# Asymptotic expansion of Feynman integrals in higher dimensions

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in preparation

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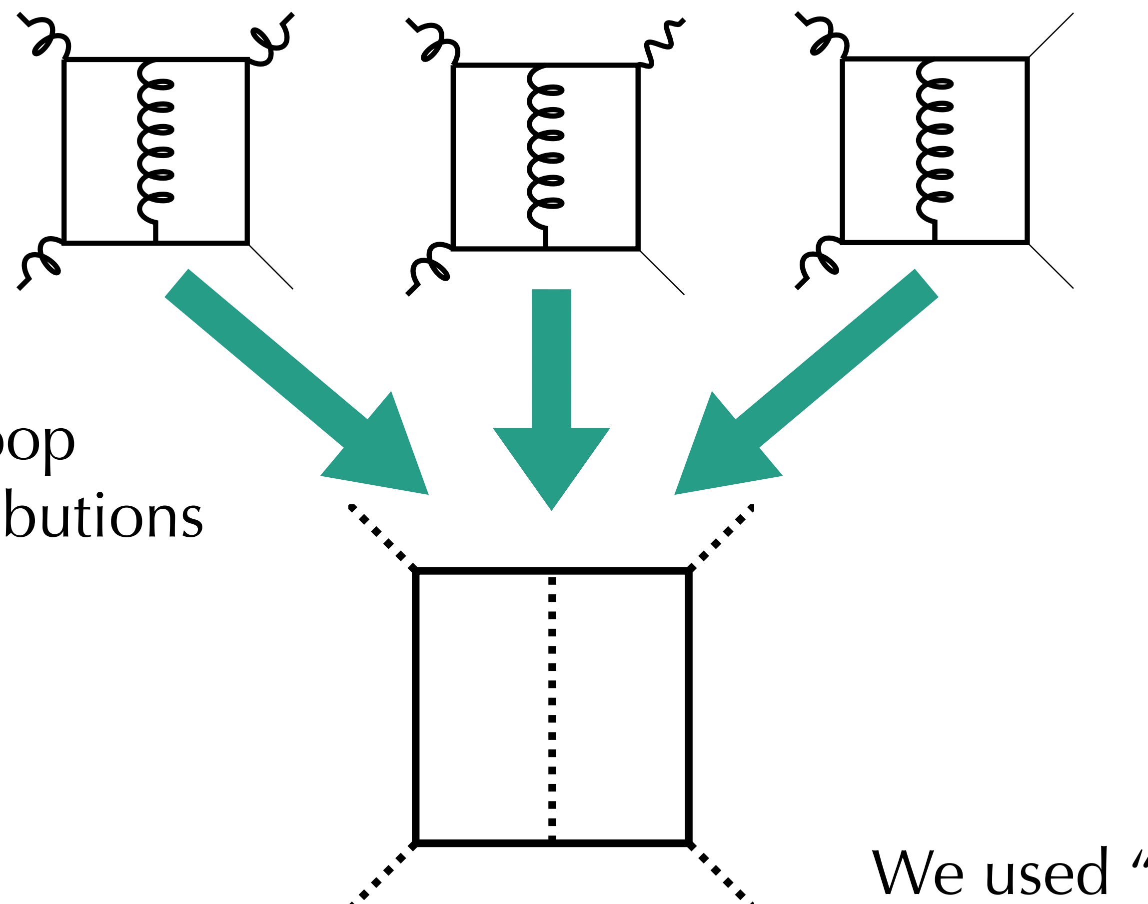
# Asymptotic expansion of Feynman integral

[Smirnov `90, Beneke, Smirnov '97, Smirnov `02, Jantzen `11]

is useful when (i) the integral is hard to solve due to multi-scale complexity
(ii) certain hierarchy in dimensionful parameters makes sense

Some phenomenologically interesting processes reduce to
the diagrams with massless external lines when $m_Z^2 \sim m_H^2 \sim m_t^2 \ll S \sim T \sim U$

$gg \to gH$ $\quad gg \to ZH$ $\quad gg \to HH$



top-loop contributions

i.e. expansion in the masses of **external** particles is usual Taylor expansion.
(not the case with internal particles)

expansion in $\dfrac{x}{X} \ll 1$

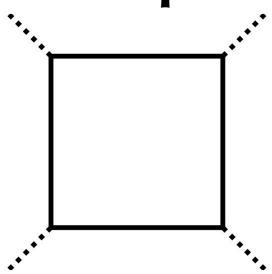
$$x = \{m_Z^2, m_H^2, m_t^2\}$$

$$X = \{S, T, U\}$$

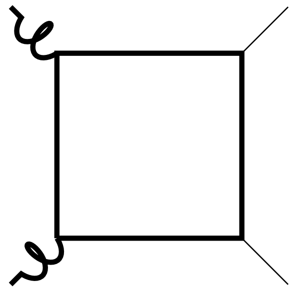
We used "asy.m" to verify this fact.
[Pak, Smirnov '10, Jantzen, Smirnov, Smirnov '12]

# Outline

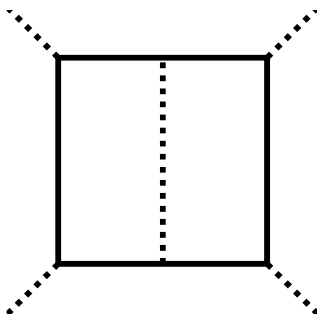
(1)Asymptotic expansion in higher dimensions: one loop case



(2)Application to physical process

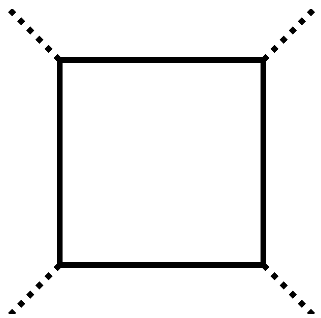


(3)two loop case



(4)Summary and outlook

# One loop case



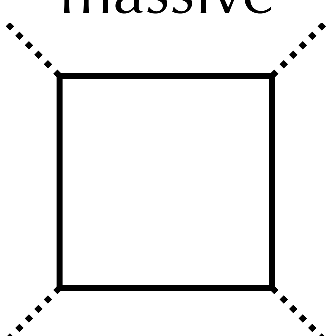
$$= \int Dk \frac{1}{(k^2 - m^2)\left((k+p_1)^2 - m^2\right)\left((k+p_1+p_2)^2 - m^2\right)\left((k+p_3)^2 - m^2\right)}$$

$$= \sum_{n=0}^{\infty} (m^2)^n f_n(s, t, \log m^2)$$

Naive expansion of the integrand like

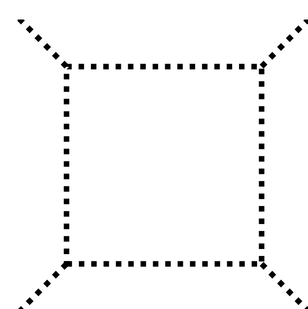
$$\frac{1}{k^2 - m^2} = \frac{1}{k^2} + \frac{m^2}{(k^2)^2} + \cdots$$ gives wrong result.

massive



is finite.

massless

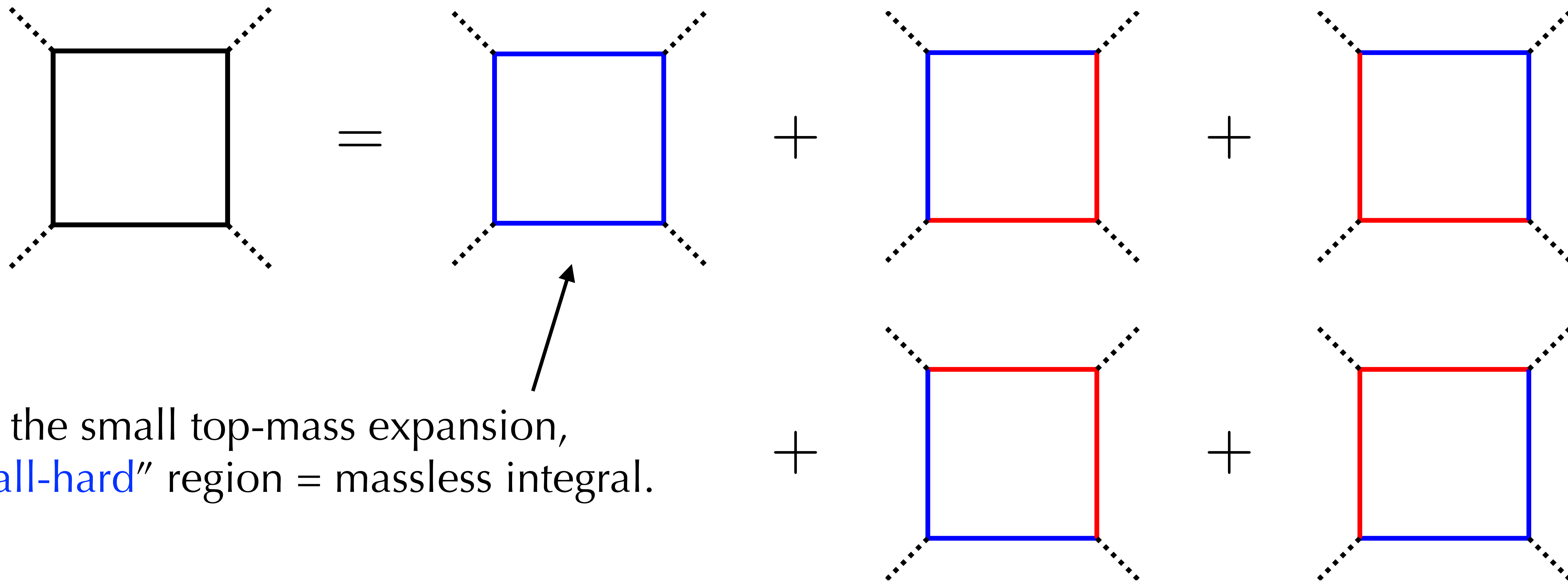


$$= \frac{1}{st}\left(\frac{4}{\varepsilon^2} - \frac{2\log st}{\varepsilon} + 2\log s \log t - \frac{4\pi^2}{3}\right)$$

# One loop case: Expansion by region

[Beneke, Smirnov '97, Smirnov `02, Jantzen `11]

blue: hard-scaling propagator
red: soft-scaling propagators



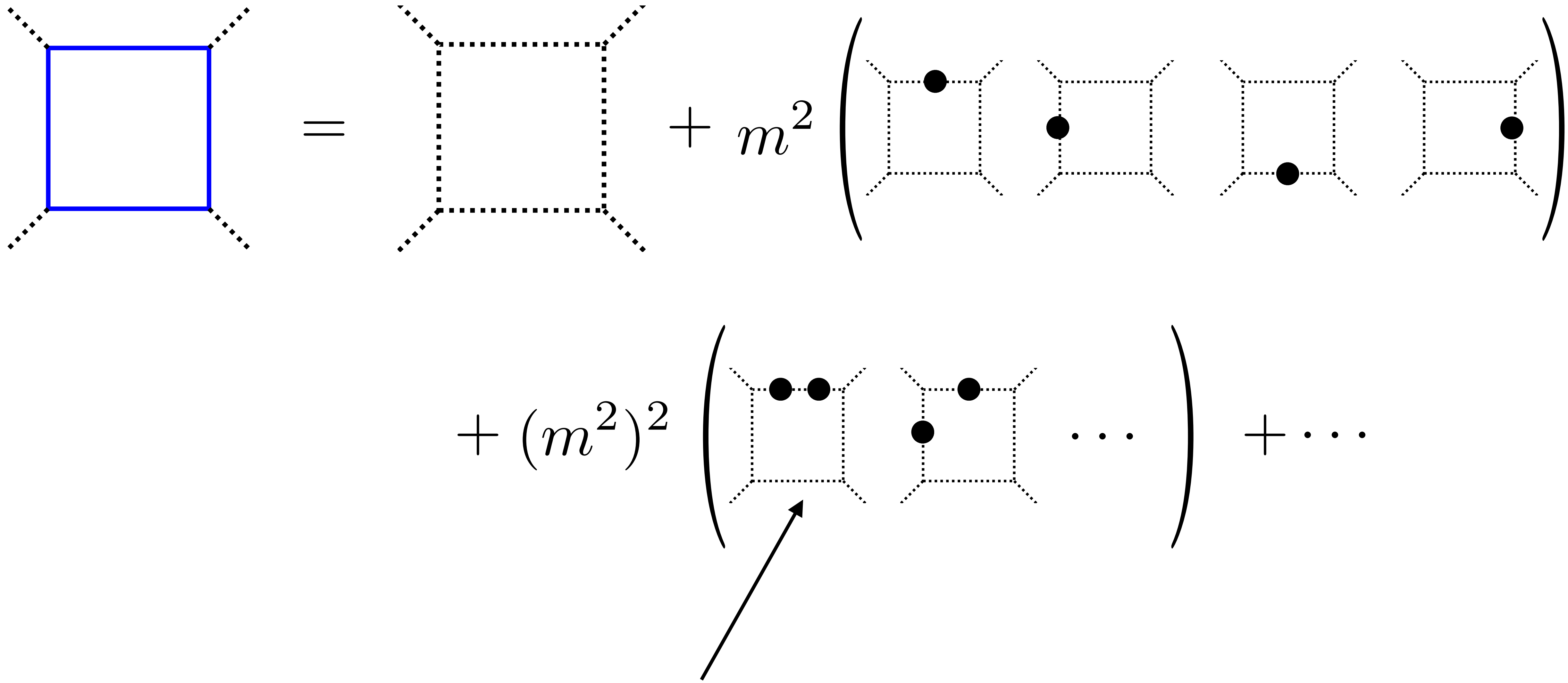
In the small top-mass expansion,
``all-hard" region = massless integral.

the scaling of propagators in terms of alpha-parameter representation

$$\int_0^\infty \left( \prod_{n=1}^{4} d\alpha_n \right) \alpha_{1234}^{-d/2} \, \mathrm{e}^{-m^2 \alpha_{1234} - (s\alpha_1\alpha_3 + t\alpha_2\alpha_4)/\alpha_{1234}}$$

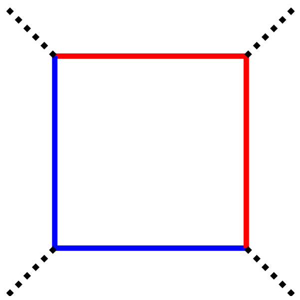
$$\alpha_{1234} = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$$

# One loop case: ``all-hard” region



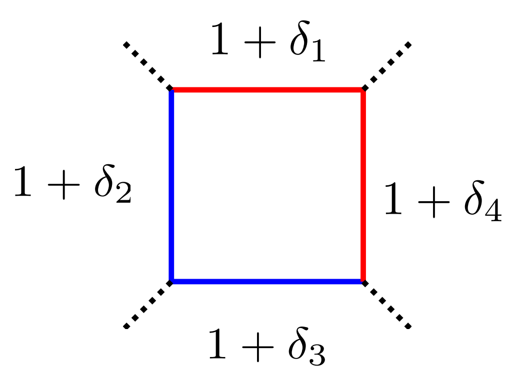

We can use integration by parts (IBP) reduction.

# One loop case: soft regions



$$= \int_0^\infty \left(\prod_{n=1}^{4} d\alpha_n\right) \begin{array}{l} \alpha_{12}^{-d/2}\,\mathrm{e}^{-m^2\alpha_{12}-(s\alpha_1\alpha_3+t\alpha_2\alpha_4)/\alpha_{12}} \\ -\,\alpha_{12}^{-d/2-2}(\alpha_3+\alpha_4)((d/2)\alpha_{12}+m^2(\alpha_{12})^2-s\alpha_1\alpha_3-t\alpha_2\alpha_4) \\ \times\,\mathrm{e}^{-m^2\alpha_{12}-(s\alpha_1\alpha_3+t\alpha_2\alpha_4)/\alpha_{12}} \\ +\cdots \end{array}$$

Usual momentum representation is not always possible...




The integrals are ill-defined,
so we have to introduce
analytic regularization
of the exponent of propagators.

$$f_0^{(2)} = \frac{(m^2)^{-\varepsilon}}{st}\left[\frac{1}{\varepsilon}\left(-\frac{1}{\delta_3}-\frac{1}{\delta_4}+\log st\right)\right]$$

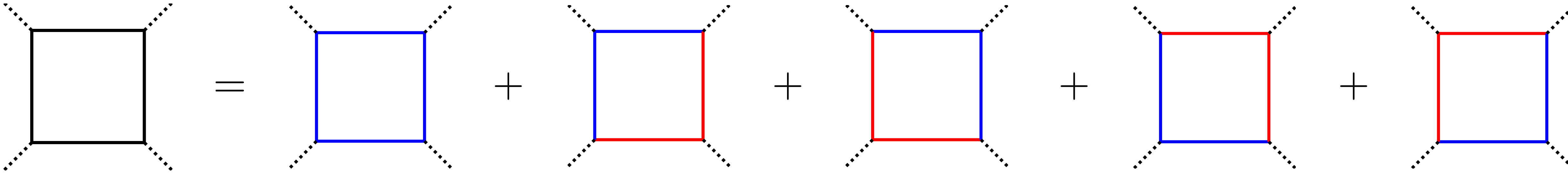
$$f_0^{(3)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{1}{\delta_3}-\frac{1}{\delta_4}+\log t/m^2\right)+\frac{\pi^2}{12}\right]$$

$$f_0^{(4)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(-\frac{1}{\delta_3}+\frac{1}{\delta_4}+\log s/m^2\right)+\frac{\pi^2}{12}\right]$$

$$f_0^{(5)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{2}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{1}{\delta_3}+\frac{1}{\delta_4}-2\log m^2\right)+\frac{\pi^2}{6}\right]$$

Cancellation of auxiliary parameters between soft regions occurs.

# One loop case: Expansion by region



$$f_0^{(1)} = \frac{1}{st}\left(\frac{4}{\varepsilon^2} - \frac{2\log st}{\varepsilon} + 2\log s \log t - \frac{4\pi^2}{3}\right)$$

$$f_0^{(2)} = \frac{(m^2)^{-\varepsilon}}{st}\left[\frac{1}{\varepsilon}\left(-\frac{1}{\delta_3} - \frac{1}{\delta_4} + \log st\right)\right]$$

$$f_0^{(3)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{1}{\delta_3} - \frac{1}{\delta_4} + \log t/m^2\right) + \frac{\pi^2}{12}\right]$$

$$f_0^{(4)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(-\frac{1}{\delta_3} + \frac{1}{\delta_4} + \log s/m^2\right) + \frac{\pi^2}{12}\right]$$

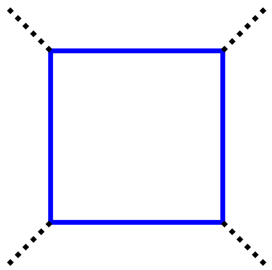
$$f_0^{(5)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{1}{\delta_3} + \frac{1}{\delta_4} - 2\log m^2\right) + \frac{\pi^2}{6}\right]$$

$$I = \sum_{n=0}^{\infty}(m^2)^n f_n$$

$$f_0 = \frac{1}{st}\left(2\log\frac{s}{m^2}\log\frac{t}{m^2} - \pi^2\right)$$

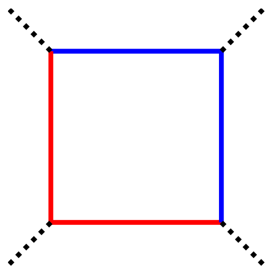
Cancellation of auxiliary parameters between soft regions occurs.

# One loop case



all-hard region

Each integral is well defined.
We can use IBP reduction.

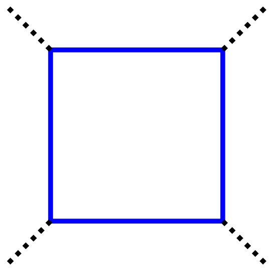


soft regions

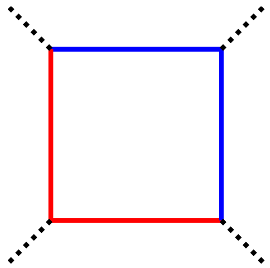
We need analytic regularization.
Conventional IBP reduction cannot be used.
(sometimes these disadvantages don't appear)

| | $(m^2)^0$ | $(m^2)^1$ | $(m^2)^2$ | $(m^2)^3$ |
|---|---|---|---|---|
| # of integrals in each soft region | 1 | 8 | 39 | 128 |

# One loop case: Why there are soft contributions?



all-hard region $\propto \dfrac{(m^2)^n}{S^{n+4-d/2}}$



soft regions $\propto \dfrac{(m^2)^{n-2+d/2}}{S^{n+2}}$

Each contribution is homogenous in $m$.
(a consequence of expansion by regions)

In $d = 4 - 2\epsilon$ dimension,

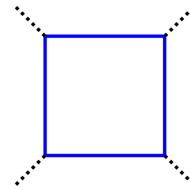
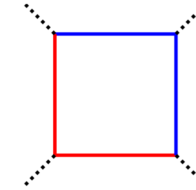
$$f^{(1)} = (m^2)^0 f_0^{(1)} + (m^2)^1 f_1^{(1)} + (m^2)^2 f_2^{(1)} + \cdots$$

$$f^{(2)} = (m^2)^0 f_0^{(2)} + (m^2)^1 f_1^{(2)} + (m^2)^2 f_2^{(2)} + \cdots$$

$$\vdots$$

$$f^{(5)} = (m^2)^0 f_0^{(5)} + (m^2)^1 f_1^{(5)} + (m^2)^2 f_2^{(5)} + \cdots$$

# One loop case:

 $\propto \frac{(m^2)^n}{S^{n+4-d/2}}$  $\propto \frac{(m^2)^{n-2+d/2}}{S^{n+2}}$

In $d = 4 - 2\epsilon$ dimension,

all-hard region $= f_0^{(1)} \quad + m^2 f_1^{(1)} \quad + (m^2)^2 f_2^{(1)}$

soft regions $= f_0^{(5)} \quad + m^2 f_1^{(5)} \quad + (m^2)^2 f_2^{(5)}$

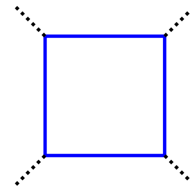
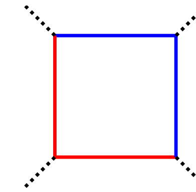
In $d = 6 - 2\epsilon$ dimension,

all-hard region $= f_0^{(1)} \quad + m^2 f_1^{(1)} \quad + (m^2)^2 f_2^{(1)}$

soft regions $= \quad + m^2 f_1^{(5)} \quad + (m^2)^2 f_2^{(5)}$

related idea: quasi-finite basis
**[Panzer '14, Manteuffel, Panzer, Schabinger, '14]**

# One loop case:

 $\propto \frac{(m^2)^n}{S^{n+4-d/2}}$  $\propto \frac{(m^2)^{n-2+d/2}}{S^{n+2}}$

In $d = 4 - 2\epsilon$ dimension,

all-hard region $= f_0^{(1)} \quad + m^2 f_1^{(1)} \quad + (m^2)^2 f_2^{(1)}$

soft regions $= f_0^{(5)} \quad + m^2 f_1^{(5)} \quad + (m^2)^2 f_2^{(5)}$

In $d = 6 - 2\epsilon$ dimension,

all-hard region $= f_0^{(1)} \quad + m^2 f_1^{(1)} \quad + (m^2)^2 f_2^{(1)}$

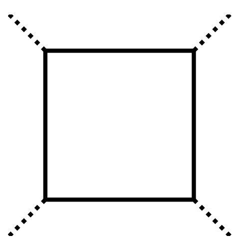
soft regions $= \quad + m^2 f_1^{(5)} \quad + (m^2)^2 f_2^{(5)}$

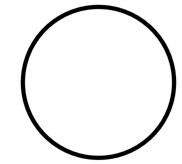
The leading contribution comes only from all-hard region!

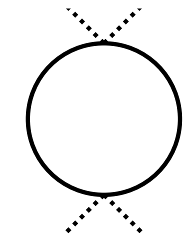
related idea: quasi-finite basis
**[Panzer ’14, Manteuffel, Panzer, Schabinger, ’14]**

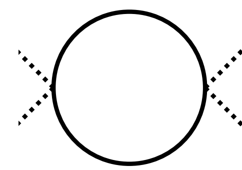
# One loop case: use of dim-recurrence relation

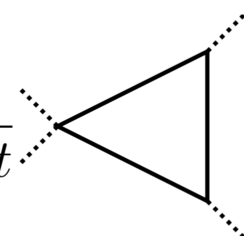
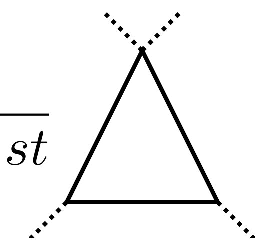
[Tarasov '96]

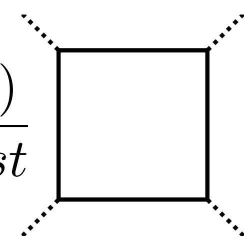
$\Big[$  $\Big]_{d=4-2\epsilon}$ $=$ $\Bigg[$

$$-\frac{2(d-2)\left(2m^2(s+t)+st\right)}{m^4\left(4m^2+s\right)\left(4m^2+t\right)\left(4m^2(s+t)+st\right)}$$ 

$$-\frac{2(d-3)t}{m^2\left(4m^2+t\right)\left(4m^2(s+t)+st\right)}$$ 

$$-\frac{2(d-3)s}{m^2\left(4m^2+s\right)\left(4m^2(s+t)+st\right)}$$ 

$$-\frac{(d-4)s}{4m^4(s+t)+m^2st}$$  $$-\frac{(d-4)t}{4m^4(s+t)+m^2st}$$ 

$$-\frac{2(d-5)(s+t)}{4m^2(s+t)+st}$$ 

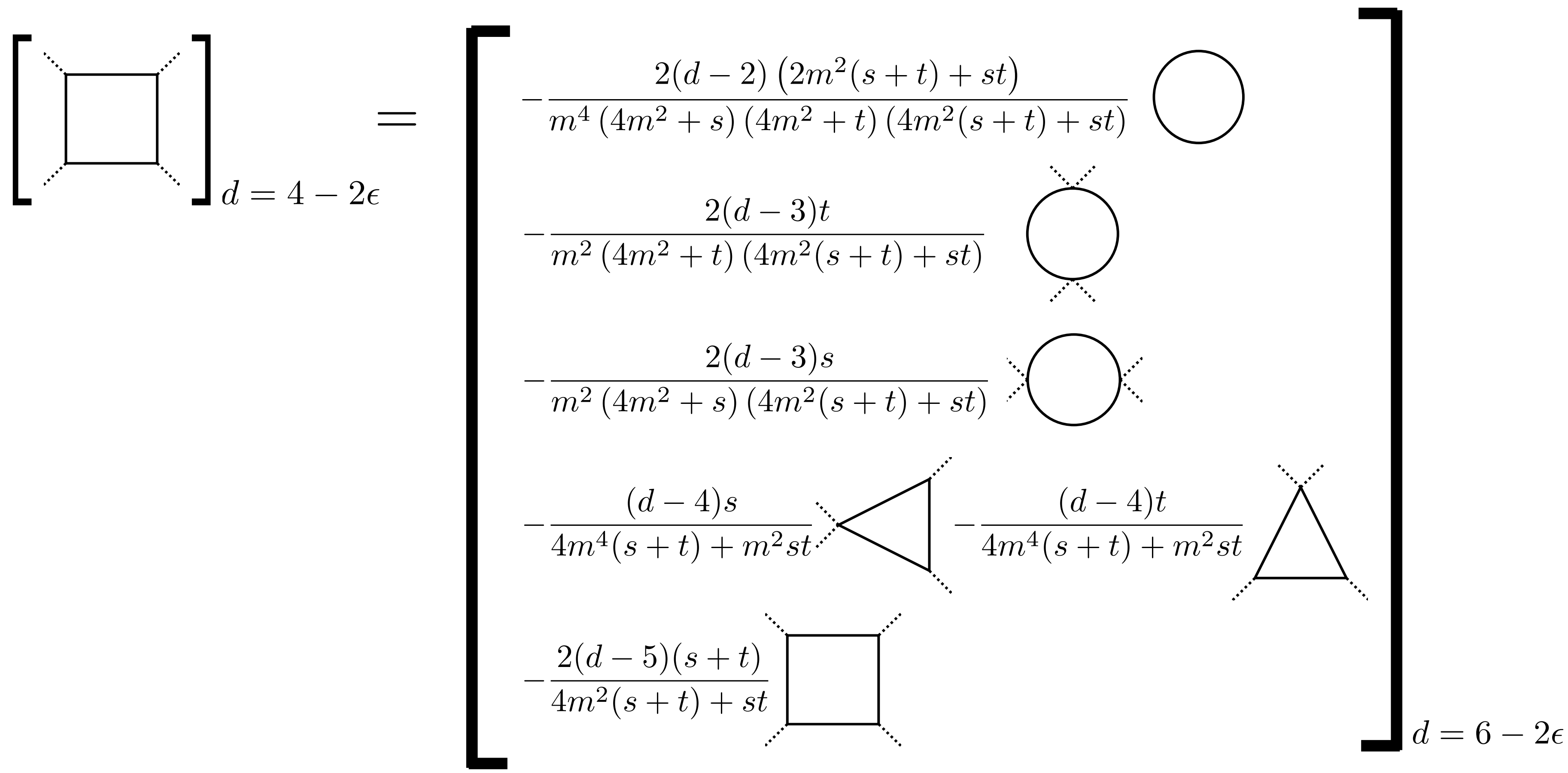
$\Bigg]_{d=6-2\epsilon}$

We used LiteRed **[Lee '13]** for obtaining the dim-rec. relation.

When we consider the leading $(m^2)^0$ contribution, we do not have to calculate the soft regions of the box diagram!

# One loop case: the recipe to avoid soft

(i) go to higher dimension using dimensional recurrence relation




(ii) soft contributions are suppressed in the higher dimensions

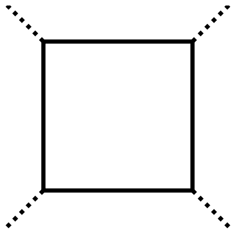
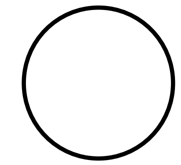
| | | | |
|---|---|---|---|
| all-hard region | $= f_0^{(1)}$ | $+m^2 f_1^{(1)}$ | $+(m^2)^2 f_2^{(1)}$ |
| soft regions | $=$ | $+m^2 f_1^{(5)}$ | $+(m^2)^2 f_2^{(5)}$ |

# One loop case: higher order of (m^2)

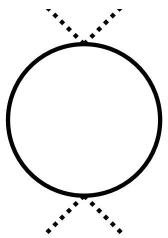
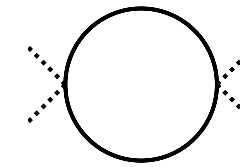
| $d = 4 - 2\epsilon$ | $(m^2)^0$ | $(m^2)^1$ | $(m^2)^2$ | $(m^2)^3$ | $(m^2)^4$ | $(m^2)^5$ | $\cdots$ |
|---|---|---|---|---|---|---|---|
| all-hard region | ● | ● | ● | ● | ● | ● | $\cdots$ |
| soft regions | ● | ● | ● | ● | ● | ● | $\cdots$ |
| $d = 6 - 2\epsilon$ | | | | | | | |
| all-hard region | ● | ● | ● | ● | ● | ● | $\cdots$ |
| soft regions | | ● | ● | ● | ● | ● | $\cdots$ |
| $d = 8 - 2\epsilon$ | | | | | | | |
| all-hard region | ● | ● | ● | ● | ● | ● | $\cdots$ |
| soft regions | | | ● | ● | ● | ● | $\cdots$ |
| $d = 10 - 2\epsilon$ | | | | | | | |
| all-hard region | ● | ● | ● | ● | ● | ● | $\cdots$ |
| soft regions | | | | ● | ● | ● | $\cdots$ |

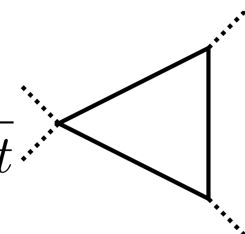
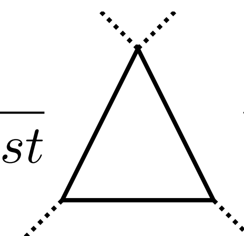
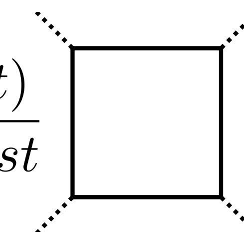
# One loop case: the use of differential equation

[Kotikov '91]

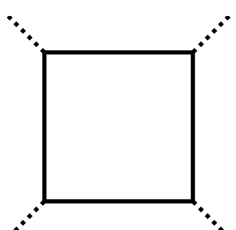
$$\frac{\partial}{\partial(m^2)}$$  $$= -\frac{2(d-2)\left(2m^2(s+t)+st\right)}{m^4\,(4m^2+s)\,(4m^2+t)\,(4m^2(s+t)+st)}$$ 

We used LiteRed [Lee '13] for obtaining the diff.-eq.

$$-\frac{2(d-3)t}{m^2\,(4m^2+t)\,(4m^2(s+t)+st)}$$  $$-\frac{2(d-3)s}{m^2\,(4m^2+s)\,(4m^2(s+t)+st)}$$ 

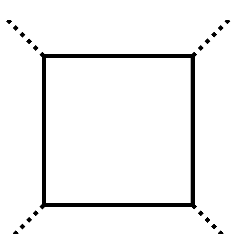
$$-\frac{(d-4)s}{4m^4(s+t)+m^2st}$$  $$-\frac{(d-4)t}{4m^4(s+t)+m^2st}$$  $$-\frac{2(d-5)(s+t)}{4m^2(s+t)+st}$$ 

Substituting the form,

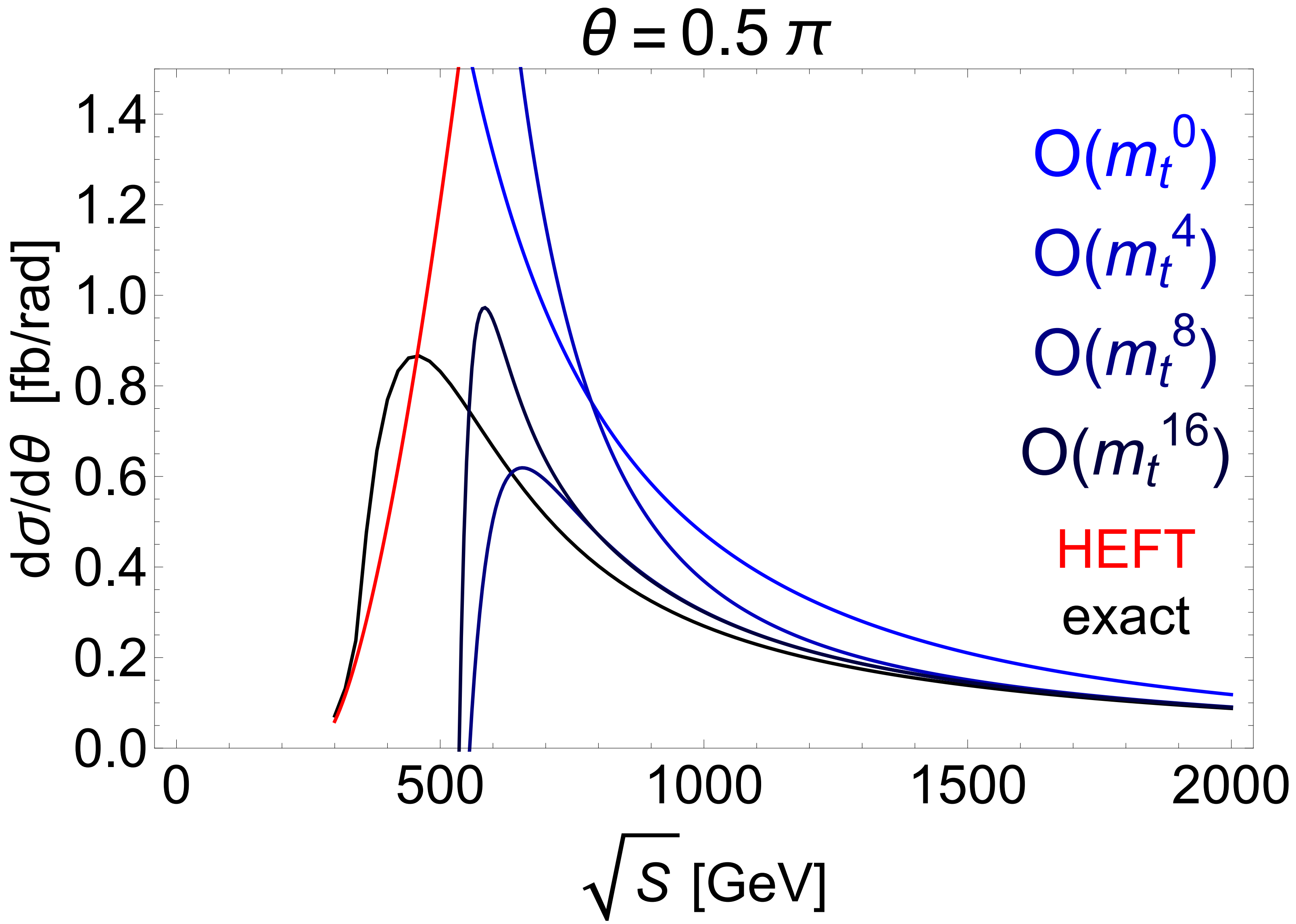
 $$= \sum_{n_1,n_2} c_{n_1,n_2}(m^2)^{n_1}(\log m^2)^{n_2}$$

we obtain recursive relations of C_n's.

See also
[Melnikov, Tancredi, Wever '16]

 $$= (m^2)^0 f_0 + (m^2)^1 f_1 + (m^2)^2 f_2 + \cdots$$

# Application to physical process gg->HH @LO




exact analytic@LO
**[Eboli, Marques, Novaes, Natale, '87, Glover, van der Bij '88]**

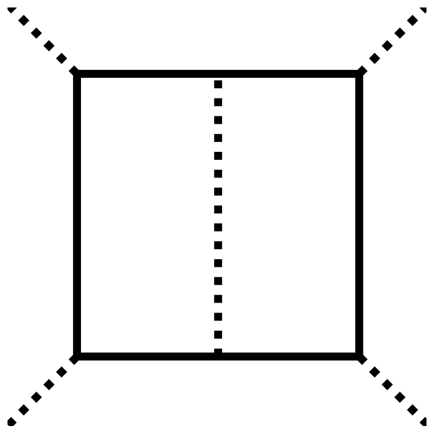
Born-improved HEFT@NLO
**[Dawson, Dittmaier Spira, '98]**

$FT_{approx}$, $FT'_{approx}$
**[Maltoni, Vryonidou, Zaro, '14]**

HEFT@NNLO with 1/mt corr.
**[Grigo, Hoff, Melnikov, Steinhauser, '13, Grigo, Melnikov, Steinhauser, '14, Grigo, Hoff, Steinhauser, '15, Degrassi, Giardino, Grber, '16]**

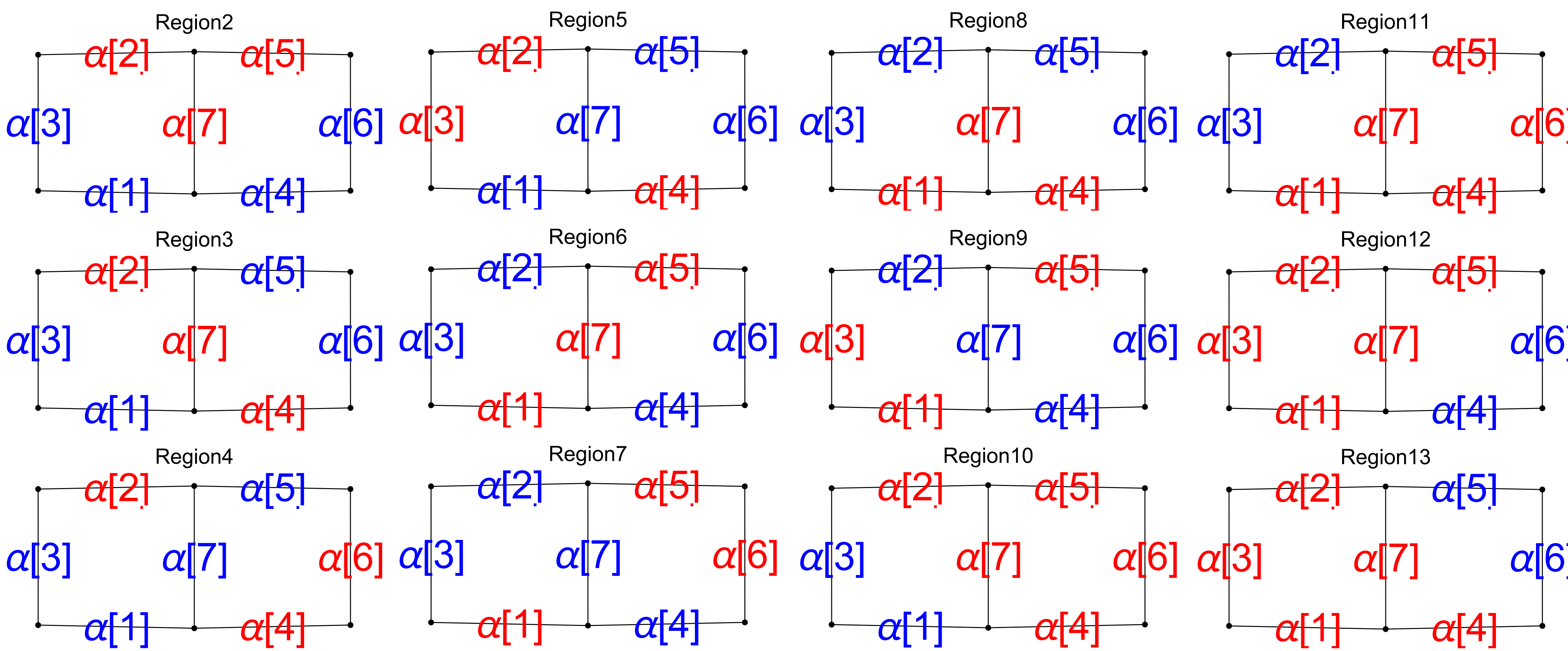
exact numerical@NLO
**[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zicke, '16]**

# Two loop case: massive double box



There are 13 regions. (all-hard region + 12 soft regions)

soft regions (i) 2~9
(# of soft propagator =3)
soft regions (ii) 10~13
(# of soft propagator =5)

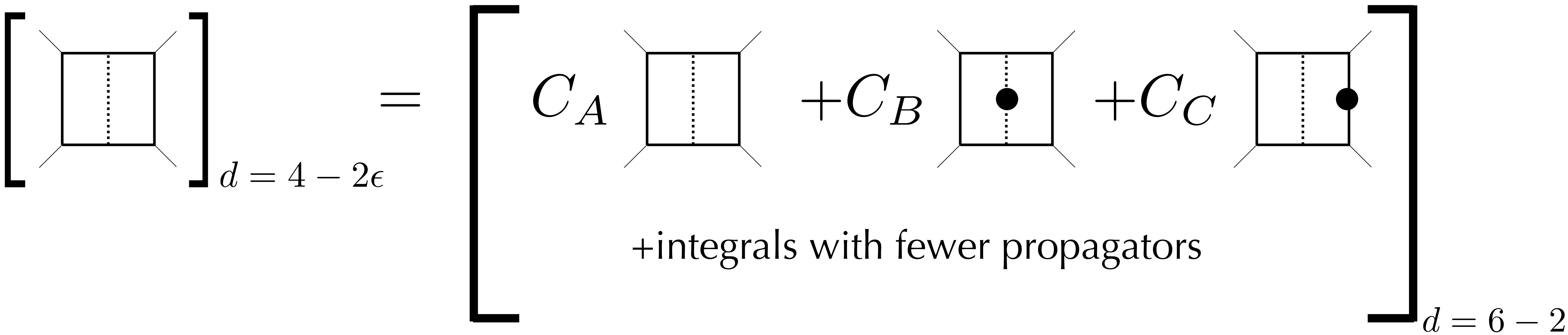


# Suppression of soft contributions in 6-dim.

| $d = 4 - 2\epsilon$ | $(m^2)^0$ | $(m^2)^1$ | $(m^2)^2$ | $\cdots$ |
|---|---|---|---|---|
| all-hard region | ● | ● | ● | $\cdots$ |
| soft regions (i) | ● | ● | ● | $\cdots$ |
| soft regions (ii) | ● | ● | ● | $\cdots$ |

| $d = 6 - 2\epsilon$ | $(m^2)^0$ | $(m^2)^1$ | $(m^2)^2$ | $\cdots$ |
|---|---|---|---|---|
| all-hard region | ● | ● | ● | $\cdots$ |
| soft regions (i) | | | ● | $\cdots$ |
| soft regions (ii) | | ● | ● | $\cdots$ |

# Dimensional recurrence relation

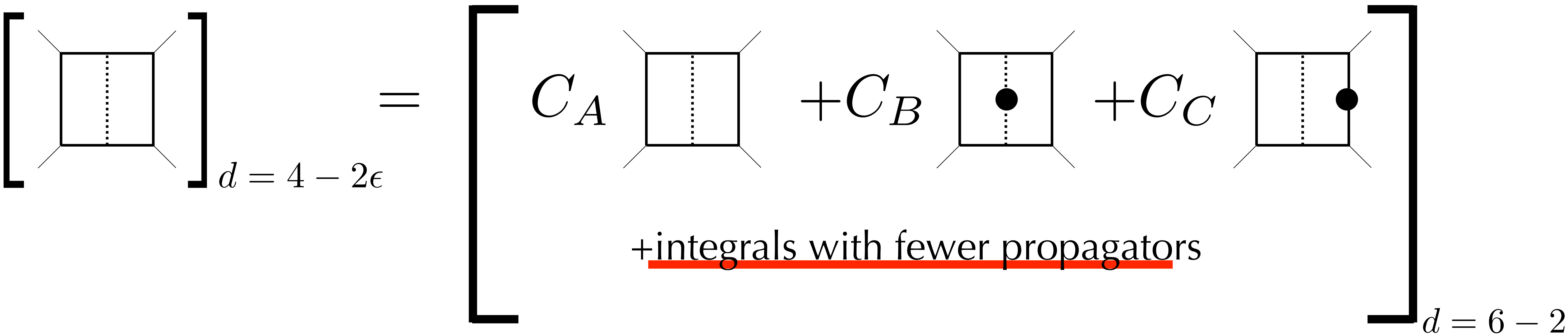


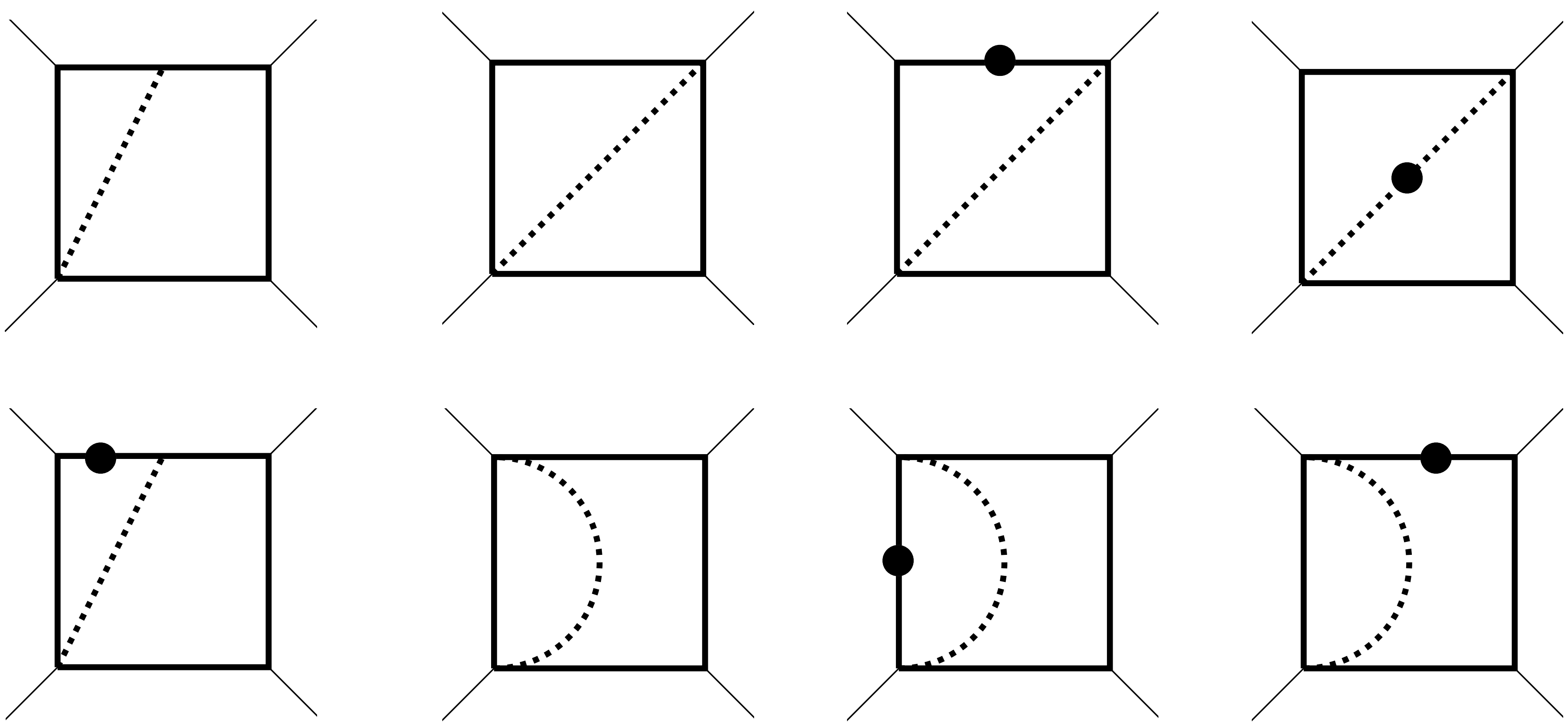

$$C_A, C_B, C_C = \mathcal{O}((m^2)^0)$$

( If e.g. $C_A = \mathcal{O}((m^2)^{-1})$, then we have to calculate up to (m^2)^1 term to obtain the (m^2)^0 term of the left hand side.
-> not so efficient )

We used LiteRed **[Lee '13]**
for obtaining the dim-rec. relation.

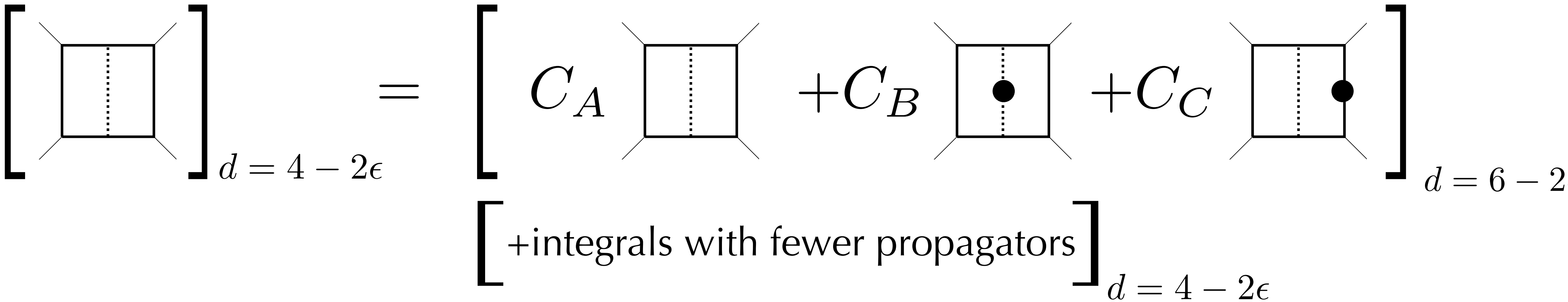
# Drawback

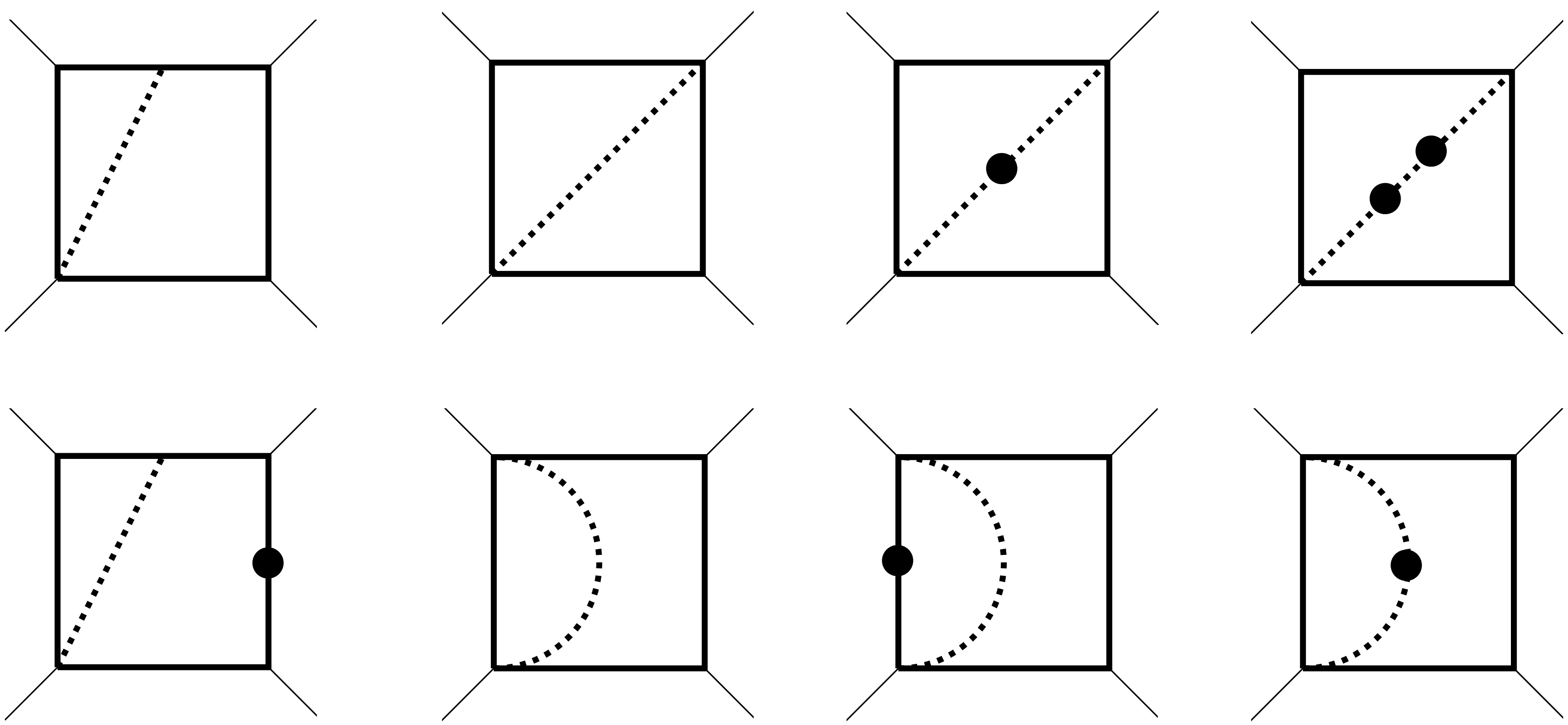


We need higher order $\sim \mathcal{O}((m^2)^3)$ terms to obtain the LO term.

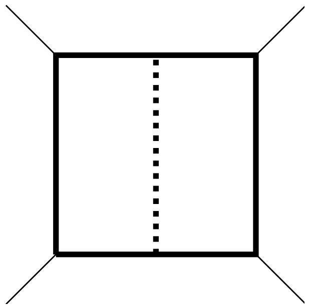
# Overcoming the drawback



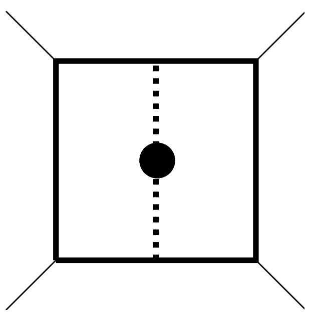



It is sufficient to know $\sim \mathcal{O}((m^2)^0)$ terms to obtain the LO term.

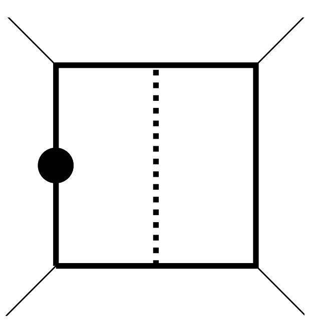
# Soft contributions in 6-dim. in dotted diagrams



| $d=6-2\epsilon$ | $(m^2)^0$ | $(m^2)^1$ | $(m^2)^2$ | $\cdots$ |
|---|---|---|---|---|
| all-hard region | ● | ● | ● | $\cdots$ |
| soft regions (2~9) | | | ● | $\cdots$ |
| soft regions (10~13) | | ● | ● | $\cdots$ |



| | | | | |
|---|---|---|---|---|
| all-hard region | ● | ● | ● | $\cdots$ |
| soft regions (2~9) | | ○ | ● | $\cdots$ |
| soft regions (10~13) | | ● | ● | $\cdots$ |



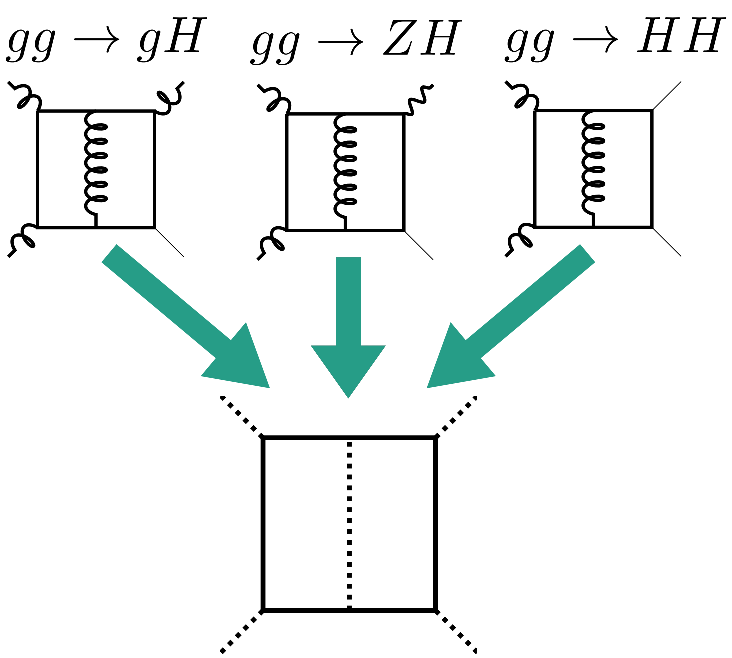
| | | | | |
|---|---|---|---|---|
| all-hard region | ● | ● | ● | $\cdots$ |
| soft regions (2~9) | | ○ | ● | $\cdots$ |
| soft regions (10,11) | ● | ● | ● | $\cdots$ |
| soft regions (12,13) | | ● | ● | |

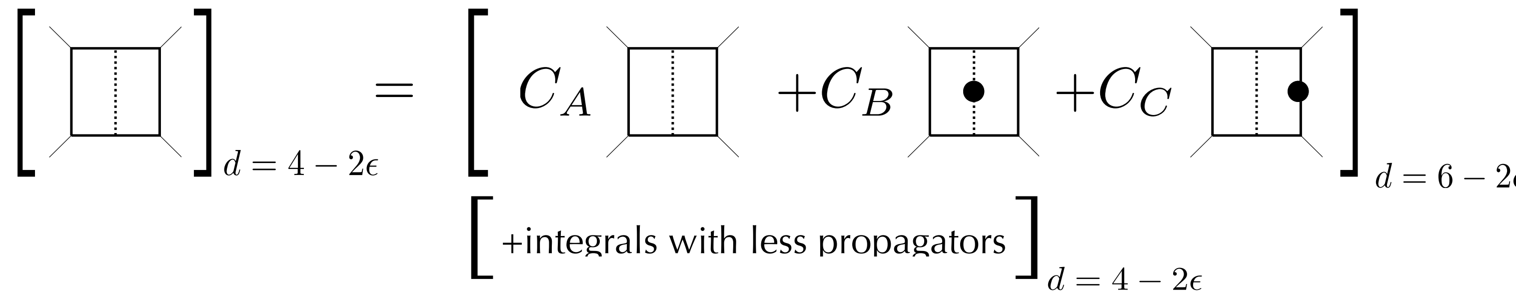
# Analytic result of massive double box diagram

$$f_0 = \frac{1}{s^2 t^2}\Bigg(\frac{7\pi^4 t}{15} + \frac{2}{3}\pi^2 t \,\mathrm{Log}\left[\frac{m2}{s}\right]^2 - t\,\mathrm{Log}\left[\frac{m2}{s}\right]^4 + \frac{1}{3}\pi^2 t\,\mathrm{Log}\left[1+\frac{s}{t}\right]^2 - \frac{1}{3} t\,\mathrm{Log}\left[1+\frac{s}{t}\right]^4 + \frac{2}{3} t\,\mathrm{Log}\left[1+\frac{s}{t}\right]^3 \mathrm{Log}\left[\frac{s}{t}\right] -$$
$$\frac{8}{3}\pi^2 t\,\mathrm{Log}\left[\frac{m2}{s}\right]\mathrm{Log}\left[\frac{t}{s}\right] + \frac{8}{3} t\,\mathrm{Log}\left[\frac{m2}{s}\right]^3 \mathrm{Log}\left[\frac{t}{s}\right] + 2\pi^2 t\,\mathrm{Log}\left[\frac{t}{s}\right]^2 - 2t\,\mathrm{Log}\left[\frac{m2}{s}\right]^2 \mathrm{Log}\left[\frac{t}{s}\right]^2 + \frac{8}{3} s\,\mathrm{Log}\left[\frac{t}{s}\right]^3 +$$
$$\frac{8}{3} t\,\mathrm{Log}\left[\frac{t}{s}\right]^3 - \frac{5}{3} t\,\mathrm{Log}\left[\frac{t}{s}\right]^4 + \frac{2}{3}\pi^2 t\,\mathrm{Log}\left[\frac{t}{s}\right]\mathrm{Log}\left[1+\frac{t}{s}\right] + \frac{2}{3} t\,\mathrm{Log}\left[\frac{t}{s}\right]^3 \mathrm{Log}\left[1+\frac{t}{s}\right] - \frac{1}{3}\pi^2 t\,\mathrm{Log}\left[1+\frac{t}{s}\right]^2 -$$
$$\frac{2}{3} t\,\mathrm{Log}\left[\frac{t}{s}\right]\mathrm{Log}\left[1+\frac{t}{s}\right]^3 + \frac{1}{3} t\,\mathrm{Log}\left[1+\frac{t}{s}\right]^4 + 8 s\,\mathrm{Log}\left[\frac{t}{s}\right]\mathrm{PolyLog}\left[2,-\frac{s}{t}\right] + 8t\,\mathrm{Log}\left[\frac{t}{s}\right]\mathrm{PolyLog}\left[2,-\frac{s}{t}\right] -$$
$$6t\,\mathrm{Log}\left[\frac{t}{s}\right]^2 \mathrm{PolyLog}\left[2,-\frac{s}{t}\right] + 4\pi^2 t\,\mathrm{PolyLog}\left[2,-\frac{t}{s}\right] + 8s\,\mathrm{Log}\left[\frac{t}{s}\right]\mathrm{PolyLog}\left[2,-\frac{t}{s}\right] +$$
$$8t\,\mathrm{Log}\left[\frac{t}{s}\right]\mathrm{PolyLog}\left[2,-\frac{t}{s}\right] - 2t\,\mathrm{Log}\left[\frac{t}{s}\right]^2 \mathrm{PolyLog}\left[2,-\frac{t}{s}\right] + 8s\,\mathrm{PolyLog}\left[3,-\frac{s}{t}\right] + 8t\,\mathrm{PolyLog}\left[3,-\frac{s}{t}\right] +$$
$$4t\,\mathrm{Log}\left[1+\frac{s}{t}\right]\mathrm{PolyLog}\left[3,-\frac{s}{t}\right] - 4t\,\mathrm{Log}\left[\frac{t}{s}\right]\mathrm{PolyLog}\left[3,-\frac{s}{t}\right] - 4t\,\mathrm{Log}\left[1+\frac{t}{s}\right]\mathrm{PolyLog}\left[3,-\frac{s}{t}\right] -$$
$$8s\,\mathrm{PolyLog}\left[3,-\frac{t}{s}\right] - 8t\,\mathrm{PolyLog}\left[3,-\frac{t}{s}\right] - 8t\,\mathrm{Log}\left[\frac{t}{s}\right]\mathrm{PolyLog}\left[3,-\frac{t}{s}\right] + 24t\,\mathrm{PolyLog}\left[4,-\frac{t}{s}\right] +$$
$$4t\,\mathrm{Log}\left[\frac{m2}{s}\right]\mathrm{Zeta}[3] - 4t\,\mathrm{Log}\left[1+\frac{s}{t}\right]\mathrm{Zeta}[3] - 8t\,\mathrm{Log}\left[\frac{t}{s}\right]\mathrm{Zeta}[3] + 4t\,\mathrm{Log}\left[1+\frac{t}{s}\right]\mathrm{Zeta}[3]\Bigg)$$

Complexity of function is at most PolyLog[4,z],
so the analytic continuation is very easy.

# Summary



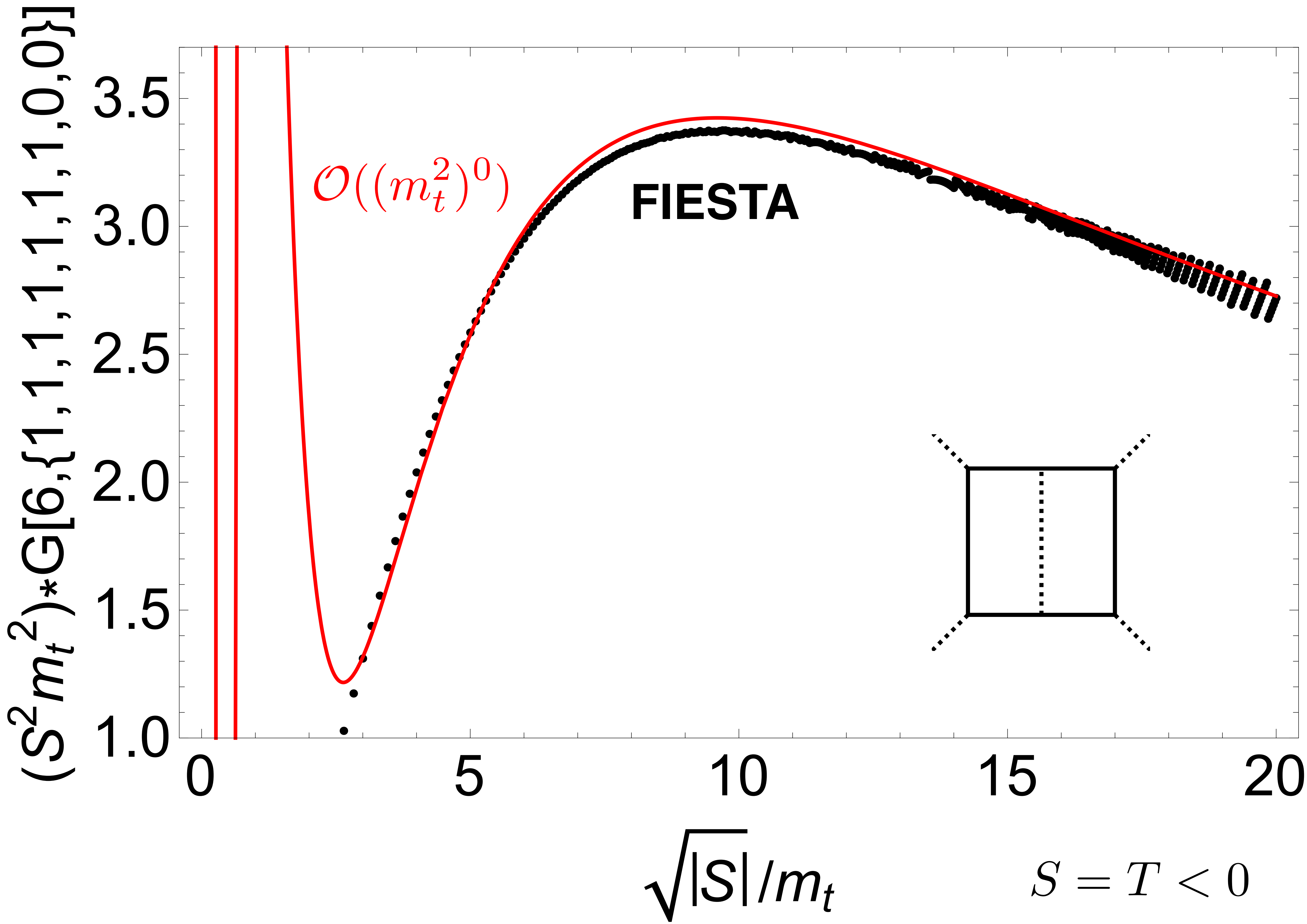




| $d = 4 - 2\epsilon$ | $(m^2)^0$ | $(m^2)^1$ | $(m^2)^2$ | $\cdots$ |
|---|---|---|---|---|
| all-hard region | ● | ● | ● | $\cdots$ |
| soft regions (i) | ● | ● | ● | $\cdots$ |
| soft regions (ii) | ● | ● | ● | $\cdots$ |

| $d = 6 - 2\epsilon$ | $(m^2)^0$ | $(m^2)^1$ | $(m^2)^2$ | $\cdots$ |
|---|---|---|---|---|
| all-hard region | ● | ● | ● | $\cdots$ |
| soft regions (i) | | | ● | $\cdots$ |
| soft regions (ii) | | ● | ● | $\cdots$ |

# Next step

Apply diff. eq. to obtain higher order of $m^2$.
Non-planar diagrams?


$(S^2 m_t^2)*G[6,\{1,1,1,1,1,1,1,0,0\}]$
3.5
3.0
2.5
2.0
1.5
1.0
0
5
10
15
20
$\mathcal{O}((m_t^2)^0)$
FIESTA
$\sqrt{|S|}/m_t$
$S = T < 0$

# Application to physical process gg->HH @LO