

A new regularization scheme from the loop-tree duality

Germán Rodrigo



EXCELENCIA
SEVERO
OCHOA



CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

VNIVERSITAT
ID VALÈNCIA

LOOPFEST XVI

Radiative Corrections for the
LHC and Future Colliders

May 31 - June 2, 2017
Argonne National Laboratory



- S. Catani, T. Gleisberg, F. Krauss, G. Rodrigo and J. C. Winter, “*From loops to trees by-passing Feynman’s theorem*,” JHEP **0809** (2008) 065 [arXiv:0804.3170 [hep-ph]].
- I. Bierenbaum, S. Catani, P. Draggiotis and G. Rodrigo, “*A Tree-Loop Duality Relation at Two Loops and Beyond*,” JHEP **1010** (2010) 073 [arXiv:1007.0194 [hep-ph]].
- I. Bierenbaum, S. Buchta, P. Draggiotis, I. Malamos and G. Rodrigo, “*Tree-Loop Duality Relation beyond simple poles*,” JHEP **1303** (2013) 025 [arXiv:1211.5048 [hep-ph]].
- S. Buchta, G. Chachamis, P. Draggiotis, I. Malamos and G. Rodrigo, “*On the singular behaviour of scattering amplitudes in quantum field theory*,” JHEP **1411** (2014) 014 [arXiv:1405.7850 [hep-ph]].
- S. Buchta, “*Theoretical foundations and applications of the Loop-Tree Duality in Quantum Field Theories*,” **PhD thesis**, Universitat de València, 2015, arXiv:1509.07167 [hep-ph].
- S. Buchta, G. Chachamis, P. Draggiotis and G. Rodrigo, “*Numerical implementation of the Loop-Tree Duality method*,” EPJC **77** (2017) 274 [arXiv:1510.00187 [hep-ph]].
- R. J. Hernández-Pinto, G. F. R. Sborlini and G. Rodrigo, “***Towards gauge theories in four dimensions***,” JHEP **1602** (2016) 044 [arXiv:1506.04617 [hep-ph]].
- G. F. R. Sborlini, F. Driencourt-Mangin, J. Hernández-Pinto and G. Rodrigo, “***Four dimensional unsubtraction from the loop-tree duality***,” JHEP **1608** (2016) 160 [arXiv:1604.06699 [hep-ph]].
- G. F. R. Sborlini, F. Driencourt-Mangin and G. Rodrigo, “***Four dimensional unsubtraction with massive particles***,” JHEP **1610** (2016) 162 [arXiv:1608.01584 [hep-ph]].
- F. Driencourt-Mangin, G. Rodrigo and G.F.R. Sborlini, “***Universal dual amplitudes and asymptotic expansions for $gg \rightarrow H$ and $H \rightarrow \gamma \gamma$*** ,” arXiv:1702.07581 [hep-ph].

QFT is poorly defined

QFT is poorly defined

1. QFT extrapolated to **infinite energy** in loop corrections

QFT is poorly defined

1. QFT extrapolated to **infinite energy** in loop corrections
2. particles with **zero energy** \neq **zero emission**

QFT is poorly defined

1. QFT extrapolated to **infinite energy** in loop corrections
2. particles with **zero energy** \neq **zero emission**
3. **Parallel** particles look like one single particle

QFT is poorly defined

1. QFT extrapolated to **infinite energy** in loop corrections
2. particles with **zero energy** \neq **zero emission**
3. **Parallel** particles look like one single particle



in **four** space-time dimensions

QFT is poorly defined

1. QFT extrapolated to **infinite energy** in loop corrections
2. particles with **zero energy** \neq **zero emission**
3. **Parallel** particles look like one single particle

Ultraviolet singularities (UV)



in **four** space-time dimensions

QFT is poorly defined

1. QFT extrapolated to **infinite energy** in loop corrections
2. particles with **zero energy** \neq **zero emission**
3. **Parallel** particles look like one single particle

soft singularities (IR)

Ultraviolet singularities (UV)



in **four** space-time dimensions

QFT is poorly defined

1. QFT extrapolated to **infinite energy** in loop corrections
2. particles with **zero energy** \neq **zero emission**
3. **Parallel** particles look like one single particle

soft singularities (IR)

collinear singularities (IR)

Ultraviolet singularities (UV)



in **four** space-time dimensions

QFT is poorly defined

1. QFT extrapolated to **infinite energy** in loop corrections
2. particles with **zero energy** \neq **zero emission**
3. **Parallel** particles look like one single particle

soft singularities (IR)

collinear singularities (IR)

Ultraviolet singularities (UV)

and **threshold** singularities,
integrable but numerically unstable



in **four** space-time dimensions

DREG

- Modify the dimensions of the space-time to **$d = 4-2e$**

LTD / FDU

DREG	LTD / FDU
<ul style="list-style-type: none">■ Modify the dimensions of the space-time to $d = 4 - 2\epsilon$	<ul style="list-style-type: none">■ Computations without altering the $d=4$ space-time dimensions¹

¹ Gnendiger et al., *To d , or not to d : Recent developments and comparisons of regularization schemes*, arXiv:1705.01827

DREG	LTD / FDU
<ul style="list-style-type: none"> ■ Modify the dimensions of the space-time to $d = 4 - 2\epsilon$ 	<ul style="list-style-type: none"> ■ Computations without altering the $d=4$ space-time dimensions¹
<ul style="list-style-type: none"> ■ Singularities manifest after integration as $1/\epsilon$ poles: <ul style="list-style-type: none"> ■ IR cancelled through suitable subtraction terms, which need to be integrated over the unresolved phase-space ■ UV renormalized 	

¹ Gnendiger et al., *To d , or not to d : Recent developments and comparisons of regularization schemes*, arXiv:1705.01827

DREG	LTD / FDU
<ul style="list-style-type: none"> ■ Modify the dimensions of the space-time to $d = 4 - 2\epsilon$ 	<ul style="list-style-type: none"> ■ Computations without altering the $d=4$ space-time dimensions¹
<ul style="list-style-type: none"> ■ Singularities manifest after integration as $1/\epsilon$ poles: <ul style="list-style-type: none"> ■ IR cancelled through suitable subtraction terms, which need to be integrated over the unresolved phase-space ■ UV renormalized 	<ul style="list-style-type: none"> ■ Singularities killed before integration: <ul style="list-style-type: none"> ■ Unsubtracted summation over degenerate IR states at integrand level through a suitable momentum mapping ■ UV through local counter-terms

¹ Gnendiger et al., *To d , or not to d : Recent developments and comparisons of regularization schemes*, arXiv:1705.01827

DREG	LTD / FDU
<ul style="list-style-type: none"> ■ Modify the dimensions of the space-time to $d = 4 - 2\epsilon$ 	<ul style="list-style-type: none"> ■ Computations without altering the $d=4$ space-time dimensions¹
<ul style="list-style-type: none"> ■ Singularities manifest after integration as $1/\epsilon$ poles: <ul style="list-style-type: none"> ■ IR cancelled through suitable subtraction terms, which need to be integrated over the unresolved phase-space ■ UV renormalized 	<ul style="list-style-type: none"> ■ Singularities killed before integration: <ul style="list-style-type: none"> ■ Unsubtracted summation over degenerate IR states at integrand level through a suitable momentum mapping ■ UV through local counter-terms
<ul style="list-style-type: none"> ■ Virtual and real contributions are considered separately: phase-space with different number of final-state particles 	

¹ Gnendiger et al., *To d , or not to d : Recent developments and comparisons of regularization schemes*, arXiv:1705.01827

DREG	LTD / FDU
<ul style="list-style-type: none"> Modify the dimensions of the space-time to $d = 4 - 2\epsilon$ 	<ul style="list-style-type: none"> Computations without altering the $d=4$ space-time dimensions¹
<ul style="list-style-type: none"> Singularities manifest after integration as $1/\epsilon$ poles: <ul style="list-style-type: none"> IR cancelled through suitable subtraction terms, which need to be integrated over the unresolved phase-space UV renormalized 	<ul style="list-style-type: none"> Singularities killed before integration: <ul style="list-style-type: none"> Unsubtracted summation over degenerate IR states at integrand level through a suitable momentum mapping UV through local counter-terms
<ul style="list-style-type: none"> Virtual and real contributions are considered separately: phase-space with different number of final-state particles 	<ul style="list-style-type: none"> Virtual and real contributions are considered simultaneously: more efficient Monte Carlo implementation and fully differential

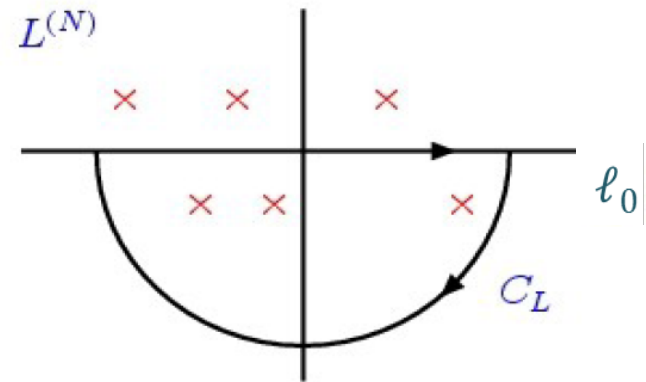
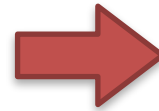
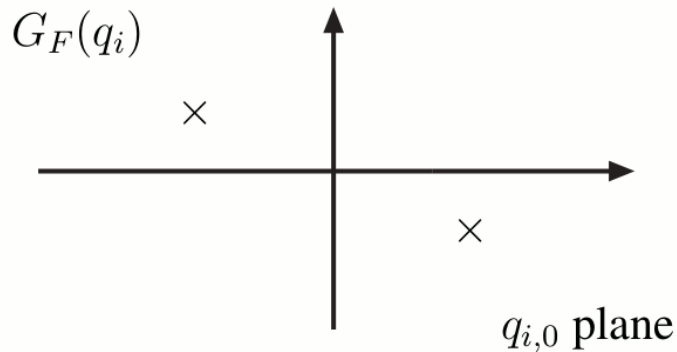
¹ Gnendiger et al., *To d , or not to d : Recent developments and comparisons of regularization schemes*, arXiv:1705.01827

The loop-tree duality theorem

[Catani et al. 2008]

Cauchy residue theorem

in the loop energy complex plane



Feynman Propagator $+i0$:

positive frequencies are propagated forward in time, and negative backward

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0}$$

selects residues with definite **positive energy and negative imaginary part** (indeed in any coordinate system)

The loop-tree duality theorem

[Catani et al. 2008]

One-loop integrals (or scattering amplitudes in any relativistic, local and unitary QFT) represented as a linear combination of N **single-cut phase-space** integrals

$$\int_{\ell} \prod G_F(q_i) = - \sum \int_{\ell} \tilde{\delta}(q_i) \prod_{j \neq i} G_D(q_i; q_j)$$

- $\tilde{\delta}(q_i) = i 2\pi \theta(q_{i,0}) \delta(q_i^2 - m_i^2)$ sets internal line on-shell, positive energy mode
- $G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0 \eta k_{ji}}$ **dual propagator**, $k_{ji} = q_j - q_i$

The loop-tree duality theorem

[Catani et al. 2008]

One-loop integrals (or scattering amplitudes in any relativistic, local and unitary QFT) represented as a linear combination of N **single-cut phase-space** integrals

$$\int_{\ell} \prod G_F(q_i) = - \sum \int_{\ell} \tilde{\delta}(q_i) \prod_{j \neq i} G_D(q_i; q_j)$$

- $\tilde{\delta}(q_i) = i 2\pi \theta(q_{i,0}) \delta(q_i^2 - m_i^2)$ sets internal line on-shell, positive energy mode
- $G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0 \eta k_{ji}}$ **dual propagator**, $k_{ji} = q_j - q_i$
- LTD realised by **modifying the customary +i0 prescription** of the Feynman propagators, it compensates for the absence of **multiple-cut** contributions that appear in the **Feynman Tree Theorem**

The loop-tree duality theorem

[Catani et al. 2008]

One-loop integrals (or scattering amplitudes in any relativistic, local and unitary QFT) represented as a linear combination of N **single-cut phase-space** integrals

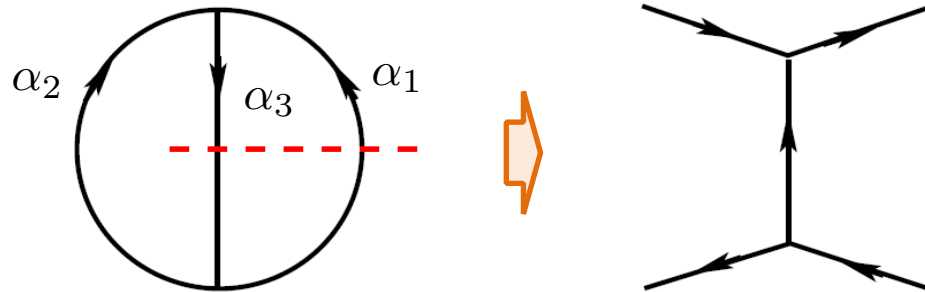
$$\int_{\ell} \prod G_F(q_i) = - \sum \int_{\ell} \tilde{\delta}(q_i) \prod_{j \neq i} G_D(q_i; q_j)$$

- $\tilde{\delta}(q_i) = i 2\pi \theta(q_{i,0}) \delta(q_i^2 - m_i^2)$ sets internal line on-shell, positive energy mode
- $G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0 \eta k_{ji}}$ **dual propagator**, $k_{ji} = q_j - q_i$
- LTD realised by **modifying the customary +i0 prescription** of the Feynman propagators, it compensates for the absence of **multiple-cut** contributions that appear in the **Feynman Tree Theorem**
- **Lorentz-covariant dual prescription** with η a **future-like** vector; from now $\eta^\mu = (1, \mathbf{0})$ only the **sign** matters

LTD at two-loops and beyond

- Iterative application of LTD at higher orders

$$\int_{\ell_1} \int_{\ell_2} G_F(\alpha_1 \cup \alpha_2 \cup \alpha_3) = \int_{\ell_1} \int_{\ell_2} [G_D(\alpha_1)G_D(\alpha_2 \cup \alpha_3) + G_D(-\alpha_1 \cup \alpha_2)G_D(\alpha_3) - G_D(\alpha_1)G_F(\alpha_2)G_D(\alpha_3)]$$



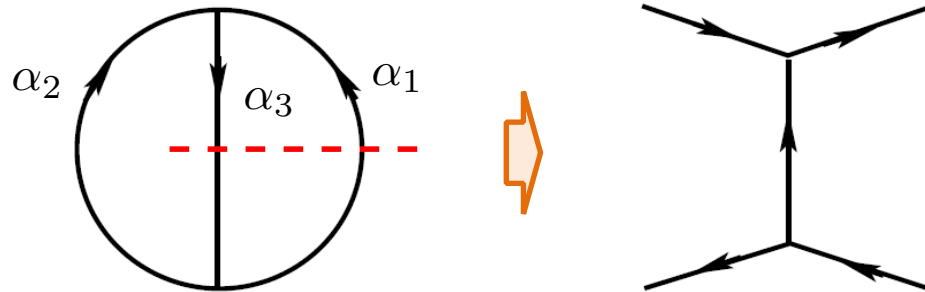
$$G_F(\alpha_k) = \sum_{i \in \alpha_k} G_F(q_i) , \quad G_D(\alpha_k) = \sum_{i \in \alpha_k} \tilde{\delta}(q_i) \prod_{j \neq i} G_D(q_i; q_j) ,$$

- With a **number of cuts equal to the number of loops** the loop amplitude opens to a tree-level like object

LTD at two-loops and beyond

- Iterative application of LTD at higher orders

$$\int_{\ell_1} \int_{\ell_2} G_F(\alpha_1 \cup \alpha_2 \cup \alpha_3) = \int_{\ell_1} \int_{\ell_2} [G_D(\alpha_1)G_D(\alpha_2 \cup \alpha_3) + G_D(-\alpha_1 \cup \alpha_2)G_D(\alpha_3) - G_D(\alpha_1)G_F(\alpha_2)G_D(\alpha_3)]$$

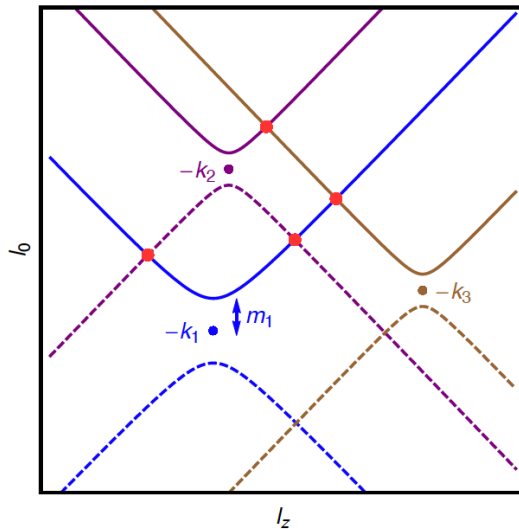


$$G_F(\alpha_k) = \sum_{i \in \alpha_k} G_F(q_i) , \quad G_D(\alpha_k) = \sum_{i \in \alpha_k} \tilde{\delta}(q_i) \prod_{j \neq i} G_D(q_i; q_j) ,$$

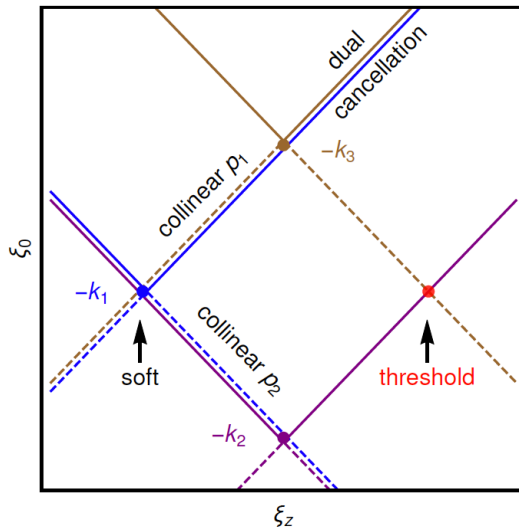
- With a **number of cuts equal to the number of loops** the loop amplitude opens to a tree-level like object
- However, the on-shell loop momenta still **unconstrained**

Singularities of the loop integrand

Buchta et al, arXiv:1405.7850

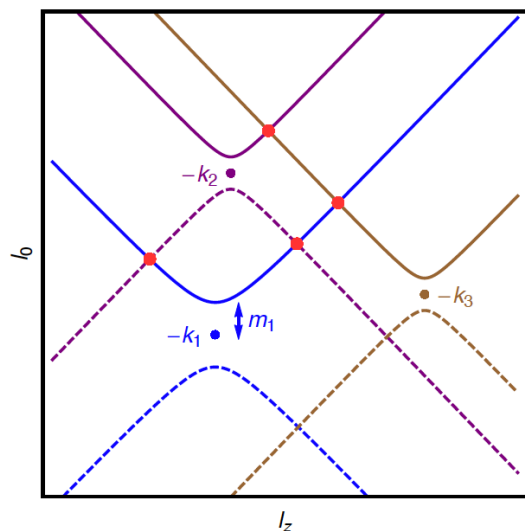


- **LTD:** equivalent to integrate along the **forward** on-shell hyperboloids / light-cones (positive energy modes)

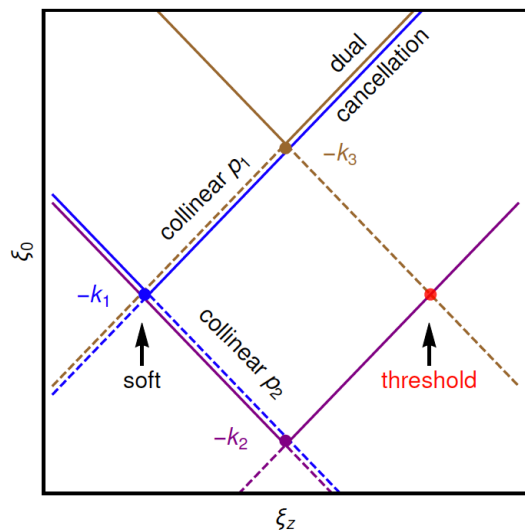


Singularities of the loop integrand

Buchta et al, arXiv:1405.7850

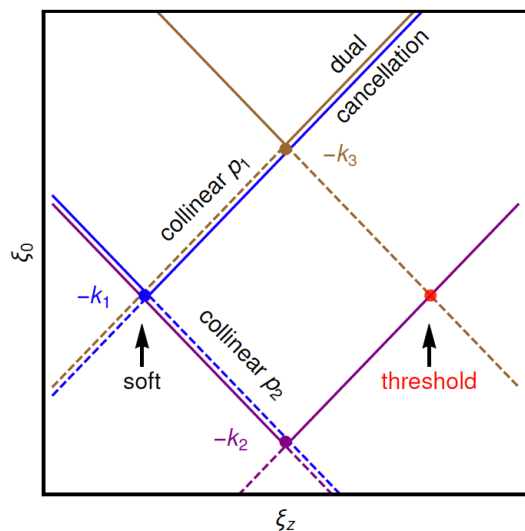
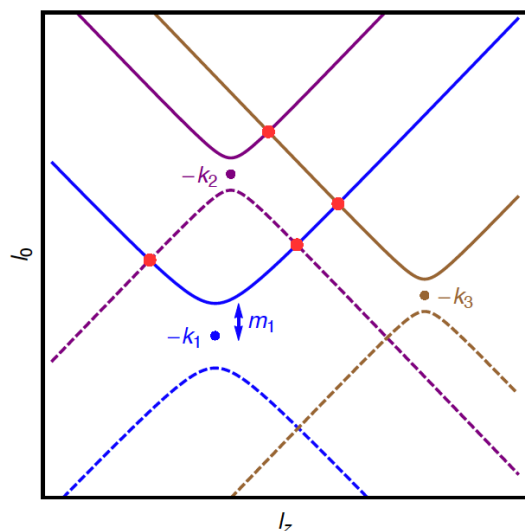


- **LTD:** equivalent to integrate along the **forward** on-shell hyperboloids / light-cones (positive energy modes)
- The dual loop integrand becomes singular when subsets (≥ 2) of internal propagators go on-shell



Singularities of the loop integrand

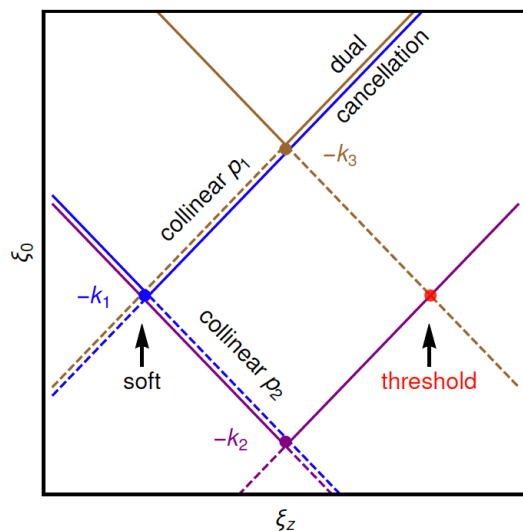
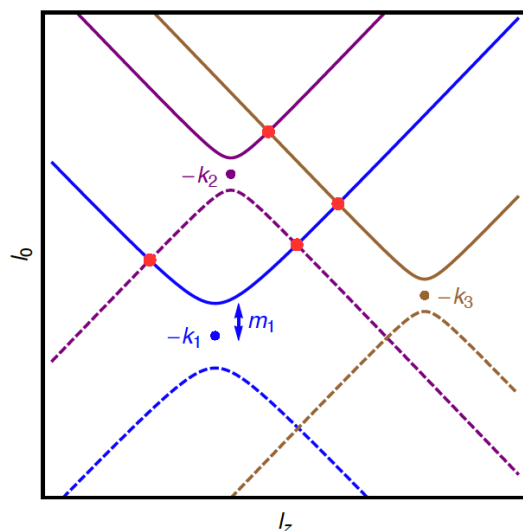
Buchta et al, arXiv:1405.7850



- **LTD**: equivalent to integrate along the **forward** on-shell hyperboloids / light-cones (positive energy modes)
- The dual loop integrand becomes singular when subsets (≥ 2) of internal propagators go on-shell
- **Cancellation** of singularities among dual amplitudes at **forward-forward intersections**: dual $+i0$ prescription changes sign, proof of consistency

Singularities of the loop integrand

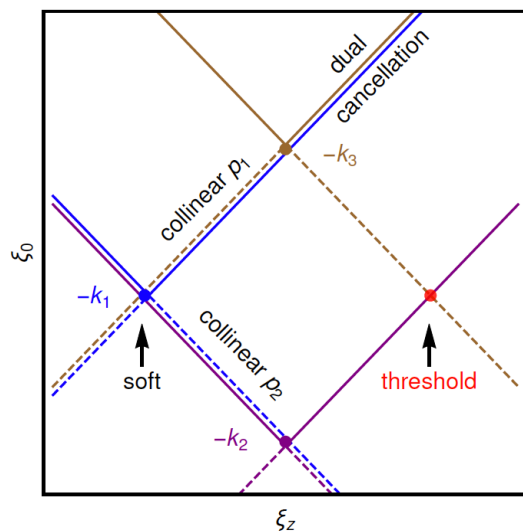
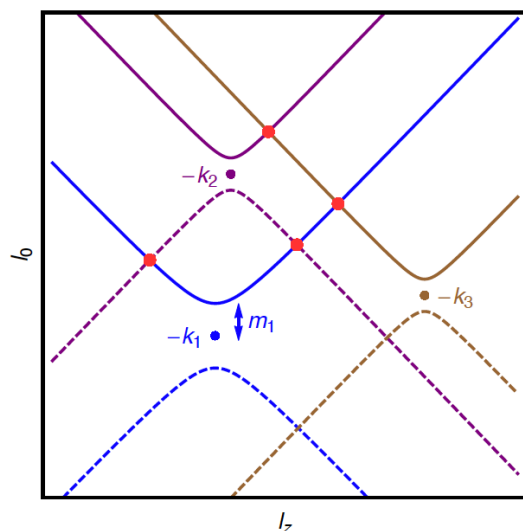
Buchta et al, arXiv:1405.7850



- **LTD**: equivalent to integrate along the **forward** on-shell hyperboloids / light-cones (positive energy modes)
- The dual loop integrand becomes singular when subsets (≥ 2) of internal propagators go on-shell
- **Cancellation** of singularities among dual amplitudes at **forward-forward intersections**: dual $+i0$ prescription changes sign, proof of consistency
- Only backward (negative energy) with forward IR and threshold singularities remain: **time-like** separated propagators with lower energy **causally connected**

Singularities of the loop integrand

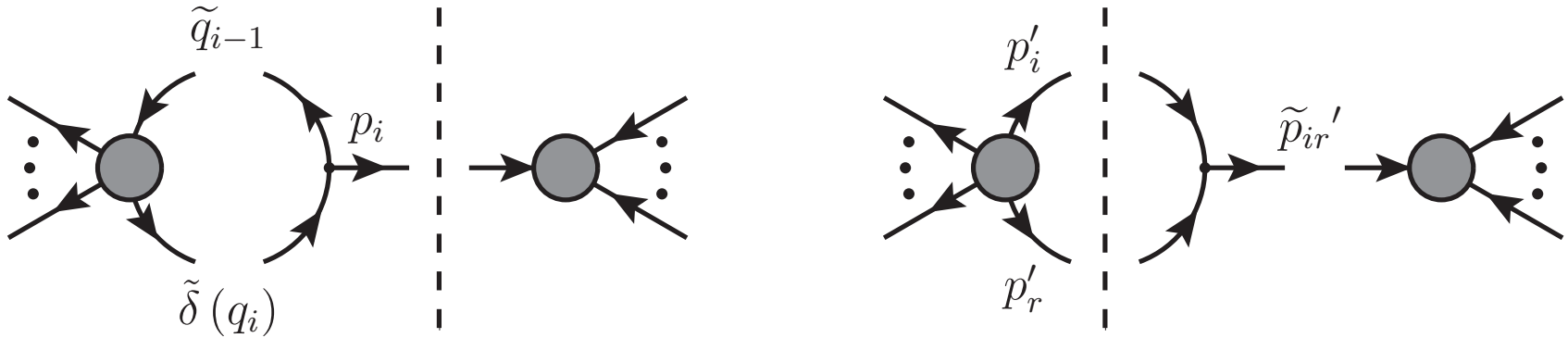
Buchta et al, arXiv:1405.7850



- **LTD**: equivalent to integrate along the **forward** on-shell hyperboloids / light-cones (positive energy modes)
- The dual loop integrand becomes singular when subsets (≥ 2) of internal propagators go on-shell
- **Cancellation** of singularities among dual amplitudes at **forward-forward intersections**: dual $+i0$ prescription changes sign, proof of consistency
- Only backward (negative energy) with forward IR and threshold singularities remain: **time-like** separated propagators with lower energy **causally connected**

IR and threshold singularities are restricted to a **compact region** of the loop three-momentum

Momentum mapping



- Motivated by the **factorization properties of QCD**: assuming q_i^μ on-shell, and close to collinear with p_i^μ , we define the momentum mapping

$$\begin{aligned}
 p_r'^\mu &= q_i^\mu, & q_{i,0} &< p_{i,0} \\
 p_i'^\mu &= p_i^\mu - q_i^\mu + \alpha_i p_j^\mu, & \alpha_i &= \frac{(q_i - p_i)^2}{2p_j \cdot (q_i - p_i)}, \\
 p_j'^\mu &= (1 - \alpha_i) p_j^\mu, & p_k'^\mu &= p_k^\mu, \quad k \neq i, j
 \end{aligned}$$

- All the primed momenta (real process) **on-shell and momentum conservation**
- $p_i'^\mu$ is the **emitter**, $p_j'^\mu$ the **spectator** needed to absorb momentum recoil

Massive particles

Sborlini, Driencourt-Mangin,GR, arXiv:**1608.01584**

- Rewrite **emitter** and **spectator** in terms of two massless momenta

$$p_i^\mu = \beta_+ \hat{p}_i^\mu + \beta_- \hat{p}_j^\mu$$

$$p_j^\mu = (1 - \beta_+) \hat{p}_i^\mu + (1 - \beta_-) \hat{p}_j^\mu \quad \hat{p}_i^\mu + \hat{p}_j^\mu = p_i^\mu + p_j^\mu$$

- Mapping and phase-space partition formally equal to the massless case:
determine **mapping parameters from on-shell conditions**

$$p_r'^\mu = q_i^\mu ,$$

$$p_i'^\mu = (1 - \alpha_i) \hat{p}_i^\mu + (1 - \gamma_i) \hat{p}_j^\mu - q_i^\mu ,$$

$$p_j'^\mu = \alpha_i \hat{p}_i^\mu + \gamma_i \hat{p}_j^\mu , \quad p_k'^\mu = p_k^\mu , \quad k \neq i, j$$

Massive particles

Sborlini, Driencourt-Mangin,GR, arXiv:**1608.01584**

- Rewrite **emitter** and **spectator** in terms of two massless momenta

$$p_i^\mu = \beta_+ \hat{p}_i^\mu + \beta_- \hat{p}_j^\mu$$

$$p_j^\mu = (1 - \beta_+) \hat{p}_i^\mu + (1 - \beta_-) \hat{p}_j^\mu \quad \hat{p}_i^\mu + \hat{p}_j^\mu = p_i^\mu + p_j^\mu$$

- Mapping and phase-space partition formally equal to the massless case:
determine **mapping parameters from on-shell conditions**

$$p_r'^\mu = q_i^\mu ,$$

$$p_i'^\mu = (1 - \alpha_i) \hat{p}_i^\mu + (1 - \gamma_i) \hat{p}_j^\mu - q_i^\mu ,$$

$$p_j'^\mu = \alpha_i \hat{p}_i^\mu + \gamma_i \hat{p}_j^\mu , \quad p_k'^\mu = p_k^\mu , \quad k \neq i, j$$

- **Quasi-collinear configurations** are conveniently mapped such that the massless limit is smooth

UV renormalisation: local subtraction

- Expand propagators and numerators around a UV propagator [Weinzierl et al.]

$$G_F(q_i) = \frac{1}{q_{UV}^2 - \mu_{UV}^2 + i0} + \dots \quad q_{UV} = \ell + k_{UV}$$

- and adjust **subleading** terms to subtract only the pole ($\overline{\text{MS}}$ **scheme**), or to define any other renormalisation scheme. For the scalar two point function

$$I_{UV}^{\text{cnt}} = \int_{\ell} \frac{1}{(q_{UV}^2 - \mu_{UV}^2 + i0)^2}$$

UV renormalisation: local subtraction

- Expand propagators and numerators around a UV propagator [Weinzierl et al.]

$$G_F(q_i) = \frac{1}{q_{UV}^2 - \mu_{UV}^2 + i0} + \dots \quad q_{UV} = \ell + k_{UV}$$

- and adjust **subleading** terms to subtract only the pole ($\overline{\text{MS}}$ **scheme**), or to define any other renormalisation scheme. For the scalar two point function

$$I_{UV}^{\text{cnt}} = \int_{\ell} \frac{1}{(q_{UV}^2 - \mu_{UV}^2 + i0)^2}$$

- Dual representation needs to deal with **multiple poles** [Bierenbaum et al.]

$$I_{UV}^{\text{cnt}} = \int_{\ell} \frac{\tilde{\delta}(q_{UV})}{2 \left(q_{UV,0}^{(+)} \right)^2}$$
$$q_{UV,0}^{(+)} = \sqrt{\mathbf{q}_{UV}^2 + \mu_{UV}^2 - i0}$$

Hernández-Pinto, Sborlini, GR, arXiv:1506.04617

UV renormalisation: local subtraction

- Expand propagators and numerators around a UV propagator [Weinzierl et al.]

$$G_F(q_i) = \frac{1}{q_{UV}^2 - \mu_{UV}^2 + i0} + \dots \quad q_{UV} = \ell + k_{UV}$$

- and adjust **subleading** terms to subtract only the pole ($\overline{\text{MS}}$ **scheme**), or to define any other renormalisation scheme. For the scalar two point function

$$I_{UV}^{\text{cnt}} = \int_{\ell} \frac{1}{(q_{UV}^2 - \mu_{UV}^2 + i0)^2}$$

- Dual representation needs to deal with **multiple poles** [Bierenbaum et al.]

$$I_{UV}^{\text{cnt}} = \int_{\ell} \frac{\tilde{\delta}(q_{UV})}{2 \left(q_{UV,0}^{(+)} \right)^2}$$
$$q_{UV,0}^{(+)} = \sqrt{\mathbf{q}_{UV}^2 + \mu_{UV}^2 - i0}$$

Hernández-Pinto, Sborlini, GR, arXiv:1506.04617

- Integration on the UV on-shell hyperboloid: loop three-momentum unconstrained, but **loop contributions suppressed** for loop energies larger than μ_{UV}

Self-energy corrections

- **Wave function** corrections usually **ignored for massless partons**, but they feature non-trivial IR/UV behaviour, **required to disentangle both regions**, indeed necessary to map the squares of the real amplitudes in the IR

Self-energy corrections

- **Wave function** corrections usually **ignored for massless partons**, but they feature non-trivial IR/UV behaviour, **required to disentangle both regions**, indeed necessary to map the squares of the real amplitudes in the IR

- **Unintegrated wave-function and mass renormalisation: e.g. quarks**

$$\Delta Z_2(p_1) = -g_S^2 C_F \int_{\ell} G_F(q_1) G_F(q_3) \left((d-2) \frac{q_1 \cdot p_2}{p_1 \cdot p_2} + 4M^2 \left(1 - \frac{q_1 \cdot p_2}{p_1 \cdot p_2} \right) G_F(q_3) \right)$$
$$\Delta Z_M^{\text{OS}}(p_1) = -g_S^2 C_F \int_{\ell} G_F(q_1) G_F(q_3) \left((d-2) \frac{q_1 \cdot p_2}{p_1 \cdot p_2} + 2 \right)$$

- smooth massless limit

Self-energy corrections

- **Wave function** corrections usually **ignored for massless partons**, but they feature non-trivial IR/UV behaviour, **required to disentangle both regions**, indeed necessary to map the squares of the real amplitudes in the IR
- **Unintegrated wave-function and mass renormalisation: e.g. quarks**

$$\Delta Z_2(p_1) = -g_S^2 C_F \int_{\ell} G_F(q_1) G_F(q_3) \left((d-2) \frac{q_1 \cdot p_2}{p_1 \cdot p_2} + 4M^2 \left(1 - \frac{q_1 \cdot p_2}{p_1 \cdot p_2} \right) G_F(q_3) \right)$$
$$\Delta Z_M^{\text{OS}}(p_1) = -g_S^2 C_F \int_{\ell} G_F(q_1) G_F(q_3) \left((d-2) \frac{q_1 \cdot p_2}{p_1 \cdot p_2} + 2 \right)$$

- smooth massless limit
- and **subtract the UV**

$$\Delta Z_2^{\text{UV}}(p_1) = -(d-2)g_S^2 C_F \int_{\ell} (G_F(q_{\text{UV}}))^2 \left(1 + \frac{q_{\text{UV}} \cdot p_2}{p_1 \cdot p_2} \right) \times (1 - G_F(q_{\text{UV}})(2q_{\text{UV}} \cdot p_1 + \mu_{\text{UV}}^2))$$

LTD unsubtraction: multi-leg

Sborlini, Driencourt-Mangin, Hernández-Pinto, GR, arXiv: **1604.06699**

- The **dual representation** of the renormalised loop cross-section: one single integral in the loop three-momentum

$$\int_m d\sigma_V^{(1,R)} = \sum_{i=1}^N \int_m \int_\ell 2 \operatorname{Re} \langle \mathcal{M}_N^{(0)} | \mathcal{M}_N^{(1,R)}(\tilde{\delta}(q_i)) \rangle \mathcal{O}_N(\{p_j\})$$

LTD unsubtraction: multi-leg

Sborlini, Driencourt-Mangin, Hernández-Pinto, GR, arXiv: **1604.06699**

- The **dual representation** of the renormalised loop cross-section: one single integral in the loop three-momentum

$$\int_m d\sigma_V^{(1,R)} = \sum_{i=1}^N \int_m \int_\ell 2 \operatorname{Re} \langle \mathcal{M}_N^{(0)} | \mathcal{M}_N^{(1,R)}(\tilde{\delta}(q_i)) \rangle \mathcal{O}_N(\{p_j\})$$

- A **partition** of the real phase-space

$$\sum \mathcal{R}_i(q_i, p_i) = \sum \prod_{jk \neq ir} \theta(y'_{jk} - y'_{ir}) = 1$$

LTD unsubtraction: multi-leg

Sborlini, Driencourt-Mangin, Hernández-Pinto, GR, arXiv: **1604.06699**

- The **dual representation** of the renormalised loop cross-section: one single integral in the loop three-momentum

$$\int_m d\sigma_V^{(1,R)} = \sum_{i=1}^N \int_m \int_\ell 2 \operatorname{Re} \langle \mathcal{M}_N^{(0)} | \mathcal{M}_N^{(1,R)}(\tilde{\delta}(q_i)) \rangle \mathcal{O}_N(\{p_j\})$$

- A **partition** of the real phase-space

$$\sum \mathcal{R}_i(q_i, p_i) = \sum \prod_{jk \neq ir} \theta(y'_{jk} - y'_{ir}) = 1$$

- The real contribution **mapped** to the **Born kinematics + loop three-momentum**

$$\int_{m+1} d\sigma_R^{(1)} = \sum_{i=1}^N \int_{m+1} |\mathcal{M}_{N+1}^{(0)}(q_i, p_i)|^2 \mathcal{R}_i(q_i, p_i) \mathcal{O}_{N+1}(\{p'_j\})$$

- with

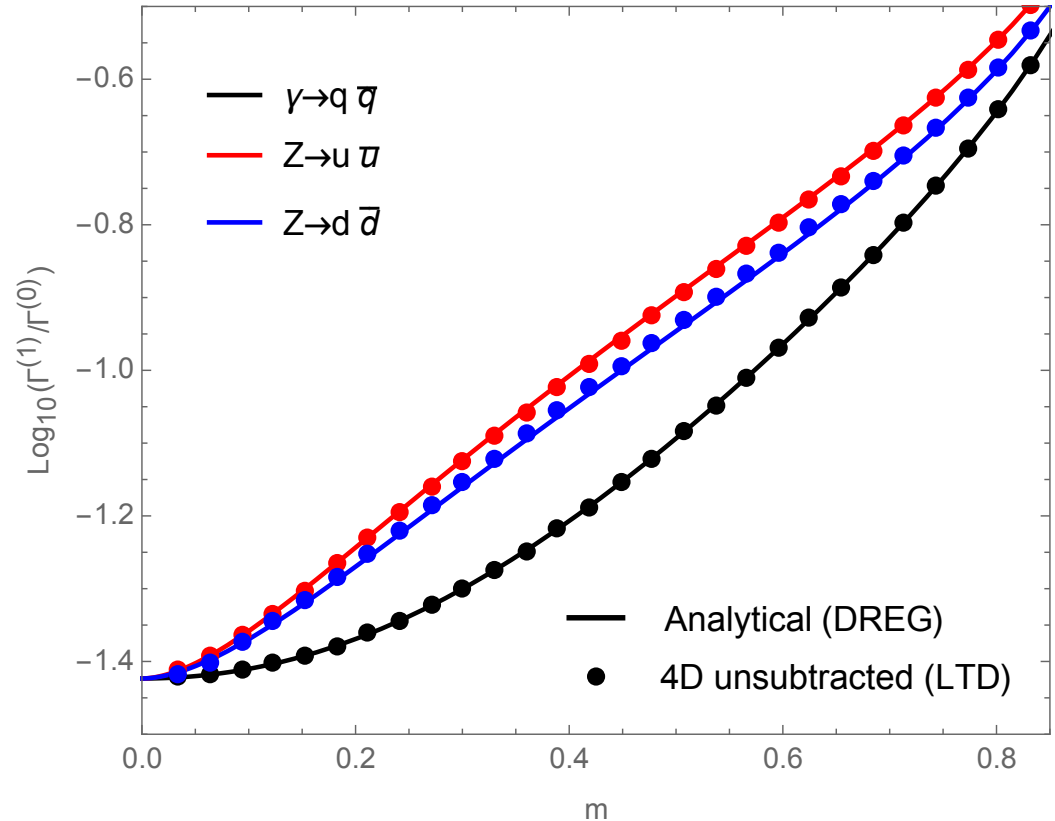
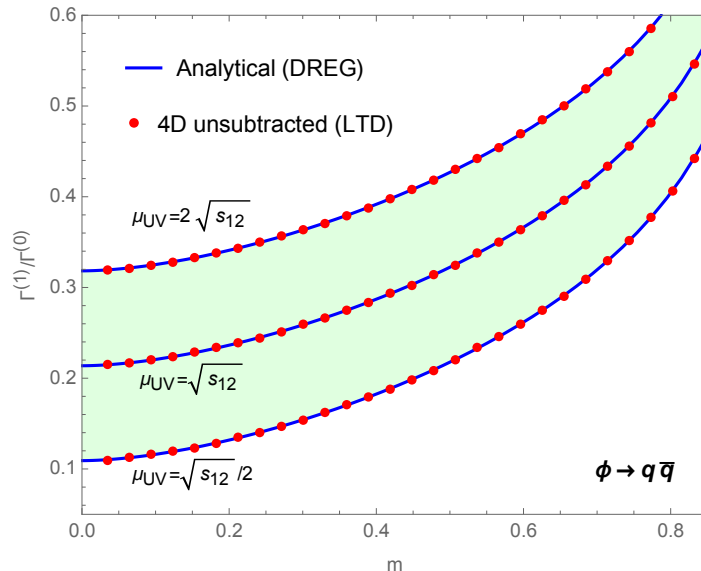
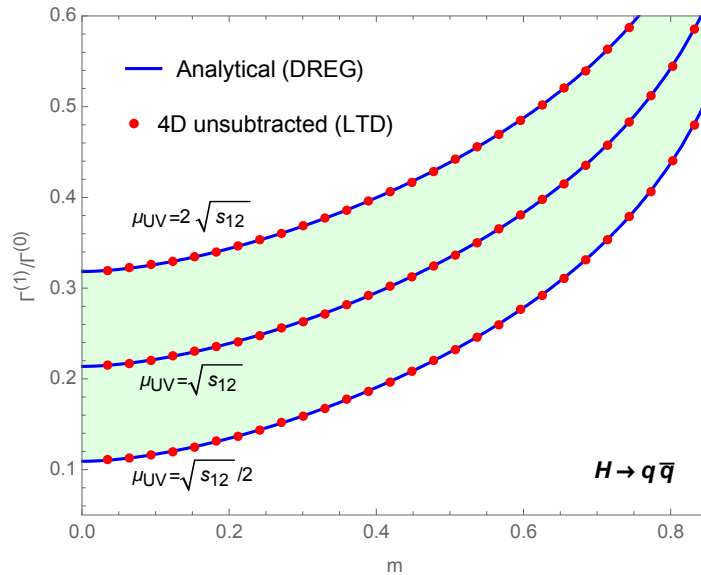
$$p_r'^\mu = q_i^\mu ,$$

$$p_i'^\mu = p_i^\mu - q_i^\mu + \alpha_i p_j^\mu , \quad \alpha_i = \frac{(q_i - p_i)^2}{2p_j \cdot (q_i - p_i)} ,$$

$$p_j'^\mu = (1 - \alpha_i) p_j^\mu , \quad p_k'^\mu = p_k^\mu , \quad k \neq i, j$$

Benchmark application: $A^* \rightarrow q\bar{q}(g)$

Sborlini, Driencourt-Mangin, GR, arXiv:**1608.01584**



- Excellent agreement with analytic DREG
- Efficient numerical implementation
- Smooth massless limit

Higgs boson interactions to gg and $\gamma\gamma$

Driencourt-Mangin,GR, Sborlini, arXiv:**1702.07581**

- **Golden channels** for production and decay of the Higgs boson

Higgs boson interactions to gg and $\gamma\gamma$

Driencourt-Mangin,GR, Sborlini, arXiv:**1702.07581**

- **Golden channels** for production and decay of the Higgs boson
- One-loop corrections are UV and IR finite due to the **absence of a direct interaction** at tree-level in the SM

Higgs boson interactions to gg and $\gamma\gamma$

Driencourt-Mangin,GR, Sborlini, arXiv:**1702.07581**

- **Golden channels** for production and decay of the Higgs boson
- One-loop corrections are UV and IR finite due to the **absence of a direct interaction** at tree-level in the SM
- However, DREG or another **regularisation/renormalisation scheme still required** for their correct evaluation

Hgg [Wilczek, 1977; Georgi, Glashow, Machacek, Nanopoulos, 1978; Rizzo, 1980]

$H\gamma\gamma$ [Ellis, Gaillard, Nanopoulos, 1976; Ioffe, Khoze, 1978; Shifman, Vainshtein, Voloshin, Zakharov, 1979]

Summary Report of the Regularization Scheme Workstop/
Thinkstart, 13-16 Sep 2016, Zurich

Universality of dual amplitude

Driencourt-Mangin,GR, Sborlini, arXiv:**1702.07581**

● **Universality** and **compactness** of the dual representation

$$\mathcal{A}_1^{(1,f)} = g_f \int_{\ell} \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{2,0}^{(+)}} + \frac{2(2\ell \cdot p_{12})^2}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} \right) \frac{s_{12} M_f^2}{(2\ell \cdot p_1)(2\ell \cdot p_2)} c_1^{(f)} \right. \\ \left. + \frac{2s_{12}^2}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} c_{23}^{(f)} \right], \quad q_{i,0}^{(+)} = \sqrt{\mathbf{q}_i^2 + M_f^2}, \quad f = \phi, t, W \\ c_{23}^{(f)} = \frac{d-4}{d-2} \left(1, 4, 2(d-1) + \frac{s_{12}}{M_W^2} \right)$$

Universality of dual amplitude

Driencourt-Mangin,GR, Sborlini, arXiv:**1702.07581**

- **Universality** and **compactness** of the dual representation

$$\mathcal{A}_1^{(1,f)} = g_f \int_{\ell} \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{2,0}^{(+)}} + \frac{2(2\ell \cdot p_{12})^2}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} \right) \frac{s_{12} M_f^2}{(2\ell \cdot p_1)(2\ell \cdot p_2)} c_1^{(f)} \right. \\ \left. + \frac{2s_{12}^2}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} c_{23}^{(f)} \right], \quad q_{i,0}^{(+)} = \sqrt{\mathbf{q}_i^2 + M_f^2}, \quad f = \phi, t, W \\ c_{23}^{(f)} = \frac{d-4}{d-2} \left(1, 4, 2(d-1) + \frac{s_{12}}{M_W^2} \right)$$

- **Naïve power counting:** unintegrated W amplitude much more singular in the UV than quark and scalar amplitudes

Universality of dual amplitude

Driencourt-Mangin, GR, Sborlini, arXiv:1702.07581

- **Universality** and **compactness** of the dual representation

$$\mathcal{A}_1^{(1,f)} = g_f \int_{\ell} \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{2,0}^{(+)}} + \frac{2(2\ell \cdot p_{12})^2}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} \right) \frac{s_{12} M_f^2}{(2\ell \cdot p_1)(2\ell \cdot p_2)} c_1^{(f)} \right. \\ \left. + \frac{2s_{12}^2}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} c_{23}^{(f)} \right], \quad q_{i,0}^{(+)} = \sqrt{\mathbf{q}_i^2 + M_f^2}, \quad f = \phi, t, W \\ c_{23}^{(f)} = \frac{d-4}{d-2} \left(1, 4, 2(d-1) + \frac{s_{12}}{M_W^2} \right)$$

- **Naïve power counting:** unintegrated W amplitude much more singular in the UV than quark and scalar amplitudes
- **Local renormalization:**

$$\mathcal{A}_{1,\text{UV}}^{(1,f)} = -g_f \int_{\ell} \frac{\tilde{\delta}(\ell) \ell_0^{(+)} s_{12}}{2(q_{\text{UV},0}^{(+)})^3} \left(1 + \frac{1}{(q_{\text{UV},0}^{(+)})^2} \frac{3\mu_{\text{UV}}^2}{d-4} \right) c_{23}^{(f)} = 0$$

Universality of dual amplitude

Driencourt-Mangin, GR, Sborlini, arXiv:1702.07581

- **Universality** and **compactness** of the dual representation

$$\mathcal{A}_1^{(1,f)} = g_f \int_{\ell} \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)} }{q_{1,0}^{(+)} } + \frac{\ell_0^{(+)} }{q_{2,0}^{(+)} } + \frac{2(2\ell \cdot p_{12})^2}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} \right) \frac{s_{12} M_f^2}{(2\ell \cdot p_1)(2\ell \cdot p_2)} c_1^{(f)} \right. \\ \left. + \frac{2s_{12}^2}{s_{12}^2 - (2\ell \cdot p_{12} - i0)^2} c_{23}^{(f)} \right], \quad q_{i,0}^{(+)} = \sqrt{\mathbf{q}_i^2 + M_f^2}, \quad f = \phi, t, W \\ c_{23}^{(f)} = \frac{d-4}{d-2} \left(1, 4, 2(d-1) + \frac{s_{12}}{M_W^2} \right)$$

- **Naïve power counting:** unintegrated W amplitude much more singular in the UV than quark and scalar amplitudes
- **Local renormalization:** smooth four dimensional limit

$$\mathcal{A}_{1,\text{UV}}^{(1,f)} = -g_f \int_{\ell} \frac{\tilde{\delta}(\ell) \ell_0^{(+)} s_{12}}{2(q_{\text{UV},0}^{(+)})^3} \left(1 + \frac{1}{(q_{\text{UV},0}^{(+)})^2} \frac{3\mu_{\text{UV}}^2}{d-4} \right) c_{23}^{(f)} = 0$$



$$\left. \mathcal{A}_{1,\text{R}}^{(1,f)} \right|_{d=4} = \left(\mathcal{A}_1^{(1,f)} - \mathcal{A}_{1,\text{UV}}^{(1,f)} \right)_{d=4}$$

Direct asymptotic expansion

Driencourt-Mangin, GR, Sborlini, arXiv:**1702.07581**

- Integration domain is an **Euclidean** space (loop three-momentum)

Direct asymptotic expansion

Driencourt-Mangin,GR, Sborlini, arXiv:**1702.07581**

- Integration domain is an **Euclidean** space (loop three-momentum)
- Asymptotic expansions** (heavy or light internal mass) more direct at integrand level than **Minkowsky**

$$\frac{\delta(\ell^2 - M^2)}{s_{12} + 2\ell \cdot p_{12}} = \frac{\delta(\ell^2 - M^2)}{2\ell \cdot p_{12}} \sum_{n=0} \left(\frac{-s_{12}}{2\ell \cdot p_{12}} \right)^n$$

Direct asymptotic expansion

Driencourt-Mangin,GR, Sborlini, arXiv:**1702.07581**

- Integration domain is an **Euclidean** space (loop three-momentum)
- Asymptotic expansions** (heavy or light internal mass) more direct at integrand level than **Minkowsky**

$$\frac{\delta(\ell^2 - M^2)}{s_{12} + 2\ell \cdot p_{12}} = \frac{\delta(\ell^2 - M^2)}{2\ell \cdot p_{12}} \sum_{n=0} \left(\frac{-s_{12}}{2\ell \cdot p_{12}} \right)^n$$

- Each term of the integrand expansion less UV singular than the previous one

Direct asymptotic expansion

Driencourt-Mangin,GR, Sborlini, arXiv:**1702.07581**

- Integration domain is an **Euclidean** space (loop three-momentum)
- Asymptotic expansions** (heavy or light internal mass) more direct at integrand level than **Minkowsky**

$$\frac{\delta(\ell^2 - M^2)}{s_{12} + 2\ell \cdot p_{12}} = \frac{\delta(\ell^2 - M^2)}{2\ell \cdot p_{12}} \sum_{n=0} \left(\frac{-s_{12}}{2\ell \cdot p_{12}} \right)^n$$

- Each term of the integrand expansion less UV singular than the previous one
- Circumvent **expansion by regions** [Smirnov, Beneke]

Conclusions

- **New algorithm/regularization scheme** for higher-orders in perturbative QFT based on LTD: summation over degenerate soft, final-state collinear singularities and quasi-collinear configurations achieved through a **mapping of momenta** between real and virtual kinematics.

Conclusions

- **New algorithm/regularization scheme** for higher-orders in perturbative QFT based on LTD: summation over degenerate soft, final-state collinear singularities and quasi-collinear configurations achieved through a **mapping of momenta** between real and virtual kinematics.
- **IR unsubtracted and four-dimensional:** fully **local cancellation** of IR and UV singularities.

Conclusions

- **New algorithm/regularization scheme** for higher-orders in perturbative QFT based on LTD: summation over degenerate soft, final-state collinear singularities and quasi-collinear configurations achieved through a **mapping of momenta** between real and virtual kinematics.
- **IR unsubtracted and four-dimensional:** fully **local cancellation** of IR and UV singularities.
- **Smooth massless limit** due to proper treatment of quasi-collinear configurations

Conclusions

- **New algorithm/regularization scheme** for higher-orders in perturbative QFT based on LTD: summation over degenerate soft, final-state collinear singularities and quasi-collinear configurations achieved through a **mapping of momenta** between real and virtual kinematics.
- **IR unsubtracted and four-dimensional:** fully **local cancellation** of IR and UV singularities.
- **Smooth massless limit** due to proper treatment of quasi-collinear configurations
- **Threshold singularities** through contour deformation in the loop three-momentum.

Conclusions

- **New algorithm/regularization scheme** for higher-orders in perturbative QFT based on LTD: summation over degenerate soft, final-state collinear singularities and quasi-collinear configurations achieved through a **mapping of momenta** between real and virtual kinematics.
- **IR unsubtracted and four-dimensional:** fully **local cancellation** of IR and UV singularities.
- **Smooth massless limit** due to proper treatment of quasi-collinear configurations
- **Threshold singularities** through contour deformation in the loop three-momentum.
- **Simultaneous generation** of real and virtual corrections **advantageous**, particularly for multi-leg processes.

Conclusions

- **New algorithm/regularization scheme** for higher-orders in perturbative QFT based on LTD: summation over degenerate soft, final-state collinear singularities and quasi-collinear configurations achieved through a **mapping of momenta** between real and virtual kinematics.
- **IR unsubtracted and four-dimensional:** fully **local cancellation** of IR and UV singularities.
- **Smooth massless limit** due to proper treatment of quasi-collinear configurations
- **Threshold singularities** through contour deformation in the loop three-momentum.
- **Simultaneous generation** of real and virtual corrections **advantageous**, particularly for multi-leg processes.
- **Universality** for EW corrections, and direct **asymptotic expansions**.

Conclusions

- **New algorithm/regularization scheme** for higher-orders in perturbative QFT based on LTD: summation over degenerate soft, final-state collinear singularities and quasi-collinear configurations achieved through a **mapping of momenta** between real and virtual kinematics.
- **IR unsubtracted and four-dimensional:** fully **local cancellation** of IR and UV singularities.
- **Smooth massless limit** due to proper treatment of quasi-collinear configurations
- **Threshold singularities** through contour deformation in the loop three-momentum.
- **Simultaneous generation** of real and virtual corrections **advantageous**, particularly for multi-leg processes.
- **Universality** for EW corrections, and direct **asymptotic expansions**.

Outlook: fully differential multi-leg at **NNLO (and beyond)**