Mass Dependence of Higgs Production at large P_T

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In collaboration with ***Eric Braaten*** and ***Jia-Wei Zhang***

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Higgs Effective Field Theory

Standard Model $\mathcal{L} = -\frac{m_t}{v}\bar{t}th$ HEFT $\mathcal{L}_{\rm eff} = \frac{\alpha_s}{12\pi v} G^A_{\mu\nu} G^{\mu\nu,A} h$

- Eliminate the scale m_t
- Reduce the number of loops by 1.
- Inclusive Higgs production cross section from gluon fusion has been calculated to **NNNLO** with Higgs EFT.
  Anastasiou, Duhr, Dulat, Herzog, Mistlberger, PRL 2015
- Higgs + 1 jet, **NNLO**
  Boughezal, Focke, Giele, Liu, Petriello, PLB 2015
- Higgs + ≥ 2 jet, **NLO**
  Campbell, Ellis, Williams, PRD 2010

Higgs P_T Distribution

- Higgs EFT cannot be applied to large P_T

  Large $\sqrt{s}$ and P_T can resolve the t-quark

- Higgs at large P_T is an important probe of BSM physics.

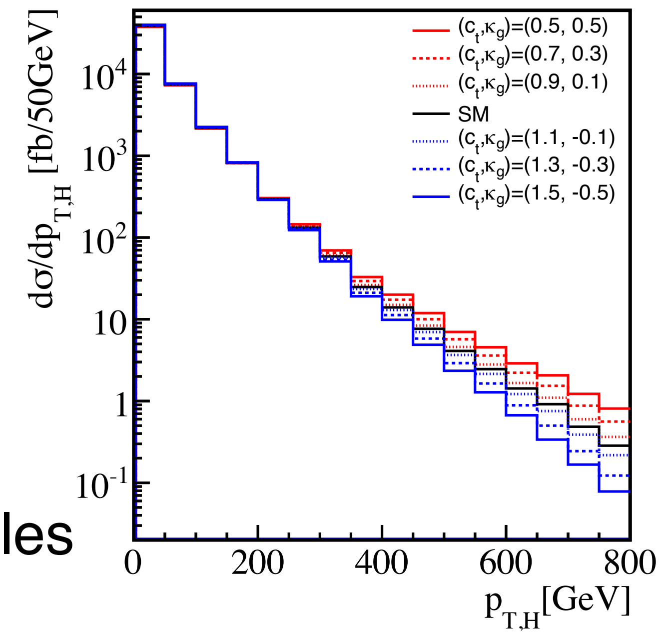
  e.g. Schlaffer et al, Eur.Phys.J C 2014

- Calculation is very difficult with many scales $(\sqrt{s}, P_T, m_t, m_H)$

  Higgs P_T distribution with physical top mass is available only at LO.

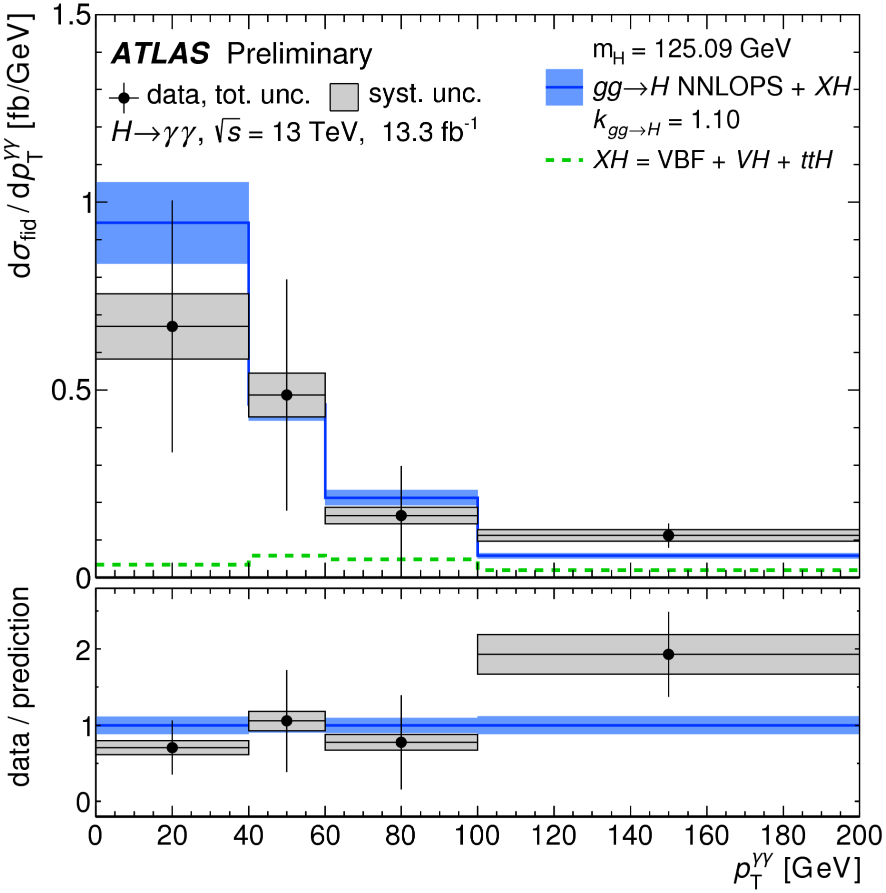
  Ellis et.al., NPB **1988**; Baur et.al., NPB **1990**

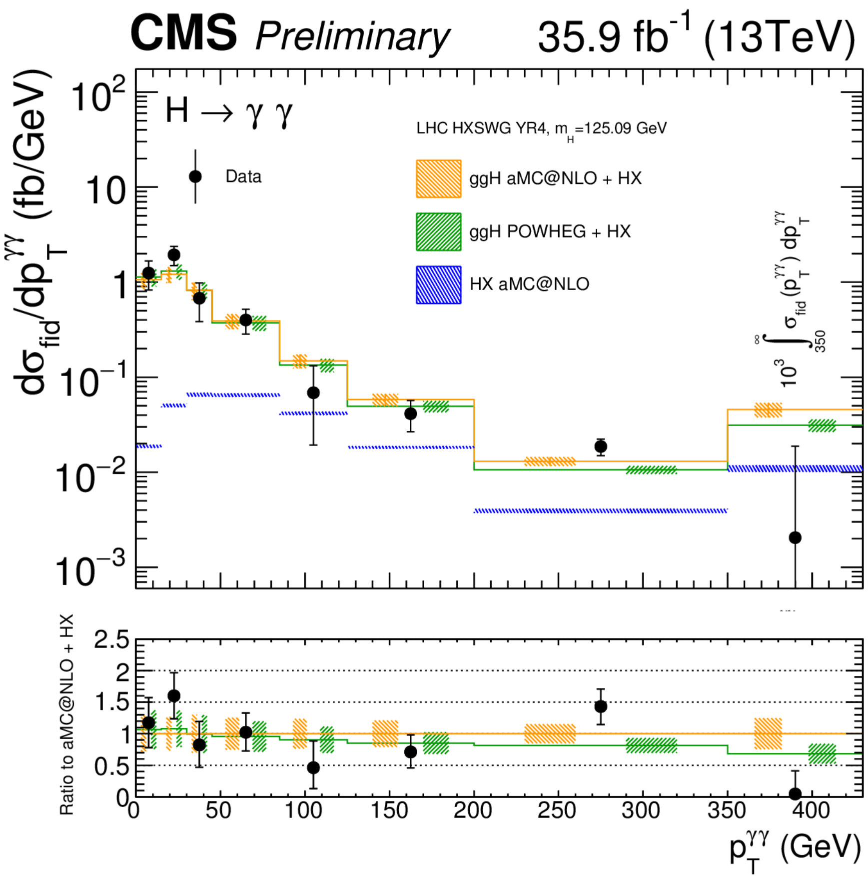
- Lack of reliable SM prediction may compromise the search for new physics.



Data Are Coming!

See Ferrari's talk for more details





(a) $p_{T,\gamma\gamma}$ differential cross-section

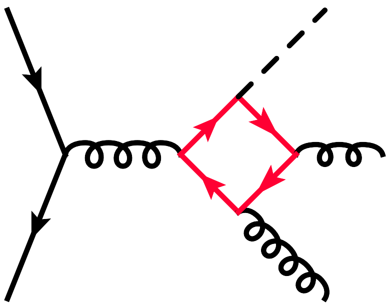
- ATLAS and CMS start to measure Higgs production at large P_T.

ATLAS-CONF-2016-067 CMS-HIG-17-015

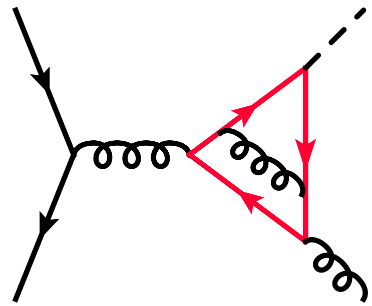
NLO Calculation with Massive Top

- Include both real and virtual corrections (e.g. $q\bar{q} \to H + g$)

  Relevant scales: $\sqrt{s}, m_t, m_H$



1-loop



2-loop

- Separate scales to simplify NLO calculation

Limit 1: $2m_t \gg \sqrt{s}, m_H$ Expand in $s/4m_t^2$ and $m_H^2/4m_t^2$ ***HEFT***

Limit 2: $\sqrt{s} \gg 2m_t, m_H$ Expand in m_H^2/s and m_t^2/s ***Double Expansion***

Limit 3: m_t is arbitrary and $\sqrt{s} \gg m_H$ Expand in m_H^2/s ***Single Expansion***

An “EFT” complementary to HEFT!

Mass Singularity

- Expansion is nontrivial due to mass singularity

  Relevant scales: $\sqrt{s}, m_t, m_H$ Expand in m_H^2/s and m_t^2/s

  - Non-analytic, e.g. $\log(s/m_H^2)$
  - Ratio of mass scales, e.g. $m_H^2/4m_t^2$

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  - Non-analytic, e.g. $\log(s/m_H^2)$
  - Ratio of mass scales, e.g. $m_H^2/4m_t^2$

  Keep them untouched in the expansion

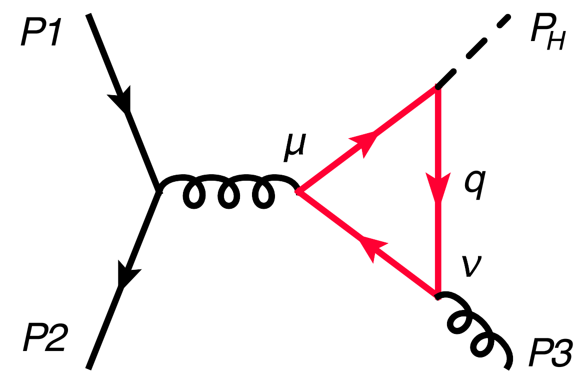
- More ambitiously, we want the expansion (scale separation) before calculating Feynman diagrams.

  Each diagram is simpler to calculate due to fewer scales.

- QCD factorization is a systematic framework to remove the small mass scales.

Setup of the Calculation

- As an example, we have calculated $q\bar{q} \to H + g$ at LO
  - Easy to calculate.
  - Can compare with the full LO analytically.



- One relevant form factor at LO

$$\frac{1}{4N_c^2}\sum|\mathcal{M}|^2 = \frac{2(N_c^2-1)g_s^2 m_t^2}{N_c^2}\,\frac{t^2+u^2}{s(s-m_H^2)^2}\,|\mathcal{F}(s,m_t^2,m_H^2)|^2$$

$$\mathcal{F}(s,m_t^2,m_H^2) = \frac{1}{(D-2)4m_t}\left(g_{\mu\nu} - \frac{p_{3\mu}(P_H+p_3)_\nu}{P_H.p_3}\right)\mathcal{T}^{\mu\nu}(P_H,p_3)$$

LP Form Factor

- The leading terms in the expansion of the full form factor are called "leading power terms (LP)".

  First consider the double expansion

- Expand the LO form factor in m_H^2/s and m_t^2/s, keeping all mass singularities.

$$\mathcal{F}^{\mathrm{LP}}(s, m_t^2, m_H^2) = \frac{g_s^2 y_t}{16\pi^2}\left(-\frac{1}{2}\log^2\frac{-s-i\epsilon}{m_t^2} + 2\log\frac{-s-i\epsilon}{m_t^2} - 2\arcsin^2 r - \frac{4\sqrt{1-r^2}}{r}\arcsin r - 2\right),$$

$$r \equiv \frac{m_H}{2m_t}$$

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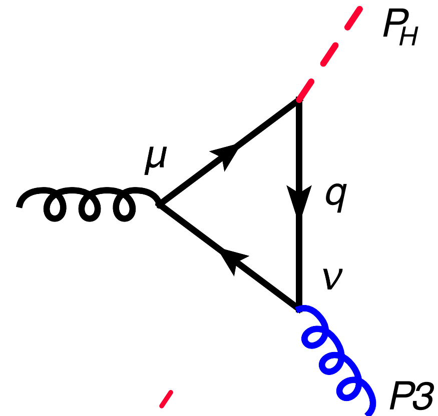
$$r \equiv \frac{m_H}{2m_t}$$

Mass singularities

- The dependence of mass ratio is non-trivial.
- All these terms must be reproduced with factorization formula

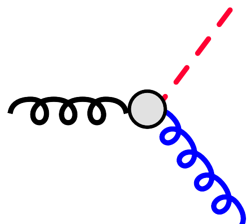
Leading Regions

- The regions of loop momentum integral giving LP terms are called "leading regions".

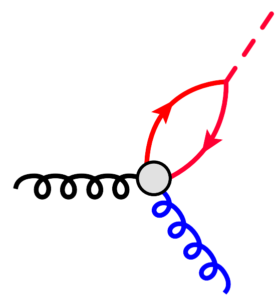


- Four leading regions: $(\sqrt{s} \gg m_t, m_H)$

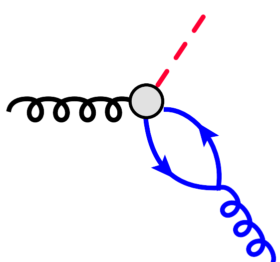
Hard
$(\sqrt{s})$



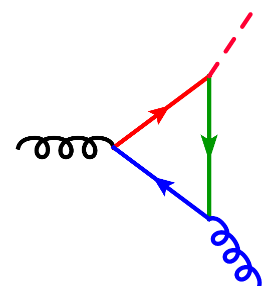
Higgs Collinear
$(\sqrt{s}, m_t, m_H)$



Gluon Collinear
$(\sqrt{s}, m_t)$

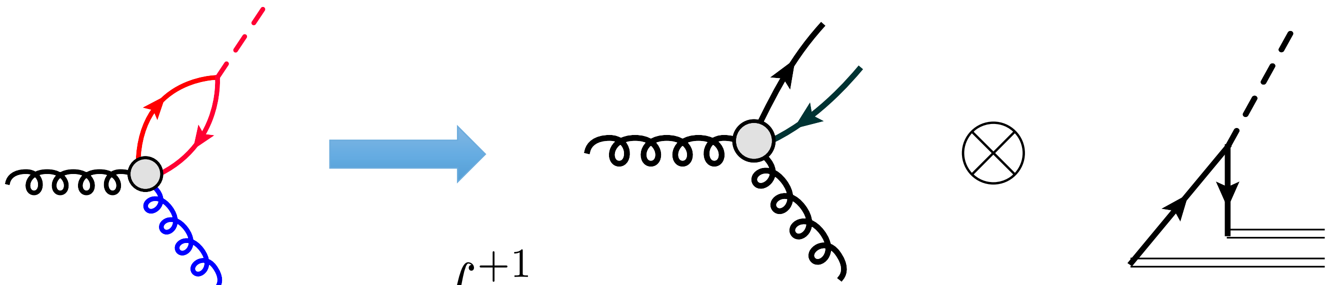


Soft
(m_t)



Factorization of Collinear Regions

- Higgs Collinear Region



$$\int_{-1}^{+1} d\zeta \quad \widetilde{\mathcal{F}}_{t\bar{t}_{1V}+g}(\zeta) \quad \otimes \quad d_{t\bar{t}_{1V}\to H}(\zeta; m_t^2, m_H^2, P.n)$$

  - Hard production of massless $t\bar{t}$ in color-singlet Lorentz-vector channel

    Only depends on hard scale $\sqrt{s}$

  - Collinear $t\bar{t}$ interact and produce the Higgs

    Only depends on soft scales m_t, m_H

  - Integrate over the relative longitudinal momentum of $t\bar{t}$ pair

- After factorization, we separate the scales before the calculation of Feynman diagrams

Factorization Formula

- LP factorization formula (Expand by both m_H^2/s and m_t^2/s)

$$\mathcal{F}^{\rm LP}(s,m_t^2,m_H^2) = \widetilde{\mathcal{F}}_{H+g}(s) + \int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{t\bar{t}_{1V}+g}(\zeta)\, d_{t\bar{t}_{1V}\to H}(\zeta;m_t^2,m_H^2,P_H\cdot n)$$
$$+ \int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{H+t\bar{t}_{8T}}(\zeta)\, d_{t\bar{t}_{8T}\to g}(\zeta;m_t^2,p_3\cdot\bar{n}) + \mathcal{F}_{\rm soft}(m_t^2)$$

- All pieces can be calculated directly from Feynman diagrams with fewer scales.

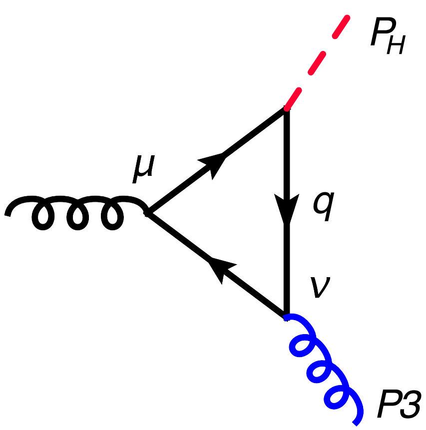
- Although the full form factor is finite, each region is divergent. Dimensional regularization and rapidity regularization are used.

For more details, see Braaten, HZ and Zhang, arXiv:1704.06620

Hard Region

- LP factorization formula (Expand by both m_H^2/s and m_t^2/s)

$$\mathcal{F}^{\rm LP}(s,m_t^2,m_H^2) = \boxed{\widetilde{\mathcal{F}}_{H+g}(s)} + \int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{t\bar{t}_{1V}+g}(\zeta)\, d_{t\bar{t}_{1V}\to H}(\zeta; m_t^2, m_H^2, P_H\cdot n)$$
$$+ \int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{H+t\bar{t}_{8T}}(\zeta)\, d_{t\bar{t}_{8T}\to g}(\zeta; m_t^2, p_3\cdot\bar{n}) + \mathcal{F}_{\rm soft}(m_t^2)$$



$$m_H = m_t = 0$$

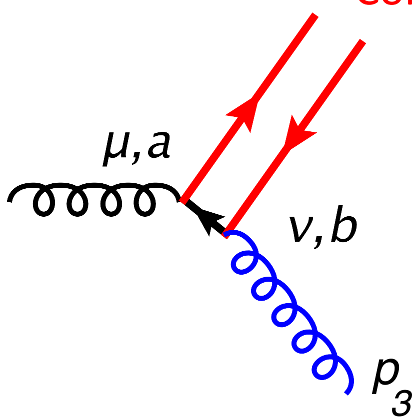
$$\widetilde{\mathcal{F}}_{H+g}(s) \propto \int_q \frac{\mathrm{Tr}[\ldots]}{[(q+P_H)^2+i\epsilon]\,[q^2+i\epsilon]\,[(q-p_3)^2+i\epsilon]}$$
$$\propto \left[\frac{-s-i\epsilon}{\mu^2}\right]^{-\epsilon}\left(\frac{1}{\epsilon^2}+\frac{2}{\epsilon}+6-\frac{\pi^2}{6}\right)$$

- Only depends on s , no mass scale.

Higgs Collinear Region: Hard Coeff.

- LP factorization formula (Expand by both m_H^2/s and m_t^2/s)

$$\mathcal{F}^{\rm LP}(s,m_t^2,m_H^2) = \widetilde{\mathcal{F}}_{H+g}(s) + \int_{-1}^{+1} d\zeta\, \boxed{\widetilde{\mathcal{F}}_{t\bar{t}_{1V}+g}(\zeta)}\, d_{t\bar{t}_{1V}\to H}(\zeta; m_t^2, m_H^2, P_H\cdot n) + \int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{H+t\bar{t}_{8T}}(\zeta)\, d_{t\bar{t}_{8T}\to g}(\zeta; m_t^2, p_3\cdot\bar{n}) + \mathcal{F}_{\rm soft}(m_t^2)$$



$$m_H = m_t = 0$$

$$p_t = \frac{1}{2}(1+\zeta)P_H,$$

$$p_{\bar{t}} = \frac{1}{2}(1-\zeta)P_H$$

$$\widetilde{\mathcal{F}}_{t\bar{t}_{1V}+g}(\zeta) \propto \mathrm{Tr}[\ldots \not{P}_H]\mathrm{Tr}_c[T^a T^b 1]$$

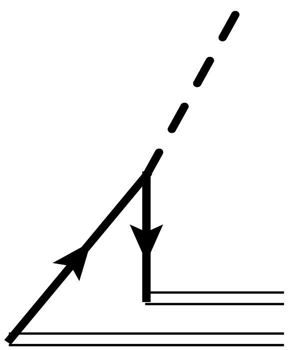
$$\propto \zeta$$

➢ Only depends on s , no mass scale.

Higgs Collinear Region: Distribution

- LP factorization formula (Expand by both m_H^2/s and m_t^2/s)

$$\mathcal{F}^{\rm LP}(s,m_t^2,m_H^2) = \widetilde{\mathcal{F}}_{H+g}(s) + \int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{t\bar{t}_{1V}+g}(\zeta) \boxed{d_{t\bar{t}_{1V}\to H}(\zeta; m_t^2, m_H^2, P_H\cdot n)}$$
$$+ \int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{H+t\bar{t}_{8T}}(\zeta)\, d_{t\bar{t}_{8T}\to g}(\zeta; m_t^2, p_3\cdot\bar{n}) + \mathcal{F}_{\rm soft}(m_t^2)$$



Vector
Color-singlet

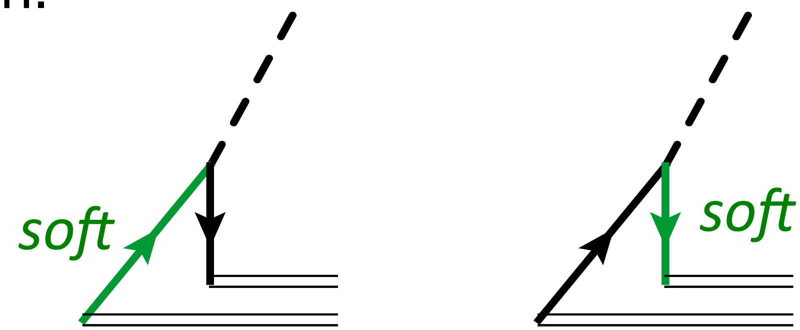
Physical m_H and m_t , $p_t = \frac{1}{2}P_H + q, \quad p_{\bar{t}} = \frac{1}{2}P_H - q$

$$d_{t\bar{t}_{1V}\to H} \propto \int_q \frac{\delta(\zeta - 2q\cdot n/P_H\cdot n)\,{\rm Tr}[\ldots \not{n}]{\rm Tr}_c[\ldots 1]}{[(\frac{1}{2}P_H - q)^2 - m_t^2 + i\epsilon]\,[(\frac{1}{2}P_H + q)^2 - m_t^2 + i\epsilon]}$$
$$\times \left[\frac{|q\cdot n|}{\nu_+}\right]^{-\eta} \left[\frac{|(q+P_H)\cdot n|}{\nu_+}\right]^{-\eta}$$

- Only depends on m_H and m_t
- Has rapidity divergence

Zero-Bin Subtraction

- Need zero-bin subtraction to remove the double counting of the collinear region and the soft region.
- Either quark line can be soft.



- After zero-bin subtraction

$$d_{t\bar{t}_{1V\to H}} \propto \Big[-\frac{1}{2\epsilon\eta_{\rm uv}}\delta(1-\zeta^2) + \frac{1}{\epsilon}\frac{\zeta}{(1-\zeta^2)_+} - \zeta\frac{\log\big(1-(1-\zeta^2)r^2\big)}{1-\zeta^2}\Big]$$

  - $\eta_{\rm uv}$ is the rapidity regulator
  - Simple dependence on mass ratio $r \equiv \frac{m_H}{2m_t}$

For more details, see Braaten, HZ and Zhang, arXiv:1704.06620

Higgs Collinear Region: Integration

- LP factorization formula (Expand by both m_H^2/s and m_t^2/s)

$$\mathcal{F}^{\rm LP}(s,m_t^2,m_H^2) = \widetilde{\mathcal{F}}_{H+g}(s) + \boxed{\int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{t\bar{t}_{1V}+g}(\zeta)\, d_{t\bar{t}_{1V}\to H}(\zeta;m_t^2,m_H^2,P_H\cdot n)}$$
$$+ \int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{H+t\bar{t}_{8T}}(\zeta)\, d_{t\bar{t}_{8T}\to g}(\zeta;m_t^2,p_3\cdot\bar{n}) + \mathcal{F}_{\rm soft}(m_t^2)$$

- Focus only on the mass ratio term

$$\widetilde{\mathcal{F}}_{t\bar{t}_{1V}+g}(\zeta) \propto \zeta$$

$$d_{t\bar{t}_{1V}\to H} \propto \left[\cdots - \zeta\frac{\log\left(1-(1-\zeta^2)r^2\right)}{1-\zeta^2}\right]$$

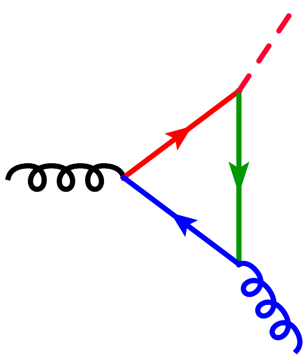
$$\int_{-1}^{+1} d\zeta\, \frac{\zeta^2}{1-\zeta^2}\log\left(1-(1-\zeta^2)r^2\right) = \boxed{-2\arcsin^2 r - \frac{4\sqrt{1-r^2}}{r}\arcsin r} + 4$$

Reproduce the mass ratio term in the LP form factor

Soft Region

- LP factorization formula (Expand by both m_H^2/s and m_t^2/s)

$$\mathcal{F}^{\rm LP}(s,m_t^2,m_H^2) = \widetilde{\mathcal{F}}_{H+g}(s) + \int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{t\bar{t}_{1V}+g}(\zeta)\, d_{t\bar{t}_{1V}\to H}(\zeta;m_t^2,m_H^2,P_H\cdot n) + \int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{H+t\bar{t}_{8T}}(\zeta)\, d_{t\bar{t}_{8T}\to g}(\zeta;m_t^2,p_3\cdot\bar{n}) + \boxed{\mathcal{F}_{\rm soft}(m_t^2)}$$



➢ Only the soft line has mass dependence

$$\mathcal{F}_{\rm soft} \propto \int_q \frac{{\rm Tr}[\dots]}{[q\cdot\bar{n}+i\epsilon]\,[q^2-m_t^2+i\epsilon]\,[q\cdot n-i\epsilon]} \left[\frac{|q\cdot(n-\bar{n})|}{\nu}\right]^{-2\eta}$$

$$\propto \left[\frac{\mu^2}{m_t^2}\right]^{\epsilon} \left[\frac{\nu}{2m}\right]^{2\eta} \frac{1}{\epsilon\eta_{\rm UV}}$$

➢ Only depends on m_t

Add All Regions

- All pieces are calculated directly from Feynman diagrams with fewer scales.
- Rapidity divergences cancel among two collinear regions and the soft region.
- $1/\epsilon$ poles cancel when all four terms are added.
- Comparing with the full form factor, all LP terms are preserved, including the mass singularities.

For more details, see Braaten, HZ and Zhang, arXiv:1704.06620

Improved Top Mass Dependence

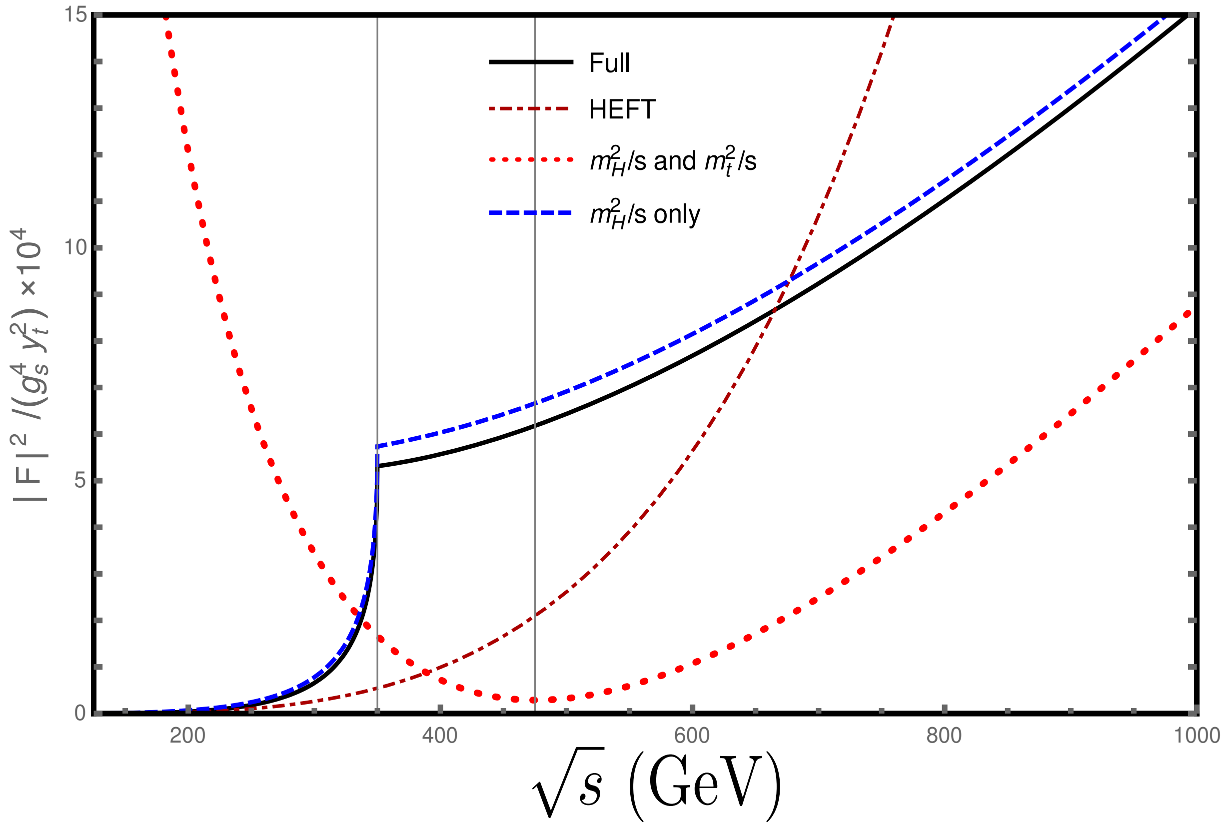
- Only expand by m_H^2/s, keep m_t an arbitrary scale

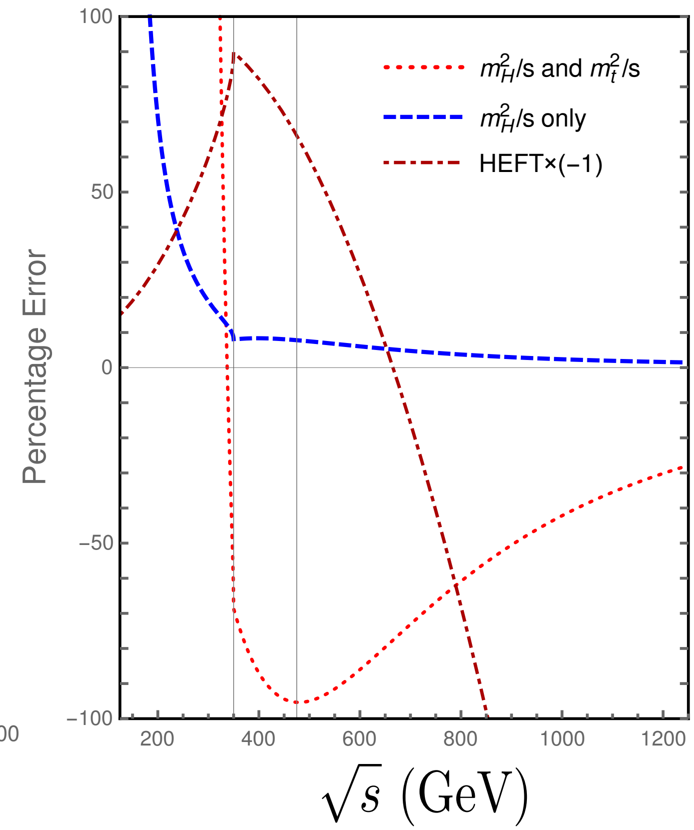
  Don't expand in m_t^2/s or m_H^2/m_t^2

- Same factorization formula, only the hard region needs to be modified
- Similar to simplified-ACOT scheme

For more details, see Braaten, HZ and Zhang, arXiv:1704.06620

Compare with Full Result





- The error of factorization formula decreases as $\sqrt{s}$ increases.
- Reliable prediction for all kinematic region can be obtained by combining the factorization formula with HEFT.

Bottom Loop Contribution

- The factorization formula can also be used to calculate the contribution from a *b*-quark loop

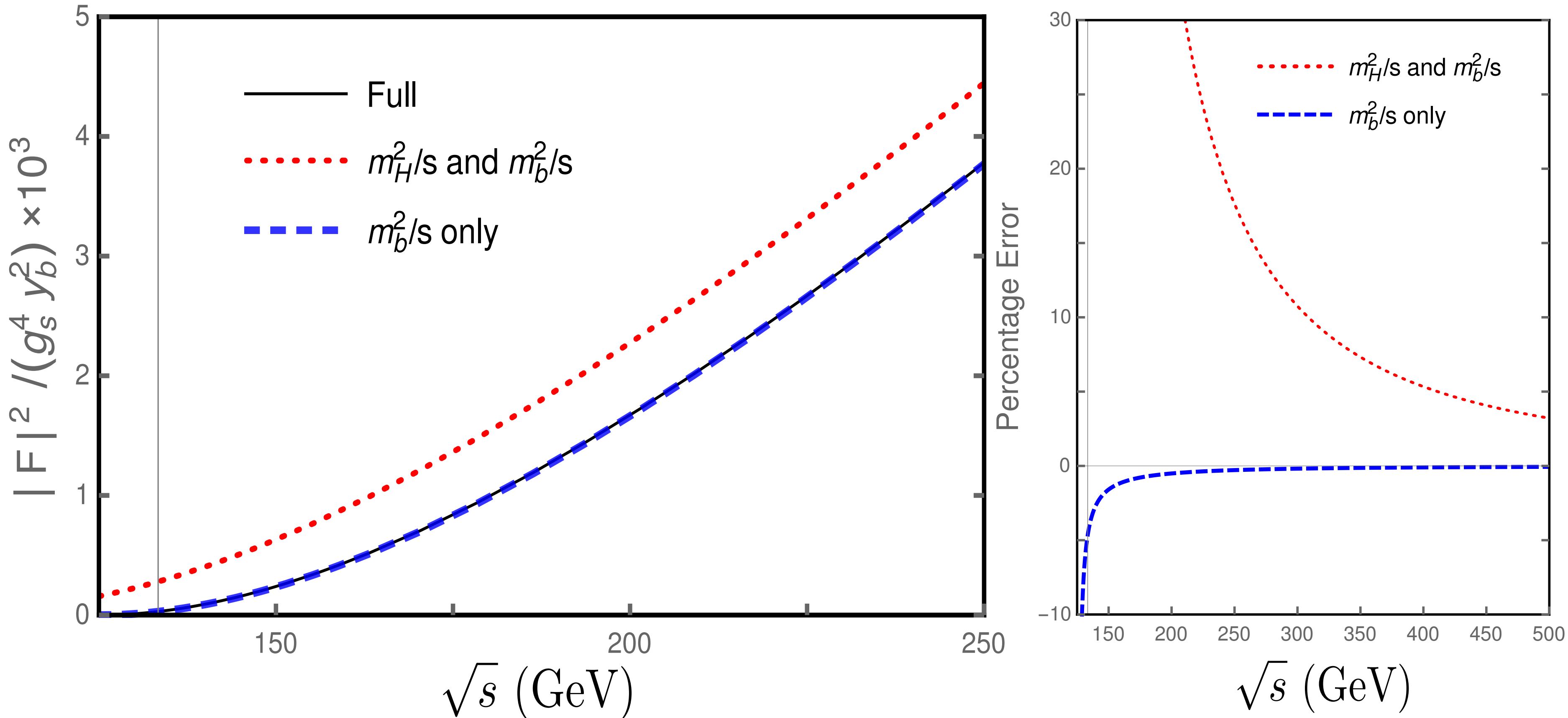
  Relevant scales: $\sqrt{s}, m_H, m_b$

  Limit 1: Expand in both m_H^2/s and m_b^2/s

  Limit 2: Expand only in m_b^2/s

- Separate the scales before calculating the Feynman diagrams. Much simpler to calculate.
- Comparing with the full form factor, all LP terms are preserved, including the mass singularities.

Compare with Full Result (b-quark loop)



- The error of factorization formula quickly decreases as $\sqrt{s}$ increases.
- The expansion in only m_b^2/s gives a very good approximation to the full result (<10% error over all kinematic region)

Next-to-leading Order (In Progress)

- Factorization formula

$$\mathcal{F}^{\mathrm{LP}}(s,m_t^2,m_H^2) = \widetilde{\mathcal{F}}_{H+g}(s) + \int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{t\bar{t}_{1V}+g}(\zeta)\, d_{t\bar{t}_{1V}\to H}(\zeta; m_t^2, m_H^2, P_H\cdot n) + \int_{-1}^{+1} d\zeta\, \widetilde{\mathcal{F}}_{H+t\bar{t}_{8T}}(\zeta)\, d_{t\bar{t}_{8T}\to g}(\zeta; m_t^2, p_3\cdot\bar{n}) + \mathcal{F}_{\mathrm{soft}}(m_t^2)$$

➢ Hard region: 3-point & 2-loop, scale $\sqrt{s}$ (and m_t)

➢ Collinear regions:

Hard coeff.: 3-point & 1-loop, scale $\sqrt{s}$

Higgs dist. amp.: 2-loop, scales m_H and m_t

Gluon dist. amp. : 2-loop, scales m_t

➢ Soft region: 3-point & 2-loop, scale m_t

Next-to-leading Order (In Progress)

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- Hard region: 3-point & 2-loop, scale $\sqrt{s}$ (and m_t)
- Collinear regions:

  Hard coeff.: 3-point & 1-loop, scale $\sqrt{s}$

  Higgs dist. amp.: 2-loop, scales m_H and m_t — 3 diagrams!

  Gluon dist. amp. : 2-loop, scales m_t — 6 diagrams!

- Soft region: 3-point & 2-loop, scale m_t

Summary

- For Higgs produced at large P_T, NLO result is still unavailable 30 years after LO.
- We have proposed a factorization formula which separates different scales before calculating Feynman diagrams.
- Each piece in the factorization formula is already available or easy to calculate.
- With the example $q\bar{q} \to H + g$, we show the factorization formula gives a very good approximation of the full result.
- Combined with HEFT, a reliable prediction of Higgs P_T distribution can be obtained at higher orders.
- The same method also shows great power to study the b-quark loop contribution.

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Thank you!

Backup slides

Percentage Error (t-loop)