Subleading Power Corrections for N-Jettiness Subtractions

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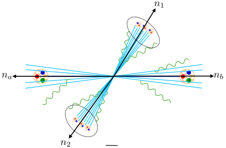
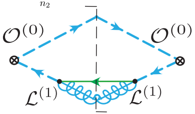
Based on 1612.00450 With Lorena Rothen, Iain Stewart, Frank Tackmann, and HuaXing Zhu

Outline

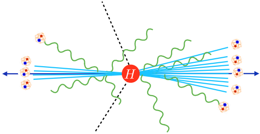
- *N*-Jettiness Subtractions and Power Corrections



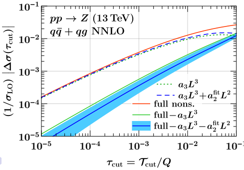
- Subleading Power Fixed Order Calculations in SCET



- Observable Dependence



- Numerical Results for Color Singlet Production

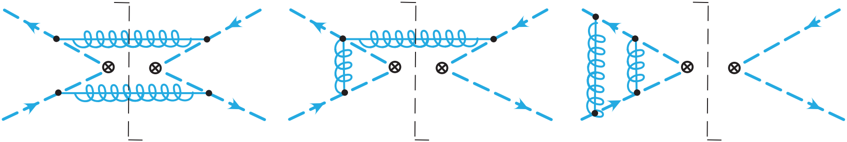


N-Jettiness Subtractions and Power Corrections

Fixed Order Calculations

- Higher order calculations require cancellation of infrared (soft and collinear) divergences between real and virtual diagrams.
- NNLO:

Real-Real Virtual-Real Virtual-Virtual



- Significant recent progress towards feasible NNLO subtractions
  - Local Subtractions: Colorful NNLO, Sector Decomposition, Antenna Subtraction, . . . [Del Duca, Duhr, Kardos, Somogyi, Trocsanyi, Tulipant] [Anastasiou, Melnikov, Petriello] [Gehrmann-De Ridder, Gehrmann, Glover et al.]
  - Global Subtractions: q_T, N-jettiness, . . . [Catani, Grazzini] [Boughezal, Focke, Petriello, Liu] [Gaunt, Stahlhofen,Tackmann, Walsh]
- See also “Higgs Differential”, “projection to Born”, . . . [Falko Dulat’s Talk] [Frederic Dreyer’s Talk]

Global Subtractions at NNLO

- Use an observable, $\mathcal{T}$, to regulate phase space.

$$\sigma(X) = \int_0 d\mathcal{T}\frac{d\sigma(X)}{d\mathcal{T}} = \int_0^{\mathcal{T}^{\text{cut}}} d\mathcal{T}\frac{d\sigma(X)}{d\mathcal{T}} + \int_{\mathcal{T}^{\text{cut}}} d\mathcal{T}\frac{d\sigma(X)}{d\mathcal{T}}$$

- Want $\mathcal{T}$ to isolate collinear and soft singularities around a Born configuration.

$$\sigma(\mathcal{T}^{\text{cut}}) = \int_0^{\mathcal{T}^{\text{cut}}} d\mathcal{T}\frac{d\sigma(X)}{d\mathcal{T}}$$

Compute using factorization in soft/collinear limits:

$$\int_{\mathcal{T}^{\text{cut}}} d\mathcal{T}\frac{d\sigma(X)}{d\mathcal{T}}$$

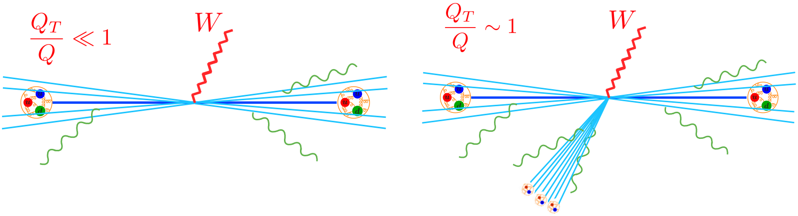
Additional jet resolved. Use NLO subtractions.

$$\frac{d\sigma}{d\mathcal{T}} = HB_a \otimes B_b \otimes S \otimes J_1 \otimes \cdots \otimes J_{N-1}$$

Q_T Subtractions

[Catani, Grazzini]

- For color singlet production can use Q_T.



- All orders factorization theorem:

[Collins, Soper, Sterman]

$$\frac{d\sigma}{dQ_T^2} = H B_a \otimes B_b \otimes S + \mathcal{O}(Q_T^2/Q^2)$$

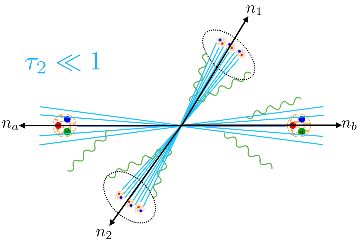
- Successfully applied to
  $pp \to H, W, Z, \gamma\gamma, WH, ZH, ZZ, W^+W^-, Z\gamma, W\gamma$

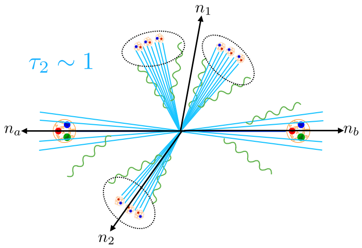
[Catani, Grazzini et al.]

N-Jettiness Subtractions

[Boughezal, Focke, Petriello, Liu]
[Gaunt, Stahlhofen, Tackmann, Walsh]

- N-jettiness: Inclusive event shape to identify N jets.





$$\boxed{\tau_N = \frac{2}{Q^2} \sum_k \min\left\{q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \cdots, q_N \cdot p_k\right\}} \quad q_i = Q n_i$$

- Factorization:

$$\frac{d\sigma}{d\tau_N} = H B_a \otimes B_b \otimes S \otimes J_1 \otimes \cdots \otimes J_{N-1} + \mathcal{O}(\tau_N) + \text{Glaubers } (\alpha_s^4)$$

[Stewart, Tackmann, Waalewijn], [Gaunt], [Zeng]

- Succesfully applied to $W/Z/H/\gamma$+jet

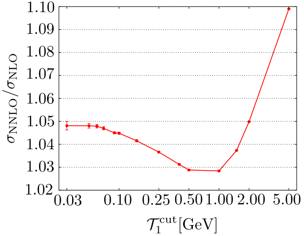
[Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello, Williams]

Power Corrections

- Standard factorization drops power corrections in $\mathcal{T}_N^{\rm cut}$.

$$\sigma(X) = \int_0 d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \int_0^{\mathcal{T}_N^{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} + \int_{\mathcal{T}_N^{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

$\mathcal{T}_1^{\rm cut}$ Dependence for NNLO $Z+$ Jet



[Boughezal, Focke, Giele, Petriello, Liu]

- Difficult numerically to go to low $\mathcal{T}^{\rm cut}$ values for the NLO calculation.

Power Corrections to N-Jettiness Subtractions

- Approximation in the singular region receives power corrections

$$\frac{d\sigma}{d\tau} = \underbrace{\sum_{n=0}^{n=\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} c^{(0)}_{nm}\left(\frac{\log^m \tau}{\tau}\right)_+}_{\text{Leading Power}} + \underbrace{\sum_{n=0}^{n=\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} c^{(2)}_{nm}\log^m \tau + \sum_{n=0}^{n=\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} c^{(2)}_{nm}\,\tau\log^m \tau + \cdots}_{\text{Power Corrections}}$$

- Gives rise to power corrections in the integrated cross section

$$\Longrightarrow \sigma(\tau_{\rm cut}) = \int_0^{\tau_{\rm cut}} d\tau \frac{d\sigma}{d\tau} = \underbrace{\sum_{n=0}^{n=\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} \tilde{c}^{(0)}_{nm}\log^m(\tau_{\rm cut})}_{\text{Leading Power}} + \underbrace{\tau_{\rm cut}\sum_{n=0}^{n=\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} \tilde{c}^{(2)}_{nm}\log^m(\tau_{\rm cut}) + \cdots}_{\text{Power Corrections}}$$

- The function $\tau_{\rm cut}\log^m(\tau_{\rm cut})$ approaches zero slowly!
  - NLO: $\tau_{\rm cut}\log(\tau_{\rm cut}) + \cdots$
  - NNLO: $\tau_{\rm cut}\log^3(\tau_{\rm cut}) + \cdots$
  - NNNLO: $\tau_{\rm cut}\log^5(\tau_{\rm cut}) + \cdots$
- Very small values of $\tau_{\rm cut}$ are required.

Power Corrections

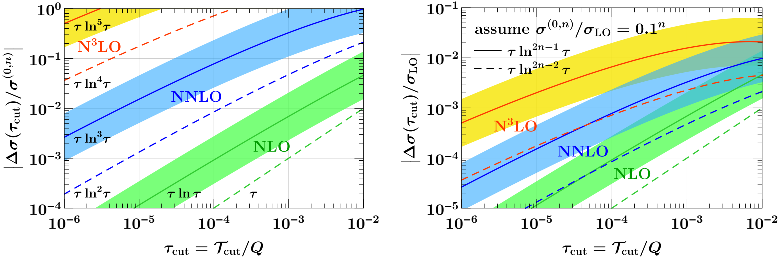
- Use functional form to estimate size of power corrections

$$\Delta\sigma(\tau_{\rm cut}) = \sigma(\tau_{\rm cut})_{\sf exact} - \sigma(\tau_{\rm cut})_{\sf approx}$$

Solid=LP Dashed=remove LL NLP

$$\sigma(\tau_{\rm cut}) = \sum_{n=0}^{n=\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} \tilde{c}_{nm}^{(0)} \log^m(\tau_{\rm cut}) + \tau_{\rm cut} \sum_{n=0}^{n=\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} \tilde{c}_{nm}^{(2)} \log^m(\tau_{\rm cut}) + \cdots$$

Estimated Missing Correction



- Factor of ~ 10 improvement by calculating leading log (LL) at NLP.

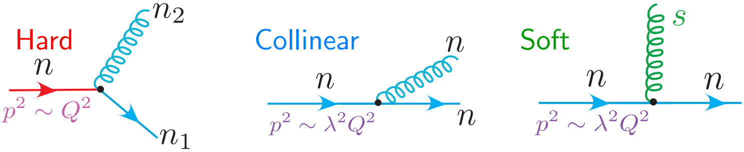
Subleading Power Fixed Order Calculations in SCET

See also calculation in direct QCD by [Boughezal, Petriello, Liu], Xiaohui Liu's Talk

Effective Field Theory

[Bauer, Fleming, Pirjol, Stewart]

- Effective theory for long wavelength dynamics of soft and collinear radiation in the presence of a hard scattering source
  $\Longrightarrow$ Soft Collinear Effective Theory



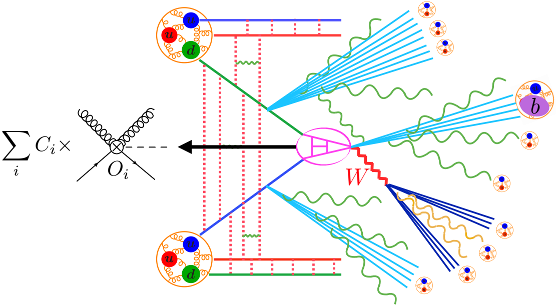
Operator	$\mathcal{B}^{\mu}_{n_i\perp}$	χ_{n_i}	$\mathcal{P}^{\mu}_{\perp}$	q_{us}	D^{μ}_{us}
Power Counting	$\sqrt{\tau}$	$\sqrt{\tau}$	$\sqrt{\tau}$	$\tau^{3/2}$	τ

- Primarily used for factorization/resummation.
- Here will use SCET to perform fixed order calculations at subleading power.

Soft Collinear Effective Theory

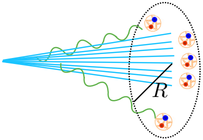
[Bauer, Fleming, Pirjol, Stewart]

- Hard scattering is described by operators in EFT



- Long wavelength dynamics of soft and collinear radiation described by Lagrangian

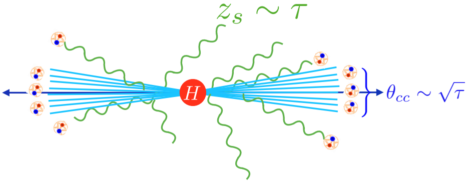
$\mathcal{L}_{\mathsf{dyn}}$:



Fixed Order Thrust at NLP

- Simple playground is Thrust in e^+e^-
- Most interested in the leading log: $\alpha_s \log(\tau)$, $\alpha_s^2 \log^3(\tau)$, $\cdots$
- More generally, interested in structure at subleading powers.

$$\tau = 1 - \max_{\hat{t}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$



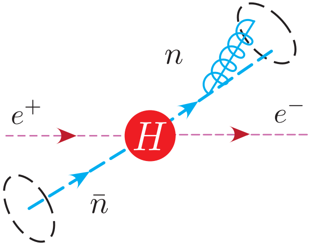
- Exact NLO result known.
- Related to color singlet production at the LHC.

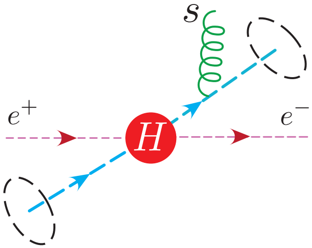
Leading Power SCET

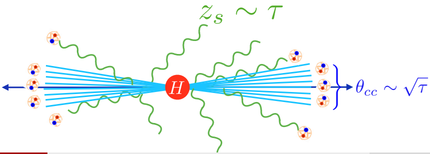
- Leading Power SCET:
  - Leading Power Hard Scattering Operators:

$$\mathcal{O} = C(Q^2)\bar{\chi}_n \Gamma \chi_{\bar{n}}$$

  - Leading power Lagrangian (eikonal/ collinear)





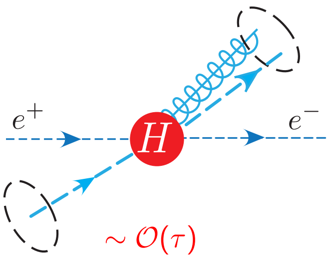


Subleading Power SCET

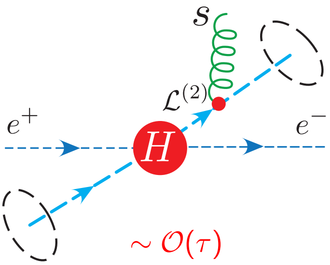
- Subleading Power in SCET:

$$\mathcal{L}_{\mathsf{SCET}} = \mathcal{L}_{\mathrm{hard}} + \mathcal{L}_{\mathrm{dyn}} = \sum_{i\geq 0} \mathcal{L}^{(i)}_{\mathrm{hard}} + \sum_{i\geq 0} \mathcal{L}^{(i)}$$

Subleading Hard Scattering Operators



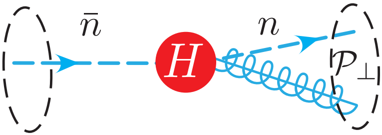
Subleading Lagrangians



Relevant Hard Scattering Operators

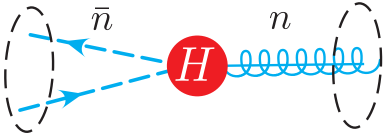
- For leading log (singularity), $\alpha_s^n \log^{2n-1}(\tau)$, two relevant hard scattering operators:

qg In Same Sector



$\bar{\chi}_n \chi_{\bar{n}} \mathcal{P}_\perp \mathcal{B}_n$

$q\bar{q}$ In Same Sector



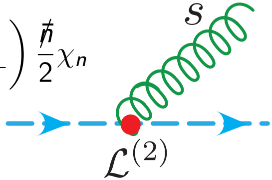
$\bar{\chi}_{\bar{n}} \chi_{\bar{n}} \mathcal{B}_n$

- $q\bar{q}$ in same sector has no LP analog.

Subleading Lagrangian

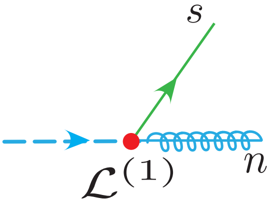
- Subleading Lagrangians are universal, and known.
- Correct the dynamics of soft and collinear particles. e.g.
  - Correction to eikonal emission:

$$\mathcal{L}^{(2)}_{\chi_n} = \bar{\chi}_n \left(i\slashed{D}_{us\perp} \frac{1}{\overline{\mathcal{P}}} i\slashed{D}_{us\perp} - i\slashed{\mathcal{D}}_{n\perp} \frac{i\bar{n}\cdot D_{us}}{(\overline{\mathcal{P}})^2} i\slashed{\mathcal{D}}_{n\perp} \right) \frac{\bar{\slashed{n}}}{2} \chi_n$$



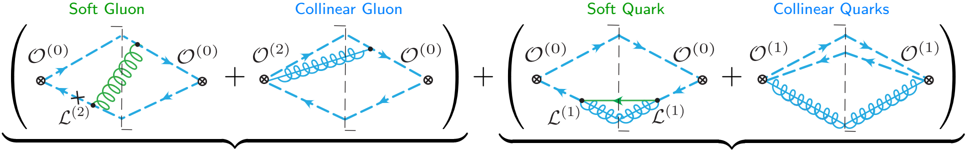
  - Emission of soft quarks:

$$\mathcal{L}^{(1)}_{\chi_n q_{us}} = \bar{\chi}_n \frac{1}{\overline{\mathcal{P}}} g\slashed{\mathcal{B}}_{n\perp} q_{us} + \text{h.c.}$$



NLO Thrust at NLP

- Sum four graphs to get NLP result:



$$-8C_F\left[\frac{1}{\epsilon}+\log\left(\frac{\mu^2}{Q^2\tau^2}\right)\right]+8C_F\left[\frac{1}{\epsilon}+\log\left(\frac{\mu^2}{Q^2\tau}\right)\right]$$

$$=8C_F\log(\tau)$$

$$4C_F\left[\frac{1}{\epsilon}+\log\left(\frac{\mu^2}{Q^2\tau^2}\right)\right]-4C_F\left[\frac{1}{\epsilon}+\log\left(\frac{\mu^2}{Q^2\tau}\right)\right]$$

$$=-4C_F\log(\tau)$$

Total

$$\boxed{\frac{1}{\sigma_0}\frac{d\sigma^{(2,1)}}{d\tau}=4C_F\left(\frac{\alpha_s}{4\pi}\right)\log(\tau)}$$

- Result gives directly (no expansions) the NLP contribution of the well known NLO thrust result.

Fixed Order Consistency Relations

- Fixed order calculations made simple by consistency relation in EFT.
- EFT modes have well defined scaling:

$$\mu_h^2 = Q^2\,, \qquad \mu_c^2 = Q^2\tau\,, \qquad \mu_s^2 = Q^2\tau^2$$

- General form of n-loop fixed order calculation at NLP:

$$\frac{\mathrm{d}\sigma^{(2,n)}}{\mathrm{d}\tau} = \sum_{\kappa}\sum_{i=0}^{2n-1}\frac{c_{\kappa,i}}{\epsilon^i}\left(\frac{\mu^{2n}}{Q^{2n}\tau^{m(\kappa)}}\right)^{\epsilon} + \sum_{\gamma}\sum_{i=0}^{2n-2}\frac{d_{\gamma,i}}{\epsilon^i}\left(\frac{\mu^{2(n-1)}}{Q^{2(n-1)}\tau^{m(\gamma)}}\right)^{\epsilon}$$

  - 1-loop:

soft:	$\kappa = s\,,$	$m(\kappa) = 2\,,$
collinear:	$\kappa = c\,,$	$m(\kappa) = 1$

  - 2-loop:

hard-collinear:	$\kappa = hc\,,$	$m(\kappa) = 1\,,$
hard-soft:	$\kappa = hs\,,$	$m(\kappa) = 2\,,$
collinear-collinear:	$\kappa = cc\,,$	$m(\kappa) = 2\,,$
collinear-soft:	$\kappa = cs\,,$	$m(\kappa) = 3\,,$
soft-soft:	$\kappa = ss\,,$	$m(\kappa) = 4$

- Pole terms must cancel $\Longrightarrow$ non-trivial constraints.

Fixed Order Consistency Relations

- Solving the set of equations, one finds:
  - 1-loop:

$$c_{s,1} = -c_{c,1}$$

  - 2-loop:

$$\begin{aligned} c_{hc,3} &= \frac{c_{cs,3}}{3} = -c_{ss,3} = -\frac{1}{3}(c_{hs,3} + c_{cc,3}) \,, \\ c_{cs,2} &= c_{hc,2} - 2c_{ss,2} + d_{c,2} \,, \\ c_{hs,2} + c_{cc,2} &= -2c_{hc,2} + c_{ss,2} - d_{c,2} \,, \\ c_{hs,1} + c_{cc,1} &= -(c_{cs,1} + c_{hc,1} + c_{ss,1} + d_{c,1} + d_{s,1}) \end{aligned}$$

- 2-loop NLP result can be written:

$$\begin{aligned} \frac{\mathrm{d}\sigma^{(2,2)}}{\mathrm{d}\tau} = {} & c_{hc,3} \ln^3 \tau + (c_{hc,2} + c_{ss,2} + d_{c,2}) \ln^2 \tau \\ & + \left(-c_{cs,1} + c_{hc,1} - 2c_{ss,1} + d_{c,1}\right) \ln \tau \\ & + d_{c,2} \ln \frac{Q^2}{\mu^2} \ln \tau + \text{const} \end{aligned}$$

- LL can be computed from only the hard-collinear contribution.
  $\Longrightarrow$ Hope can simply generalize to multi-jet final state.

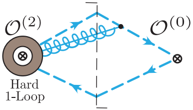
NNLO Thrust at NLP

[Ellis, Ross, Terrano]

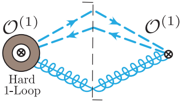
[Garland, Gehrmann, Glover, Koukoutsakis, Remiddi]

- 1 (and 2) loop results for $e^+e^- \to 3$ partons known.
- Calculation of NLP $\alpha_s^2 \log^3(\tau)$ straightforward using consistency relations: Only need hard-collinear contribution.

Collinear Gluon



Collinear Quarks



$$\frac{1}{\sigma_0}\frac{d\sigma^{(2,2)}_{\mathrm{Cat.1}}}{d\tau} = -32C_F^2\left(\frac{\alpha_s}{4\pi}\right)^2\log^3(\tau)$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{(2,2)}_{\mathrm{Cat.2}}}{d\tau} = 8C_F(C_F+C_A)\left(\frac{\alpha_s}{4\pi}\right)^2\log^3(\tau)$$

Cusp Scaling!

Not Cusp!

Total

$$\frac{1}{\sigma_0}\frac{d\sigma^{(2,2)}}{d\tau} = 8C_F(C_A-3C_F)\left(\frac{\alpha_s}{4\pi}\right)^2\log^3(\tau)$$

Extension to pp

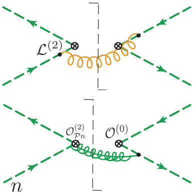
- Operators and Lagrangians also applicable to perturbative power corrections in pp.

$$\mathrm{d}\sigma = \sum_{ij} \int \mathrm{d}\xi_a \mathrm{d}\xi_b \, f_i(\xi_a)\, f_j(\xi_b)\, \mathrm{d}\hat{\sigma}_{ij}(\xi_a, \xi_b)$$

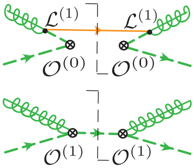
- Partonic cross section at $\mathcal{O}(\tau^0)$ written as

$$\frac{\mathrm{d}\hat{\sigma}_{ij}^{(2,n)}(\xi_a, \xi_b; X)}{\mathrm{d}Q^2\, \mathrm{d}Y\, \mathrm{d}\tau} = \sigma_{q0}(Q, X) \left(\frac{\alpha_s}{4\pi}\right)^n \sum_{m=0}^{2n-1} C_{ij,m}^{(2,n)}(\xi_a, \xi_b) \ln^m \tau$$

$q\bar{q}$ channel



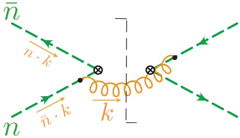
qg channel



Extension to pp

- Power corrections arise from residual momentum routed into pdfs.
- Must be expanded homogeneously:

$$f_i\left[\xi\left(1+\frac{k}{Q}\right)\right] = f_i(\xi) + \frac{k}{Q}\,\xi f_i'(\xi) + \cdots$$



- We take $\xi f_i'(\xi) \sim f_i(\xi)$.
- Coefficients of partonic cross section at $\mathcal{O}(\tau^0)$ involve δ'.
- Use the shorthand notation

$$\delta_a' \equiv x_a\,\delta'(\xi_a - x_a)\,, \qquad \delta_b' \equiv x_b\,\delta'(\xi_b - x_b)$$

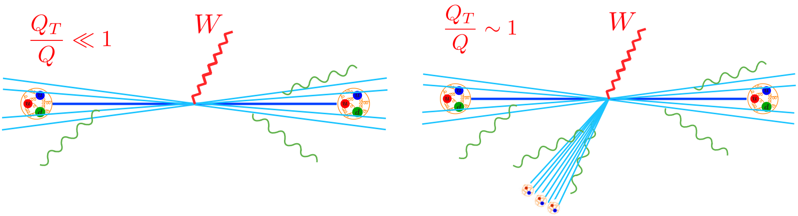
for the δ' acting on either beam direction.

Observable Dependence

Observable Dependence: Q_T Subtractions

- Highly desirable for power corrections to be independent of Born kinematics

  $\implies$ e.g. Q_T subtractions: Q_T/Q independent of rapidity.



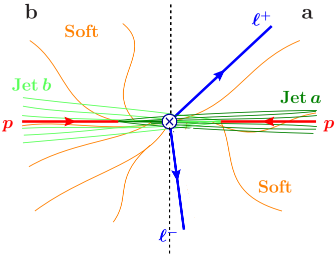
- Want to choose definition of N-jettiness such that power expansion is well behaved throughout phase space.

Observable Dependence

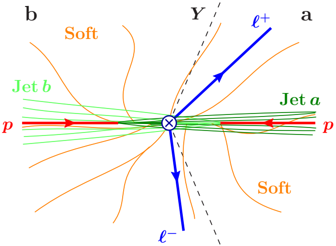
- Analytic calculation of power corrections allows understanding of observable dependence

$$\mathcal{T}_0^x = \sum_k \min\Big\{\lambda_x\, n_a \cdot k,\, \lambda_x^{-1}\, n_b \cdot k\Big\}$$

Hadronic: $\lambda_x = 1$



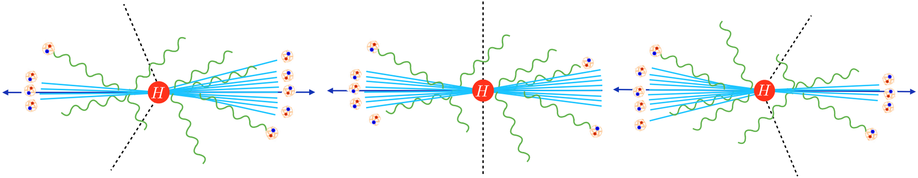
Leptonic: $\lambda_x = e^Y$



- Definition generalizes to final state jets using reconstructed momentum of jets.
- Hadronic definition currently used in most studies.

Observable Dependence

- Leptonic definition $\implies$ constraint on radiation is independent of rapidity.
- Consider boosting thrust in $e^+e^- \to$ dijets:



$$\frac{1}{\sigma}\frac{d\sigma^{\text{NLO}}}{d\tau} = \underbrace{\frac{1}{\tau}\frac{C_F\alpha_s}{4\pi}[-6-8\log(\tau)]}_{\text{Leading Power}} + \underbrace{\frac{C_F\alpha_s}{4\pi}[-4+4\log(\tau)]}_{\text{Next to Leading Power}} + \tau[\cdots] + \cdots$$

- Result invariant IF observable changes accordingly.
- Ensures power corrections independent of rapidity.

Results for Beam Thrust in Drell Yan: Leptonic Definition

$$\frac{d\hat{\sigma}_{ij}^{(2,n)}(\xi_a,\xi_b;X)}{dQ^2\,dY\,d\tau} = \sigma_{q0}(Q,X)\left(\frac{\alpha_s}{4\pi}\right)^n \sum_{m=0}^{2n-1} C_{ij,m}^{(2,n)}(\xi_a,\xi_b)\ln^m\tau$$

- NLO results for partonic cross section at $\mathcal{O}(\tau^0)$

$$C_{q\bar{q},1}^{(2,1)}(\xi_a,\xi_b) = 8C_F\left(\delta_a\delta_b + \frac{\delta_a'\delta_b}{2} + \frac{\delta_a\delta_b'}{2}\right)$$

$$C_{qg,1}^{(2,1)}(\xi_a,\xi_b) = -2T_F\,\delta_a\delta_b$$

- NNLO results obtained from hard-collinear contribution using consistency

$$C_{q\bar{q},3}^{(2,2)}(\xi_a,\xi_b) = -32C_F^2\left(\delta_a\delta_b + \frac{\delta_a'\delta_b}{2} + \frac{\delta_a\delta_b'}{2}\right)$$

$$C_{qg,3}^{(2,2)}(\xi_a,\xi_b) = 4T_F(C_F+C_A)\,\delta_a\delta_b$$

- Note no explicit dependence on rapidity.

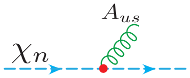
Results for Beam Thrust in Drell Yan: Hadronic Definition

- Power corrections for hadronic definition are enhanced by $e^{|Y|}$!

$$\widetilde{C}^{(2,2)}_{q\bar{q},3}(\xi_a,\xi_b) = -16C_F^2\left[e^{Y}\delta_a(\delta_b+\delta_b') + e^{-Y}(\delta_a+\delta_a')\delta_b\right]$$
$$\widetilde{C}^{(2,2)}_{qg,3}(\xi_a,\xi_b) = 4T_F(C_F+C_A)e^{Y}\delta_a\delta_b$$
$$\widetilde{C}^{(2,2)}_{gq,3}(\xi_a,\xi_b) = 4T_F(C_F+C_A)e^{-Y}\delta_a\delta_b$$

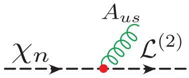
- Physical origin of enhancement:

Eikonal



$$= igT^a n^\mu$$

Next-to-Eikonal



$$= igT^a \frac{n\cdot p_{us}}{\bar{n}\cdot p_c} n^\mu$$

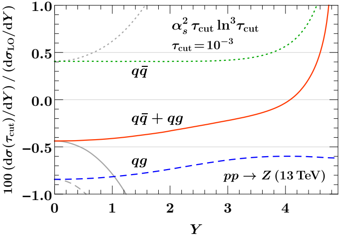
- Expansion parameter for hadronic definition is $\lambda^2 \sim \tau e^{|Y|}$.
- Breaks down away from central rapidity.

Observable Dependence

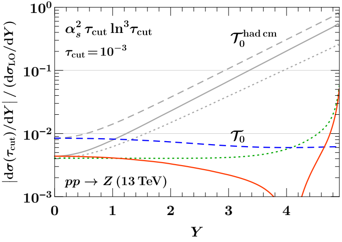
- Exponential growth of power corrections for hadronic definition.
- Power corrections for leptonic definition close to rapidity independent!
- Very important when computing differential distributions. (e.g. rapidity spectrum)

$\Longrightarrow$ Leptonic Definition Strongly Preferred!

Power Correction (Linear)



Power Correction (Log)



Numerical Results for Color Singlet Processes

Numerical Comparisons

[Campbell, Ellis, Williams]

- Exact fixed order result can be computed numerically (MCFM).
- Subtract known leading power result to obtain power corrections:

$$\overbrace{\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}}^{\text{MCFM}} - \underbrace{\sum_{n=0}^{n=\infty}\left(\frac{\alpha_s}{4\pi}\right)^n \sum_{n=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \tau}{\tau}\right)_+}_{\text{Leading Power}} = \overbrace{\sum_{n=0}^{n=\infty}\left(\frac{\alpha_s}{4\pi}\right)^n \sum_{n=0}^{2n-1} c_{nm}^{(2)} \log^m \tau + \cdots}^{\text{Power Corrections} \,\equiv\, \text{Nonsingular}}$$

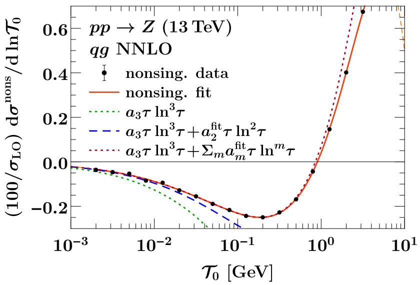
- Allows a numerical study of the size of power corrections in N-jettiness subtraction scheme.

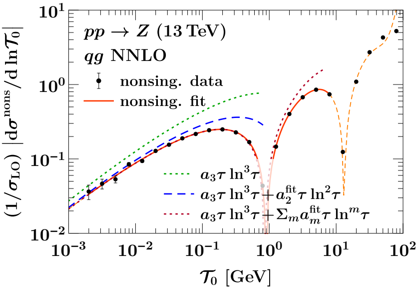
NNLO Beam Thrust at NLP

- Leading logarithm provides good approximation at NNLO.
- At NNLO there are subleading logarithms which we have not (yet) calculated.

$$\frac{d\sigma^{\text{nons}}}{d\tau} = \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg(\overbrace{\tilde{c}^{(3)}\log^3(\tau)}^{\text{Calculated}} + \tilde{c}^{(2)}\log^2(\tau) + \tilde{c}^{(1)}\log(\tau) + \tilde{c}^{(0)} + \mathcal{O}(\tau)\Bigg)$$

Nonsingular (Linear) | Nonsingular (Log)

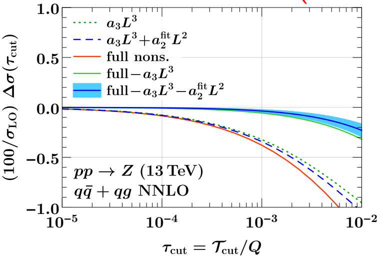




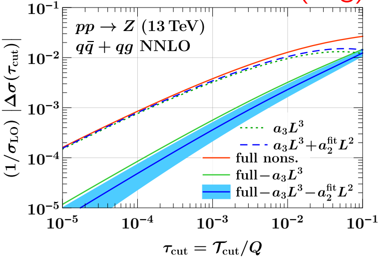
NNLO Beam Thrust at NLP

- Combined result for $\Delta\sigma(\tau_{\rm cut})$ for both channels at NNLO.

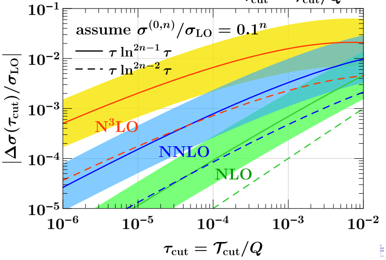
Power Correction (Linear)



Power Correction (Log)

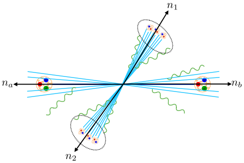


- Agrees well with scaling estimate.

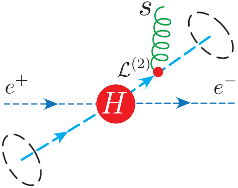


Conclusions

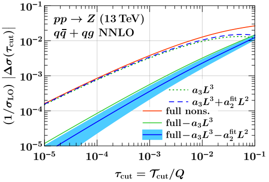
- *N*-jettiness subtractions are a general method for NNLO subtractions with jets in the final state.



- Subleading power fixed order calculations can be performed efficiently in SCET using consistency relations.



- Power corrections for *N*-jettiness subtractions can be analytically computed.



Thanks!