Universität Zürich[UZH]

Four-loop QCD renormalization group functions with different fermion representations of the gauge group

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in collaboration with K. G. Chetyrkin

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Outline

I. Motivation: Higher order RG functions

II. Extension of QCD to reducible fermion representations

III. Calculation of four-loop RG functions

IV. Results

V. Conclusion

I. Renormalization Group (RG) functions to higher orders

- RG functions are fundamental QFT quantities. Evolution of model parameters with renormalization scale used as input in many calculations.

- β-function $\rightarrow$ ingredient for Sequential extended BLM approach [Brodsky, Lepage and Mackenzie; Mikhailov; Kataev et al] and Principle of Maximum Conformality and Commensurate Scale Relations [Brodsky et al]:

$$\begin{aligned} \text{Observable} &= \sum_{n=0}^{\infty} \alpha_s^n(\mu)\, c_n, \qquad \beta(\alpha_s) = \alpha_s \sum_{n=1}^{\infty} \alpha_s^n(\mu)\, \beta_n \\ \Rightarrow c_1 &= c_1[0] \\ c_2 &= c_2[1]\, \beta_1 + c_2[0] \\ c_3 &= c_3[2]\, \beta_1^2 + c_3[0,1]\, \beta_2 + c_3[1]\, \beta_1 + c_3[0] \ldots \end{aligned}$$

  Try to unambiguously identify β-terms in a pQCD expansion, e. g. by n_f-terms (β_1 identified by $x = \frac{4}{3} T_F n_f$ and/or $y = \frac{4}{3}\frac{C_A}{2} n_{\tilde{g}}$ [Mikhailov (2017)])
  and absorb β_n into scale μ order by order in pQCD.
  $\Rightarrow$ Come as close as possible to conformal series with $\beta = 0$.

- Gauge boson and ghost propagator anomalous dimensions $\rightarrow$ ingredients in comparing momentum dependence of corresponding propagators derived in lattice calculations with perturbative results [Suman et al; Becirevic et al; von Smekal et al; Blossier et al; Bornyakov et al].

Anomalous dimensions and β-functions at higher orders

- **QCD β-function:** $\beta_{\alpha_s}(\alpha_s)$ at **5 loop** [Baikov, Chetyrkin, Kühn (2017)];
  $\beta_{\alpha_s}(\alpha_s)$ at **4 loop** [van Ritbergen, Vermaseren, Larin (1997); Czakon (2005)];
  $\beta_{\alpha_s}(\alpha_s, y_t, \lambda)$ [A. Bednyakov, A. Pikelner (2015); M.Z. (2015)]

- **Quark mass and field anomalous dimensions: general gauge group** at **5 loop** [Luthe, Maier, Marquard Schröder (2016); Baikov, Chetyrkin, Kühn (2017)];
  All QCD anomalous dimensions at **5 loop** [Luthe, Maier, Marquard Schröder, 2017];
  at **4 loop** [Chetyrkin, 2004];

- **All SM β-functions and anomalous dimensions** at **3 loop** [Mihaila, Salomon, Steinhauser (2012); Bednyakov, Pikelner, Velizhanin (2012,2013); Chetyrkin, M.Z. (2012,2013)]

Beyond the SM:

- **QCD plus gluinos** (Majorana fermions) **β-function** at **4 loop** [M.Z. (2016); Bednyakov, Pikelner (2016)];
  at **3 loop** [Clavelli, Coulter, Surguladze (1996)]

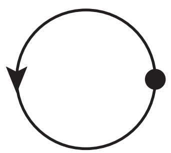
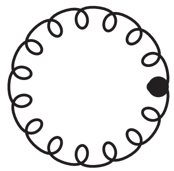
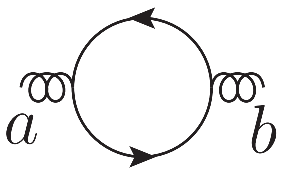
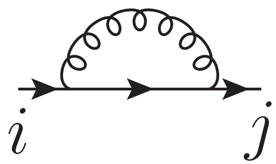
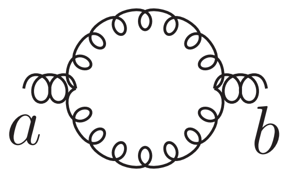
- **General gauge group with any number of fermion representations: β-function** at **4 loop** [M.Z. (2016)];
  All anomalous dimensions [Chetyrkin, M.Z. (2017)] including full ξ-dependence.

  $\Rightarrow$ arXiv:1704.04209;
  JHEP 1610 (2016) 118 [arXiv:1608.08982]
  Results as Mathematica code given in source files of arXiv:1704.04209

II. Extension of QCD to reducible fermion representations

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} - \frac{1}{2(1-\xi)} \left(\partial_\mu A^{a\,\mu}\right)^2 + \partial_\mu \bar{c}^a \partial^\mu c^a + g_s f^{abc}\, \partial_\mu \bar{c}^a A^{b\,\mu} c^c$$
$$+ \sum_{q=1}^{n_f} \left\{ \frac{i}{2} \bar{\psi}_q \overleftrightarrow{\partial\!\!\!/}\, \psi_q - m_q \bar{\psi}_q \psi_q + g_s \bar{\psi}_q A\!\!\!/^a T^a \psi_q \right\}$$

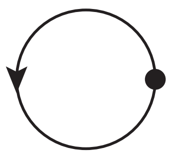
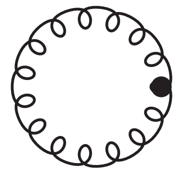
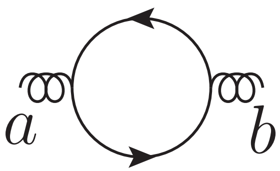
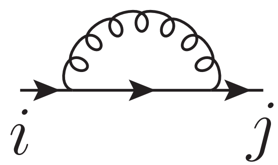
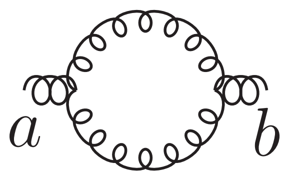
gauge group generators T^a and structure constants f^{abc} fulfill $\left[T^a, T^b\right] = i\, f^{abc}\, T^c$

	fermion rep.		adjoint rep. (gluon)	
Dimensions:		$\delta_{ii} = d_F$		$\delta_{aa} = N_A$
Traces:		**Tr** $\left(T^a\ T^b\right) = T_F\ \delta^{ab}$		
Casimir operators:		$T^a_{ik}\ T^a_{kj} = C_F\, \delta_{ij}$		$-f^{acd} f^{cdb} = C_A\, \delta_{ab}$

II. Extension of QCD to reducible fermion representations

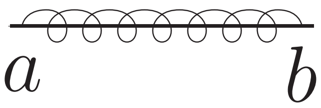
$$
\begin{aligned}
\mathcal{L}_{QCD} \;=\;& -\frac{1}{4} G^{a}_{\mu\nu} G^{a\,\mu\nu} - \frac{1}{2(1-\xi)} \left(\partial_\mu A^{a\,\mu}\right)^2 + \partial_\mu \bar{c}^a \partial^\mu c^a + g_s f^{abc}\, \partial_\mu \bar{c}^a A^{b\,\mu} c^c \\
&+ \sum_{r=1}^{N_{\text{rep}}} \sum_{q=1}^{n_{f,r}} \left\{ \frac{i}{2} \bar{\psi}_{q,r} \overleftrightarrow{\not{\partial}}\, \psi_{q,r} - m_{q,r} \bar{\psi}_{q,r} \psi_{q,r} + g_s \bar{\psi}_{q,r} \not{A}^a T^{a,r} \psi_{q,r} \right\}
\end{aligned}
$$

gauge group generators $T^{a,r}$ and structure constants f^{abc} fulfill $\left[T^{a,r}, T^{b,r}\right] = i\, f^{abc}\, T^{c,r}$

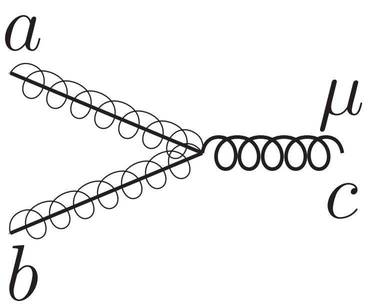
	fermion rep. $r \in \{1, \ldots, N_{\text{rep}}\}$		adjoint rep. (gluon)	
Dimensions:		$\delta_{ii} = d_{F,r}$		$\delta_{aa} = N_A$
Traces:		**Tr** $\left(T^{a,r} T^{b,r}\right) = T_{F,r}\, \delta^{ab}$		
Casimir operators:		$T^{a,r}_{ik} T^{a,r}_{kj} = C_{F,r}\, \delta_{ij}$		$-f^{acd} f^{cdb} = C_A\, \delta_{ab}$

Example with $N_{\text{rep}} = 2$: QCD plus gluinos (Majorana fermions)

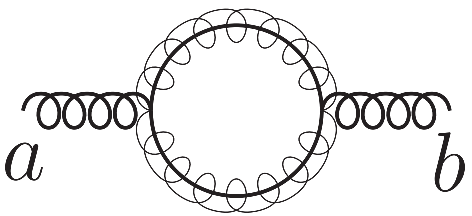
Calculation can be done with Dirac fermion propagator, provided a factor $\frac{1}{2}$ is introduced in closed gluino loops (along with the usual (-1) for fermion loops) [Denner, Eck, Hahn, Küblbeck (1992); Clavelli, Coulter, Surguladze (1996)]



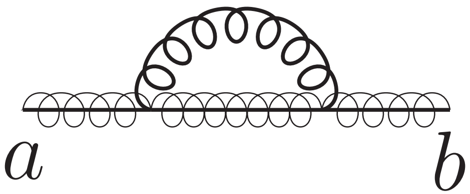
$$= \frac{i}{\not p - \underbrace{m_{\tilde g}}_{\to 0}}\,\delta^{ab}$$



$$= (i\gamma^\mu)\,(-i \underbrace{f^{abc}}_{=(T^{c,A})_{ab}})$$



$$\sim C_A\,\frac{n_{\tilde g}}{2}\,\delta^{ab}$$



$$\sim C_A\,\delta^{ab}$$

$\Rightarrow$

$$\begin{array}{lllllll} n_{f,1} & = & n_f, & \quad & n_{f,2} & = & \frac{n_{\tilde g}}{2}, \\ T_{F,1} & = & T_F, & & T_{F,2} & = & C_A, \\ C_{F,1} & = & C_F, & & C_{F,2} & = & C_A \end{array}$$

III. Renormalization Group (RG) functions in QFT

Evolution of any parameter or field Φ with the renormalization scale μ described by the anomalous dimension γ_Φ, evolution of coupling $\alpha_s = \frac{g_s^2}{4\pi}$ described by β-function:

$$\mu^2 \frac{d}{d\mu^2} \Phi = \gamma_\Phi \Phi \qquad \mu^2 \frac{d}{d\mu^2} \alpha_s = \gamma_{\alpha_s} \alpha_s = \beta(\alpha_s)$$

In $D = 4 - 2\varepsilon$ space-time dimensions:

$$\begin{aligned} \alpha_s^{\text{bare}} &= Z_{\alpha_s} \alpha_s \mu^{2\varepsilon} \quad \Rightarrow \quad \beta^{(D)}(\alpha_s) = \mu^2 \frac{d\alpha_s}{d\mu^2} = -\varepsilon \alpha_s + \beta(\alpha_s), \\ \Phi^{\text{bare}} &= Z_\Phi \Phi \quad \text{with} \quad Z_\Phi(\alpha_s, \xi) = 1 + \sum_{n=1}^{\infty} \frac{z^{(n)}(\alpha_s, \xi)}{\varepsilon^n} \end{aligned}$$

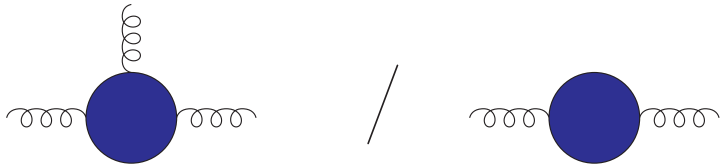
$$\left[\mu^2 \frac{d}{d\mu^2} \left\{ \alpha_s^{\text{bare}}, \Phi^{\text{bare}} \right\} = 0 \right] \Rightarrow \quad \gamma_\Phi(\alpha_s, \xi) = -\mu^2 \frac{d \log Z_\Phi(\alpha_s, \xi)}{d\mu^2} = \alpha_s \frac{\partial z^{(1)}(\alpha_s, \xi)}{\partial \alpha_s}$$

$$\begin{aligned} \beta(\alpha_s) &= \alpha_s \sum_{n=1}^{\infty} \beta_{\alpha_s}^{(n)} \left(\frac{\alpha_s}{4\pi} \right)^n, \\ \gamma_\Phi(\alpha_s, \xi) &= -\sum_{n=1}^{\infty} \gamma^{(n)}(\xi) \left(\frac{\alpha_s}{4\pi} \right)^n \end{aligned}$$

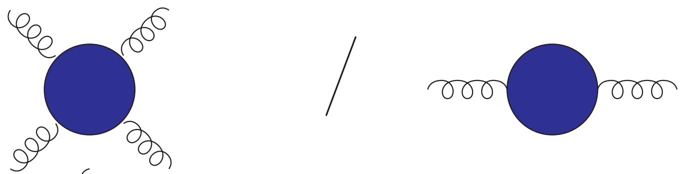
$$\mathcal{L}^{\text{bare}}_{QCD} = -\frac{1}{4} Z_3^{(2g)} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 - \frac{1}{2(1-\xi)} \left(\partial_\mu A^{a\,\mu}\right)^2 - \frac{1}{2} Z_1^{(3g)} g_s f^{abc} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right) A^b_\mu A^c_\nu$$
$$- \frac{1}{4} Z_1^{(4g)} g_s^2 \left(f^{abc} A^b_\mu A^c_\nu\right)^2 + Z_3^{(2c)} \partial_\mu \bar{c}^a \partial^\mu c^a + Z_1^{(ccg)} g_s f^{abc} \partial_\mu \bar{c}^a A^{b\,\mu} c^c$$
$$+ \sum_{r=1}^{N_{\text{rep}}} \sum_{q=1}^{n_{f,r}} \left\{ Z_2^{(q,r)} \frac{i}{2} \bar{\psi}_{q,r} \overleftrightarrow{\partial\!\!\!/} \psi_{q,r} - m_{q,r} Z_m^{(q,r)} Z_2^{(q,r)} \bar{\psi}_{q,r} \psi_{q,r} + g_s Z_1^{(q,r)} \bar{\psi}_{q,r} A\!\!\!/^a T^{a,r} \psi_{q,r} \right\},$$

Slavnov-Taylor identities ensure

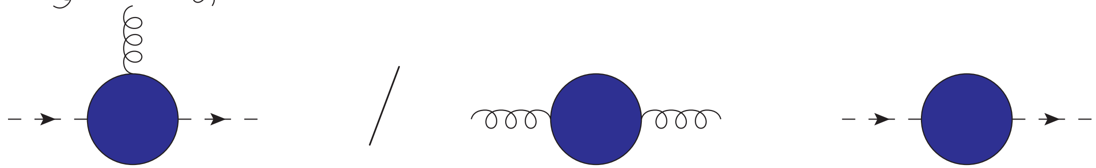
$$Z_{g_s} = Z_1^{(3g)} \left(\sqrt{Z_3^{(2g)}}\right)^{-3}$$



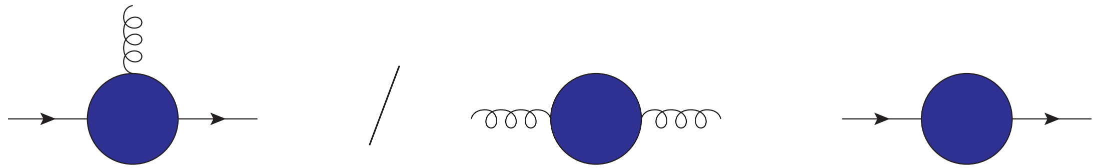
$$Z_{g_s} = \sqrt{Z_1^{(4g)}} \left(Z_3^{(2g)}\right)^{-1}$$



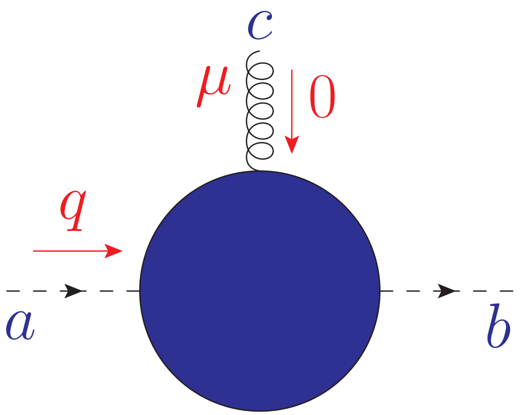
$$Z_{g_s} = Z_1^{(ccg)} \left(Z_3^{(2c)} \sqrt{Z_3^{(2g)}}\right)^{-1}$$

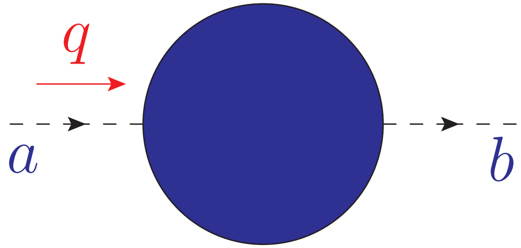


$$Z_{g_s} = Z_1^{(q,r)} \left(Z_2^{(q,r)} \sqrt{Z_3^{(2g)}}\right)^{-1}$$



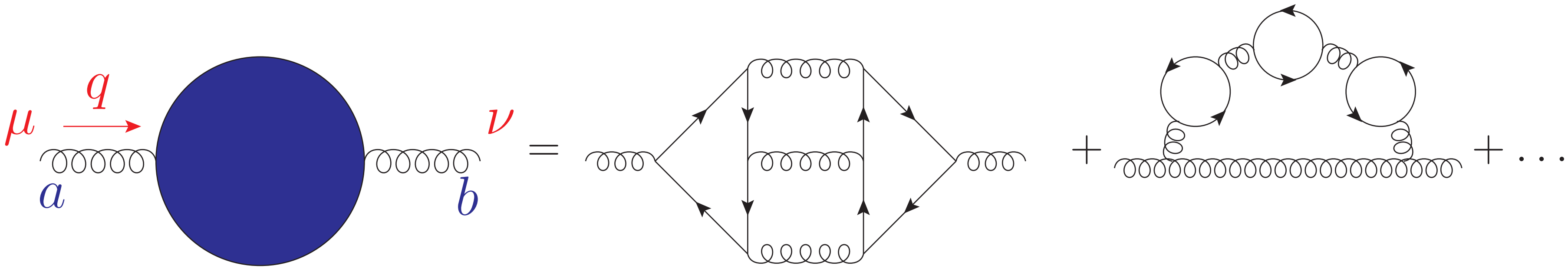
β_{α_s} at 4 loop from the ghost-gluon-vertex





projector: $\frac{q^\mu}{q^2} \quad \frac{f^{abc}}{N_A C_A}$

projector: $\frac{1}{q^2} \quad \frac{\delta^{ab}}{N_A}$



projector: $\left(\frac{1}{D-1}\, g^{\mu\nu} \cdot \mathsf{P}_{\mathsf{trans}} + q^\mu q^\nu \cdot \mathsf{P}_{\mathsf{long}}\right) \quad \frac{\delta^{ab}}{N_A}$

Computation of Renormalization constants: massive approach

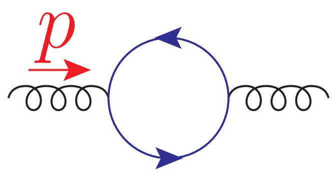
[Misiak and Münz (1995); Larin, van Ritbergen, Vermaseren (1996); Chetyrkin, Misiak and Münz (1998)]

- Introduce the same auxiliary mass M^2 in every propagator denominator:

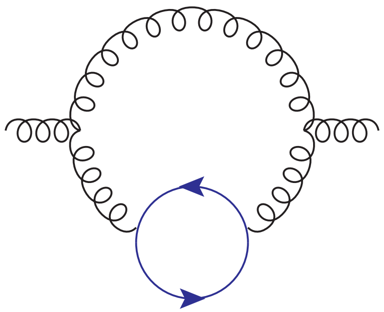
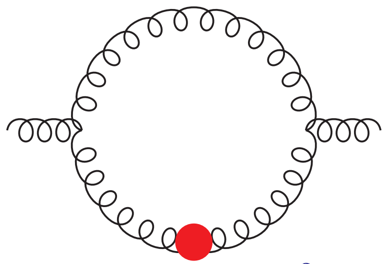
  $\frac{1}{(q+p)^2} \to \frac{1}{(q+p)^2 - M^2}$. ($p$ external, q internal momenta)

- Expand in external momenta p in order to factor out tree-level momentum structure, e. g. $\not{p},\ p^\mu,\ p^2,\ (p^\mu p^\nu - p^2 g^{\mu\nu}) \Rightarrow$ **massive tadpole integrals with one scale M^2.**

- Introduce M^2-counterterm $\boxed{\frac{M^2}{2}\,\delta Z_{M^2}\,(A^a_\mu A^{a\,\mu})}$ to cancel subdivergences $\propto M^2$:



$$= \frac{(p^\mu p^\nu - p^2 g^{\mu\nu})}{\varepsilon} C_{p^2} + \frac{M^2 g^{\mu\nu}}{\varepsilon} C_{M^2}$$
$+$ finite

$\rightarrow$  $+$ 

$$M^2 \delta Z_{M^2} = -\frac{M^2}{\varepsilon} C_{M^2}$$

$\Rightarrow$ only UV part computed correctly!

Automation

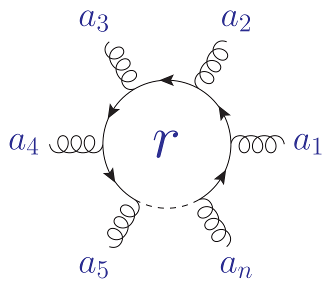
- Generation of diagrams $\rightarrow$ QGRAF [Noguira]

- gauge group factors $\rightarrow$ COLOR [van Ritbergen, Schellekens, Vermaseren] extended to allow for different fermion representations [M.Z.]

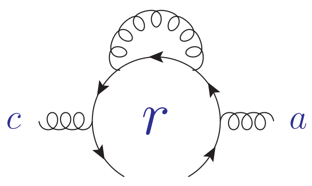
- Feynman integrals:

  - find topologies $\rightarrow$ Q2E, EXP [Seidesticker, Harlander, Steinhauser]
  - Feynman rules, projectors, counterterms, fermion traces, expansion in external momenta $\rightarrow$ FORM [Vermaseren] code [Chetyrkin, M.Z.]
  - massive tadpole integrals $\rightarrow$ up to 3 loop: MATAD [Steinhauser];
    Reduction at 4 loop: FIRE5 (C++ version) [Smirnov] (based on IBP)
    $\Rightarrow$ use 19 Master integrals [Czakon et al]
  - L-loop UV-part for fermion and ghost propagators and vertices with full ξ-dependence from the UV and finite part of massless (L-1)-loop propagators [Chetyrkin]; computed with MINCER [Vermaseren]

The COLOR algorithm [Van Ritbergen, Schellekens, Vermaseren] extended

- 

$\rightarrow$ $\mathbf{Tr}\left(T^{a_1,r}T^{a_2,r}\dots T^{a_n,r}\right)$

- simplify using relations like $\left(\right.$ 

$\left.\right) = T^{a,r}T^{b,r}T^{b,r}T^{c,r} = C_{F,r}T^{a,r}T^{c,r}$

- Write result in terms of symmetrized traces $\boxed{d_r^{a_1 a_2 \dots a_n} = \frac{1}{n!}\sum_\pi \mathrm{Tr}\left\{T^{a_{\pi(1)},r}T^{a_{\pi(2)},r}\dots T^{a_{\pi(n)},r}\right\}}$:

  $$\mathbf{Tr}\left(T^{a_1,r}\dots T^{a_n,r}\right) = \mathbf{Tr}\left(T^{a_1,r}\dots T^{a_n,r}\right) - \frac{1}{n!}\sum_\pi \underbrace{\mathrm{Tr}\left\{T^{a_{\pi(1)},r}\dots T^{a_{\pi(n)},r}\right\}} + d_r^{a_1\dots a_n}$$

  use $\left[T^{a,r},T^{b,r}\right] = i\, f^{abc}\,T^{c,r}$ to write as $\mathbf{Tr}\left(T^{a_1,r}\dots T^{a_n,r}\right)$ + shorter traces

  $\Rightarrow$ cancel $\mathbf{Tr}\left(T^{a_1,r}\dots T^{a_n,r}\right)$.

  $\Rightarrow$ Recursively substitute traces by $d_r^{a_1\dots a_n}$ and f^{abc}, e. g. $\mathbf{Tr}\left(T^{a_1,r}T^{a_2,r}T^{a_3,r}\right) = d_r^{a_1 a_2 a_3} + \frac{i}{2}f^{a_1 a_2 a_3}T_{F,r}$;
  continue to draw out $C_{F,r}, T_{F,r}, C_A$ if possible.

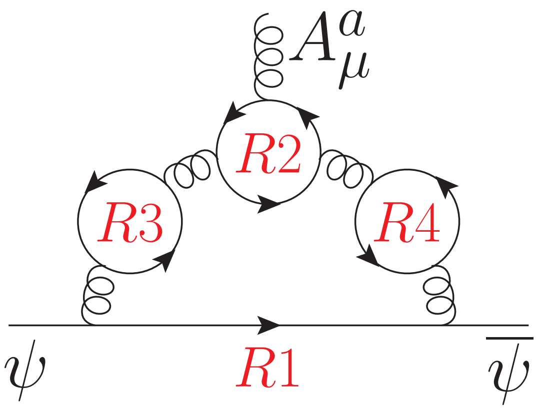
- Adjoint traces with $(T^{a,A})_{bc} = -i f^{abc}$ reduced via the same procedure and the relation

  $$f^{a_1\,ab}f^{a_2\,bc}f^{a_3\,ca} = \frac{C_A}{2}f^{a_1 a_2 a_3}.$$

$\Rightarrow$ Final result in terms of invariants $C_{F,r}, T_{F,r}, C_A$ and contractions of the $d_r^{a_1\dots a_n}$, e. g. $d_{AA}^{(4)} = \frac{d_A^{abcd}d_A^{abcd}}{N_A}$.

Color factors for several fermion representations

1. Use one field A for the adjoint representation and one field ψ for all the fermion representations in QGRAF. Number of diagrams to be computed same as in QCD!

2. 

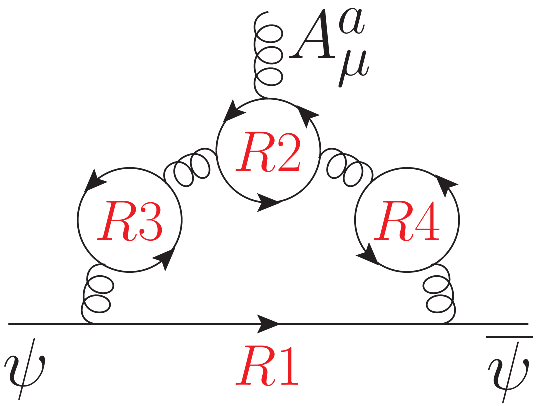
For each fermion line use a separate "line representation" $T^{a,R1}, T^{a,R2}, T^{a,R3}, T^{a,R4}$.

After evaluation with COLOR substitute color factor from each fermion loop by sum over all physical representations and flavours:

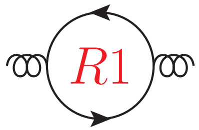
Tr $\left(T^{a_1,R1} T^{a_2,R1} \ldots T^{a_n,R1}\right) \to \sum_r n_{f,r}$ **Tr** $\left(T^{a_1,r} T^{a_2,r} \ldots T^{a_n,r}\right)$.

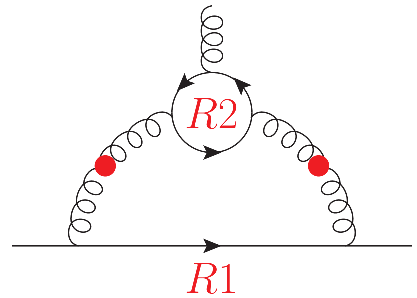
Renormalization procedure

2.a)



Trick for cancellation of subdivergences: Use explicit sum of group representations (as many as can appear in a single diagram), e. g. $\boxed{T_{F,R_i} \to n_{f,1}T_{F,1} + n_{f,2}T_{F,2} + n_{f,3}T_{F,3} + n_{f,4}T_{F,4}}$.

This ensures that e. g. 1-loop counterterm from  inserted in $\to$



cancels the subdivergencies of the $R3$ and $R4$ loop in the above diagram.

2.b) After cancellation of subdivergencies by lower-loop diagrams with counterterm insertions collect color factors in sums of terms over all physical representations r ,

e. g. $\boxed{n_{f,1}T_{F,1} \to \sum_r n_{f,r}T_{F,r} - n_{f,2}T_{F,2} - n_{f,3}T_{F,3} - n_{f,4}T_{F,4}}$.

IV. Results: β_{α_s} with different fermion representations

$$
\begin{aligned}
\beta^{(1)}_{\alpha_s} &= -\frac{11}{3}C_A + \frac{4}{3}\sum_i n_{f,i}T_{F,i} \\
&\vdots \\
\beta^{(4)}_{\alpha_s} &= -\left(\frac{150653}{486} - \frac{44}{9}\zeta_3\right)C_A^4 + \left(\frac{80}{9} - \frac{704}{3}\zeta_3\right)\frac{d_A^{abcd}d_A^{abcd}}{N_A} \\
&+ \sum_i n_{f,i}\Bigg\{T_{F,i}\Bigg[-46C_{F,i}^3 + \left(\frac{4204}{27} - \frac{352}{9}\zeta_3\right)C_AC_{F,i}^2 - \left(\frac{7073}{243} - \frac{656}{9}\zeta_3\right)C_A^2C_{F,i} \\
&\quad + \left(\frac{39143}{81} - \frac{136}{3}\zeta_3\right)C_A^3\Bigg] - \left(\frac{512}{9} - \frac{1664}{3}\zeta_3\right)\frac{d_{F,i}^{abcd}d_A^{abcd}}{N_A}\Bigg\} \\
&+ \sum_{i,j} n_{f,i}n_{f,j}\Bigg\{T_{F,i}T_{F,j}\Bigg[-\left(\frac{184}{3} - 64\zeta_3\right)C_{F,i}C_{F,j} + \left(\frac{304}{27} + \frac{128}{9}\zeta_3\right)C_{F,i}^2 \\
&\quad - \left(\frac{17152}{243}C_AC_{F,i} + \frac{448}{9}\zeta_3\right)C_AC_{F,i} - \left(\frac{7930}{81} + \frac{224}{9}\zeta_3\right)C_A^2\Bigg] \\
&\quad + \left(\frac{704}{9} - \frac{512}{3}\zeta_3\right)\frac{d_{F,i}^{abcd}d_{F,j}^{abcd}}{N_A}\Bigg\} \\
&- \sum_{i,j,k} n_{f,i}n_{f,j}n_{f,k}T_{F,i}T_{F,j}T_{F,k}\left[\frac{1232}{243}C_{F,i} + \frac{424}{243}C_A\right]
\end{aligned}
$$

i, j, k summed over all fermion representations of the gauge group.

in $\overline{\text{MS}}$-scheme

Anomalous dimension $\gamma_2^{(q,r)}$ of fermion of flavour q in representation r:

$$\left(\gamma_2^{(q,r)}\right)^{(1)} = C_{F,r}$$

$$\vdots$$

$$\begin{aligned}
\left(\gamma_2^{(q,r)}\right)^{(4)} = & -C_{F,r}^4\left(\frac{1027}{8}+400\zeta_3-640\zeta_5\right)+C_A C_{F,r}^3\left(\frac{5131}{12}+848\zeta_3-1440\zeta_5\right) \\
& -C_A^2 C_{F,r}^2\left(\frac{23777}{36}+214\zeta_3+66\zeta_4-790\zeta_5\right)+C_A^3 C_{F,r}\left(\frac{10059589}{15552}\right. \\
& \left.-\frac{1489}{24}\zeta_3+\frac{173}{4}\zeta_4-\frac{1865}{12}\zeta_5\right)-\frac{d_{F,r}^{abcd}d_A^{abcd}}{d_{F,r}}\left(66-190\zeta_3+170\zeta_5\right) \\
& +\sum_i n_{f,i}\left\{T_{F,i}C_{F,r}\left[3C_{F,i}^2+C_{F,r}C_{F,i}\left(62-48\zeta_3\right)-C_{F,r}^2\left(\frac{119}{3}+16\zeta_3\right)\right.\right. \\
& -C_A C_{F,i}\left(\frac{2945}{12}-156\zeta_3-12\zeta_4\right)+C_A C_{F,r}\left(\frac{1607}{9}-112\zeta_3+24\zeta_4\right. \\
& \left.\left.+160\zeta_5\right)-C_A^2\left(\frac{1365691}{3888}+\frac{119}{3}\zeta_3+25\zeta_4+80\zeta_5\right)\right]+128\left.\frac{d_{F,r}^{abcd}d_{F,i}^{abcd}}{d_{F,r}}\right\} \\
& -\sum_{i,j}n_{f,i}n_{f,j}T_{F,i}T_{F,j}C_{F,r}\left[\frac{92}{9}C_{F,r}-C_{F,j}\left(44-32\zeta_3\right)\right. \\
& \left.-C_A\left(\frac{6835}{243}+\frac{112}{3}\zeta_3\right)\right]+\frac{280}{81}C_{F,r}\sum_{i,j,k}n_{f,i}n_{f,j}n_{f,k}T_{F,i}T_{F,j}T_{F,k}
\end{aligned}$$

Here $\xi=0$. Full ξ-dependence (in Mathematica file) $\rightarrow$ source files of arXiv:1704.04209

V. Summary

- Complete set of 4 loop renormalization constants, anomalous dimensions and β-function for any simple gauge group model with one coupling and any number of fermion representations presented, full ξ-dependence.

- Color factors in these results make contributions of individual diagram topologies more transparent as compared to QCD: Each fermion line comes with a gauge group representation index ($C_F \to C_{F,i}$ etc) summed over in loops and free in open lines.

- Application in a large group of SM extensions, e. g. in supersymmetric models.
  Example: QCD plus gluinos.

- COLOR [van Ritbergen, Schellekens, Vermaseren] package (FORM) extended to allow for any number of fermion representations of the gauge group $\to$ Can be applied in other calculations.

BACKUP

Motivation of calculation of UV part with massive tadpoles

Exact decomposition of all propagator denominators:

$$\frac{1}{(q+p)^2} = \frac{1}{q^2 - M^2} + \frac{-p^2 - 2q{\cdot}p - M^2}{q^2 - M^2}\frac{1}{(q+p)^2}$$

(p external, q internal momenta)

Recursion:

$$\begin{aligned}\frac{1}{(q+p)^2} =& \frac{1}{q^2 - M^2} + \frac{-p^2 - 2q{\cdot}p}{(q^2 - M^2)^2} + \frac{(-p^2 - 2q{\cdot}p)^2}{(q^2 - M^2)^3} \\ & - \frac{M^2}{(q^2 - M^2)^2} + \frac{M^2(M^2 + 2p^2 + 4q{\cdot}p)}{(q^2 - M^2)^3} \\ & + \frac{(-p^2 - 2q{\cdot}p - M^2)^3}{(q^2 - M^2)^3}\frac{1}{(q+p)^2}\end{aligned}$$

Result independent of $M^2 \Rightarrow$ omit terms $\sim M^2$;

Introduce counterterms to cancel M^2 dependent subdivergences.