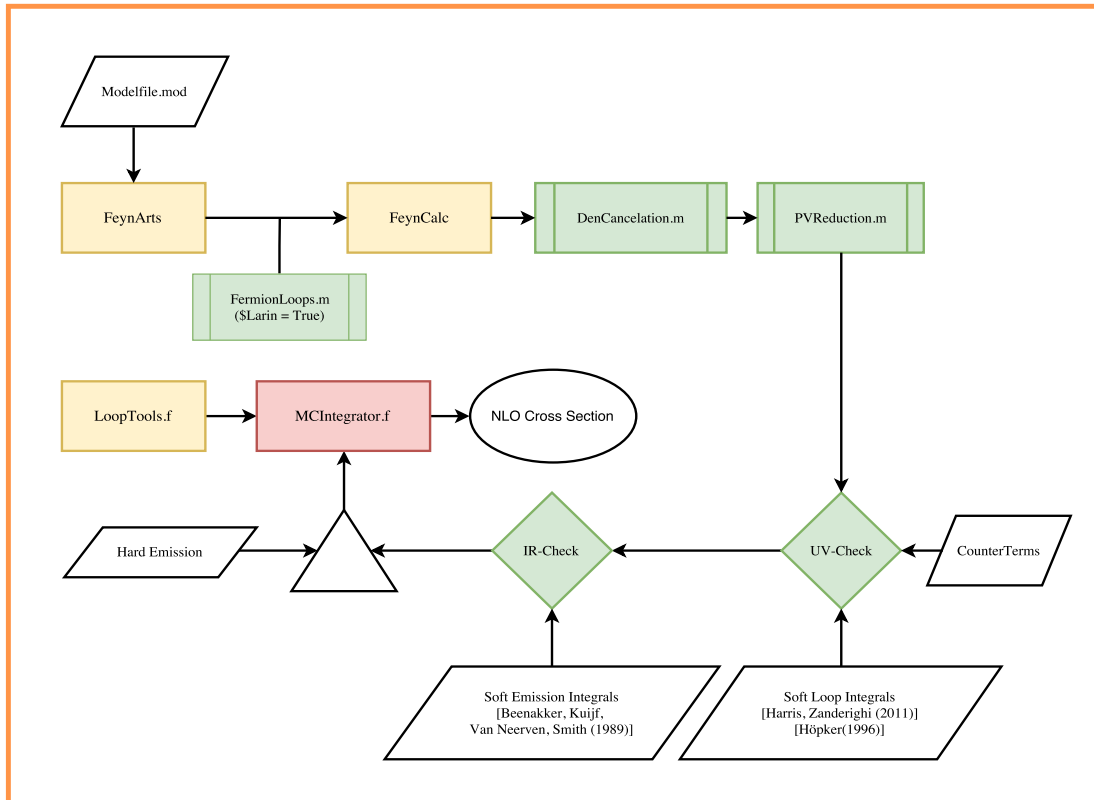
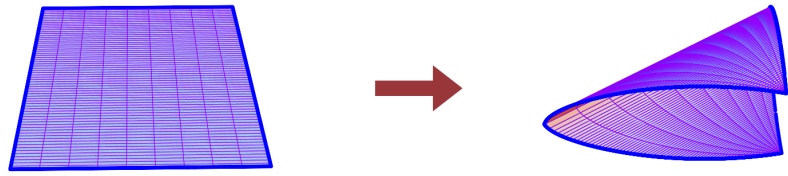
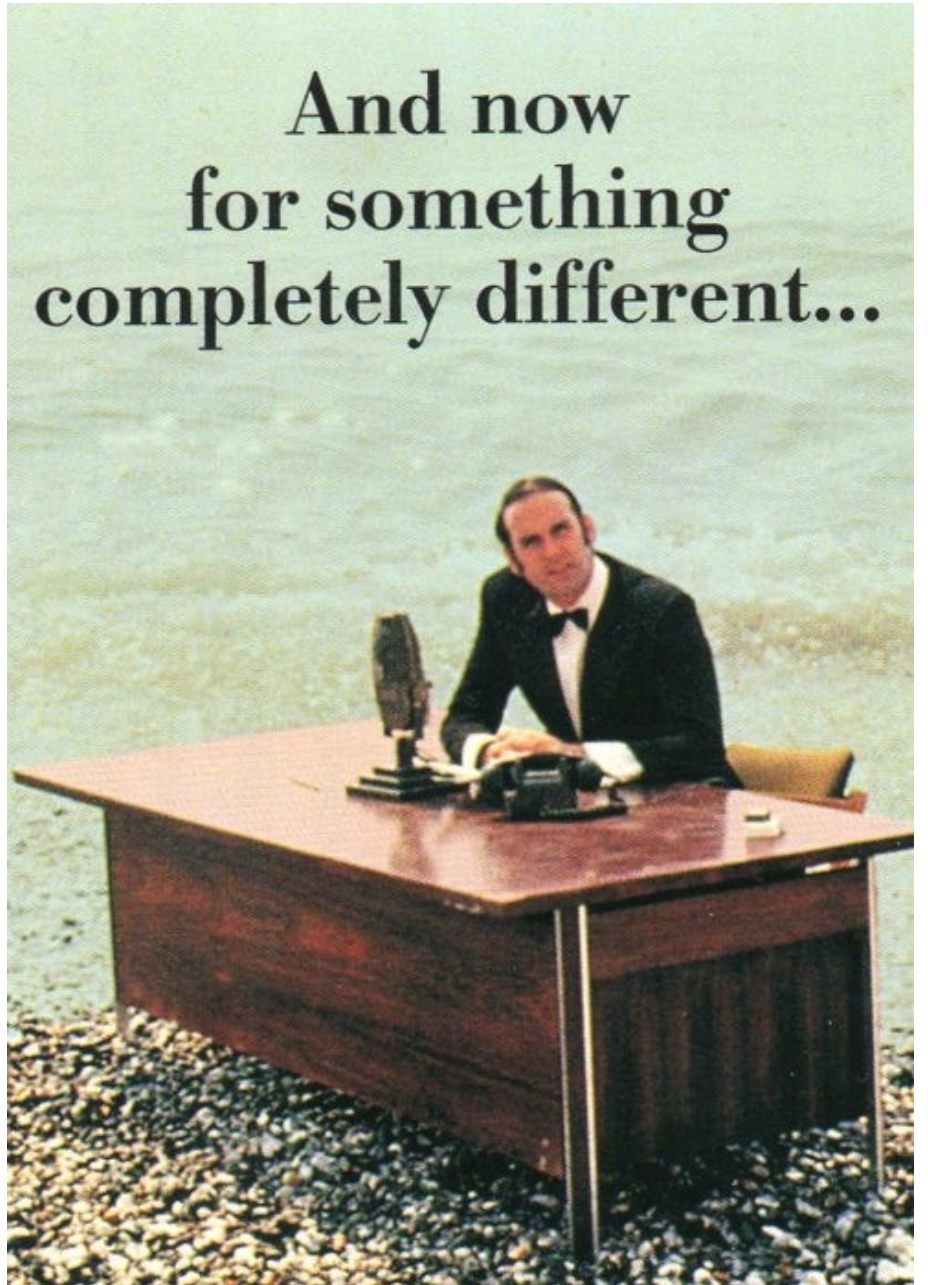
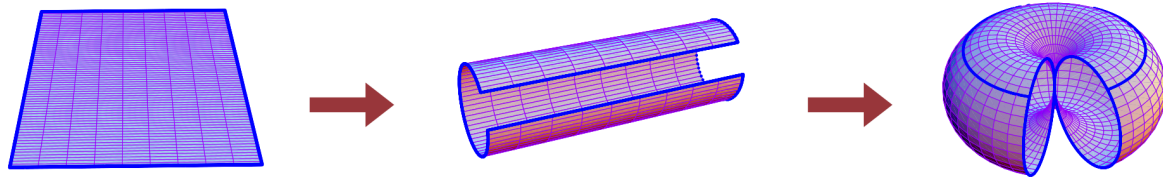


Fig.4 The production pipeline showing the implementation in the FeynArts/FeynCalc framework

- 1) T. Hahn hep-ph/0012260
- 2) T. Hahn hep-ph/1006.2231



## From Extra Dimensions to Colorons



# Universal Extra Dimensions (UED)

## Universal Extra Dimensions:

- Assume five-dimensional spacetime manifold
- To explain four-dimensional world impose boundary conditions (Kaluza Klein Compactification/**Orbifolding**)
- Fields  $\Psi(x^\mu, y)$  propagating can be decomposed into Fourier modes
- $\psi_0$  are the standard model modes,  $\psi_n$  a tower of additional (heavy) excitations of mass  $M = \frac{n}{R}$

$$\Psi(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \psi_0(x) + \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \psi_n(x) \cos \frac{ny}{R}$$

(Current Limit:  $M \geq 1400\text{GeV}$  with  $\Lambda R \sim 10^{1,2,3}$  @LO)

1) N. Deutschmann, T.Flacke, J. Kim hep-ph/1702.00401

2) K. Matchev, A.Datta et al hep-ph/1702.00413

3) ATLAS hep-ex/1501.03555

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$$A_M^a(x, y) \rightarrow A_M^a(x, y) + \partial_M \Theta^a(x, y) - g_5 f^{abc} \Theta^a(x, y) A_M^c(x, y)$$

$$\begin{aligned} \left. \begin{array}{l} \rightarrow A_\mu^{0,a} \\ \rightarrow A_\mu^{n,a} \end{array} \right\} & \rightarrow A_\mu^{0,a} + \partial_\mu \Theta^{0,a} - \frac{1}{2} \frac{g_5}{\sqrt{2\pi R}} f^{abc} \sum_m 2^{1-\delta_{m,0}} (1 + \delta_{m,0}) \Theta^{m,b} A_\mu^{m,c} \\ & \rightarrow A_\mu^{n,a} + \partial_\mu \Theta^{n,a} - \frac{1}{2} \frac{g_5}{\sqrt{2\pi R}} f^{abc} \sum_m \sqrt{2}^{1-\delta_{m,0}} \Theta^{m,b} \left( \sqrt{2}^{\delta_{m,n}} (1 + \delta_{m,n}) A_\mu^{m-n,c} + A_\mu^{m+n,c} \right) \end{aligned}$$

→ Truncation breaks gauge invariance!

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# Building a Non-Linear Sigma Model

$$SU(3)_1 \times SU(3)_2 \times SU(2)_W \times U(1)_Y$$



$$\langle \chi_a \rangle = v \mathbb{1}_a$$

$$SU(3)_C \times SU(2)_W \times U(1)_Y$$



$$SU(3)_C \times U(1)_{EM}$$

With massless gauge bosons  $A_1^\mu, A_2^\mu$

Introduce a  $\Sigma$  field that transforms **non-linearly** under the full group

$$\Sigma \rightarrow U_1 \Sigma U_2^\dagger \quad \Sigma = \exp\left(\frac{i\chi_a T^a}{f}\right)$$

With the vev of the Goldstone Boson  $\chi_a$  breaking the  $SU(3)_1 \times SU(3)_2$

For which we can build a Lagrangian that respects the symmetry

$$\mathcal{L}_\Sigma = \frac{f^2}{2} \text{Tr} \left[ (D_\mu \Sigma)^\dagger D^\mu \Sigma \right] - \frac{1}{4} \text{Tr} [F_1^{\mu\nu} F_{1,\mu\nu}] - \frac{1}{4} \text{Tr} [F_2^{\mu\nu} F_{2,\mu\nu}]$$

Additionally introduce gauge-fixing and ghosts

$$\mathcal{L}_{GF} = -\frac{1}{2\xi_g} (\partial_\mu g^\mu)^2 - \frac{1}{2} \left( \frac{1}{\sqrt{\xi_{G1}}} \partial_\mu C^\mu - \sqrt{\xi_{G2}} M \chi \right)^2$$

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"KK parity"



$\mathbb{Z}_2$  Symmetry:  $G_1^\mu \leftrightarrow G_2^\mu$  becomes  $g^\mu \rightarrow g^\mu \quad C^\mu \rightarrow -C^\mu \quad \Sigma \rightarrow \Sigma^\dagger$

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Diagonalized mass terms reveals a massive and a massless vector octet

$$C^\mu = \sin \alpha A_1^\mu + \cos \alpha A_2^\mu$$

$$g^\mu = \sin \alpha A_2^\mu - \cos \alpha A_1^\mu$$

$$\tan(\alpha) = \frac{g_2}{g_1} = 1$$

"coupling universality"

# Renormalization

Define the renormalization constants in an On-Shell scheme

$$\delta Z_{GG} = -\text{Re} \left. \frac{\partial \Sigma_{GG}^T(p^2)}{\partial p^2} \right|_{p^2=M_{kk}^2} \quad \delta Z_{gg} = -\text{Re} \left. \frac{\partial \Sigma_{gg}^T(p^2)}{\partial p^2} \right|_{p^2=0}$$

$$\delta Z_F^{L/R} = -\text{Re} \Sigma^{L/R}(M_{kk}^2) - M_{kk}^2 \left. \frac{\partial}{\partial p^2} \text{Re} [\Sigma^R(p^2) + \Sigma^L(p^2) + 2\Sigma^S(p^2)] \right|_{p^2=M_{kk}^2}$$

$$\delta M_{kk}^Q = \frac{M_{kk}}{2} \text{Re} [\Sigma^R(M_{kk}) + \Sigma^L(M_{kk}) + 2\Sigma^S(M_{kk})] \quad \delta M_{kk}^2 = -\text{Re} \Sigma_{kk}^T(M_{kk}^2)$$

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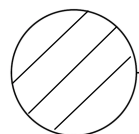
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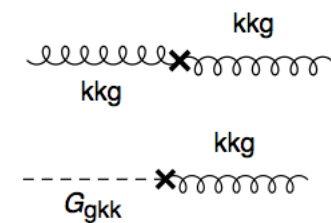
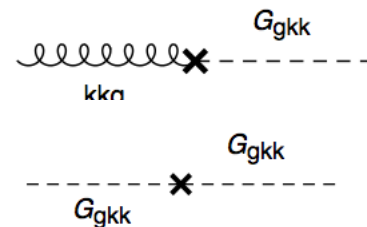
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Demand that the Pole of the propagator remains at the mass

N.B.: The tadpole Renormalization in Feynman Gauge demands special attention: We assume a Higgs-like breaking mechanism but the non-linear sigma model integrates out the radial part of the Goldstone Boson



$$= i \text{ Tad} = i \Sigma_{GB}(p^2) \Big|_{p^2=0}$$



# Running of the Couplings

- Choose a 5-flavor Scheme  $\overline{MS}$  Scheme
- Fix all couplings at  $\mu = M$
- Account for heavy particles with finite log's

$$\hat{g}_s \rightarrow g_s(\mu_R^2) \left\{ 1 + \frac{\alpha_s(\mu_R^2)}{4\pi} \left[ \left( -\frac{1}{\tilde{\epsilon}} + \log\left(\frac{\mu_R^2}{\mu^2}\right) \right) \frac{\beta_0}{2} - \frac{1}{3} \log\left(\frac{m_T^2}{\mu_R^2}\right) + \frac{23}{12} \log\left(\frac{M_{kk}^2}{\mu_R^2}\right) - \frac{2}{3} \log\left(\frac{(M_{kk} + m_T)^2}{\mu_R^2}\right) \right] \right\}$$

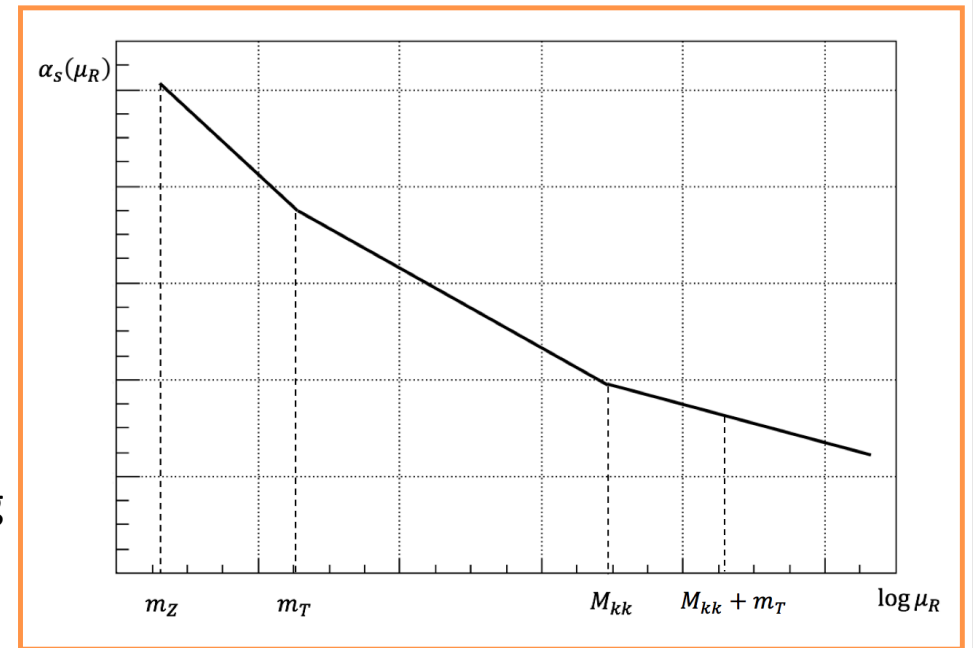
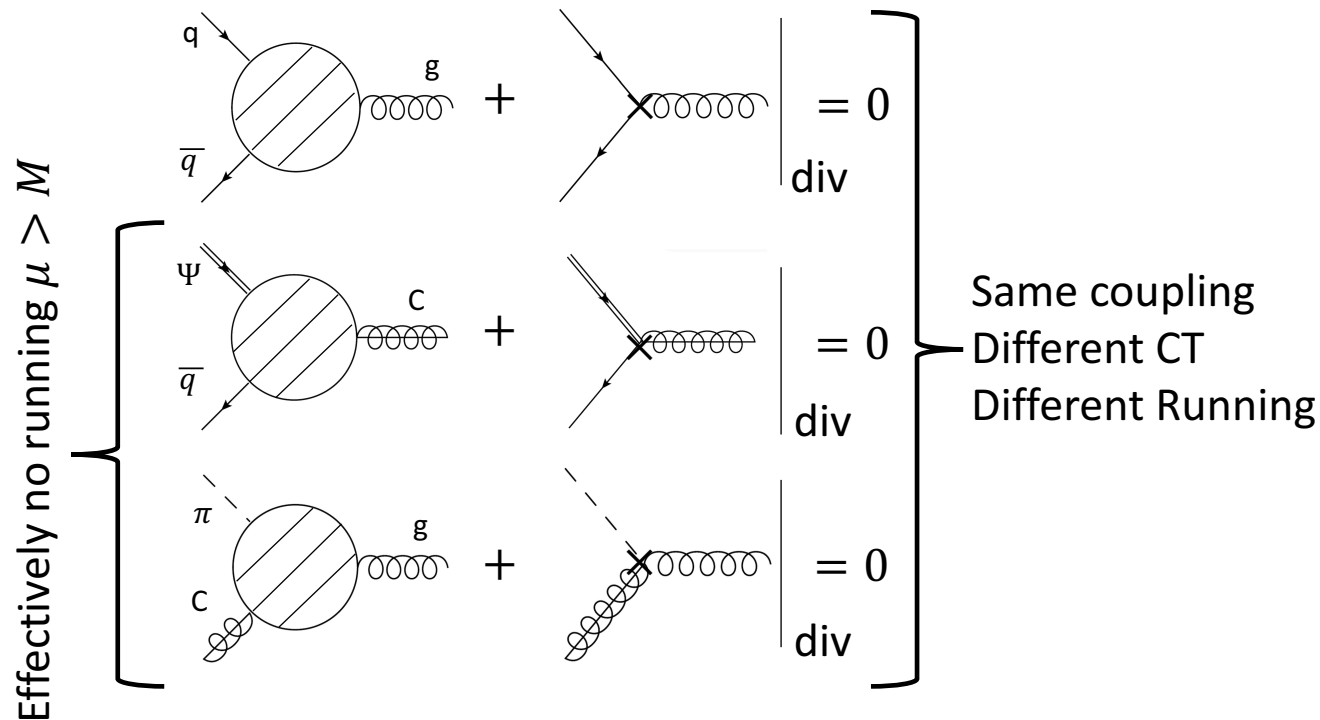
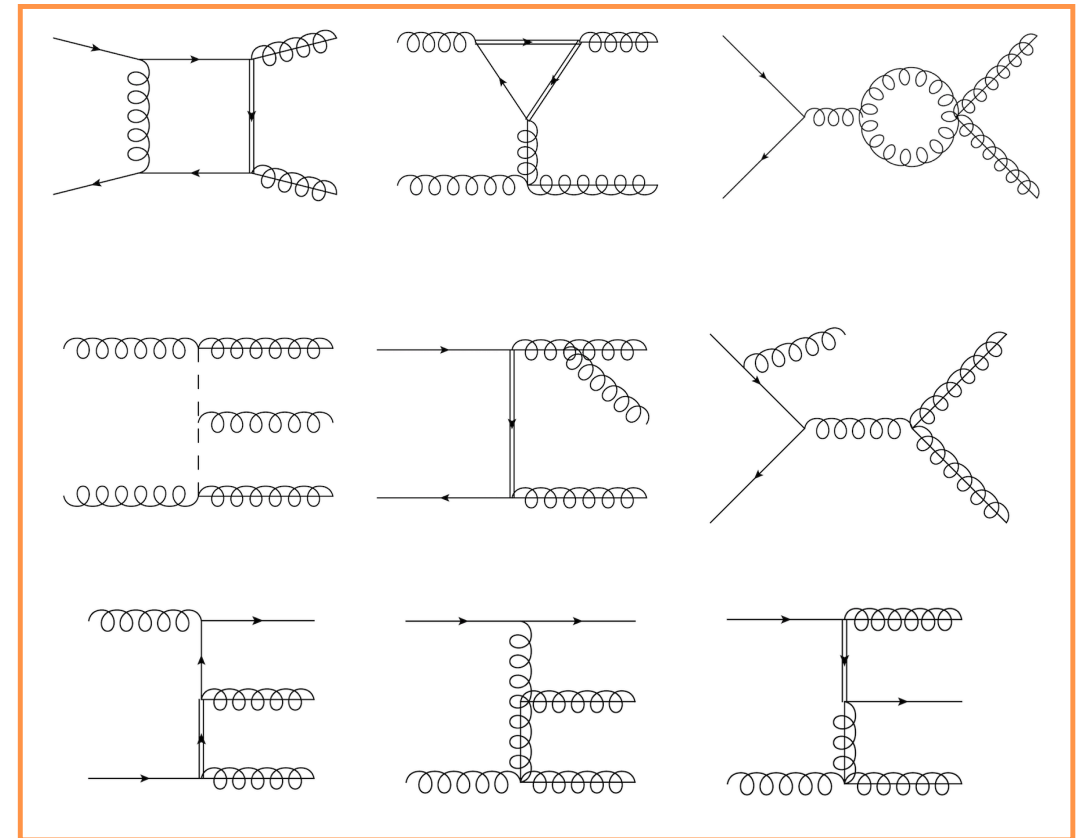
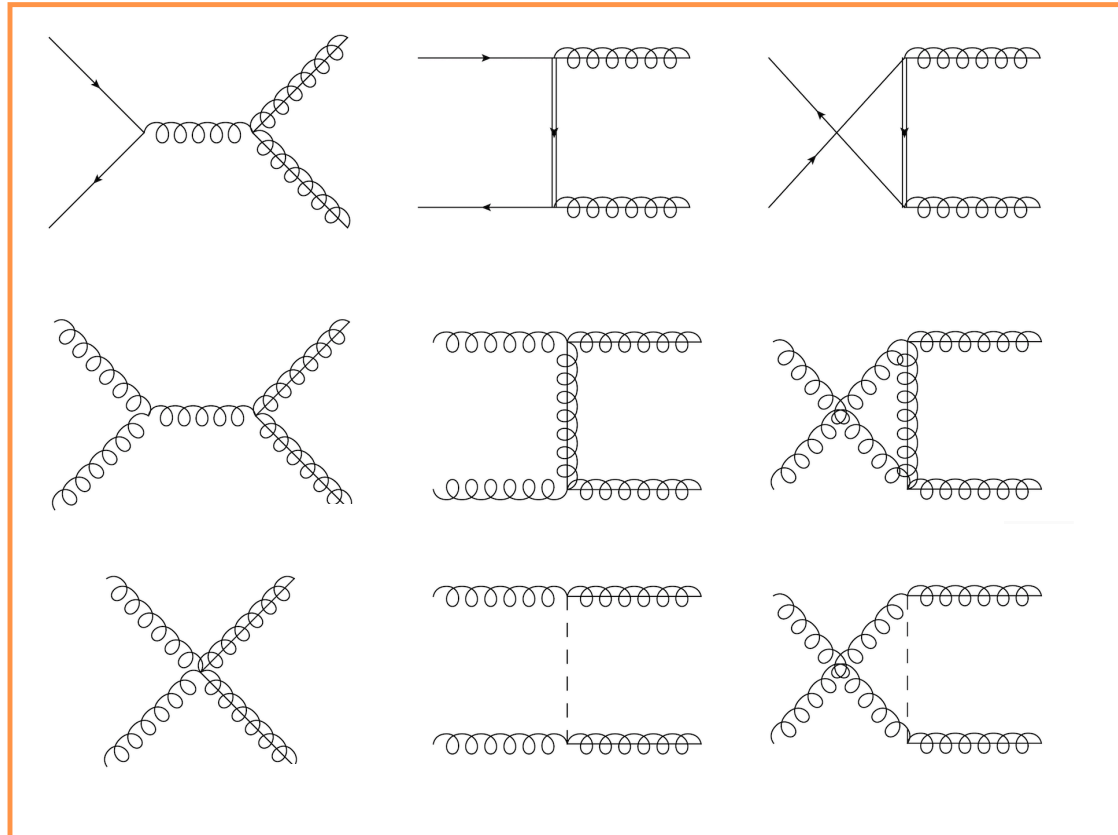


Fig.5 Decoupling of heavy particles in  $\alpha_s(\bar{q}qg)$  Vertex

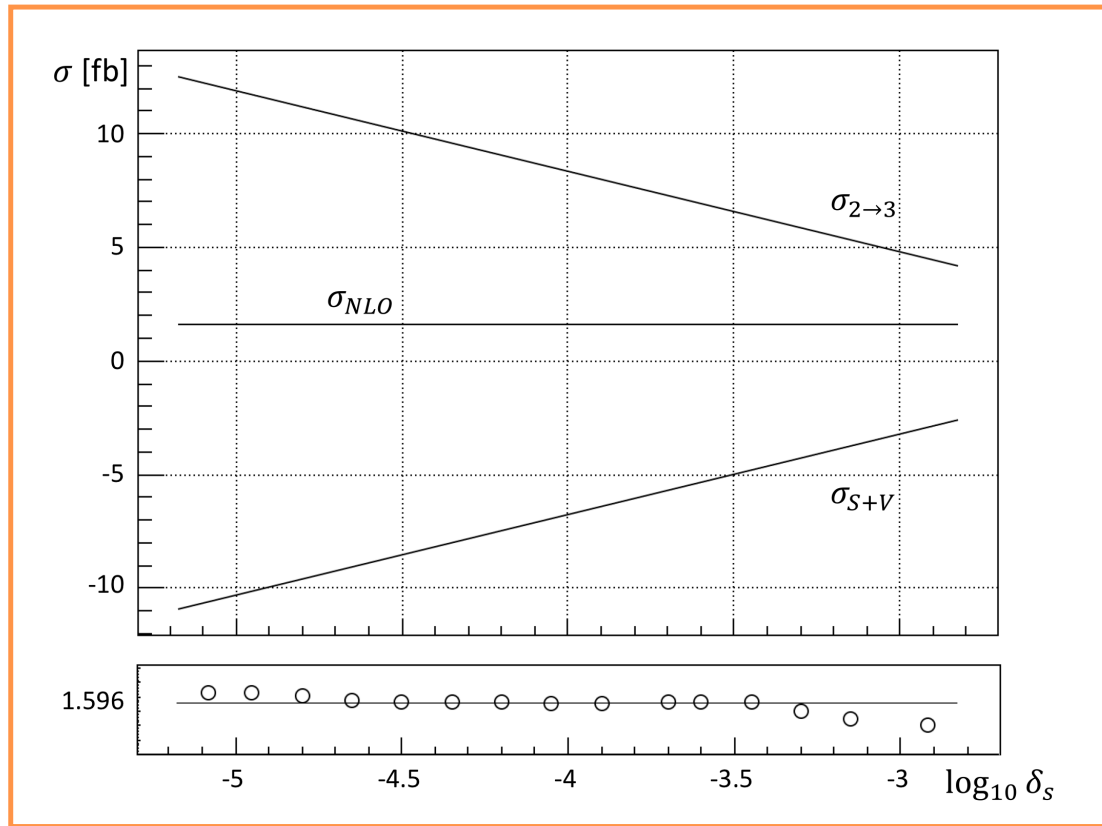
# Coloron Production at LHC

**Fig.6** Diagrams contributing to the Born cross section



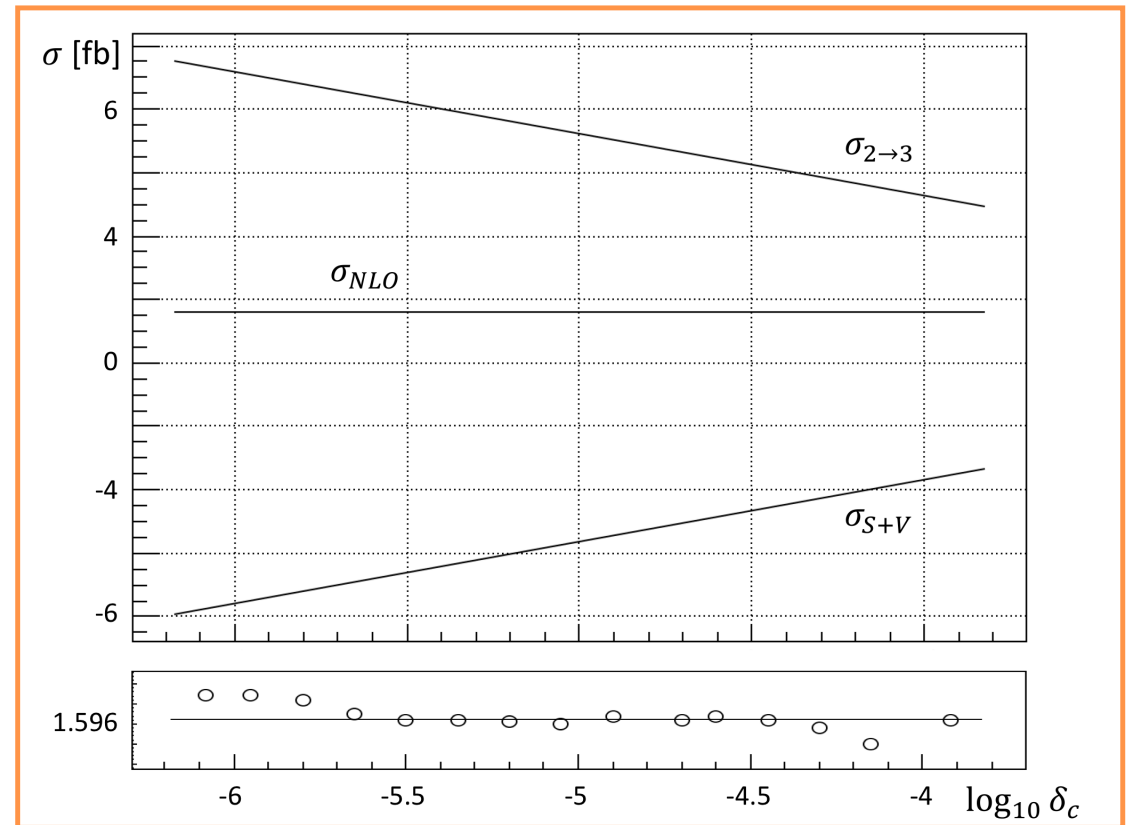
**Fig.7** Diagrams contributing to  
 A) the virtual cross section  
 B) Real emission part  
 C) quark-gluon channel

# Cutoff Dependence



**Fig.8**  $\delta_s$  dependence of  $\sigma_{NLO}$   
 $(M_{kk} = \mu_R = \mu_f = 1\text{TeV}, \delta_c = 7 \times 10^{-6}, \sqrt{s} = 14\text{TeV})$

**Fig.9**  $\delta_c$  dependence of  $\sigma_{NLO}$   
 $(M_{kk} = \mu_R = \mu_f = 1\text{TeV}, \delta_s = 7 \times 10^{-6}, \sqrt{s} = 14\text{TeV})$



# Scale Dependence

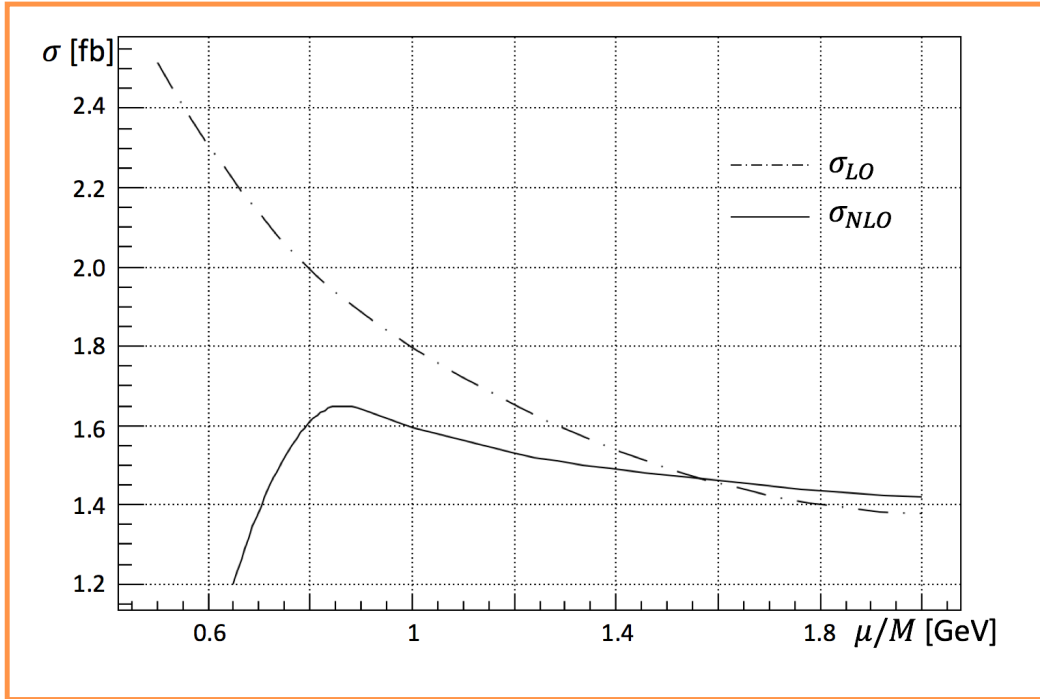
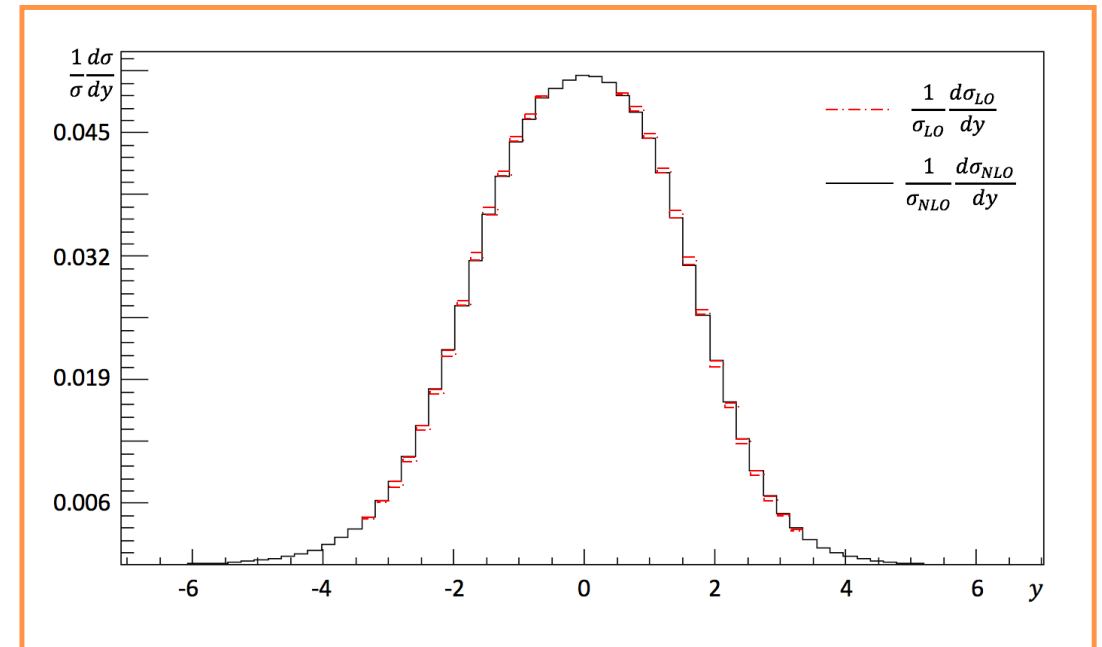


Fig.10 Variation of the scales ( $\mu_R = \mu_F$ ) for 1TeV Colorons

Fig.11 Rapidity distribution of the Colorons ( $M = \mu_{R/F} = 1$  TeV)



# Mass Dependence

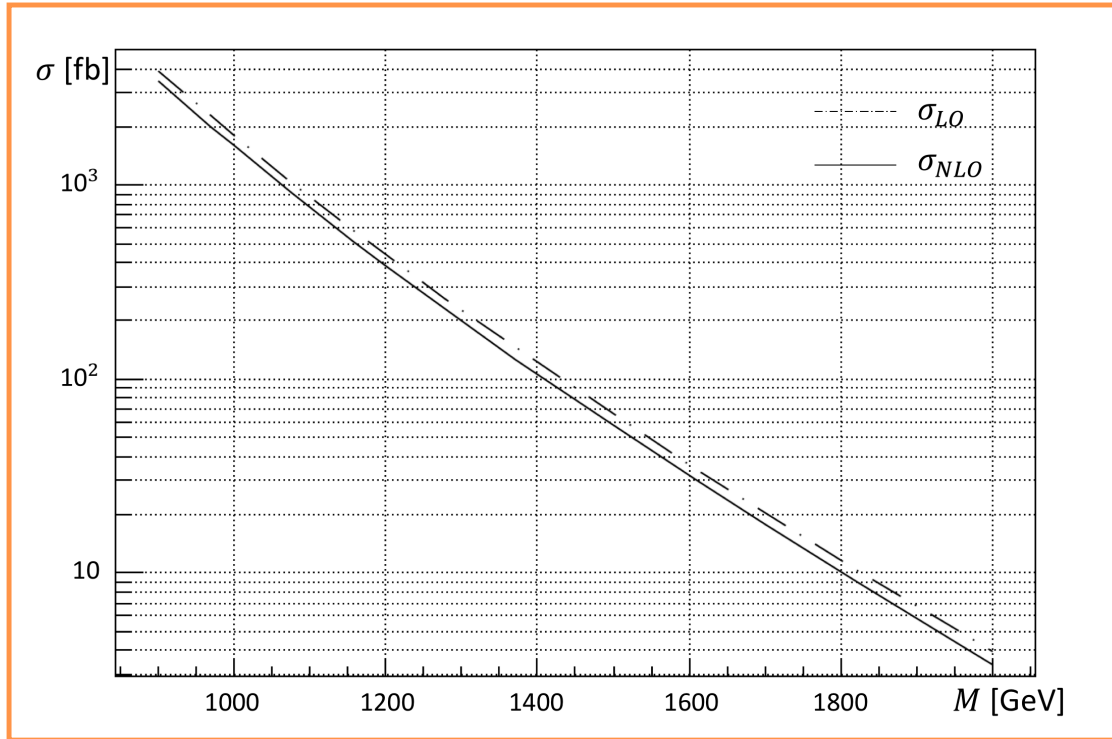
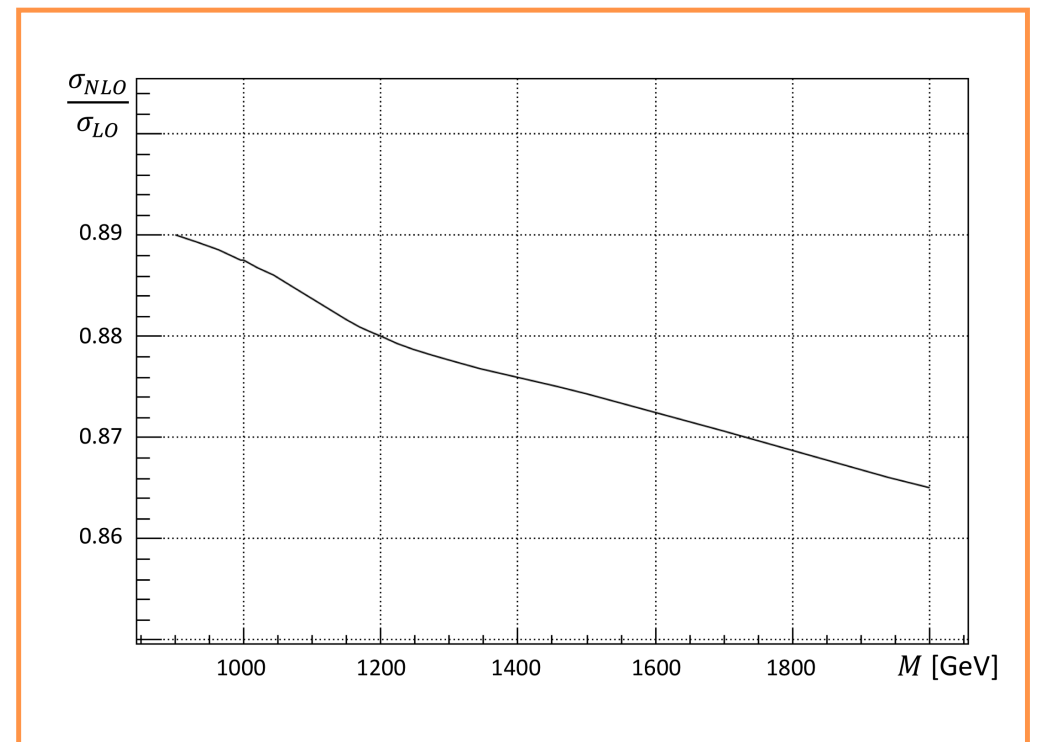


Fig.12 Mass dependence of the NLO cross section

Fig.13 Mass dependence of the  $\kappa$ -Factor



# Mass Splitting

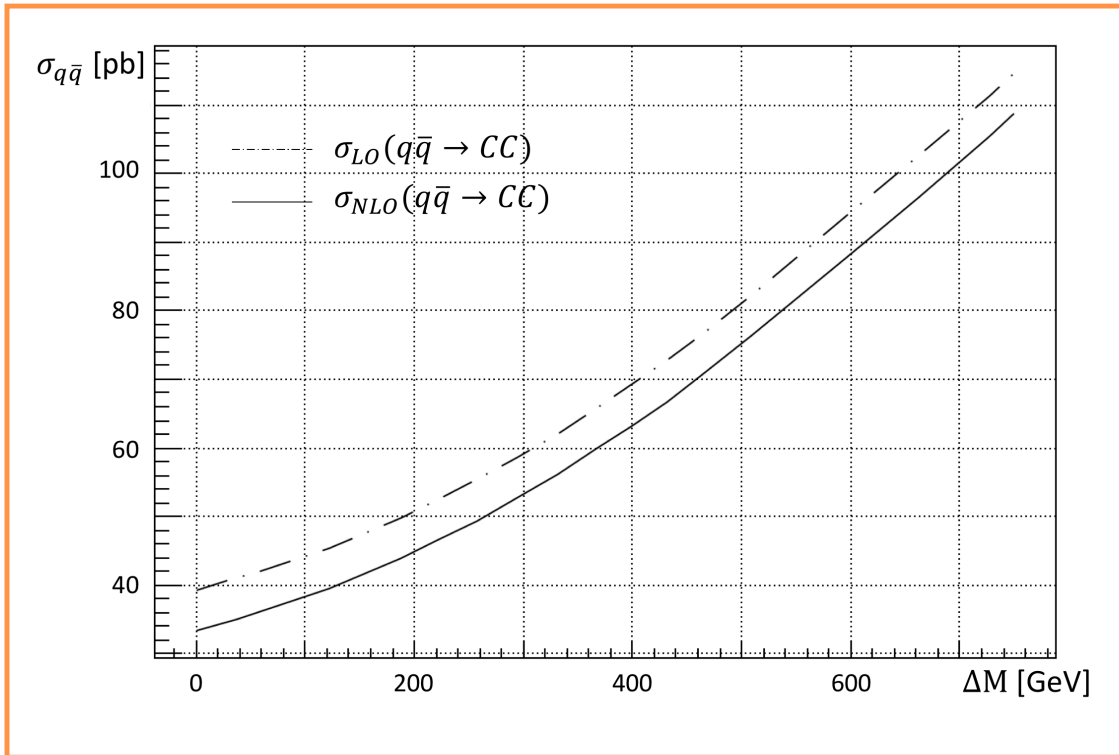
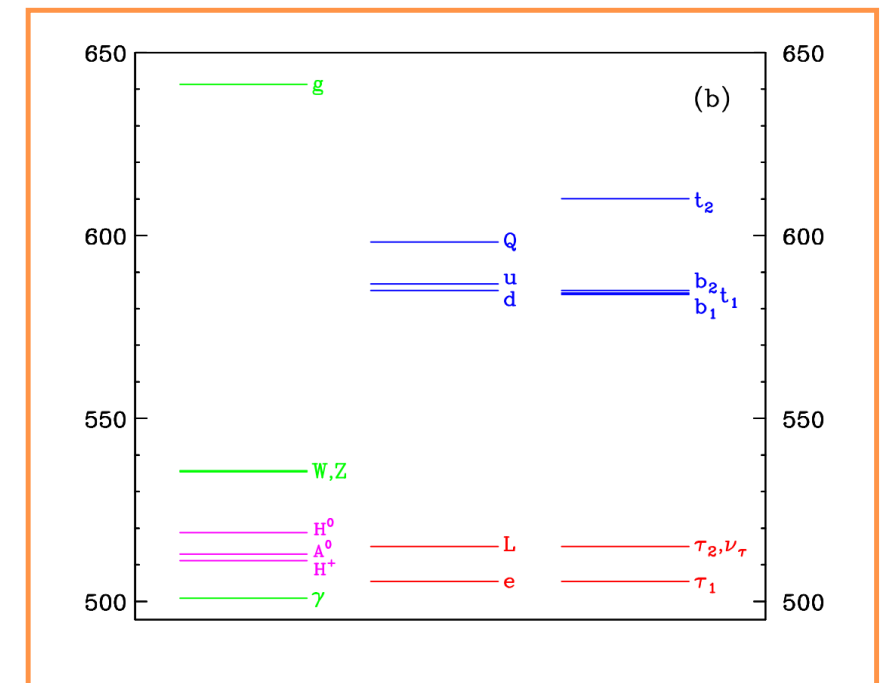


Fig.14 Quark Channel dependence on the splitting  $\Delta M$

Fig.15 Corrected mass spectrum



The degenerate modes also receive radiative corrections in UED  
 $\leftrightarrow$  All masses are independent in Coloron Models anyways

$$\Delta M = \delta M_C - \delta M_Q = \frac{\alpha_s}{4\pi} M^2 \left[ -\frac{3}{2} \frac{\xi(3)}{\pi} + \frac{17}{2} \ln \left( \frac{\Lambda^2}{\mu^2} \right) \right]^{1)}$$

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# ...and now what?

We amended the `FeynCalc/FeynArts` package to properly include QCD corrections with dimensionally regularized Soft/Collinear divergencies

## Pro

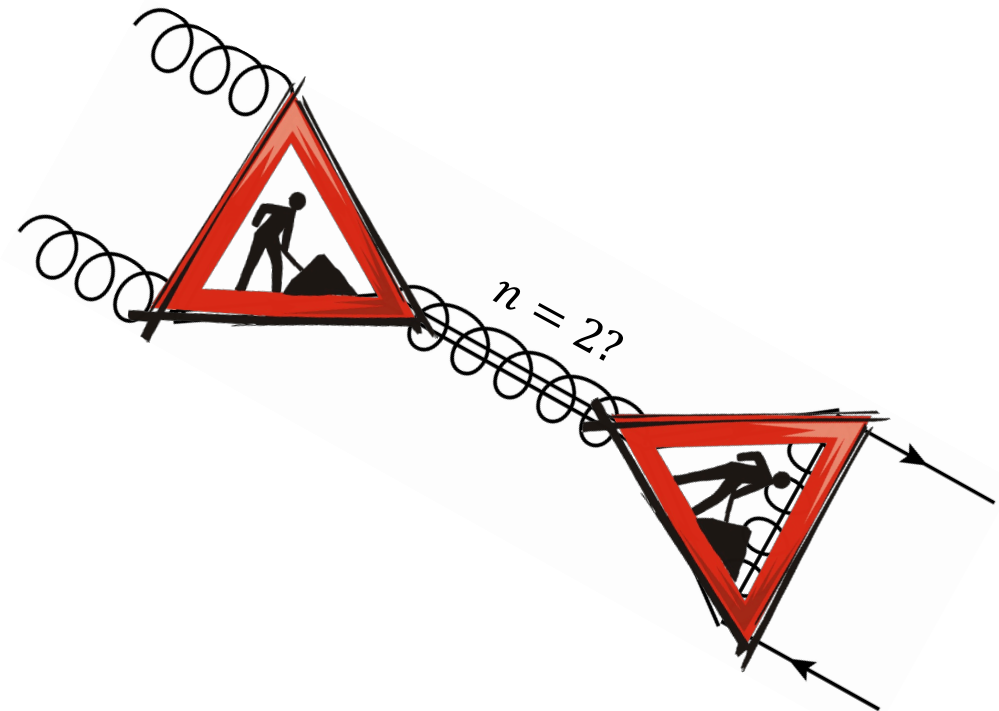
- Very modular procedure for different models and different processes

## Con

- Time intensive ME for large number of diagrams/external legs

## Follow-up/Work in Progress:

- Signature/Coloron decay? Update Limits?
- Understand higher modes/UV sensitivity?



Thanks!