IBP and differential equations from unitarity

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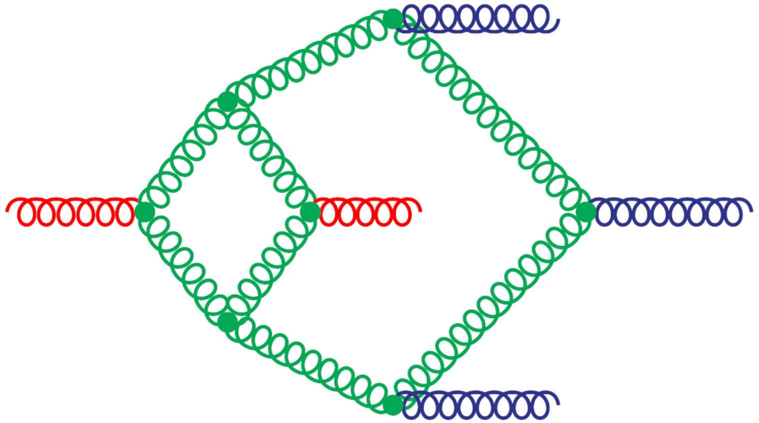


arXiv:1702.02355, *MZ*

arXiv: 1703.05273, *Abreu, Febres-Cordero, Ita, Jacquier, Page, MZ*

In progress, *Ita, Bern, MZ*

Outline

- Motivations
- Generalized unitarity: the *universal tool*
  ✓Loop Integrand ✓Integration by parts ✓Differential equations
- IBP from dual conformal symmetry
- Differential equations from unitarity cuts

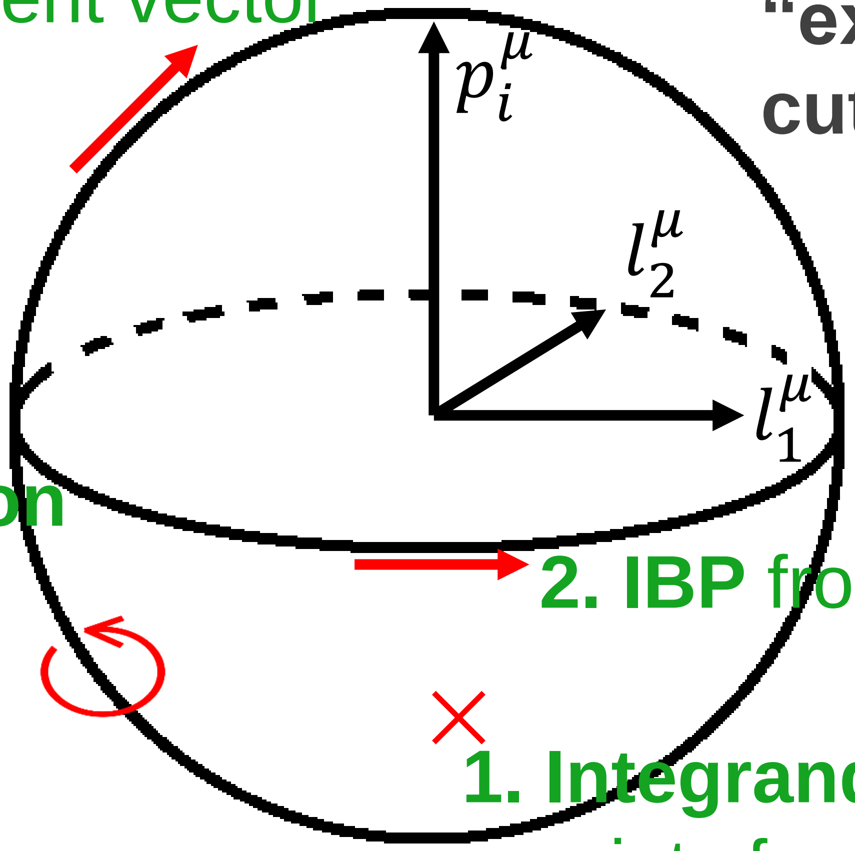
Motivations

- At colliders, NNLO QCD reaching maturity for massless $2 \to 2$.
- Current frontier: more mass scales / higher multiplicity.
- Does generalized unitarity help?
  1-loop: huge success in integrand construction.
  Higher loops: integration challenge

Unitarity beyond integrands

3. DE from tangent vector
$\chi^\mu \partial_{p^\mu} + v^\mu \partial_{l^\mu}$

MZ '17
Frellesvig, Papadopoulos '17
Basis choice: Henn '14



"extended" unitarity cut surface in (p^μ, l^μ):

$$z_i = (l - q_i)^2 = 0$$

Gudja, Kluza, Kosower '10
Ita '15
Larsen, Zhang '15

2. IBP from tangent vector $v^\mu \partial_{l^\mu}$

4. Direct integration on contours

e.g. Kosower, Larsen '11
Primo, Tancredi '16 '17
Bosma, Sogaard, Zhang '17
Abreu, Britto, Duhr, Gardi '17; Schabinger '17

1. Integrands from sampling fixed points for factorized tree amps

Integration by parts w/o doubled propagators

- Total divergences integrate to 0. Doubled propagators present.

$$\int d^d l \, \frac{\partial}{\partial l^\mu} \left(\frac{v^\mu(l,p)}{\prod_i z_i} \right) = 0$$

Software: FIRE, Reduze, LiteRed, AIR, Kira…

- $(1/z_i)^2$ killed by the special condition

$$v^\mu \frac{\partial z_i}{\partial l^\mu} = z_i \cdot f_i(l,p),$$

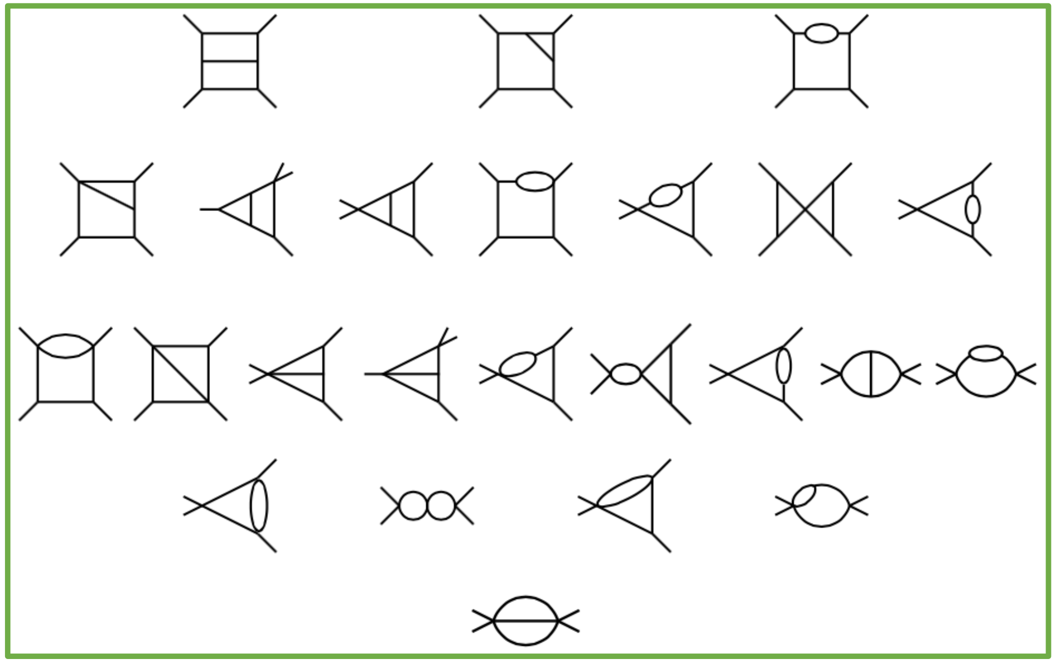
Computational algebraic geometry:
Gluza, Kadja, Kosower '10
Larsen, Zhang '15
Chen, Liu, Xie, Zhang, Zhou '15
Linear algebra: Schabinger '11
Analytic understanding: Ita '15

- $v^\mu \partial_\mu$: tangent vector to unitarity cut surface $z_i = 0$.

2-loop application: $gg \to gg$ in BlackHat 2

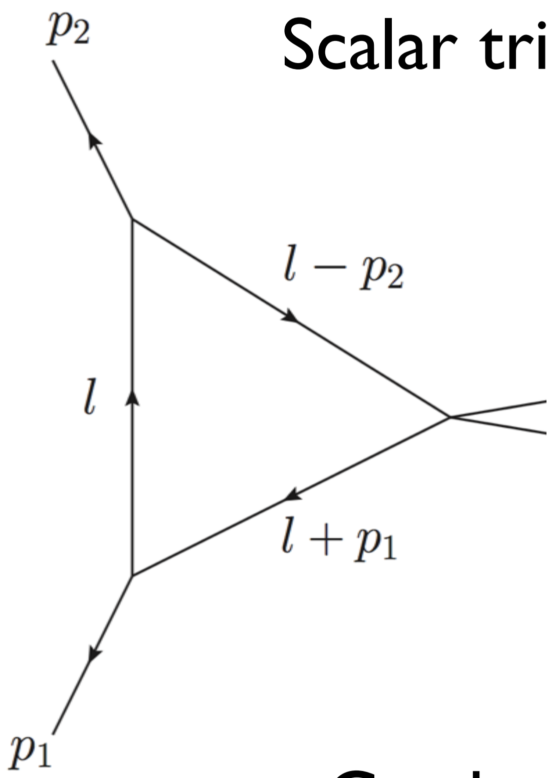
Talks by Ben Page, Harald Ita [Abreu, Febres-Cordero, Ita, Jacquier, Page, MZ '17]

- **Computational algebraic geometry** finds tangent vectors.
- Integrand = **master integrands** + **surface terms**
- Efficient **tree-amplitude engine** gives cut residues. Fit integrand by linear system (LAPACK)

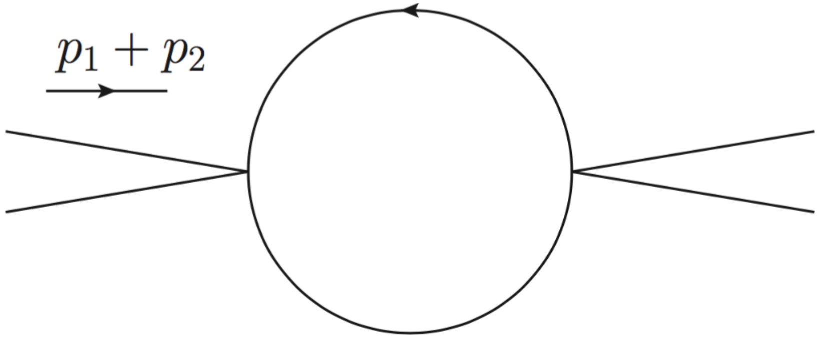


IBP from symmetry: triangle

Scalar triangle present in OPP / PV, but reducible by IBP



$$= \frac{1-2\epsilon}{\epsilon} \cdot \frac{1}{s}$$



Can be obtained from a symmetry principle

Extra symmetry: *dual conformal*

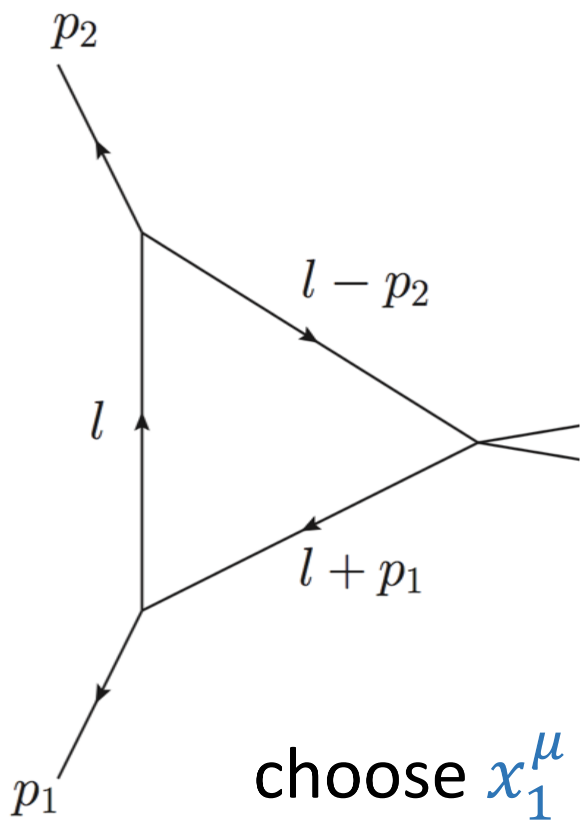
- Amplitude-Wilson loop duality in $\mathcal{N}=4$ SYM.

  [Drummond, Henn, Korchemsky, Sokatchev '07]

- Extends to many planar Feynman integrals in even-integer dimensions.
  [Drummond, Henn, Smirnov, Sokatchev '06; Henn, Naculich, Schnitzer, Spradlin '10, Henn '11, Caron-Huot, Henn '14; Broadhurst '93]

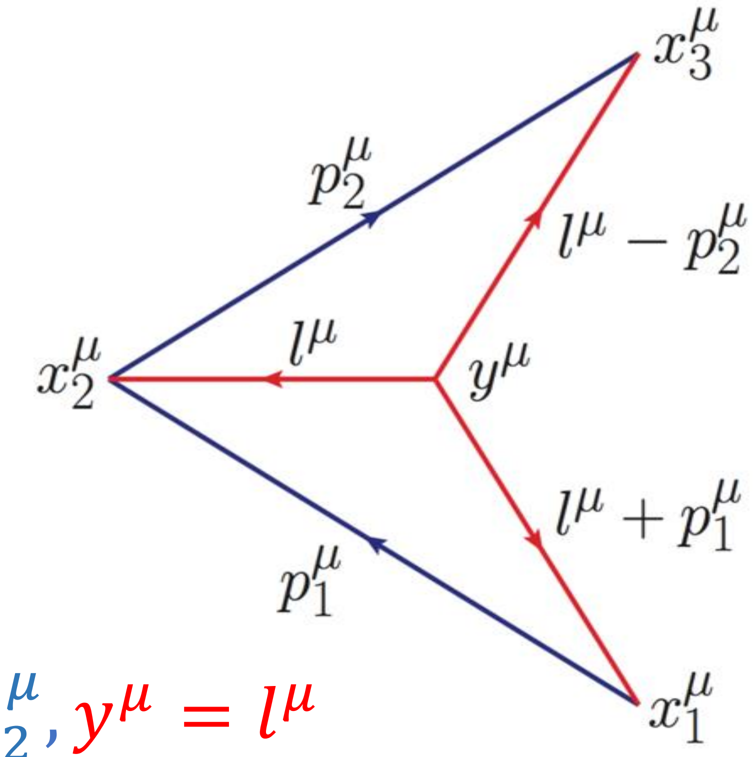
- **Subgroup leaving external p_i^μ invariant:** [Bern, Ita, MZ, in progress]
  a symmetry of planar unitarity cut surfaces in arbitrary dimensions.

Extra symmetry: *dual conformal*



$$p_i^\mu = x_{i+1}^\mu - x_i^\mu$$

$$l^\mu - q_i^\mu = x_{i+1}^\mu - x_i^\mu$$



choose $x_1^\mu = -p_1^\mu, x_2^\mu = 0^\mu, x_3^\mu = p_2^\mu, y^\mu = l^\mu$

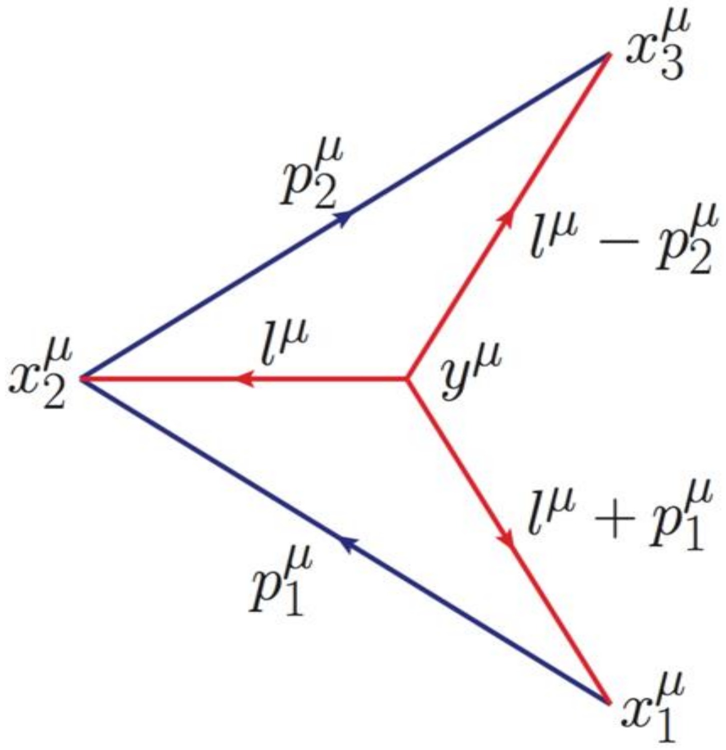
Triangle-bubble relation

- Cut condition $(l - q_i)^2 = 0 \leftrightarrow$ **null-separated** points. Preserved by (dual) conformal transformations.

- If *external momenta unchanged*, symmetry of cut surface! One solution:

$$\delta x^\mu = x^\mu + \frac{1}{2} b^\mu x^2 - (b \cdot x) x^\mu$$

$$b^\mu = \frac{2}{s}(p_1^\mu - p_2^\mu)$$

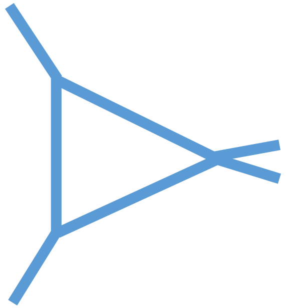
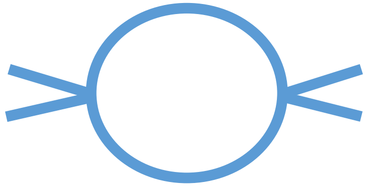


Triangle-bubble relation

- IBP vector

$$v^\mu = \frac{l^2}{s}(p_1^\mu - p_2^\mu) + l^\mu \left(1 - \frac{2}{s} l \cdot (p_1 - p_2)\right)$$

- IBP relation

$$0 = \int d^d l \, \frac{\partial v^\mu}{\partial l^\mu} = (d-4)$$  $$+ 2(d-3)$$ 

- Method extends to higher loops!

Double box IBP vectors

Computational algebraic geometry result

Gluza, Kadja, Kosower '10

$$
\begin{aligned}
v_{1;1} &= -2(k_4\cdot\ell_1 + \ell_1^2)k_1^\mu - \ell_1^2 k_2^\mu + (2k_1\cdot\ell_1 - \ell_1^2)k_4^\mu + (4k_1\cdot\ell_1 + 2k_2\cdot\ell_1 + 2k_4\cdot\ell_1 - s_{12})\ell_1^\mu\,, \\
v_{1;2} &= 2(\ell_2^2 - k_4\cdot\ell_2)k_1^\mu + \ell_2^2 k_2^\mu + (2k_1\cdot\ell_2 + \ell_2^2)k_4^\mu + (2k_3\cdot\ell_2 - 2k_1\cdot\ell_2 - s_{12})\ell_2^\mu\,; \qquad (5.8)
\end{aligned}
$$

and two solutions with coefficients of engineering dimension four,

$$
\begin{aligned}
v_{2;1} = &\,(-4k_2\cdot\ell_1 k_4\cdot\ell_1 - 4k_3\cdot\ell_2\ell_1^2 + 4k_4\cdot\ell_1\ell_1^2 - 4k_4\cdot\ell_2\ell_1^2 - 4\ell_1^2\ell_1\cdot\ell_2 - 2\ell_1^2\ell_2^2 - 2\chi_{14}\ell_1^2 s_{12})k_1^\mu \\
&+(4k_1\cdot\ell_1 k_4\cdot\ell_1 - 2k_1\cdot\ell_1\ell_1^2 - 2k_2\cdot\ell_1\ell_1^2 - 4k_3\cdot\ell_2\ell_1^2 - 4k_4\cdot\ell_2\ell_1^2 - 4\ell_1^2\ell_1\cdot\ell_2 - 2\ell_1^2\ell_2^2 \\
&+2\ell_1^2 s_{12} - 2\chi_{14}\ell_1^2 s_{12})k_2^\mu + (-4k_1\cdot\ell_1\ell_1^2 - 4k_2\cdot\ell_1\ell_1^2 + 2(\ell_1^2)^2 + 2\ell_1^2 s_{12})k_4^\mu \\
&+(4k_1\cdot\ell_1 k_2\cdot\ell_1 + 4(k_2\cdot\ell_1)^2 + 8k_1\cdot\ell_1 k_3\cdot\ell_2 + 8k_2\cdot\ell_1 k_3\cdot\ell_2 + 8k_2\cdot\ell_1 k_4\cdot\ell_1 + 8k_1\cdot\ell_1 k_4\cdot\ell_2 \\
&+8k_2\cdot\ell_1 k_4\cdot\ell_2 - 4k_4\cdot\ell_1\ell_1^2 + 8k_1\cdot\ell_1\ell_1\cdot\ell_2 + 8k_2\cdot\ell_1\ell_1\cdot\ell_2 + 4k_1\cdot\ell_1\ell_2^2 + 4k_2\cdot\ell_1\ell_2^2 \\
&-4k_1\cdot\ell_1 s_{12} - 6k_2\cdot\ell_1 s_{12} - 4k_3\cdot\ell_2 s_{12} - 2k_4\cdot\ell_1 s_{12} - 4k_4\cdot\ell_2 s_{12} + \ell_1^2 s_{12} + 2\chi_{14}\ell_1^2 s_{12} \\
&-4\ell_1\cdot\ell_2 s_{12} - 2\ell_2^2 s_{12} + 2s_{12}^2)\ell_1^\mu\,, \qquad (5.9) \\
v_{2;2} = &\,(4k_1\cdot\ell_2 k_4\cdot\ell_1 + 4k_3\cdot\ell_2 k_4\cdot\ell_1 + 4k_4\cdot\ell_1 k_4\cdot\ell_2 - 4k_4\cdot\ell_2\ell_1\cdot\ell_2 + 4k_3\cdot\ell_2\ell_2^2 - 4k_4\cdot\ell_1\ell_2^2 \\
&+6\ell_1\cdot\ell_2\ell_2^2 + 4(\ell_2^2)^2 - 2\ell_1\cdot\ell_2 s_{12} - 2\chi_{14}\ell_1\cdot\ell_2 s_{12} - 2\ell_2^2 s_{12})k_1^\mu + (4k_1\cdot\ell_2 k_4\cdot\ell_1 \\
&-4k_4\cdot\ell_2\ell_1\cdot\ell_2 + 2k_1\cdot\ell_1\ell_2^2 + 2k_2\cdot\ell_1\ell_2^2 + 4k_3\cdot\ell_2\ell_2^2 + 6\ell_1\cdot\ell_2\ell_2^2 + 4(\ell_2^2)^2 - 2\chi_{14}\ell_1\cdot\ell_2 s_{12} \\
&-2\ell_2^2 s_{12})k_2^\mu + (4k_1\cdot\ell_1\ell_2^2 + 4k_2\cdot\ell_1\ell_2^2 + 4\ell_1\cdot\ell_2\ell_2^2 + 2(\ell_2^2)^2 - 2\ell_2^2 s_{12})k_4^\mu \\
&+(-4k_3\cdot\ell_2 k_4\cdot\ell_2 - 4(k_4\cdot\ell_2)^2 + 2k_3\cdot\ell_2\ell_2^2 + 2k_4\cdot\ell_2\ell_2^2 + 2k_1\cdot\ell_2 s_{12} - 2\chi_{14}k_3\cdot\ell_2 s_{12} \\
&-2\chi_{14}k_4\cdot\ell_2 s_{12} + \ell_2^2 s_{12} + 2\chi_{14}\ell_2^2 s_{12})\ell_1^\mu + (4k_1\cdot\ell_1 k_1\cdot\ell_2 + 4k_1\cdot\ell_2 k_2\cdot\ell_1 + 4k_1\cdot\ell_1 k_3\cdot\ell_2 \\
&+4k_2\cdot\ell_1 k_3\cdot\ell_2 + 8(k_3\cdot\ell_2)^2 + 8k_1\cdot\ell_2 k_4\cdot\ell_1 + 8k_3\cdot\ell_2 k_4\cdot\ell_2 + 8k_3\cdot\ell_2\ell_1\cdot\ell_2 - 2k_1\cdot\ell_1\ell_2^2 \\
&-2k_2\cdot\ell_1\ell_2^2 + 8k_3\cdot\ell_2\ell_2^2 + 4k_4\cdot\ell_2\ell_2^2 - 2k_1\cdot\ell_1 s_{12} - 2\chi_{14}k_1\cdot\ell_1 s_{12} - 2k_2\cdot\ell_1 s_{12} \\
&-2\chi_{14}k_2\cdot\ell_1 s_{12} - 8k_3\cdot\ell_2 s_{12} + 2k_4\cdot\ell_1 s_{12} - 4k_4\cdot\ell_2 s_{12} - 6\ell_1\cdot\ell_2 s_{12} - 4\chi_{14}\ell_1\cdot\ell_2 s_{12} \\
&-2\ell_2^2 s_{12} + 2s_{12}^2)\ell_2^\mu\,;
\end{aligned}
$$

Dual conformal symmetry result

Bern, Ita, MZ, in progress

$$
v_1 = X_2^{[A}X_4^{B]}y_A\partial_B\,,
$$

$$
v_2 = X_5^{[A}X_\perp^{B]}y_A\partial_B\,,
$$

X_i: projective points in the conformal embedding formalism

[e.g. Caron-Huot, Henn '14]

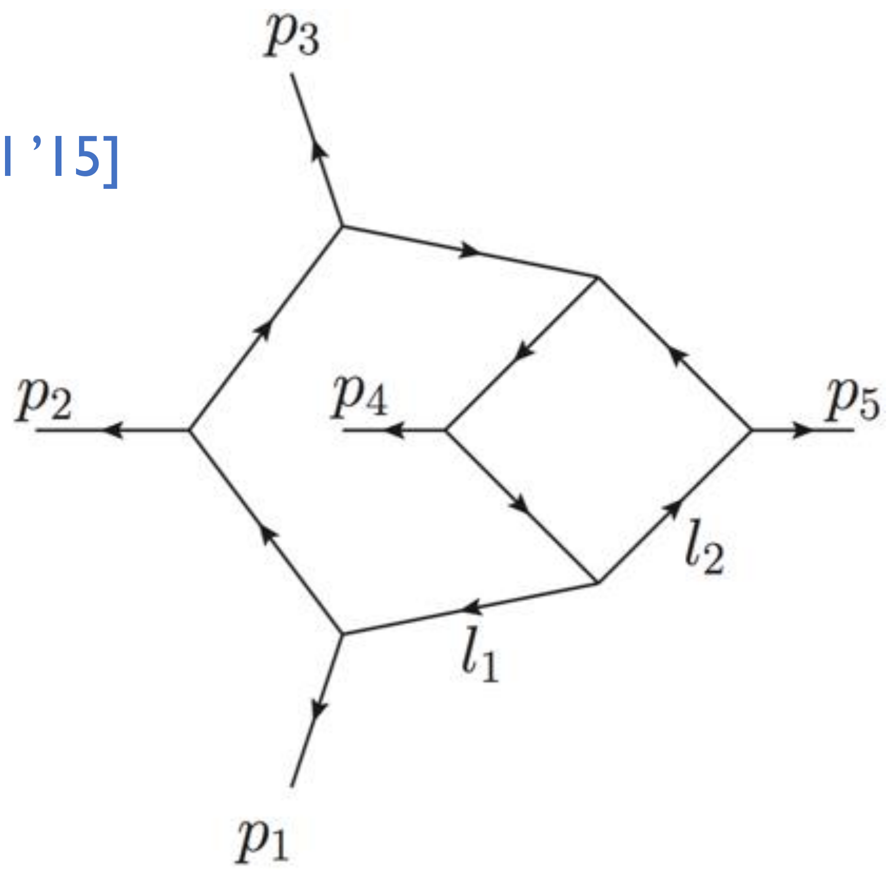
Huge simplification!

Differential Eqs:. Non-planar 5-point integral

- YM (+++++) integrand known. [Badger, Mogull, Ochirov, O'Connell '15] Analytic integrals not known.

- kinematic variables
  $s_{12},\ s_{23},\ s_{34},\ s_{45},\ s_{51}$

- Want DEs $\dfrac{\partial \vec{I}}{\partial s_{ij}} = \mathbb{M}^{(ij)}\,\vec{I}$



Unitarity-compatible differential equations

$$\beta^\mu \frac{\partial}{\partial p^\mu} \int d^d l \frac{1}{\prod_j z_j} = ?$$ Doubled propagators- IBP bottleneck?

- Free to add total derivative $$\int d^d l \left[\beta^\mu \frac{\partial}{\partial p^\mu} \frac{1}{\prod_j z_j} + \frac{\partial}{\partial l^\mu} \frac{v^\mu}{\prod_j z_j} \right]$$
- Kill doubled propagators $$\left(\beta^\mu \frac{\partial}{\partial p^\mu} + v^\mu \frac{\partial}{\partial l^\mu} \right) z_j = f_j z_j$$ [MZ '17]
- Speed up by finite field + rational reconstruction [Von Manteuffel, Schabinger '14 & '16, Peraro '16]

Canonical form: Leading singularity in Baikov rep. + *CANONICA*

[Christopher Meyer, arXiv:1705.06252]

Canonical DEs for nonplanar penta-box

$$d\vec{I} = \epsilon \sum_i \mathbb{M}_i \vec{I}\, d\log s_i$$

s_i: symbol letters

x_i: momentum twistor variables
c.f. Badger, Frellesvig, Zhang '13

$$s_1 = x_2 \qquad s_7 = -1 + x_4 + x_2 x_4 - x_2 x_5$$

$$s_2 = x_3 \qquad s_8 = 1 + x_2 x_5$$

$$s_3 = x_2 + x_3 \qquad s_9 = -x_3 + x_2 x_5 + x_3 x_5$$

$$s_4 = x_4 \qquad s_{10} = -x_3 + x_2 x_4 + x_3 x_4 + x_2 x_3 x_4 - x_2 x_3 x_5$$

$$s_5 = x_4 - x_5$$

$$s_6 = -1 + x_5 \qquad s_{11} = -x_3 + x_2 x_4 + 2 x_3 x_4 + x_2 x_3 x_4 - x_2 x_3 x_5 - x_2 x_4 x_5 - x_3 x_4 x_5$$

Canonical DEs for nonplanar penta-box

$$d\vec{I} = \epsilon \sum_i \mathbb{M}_i \vec{I}\, d\log s_i$$

$$\mathbb{M}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{pmatrix}, \quad \mathbb{M}_2 = \frac{1}{16}\begin{pmatrix} -8 & -64 & 0 \\ -3 & -24 & 0 \\ 1 & 8 & 0 \end{pmatrix}, \quad \mathbb{M}_3 = \frac{1}{16}\begin{pmatrix} -8 & 0 & 64 \\ -1 & 0 & 8 \\ 3 & 0 & -24 \end{pmatrix},$$

$$\mathbb{M}_4 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \mathbb{M}_5 = \frac{1}{4}\begin{pmatrix} 0 & -16 & 16 \\ -1 & -6 & -2 \\ 1 & -2 & -6 \end{pmatrix}, \quad \mathbb{M}_6 = \frac{1}{16}\begin{pmatrix} -8 & 64 & 0 \\ 3 & -24 & 0 \\ -1 & 8 & 0 \end{pmatrix},$$

$$\mathbb{M}_7 = \frac{1}{16}\begin{pmatrix} -8 & 0 & -64 \\ 1 & 0 & 8 \\ -3 & 0 & -24 \end{pmatrix}, \quad \mathbb{M}_8 = \frac{1}{4}\begin{pmatrix} 8 & -32 & 32 \\ -1 & 2 & -2 \\ 1 & -2 & 2 \end{pmatrix}, \quad \mathbb{M}_9 = \frac{1}{16}\begin{pmatrix} -8 & 0 & -64 \\ 1 & 0 & 8 \\ -3 & 0 & -24 \end{pmatrix},$$

$$\mathbb{M}_{10} = \frac{1}{16}\begin{pmatrix} -8 & 64 & 0 \\ 3 & -24 & 0 \\ -1 & 8 & 0 \end{pmatrix}, \quad \mathbb{M}_{11} = 2\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Outlook & Conclusions

- **Unitarity-compatible IBP:** need better algorithms & analytic understanding.
- Interesting IBP-generating vectors from **dual conformal symmetry**
- Unitarity-compatible **differential equations:** Breaking the Laporta bottleneck?
- Max. cut of **nonplanar penta-box:** multiple polylogs, **symbol alphabets**
- **Generalized unitarity** beyond integrands: at the dawn of many applications!

Thank you!

Feynman diagrams vs. Unitarity vs. Bootstrap

1. Traditional:

Feynman Diagrams → Loop integrand → Loop integral

Numerous QCD applications:
NLO revolution

2. Generalized unitarity: **abolish diagrams!**

factorization on cuts → Loop integrand

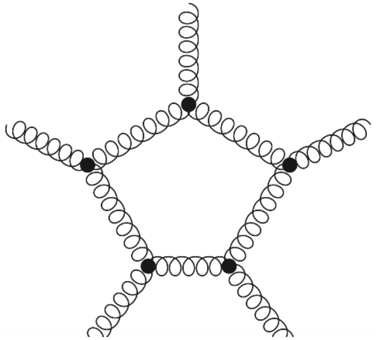
3. Bootstrap: **abolish loop integrands!**

[First application in QCD resummation: Ye Li, Duff Neill, Hua-Xing Zhu, '16]

function space + infrared limits → Loop integral

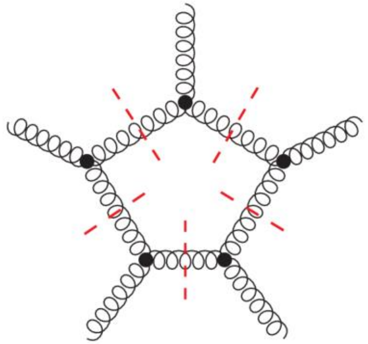
One-loop integrand from unitarity

[Bern, Dixon, Dunbar, Kosower; Britto, Cachazo, Feng; Ossola, Papadopoulos, Pittau; Ellis Kunszt, Melnikov, Zanderighi; Forde; Ita …]



$$= \int d^d l \frac{1}{z_1 z_2 z_3 z_4 z_5} \underbrace{\left[e_{12345} + D_{2345}(l) \cdot z_1(l) \ldots \right]}$$

Integrand decomposition: pentagon, box, triangle, bubble



$= e_{12345}$, integrand at $l = (l^0, l^1, l^2, l^3, \mu^2)$
s.t. $z_1 = z_2 = z_3 = z_4 = z_5 = 0$, i.e. $1/z_i \to \delta(z_i)$

Challenges beyond integrands

Loop Integrand → **Integration by parts**

$$\mathcal{I} = \sum c_i I_i$$

$$\mathcal{I} = \sum c_i I_i^{\text{master}} + \frac{\partial v^\mu}{\partial l^\mu}$$

Integrates to 0

Also applies to phase space integrals (reverse unitarity)

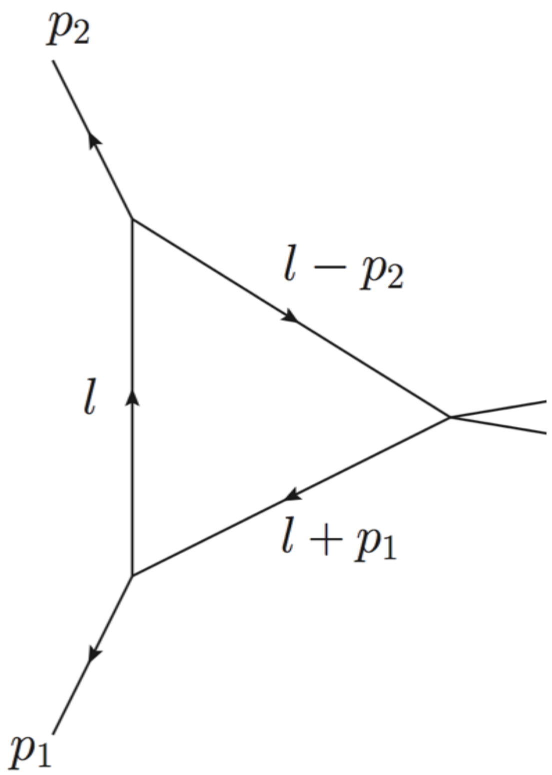
Loop integral ← **Differential equations**

$$\int d^d l \, \mathcal{I} =$$ polylogs, elliptic functions…

$$\frac{\partial}{\partial x} I_i^{\text{master}} = M_{ij} I_j^{\text{master}}$$

IBP from symmetry: triangle

[Ita'15]



- Cut surface has **rotation symmetry** in the $(d-2)$ components of l perp. to p_1 and p_2.

- **IBP vectors** = tangent vectors = rotation generators
- **Surface terms** = $\geq 1^{st}$ order spherical harmonics
- **Master integral** = 0^{th} order spherical harmonics, i.e. the scalar, 1

Triangle-bubble relation

- Conformal transformation

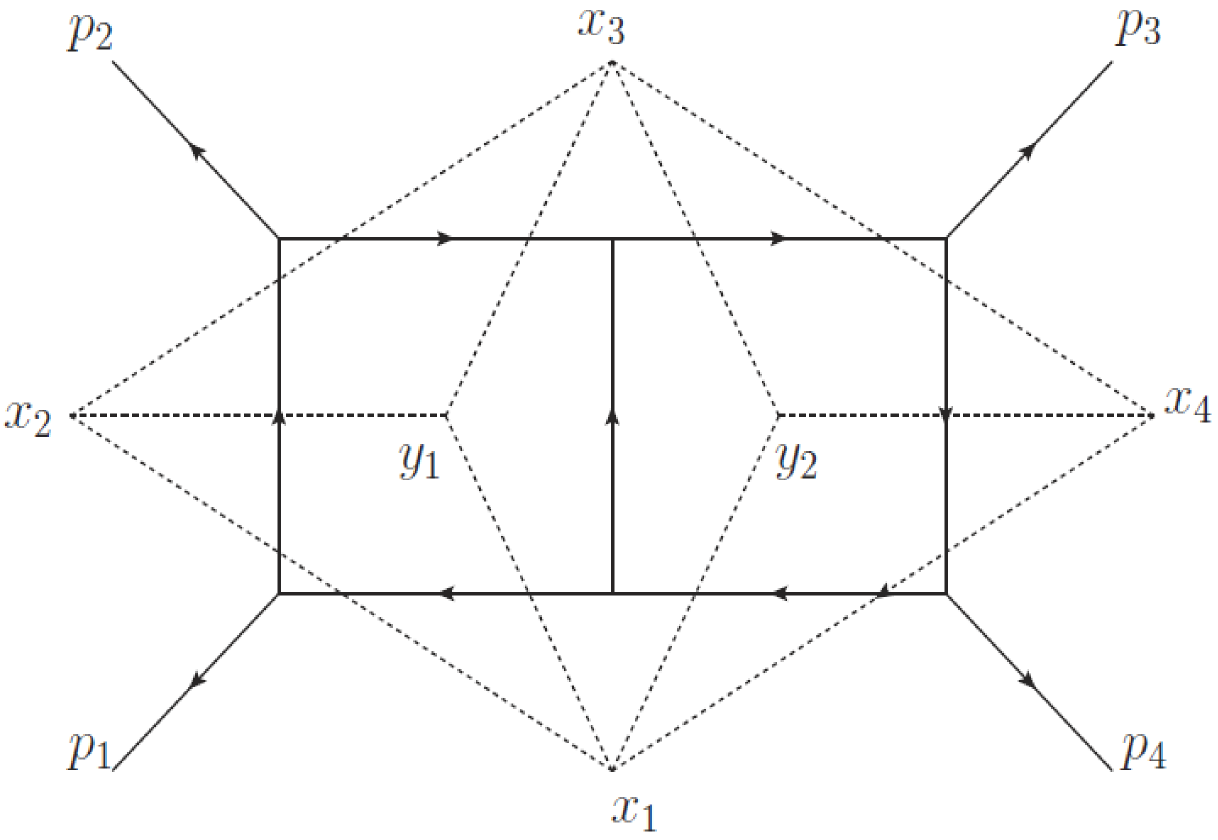
$$\delta x^\mu = x^\mu + \frac{1}{2} b^\mu x^2 - (b \cdot x) x^\mu$$

$$b^\mu = \frac{2}{s}(p_1^\mu - p_2^\mu)$$

- under which

$$\delta p_i^\mu = 0, \quad \delta l^\mu = v^\mu = \frac{l^2}{s}(p_1^\mu - p_2^\mu) + l^\mu \left(1 - \frac{2}{s} l \cdot (p_1 - p_2)\right)$$

Double box dual coordinates & embedding



$$X_i^a = \begin{pmatrix} x_i^\mu, \\ -x_i^2 \\ 1 \end{pmatrix}, I^a = \begin{pmatrix} 0^\mu \\ 1 \\ 0 \end{pmatrix}$$

$$X_\perp^a = \begin{pmatrix} l_{1\perp}^\mu \\ 0 \\ 0 \end{pmatrix}$$

X_5 : linear combination of X_i and I, s.t. $\perp$ to all X_i

DEs and the Baikov representation - box

$$\mathcal{I}_{\text{box}} = \int d^d l \frac{1}{\prod_{j=1}^{4} z_j}$$

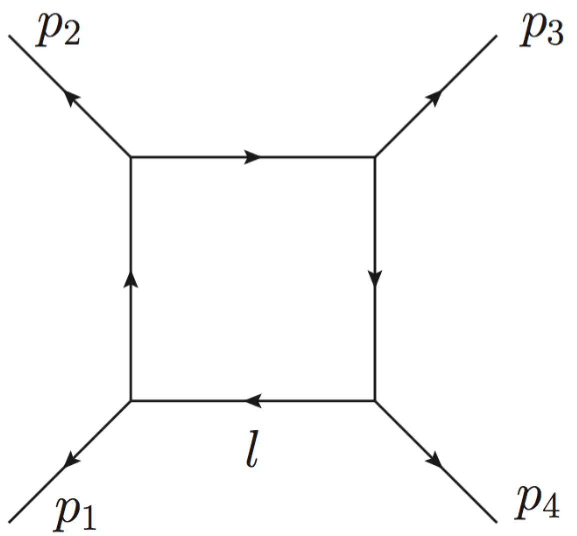
$$z_1 = l^2,$$

$$z_2 = (l - p_1)^2,$$

$$z_3 = (l - p_1 - p_2)^2,$$

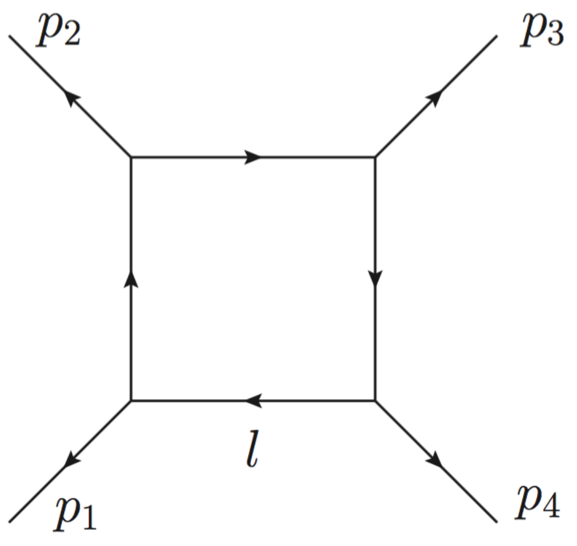
$$z_4 = (l - p_1 - p_2 - p_3)^2 = (l + p_4)^2$$

(clockwise)



DEs and the Baikov representation - box

Cutkosky: inverse propagators as loop integration variables ['60]
Further systemized by Baikov ['96]



$$\mathcal{I}_{\text{box}} \propto \frac{1}{\sqrt{st(s+t)}} \int \prod_{j=1}^{4} \frac{dz_j}{z_j} F^{(d-5)/2}$$

$$F = \frac{st}{4(s+t)} - \frac{1}{2(s+t)} \left[s(z_1 + z_3) + t(z_2 + z_4) \right] + \mathcal{O}(z_i^2)$$

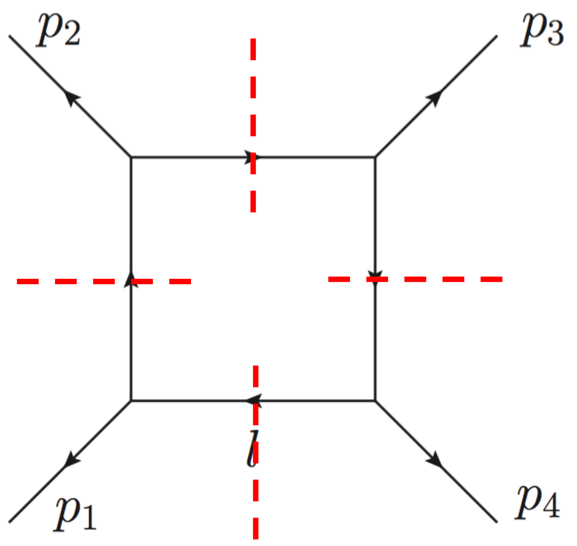
Leading singularity from the Baikov rep.

$$\mathcal{I}_{\text{box}} \propto \frac{1}{\sqrt{st(s+t)}} \int \prod_{j=1}^{4} \frac{dz_j}{z_j} F^{(d-5)/2}$$

[Henn '14] $1/z_j \to \delta(z_j)$

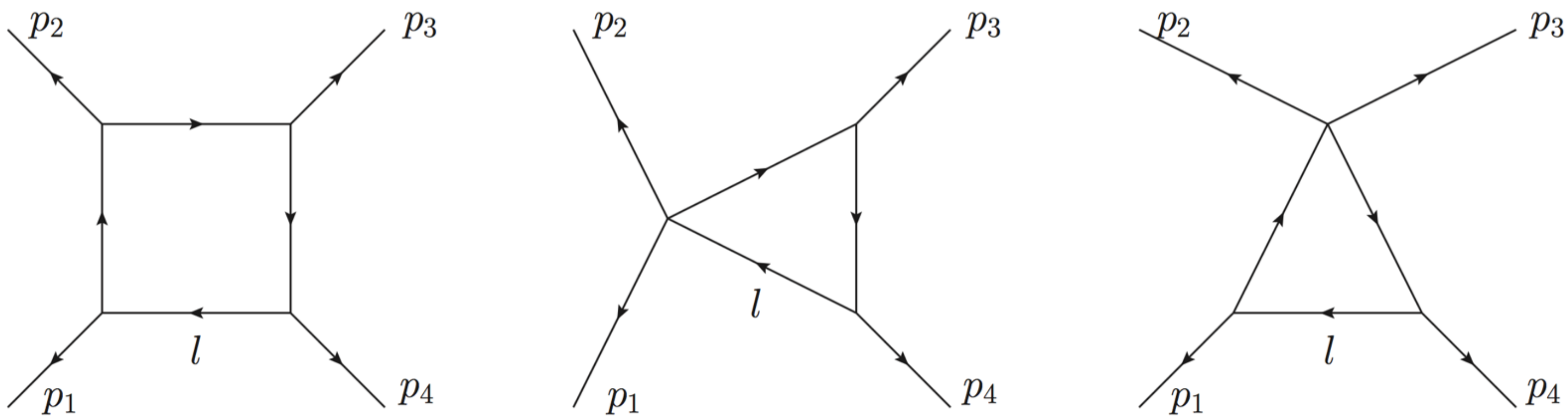
$$\mathcal{I}_{\text{box}}^{\text{cut}} \propto \frac{1}{\sqrt{st(s+t)}} [F(z_i = 0)]^{(d-5)/2}$$

$$\propto \frac{1}{st}, \quad \text{when } d = 4$$



Frellesvig, Papadopoulos '17
MZ '17
Bosma, Sogaard, Zhang '17

DEs for 1-loop box



$$\epsilon^2 st I_{\rm box} \propto \; 4 + \epsilon\left[-2\log x\right] + \epsilon^2\left[-\frac{4\pi^2}{3}\right] + \epsilon^3\left[\frac{7\pi^2}{6}\log x + \frac{1}{3}\log^3 x - \pi^2\log(1+x) - \log^2 x\log(1+x) - 2\log x\,{\rm Li}_2(-x) + 2{\rm Li}_3(-x) - \frac{34}{3}\zeta_3\right] + \mathcal{O}(\epsilon^4)\,,$$

[Henn lecture notes '14]

Maximal cut DEs

- Tangent vectors $\beta^\mu \frac{\partial}{\partial p^\mu} + v^\mu \frac{\partial}{\partial l^\mu}$ from SINGULAR.
- Uses unitarity-compatible IBP reduction.
- Computation techniques:
  Rational reconstruction: numerical $s_{ij} \to P_1(s_{ij})/P_2(s_{ij})$
  Extended Euclid algorithm: result in $\mathbb{Z}_p \longrightarrow$ result in $\mathbb{R}$
  [Von Manteuffel, Schabinger '14 & '16, Peraro '16]
- Canonical form: Leading singularity in Baikov rep. + *CANONICA*
  [Christopher Meyer, arXiv:1705.06252]

Sample results: nonplanar 5-point

$$\frac{\partial \vec{I}}{\partial s_{ij}} = \mathbb{M}^{(ij)} \vec{I}$$ + daughter integrals, for 3 top-level MIs

(28 hours × 1 CPU core) later... sample result

$$\mathbb{M}_{31}^{(23)} = \frac{(1+4\epsilon)\left(\chi_{23}-\chi_{45}+1\right)}{\chi_{45}\left(-\chi_{23}+\chi_{45}+\chi_{51}\right)} \left(\chi_{34}\chi_{23}-\chi_{23}-\chi_{34}\chi_{45}+2\chi_{45}+\chi_{45}\chi_{51}+\chi_{51}\right) / \left[\chi_{23}^2\left(\chi_{34}-1\right)^2+\left(\chi_{34}\chi_{45}-\left(\chi_{45}-1\right)\chi_{51}\right)^2+2\chi_{23} \left(-\chi_{45}\chi_{34}^2+\chi_{45}\chi_{34}+\chi_{45}\chi_{51}\chi_{34}+\chi_{51}\chi_{34}+\chi_{45}\chi_{51}-\chi_{51}\right)\right]$$

Canonical form: Leading singularity in Baikov rep. + *CANONICA*

Christopher Meyer, arXiv:1705.06252