Outlook

LoopFest 2017
May 12, 2012
Zvi Bern

UCLA The Mani L. Bhaumik Institute
for Theoretical Physics

LOOPFEST XVI

Radiative Corrections for the
LHC and Future Colliders

May 31 - June 2, 2017
Argonne National Laboratory

Outline

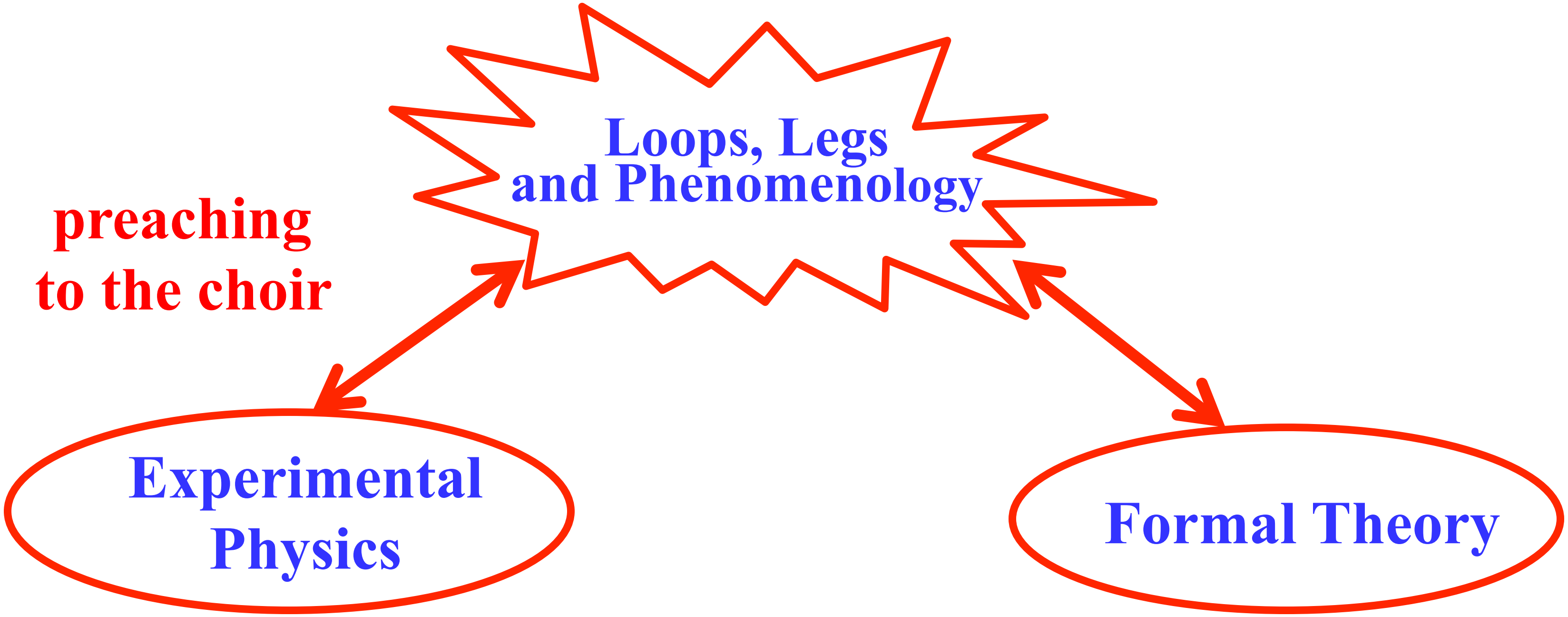
An impressive display of technical firepower
to achieve remarkable precision with experiment!

Here I want to talk about the need to keep on pushing technical firepower, not just for phenomenology, but also for theoretical issues.

I will draw examples that fit my theme from my own work as well as from talks at this conference.

Links to other fields

A healthy field should have links to other fields.



Loops, Legs and Phenomenology

preaching
to the choir

Experimental Physics

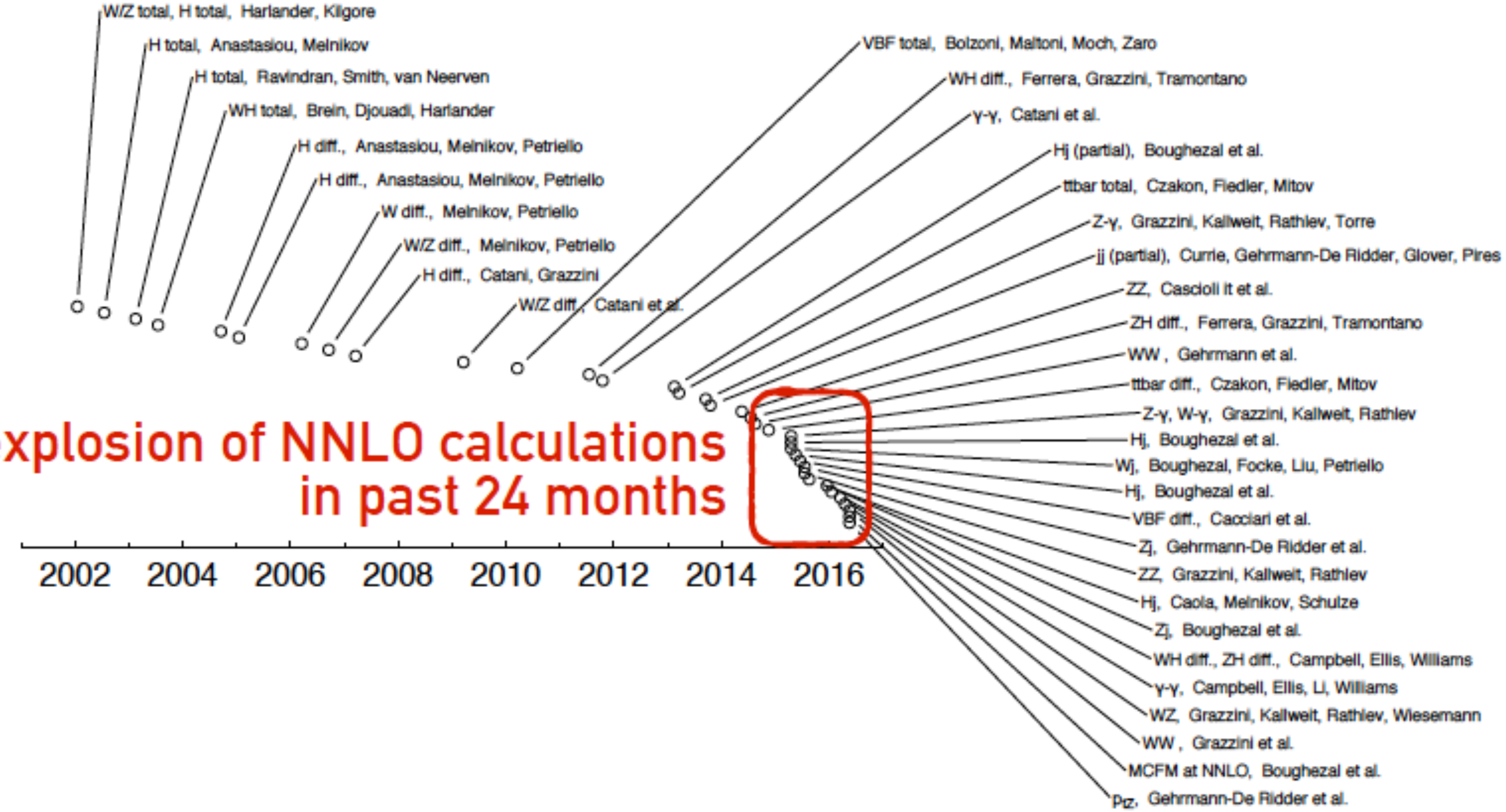
Formal Theory

- Important to exchange ideas with other subfields.
- Important impact on formal theory!

From G. Salam

NNLO hadron-collider calculations v. time

as of mid June 2016



explosion of NNLO calculations in past 24 months

Many New Results

50 Talks!

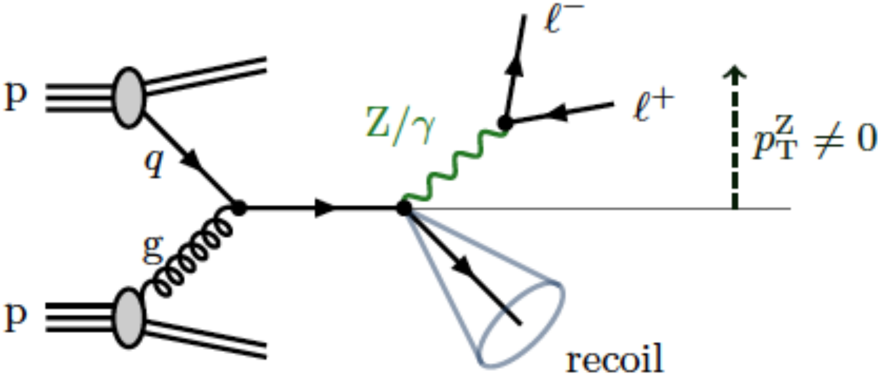
We heard about many new advances including:

1. **Continued advances in NNLO** Talks: Dulat, Dreyer, Gauld, Grazzini, Lui, Moult, Mitov, Neumann, Page, Schubert
2. **NLO including high multiplicity** Talks: Buccioni, Deutchmann, Figy, Frixione. Wiegand, Reuchele, Ringer, Vitev
3. **N^3LO calculations** Talks: Dulat, Moch
4. **N^kLL resummation.** Talks: Banfi, Monni, Museli, Theeuwes
5. **Effective field theory appoaches.** Talks: Deutschmann
6. **New ideas for multi-loop amplitudes and integrals.** Talks: Bowrowka, Ita, Li, Mishima, Schabinger, Zhang, Zeng
7. **Parton showers** Talks: Hoeche,Presetel, Soper, Re
8. **PDFs** Talks: Moch, Nadolsky
9. **4-loop β-function** Talks: Zoller
10. **Etc.**

Impressive advances

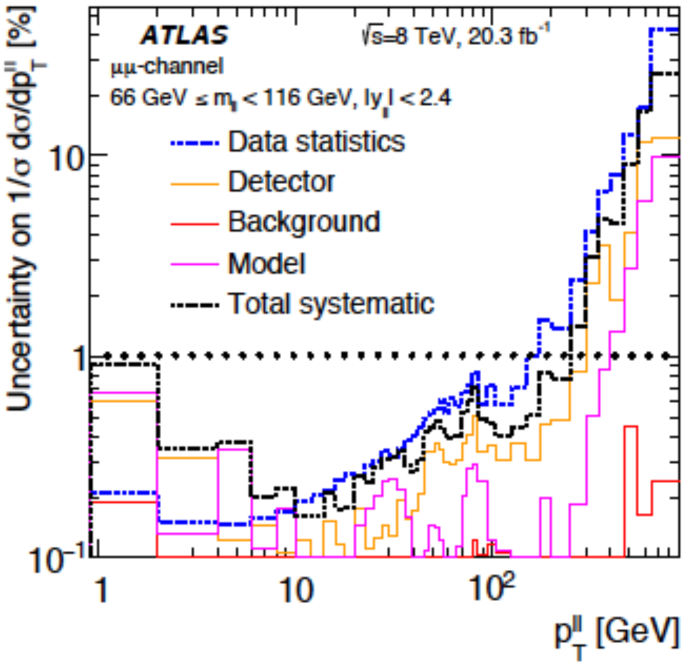
physics motivation

Talk from Guald

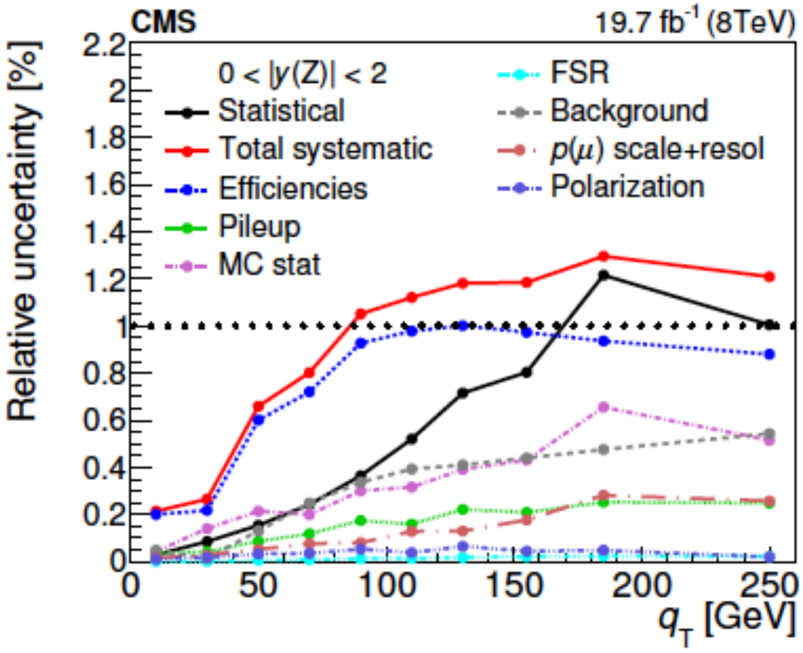


Experimental status (sub %)

Need NNLO+



ATLAS, arXiv: 1512.02192



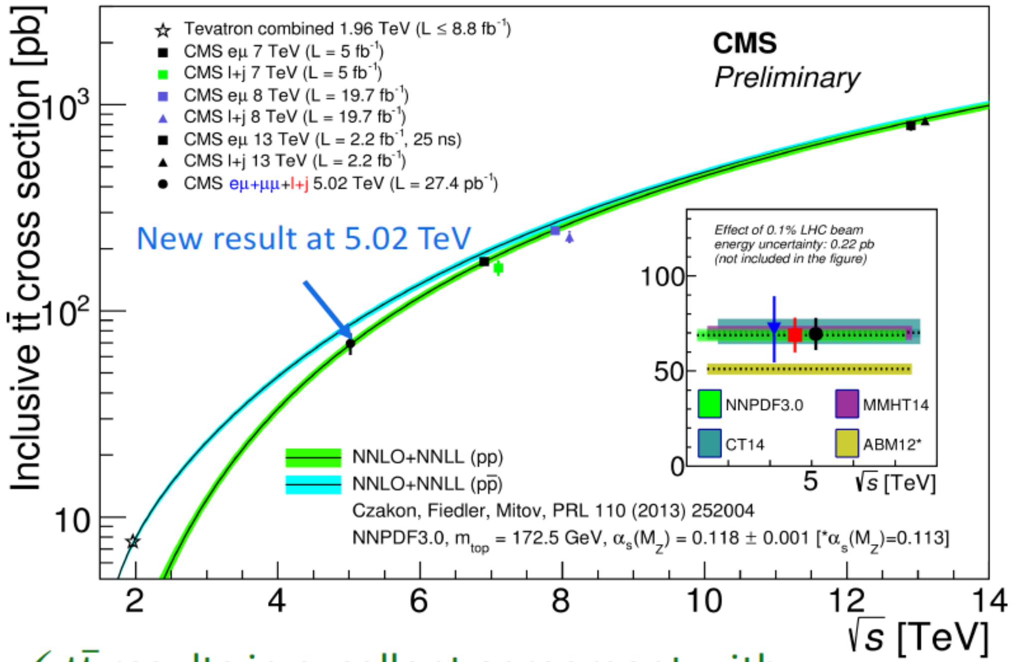
CMS, arXiv: 1504.03511

Top pair production

From Barberis' talk

Production of top quarks: tests SM, special connection t
EWSB? (heaviest know elementary particle). The LHC is a top factory, with enough statistics to perform precision measurements of differential cross-sections.

CMS-PAS-TOP-16-023



- ✓ $t\bar{t}$ results in excellent agreement with NNLO+NNLL predictions.
- ✓ Starts constraining the gluon PDF.
- ✓ α_S determination (if m_{top} fixed).

LoopFest 2017

Great example of utility of NNLO

Eur. Phys. J. C77 (2017) 299

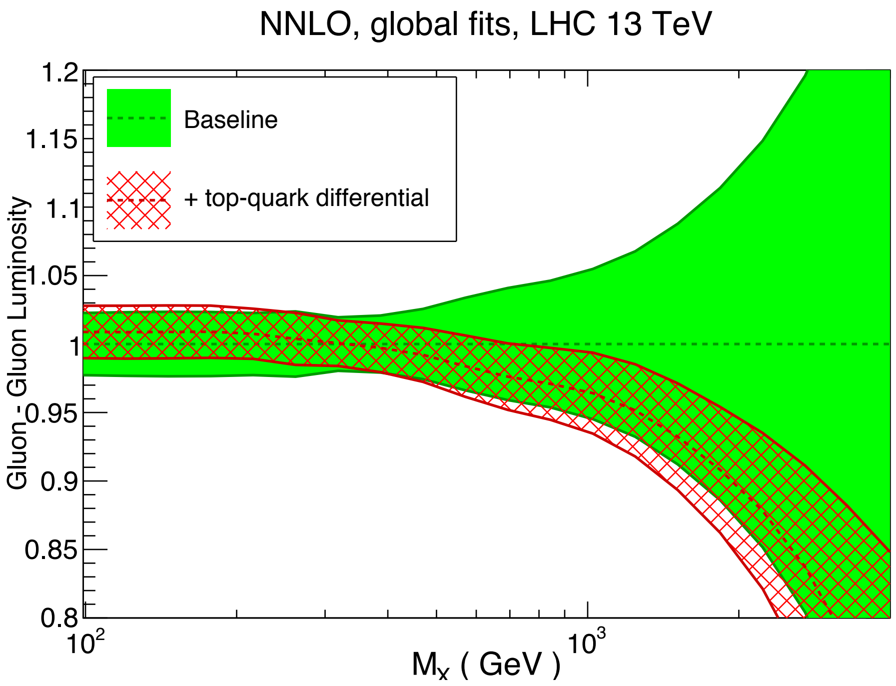
CMS-PAS-TOP-16-007

Fitting PDF from top-pair data

From Mitov's talk

Czakon, Hartland, Mitov, Nocera, Rojo 2016

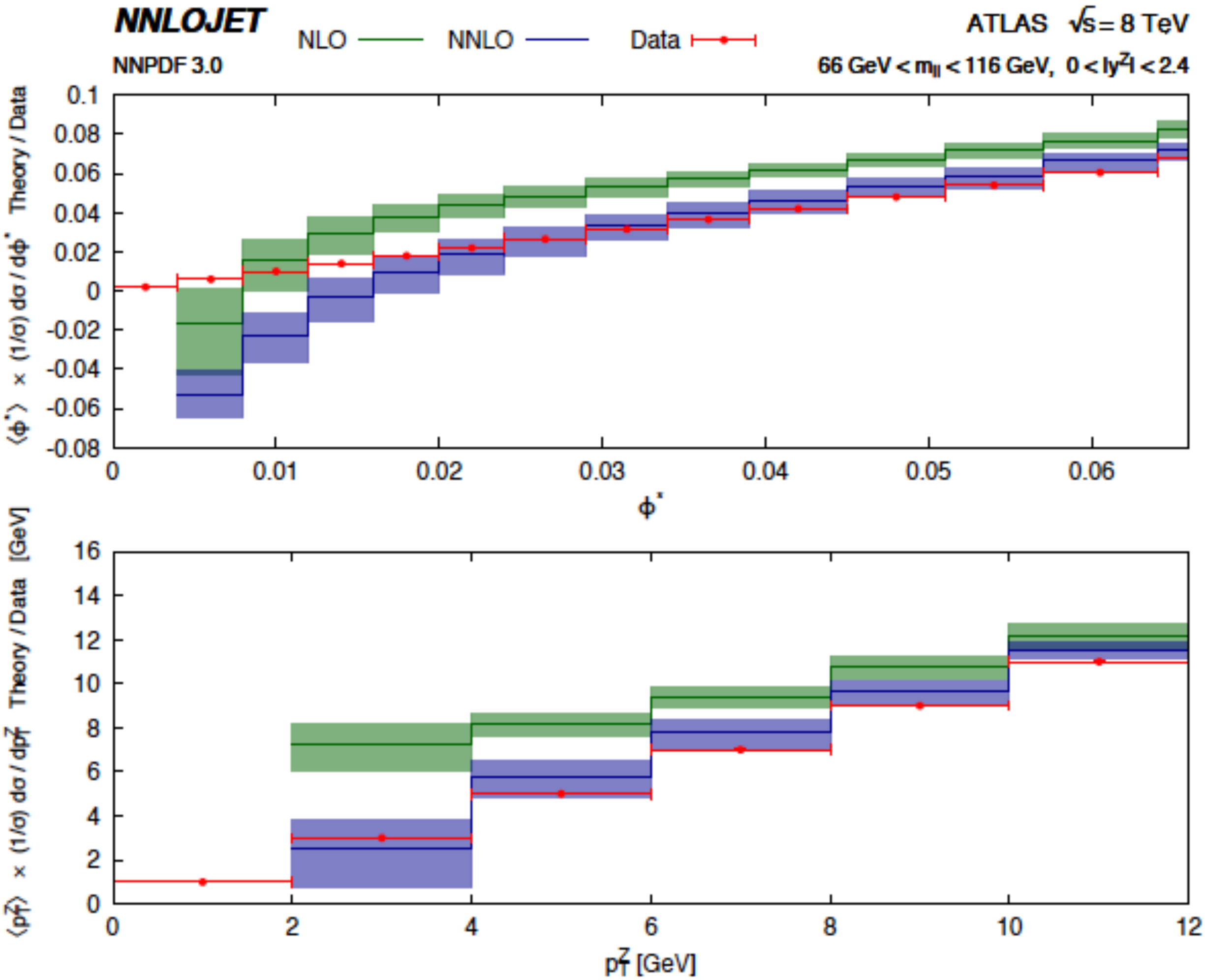
✓ **Improvement in the gluon PDF after top data is included.**



✓ **Very significant reduction of PDF error.**

NNLO $\gamma/Z+X$ ϕ^*_η observable

Talk from Gauld



$$\phi^*_\eta = \tan\left(\frac{\phi_{\mathrm{acop}}}{2}\right)\cdot\sin(\theta^*_\eta)$$

NNLO - ‘reliable’ to: $p_T^Z \sim 4$ GeV
$\phi^* \sim 0.02$

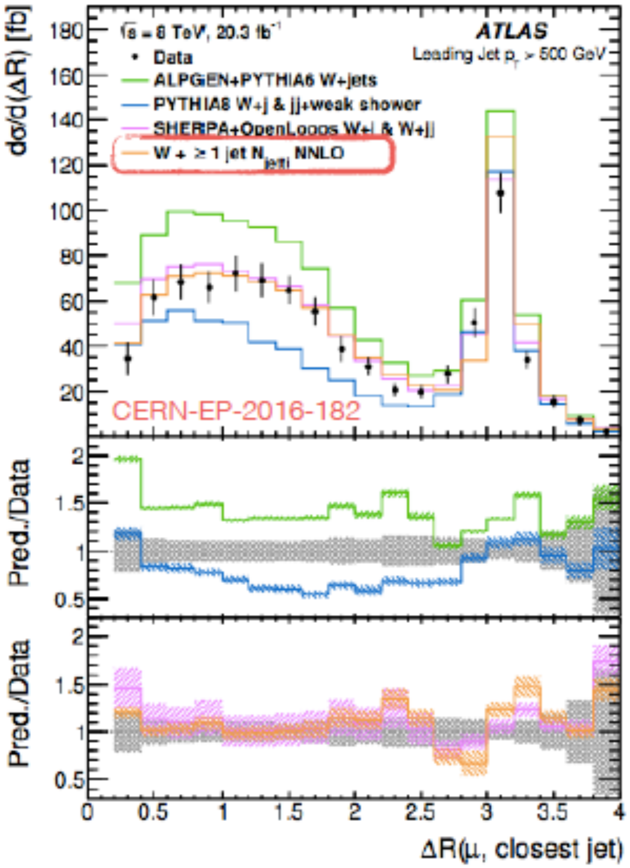
NLO - not reliable

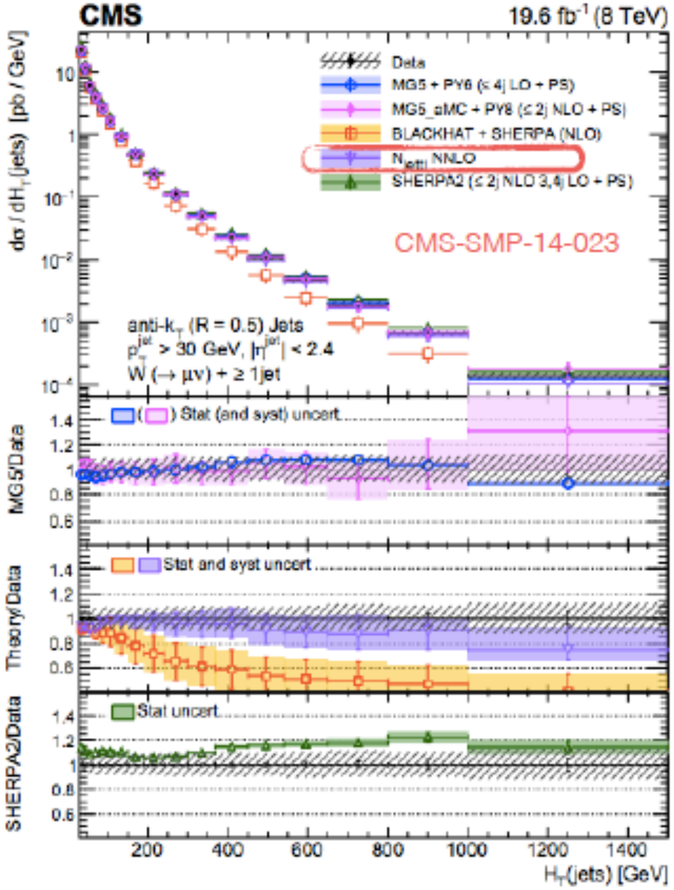
Talk from Lui

W+ ≥1 jet

Current Status of the N-Jettiness Scheme

- Good agreement with data





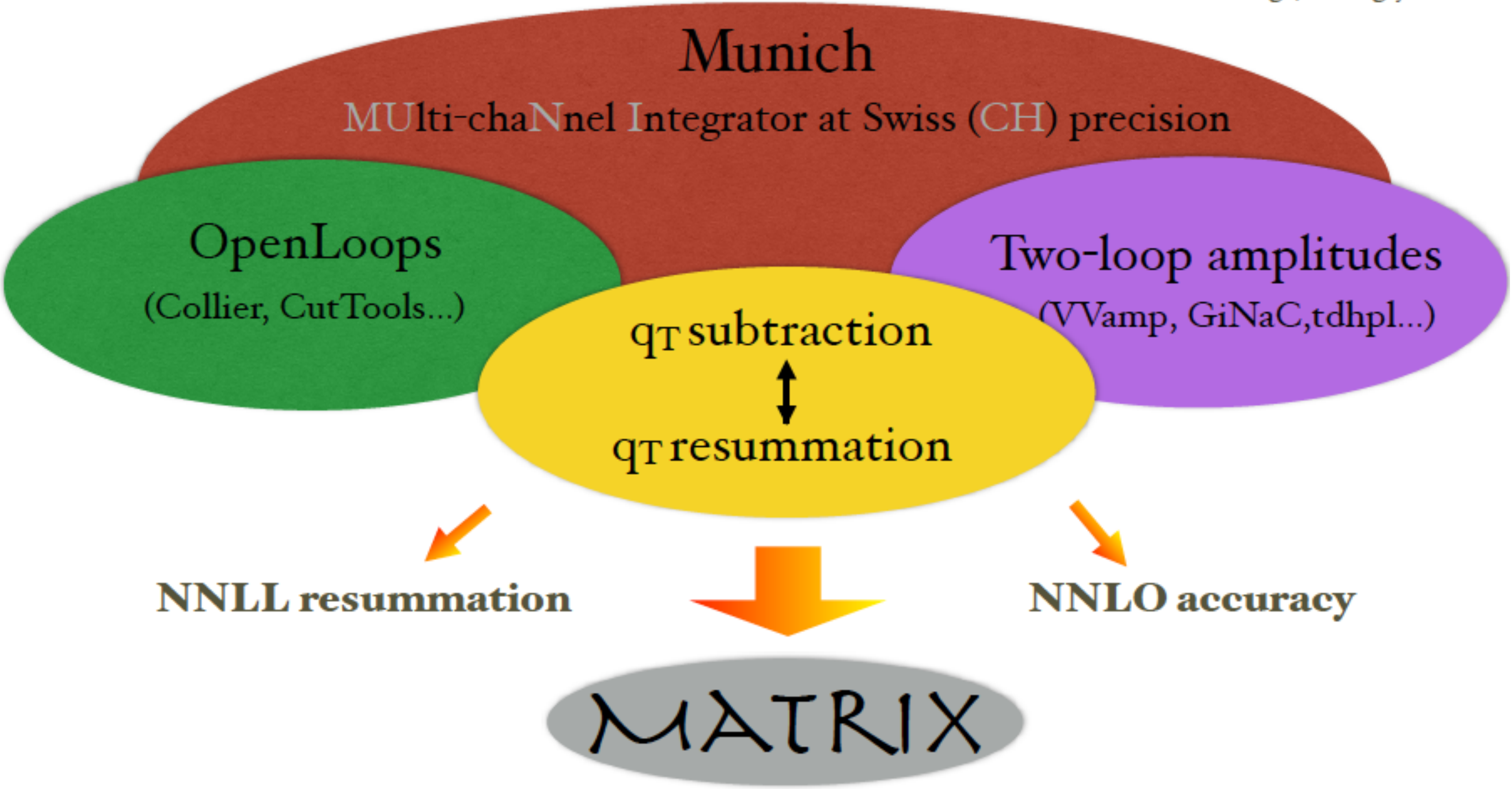
- Improved agreements with all measured distributions

Bougezhal, XL, Petriello

Talk from Grazzini

The MATRIX project

S. Kallweit, D. Rathlev, M. Wiesemann, MG
(+C.Hanga, H.Sargsyan.....)



Munich

MUlti-chaNnel Integrator at Swiss (CH) precision

OpenLoops

(Collier, CutTools...)

Two-loop amplitudes

(VVamp, GiNaC,tdhpl...)

q_T subtraction

↕

q_T resummation

NNLL resummation

NNLO accuracy

MATRIX

Munich Automates qT subtraction and Resummation to Integrate X--sections

From talks of Prestel and Hoche: NLO Parton Showers

Extending Dire beyond leading order

arXiv:1705.00742

cf. S. Höche's talk

For a shower implementation with NLO corrections, we need

- ... analytically manageable phase space for LO-like $(1 \to 2)$ and real-emission-type $1 \to 3$ transitions,
- ... algorithms that can handle negative (e.g. NLO DGLAP) kernels,
- ... cross-validation.

Ideal solution: NLO-corrected PS is a fully differential NLO calculation in the Sudakov exponent:

$$\Delta(t_0, t_1) = e^{-\int_{t_1}^{t_0} \frac{dt}{t} \int d\tilde{z} \left[\left(\mathrm{I} + \underset{\uparrow}{\frac{1}{\varepsilon}} \mathcal{P} - \mathcal{I} \right)(\tilde{z}) + \int \mathrm{d}\Phi_{+1} (\mathrm{R} \underset{\uparrow}{-} \mathrm{S})(\tilde{z}, \Phi_{+1}) \right]}$$

S-event, a.k.a. endpoint H-event

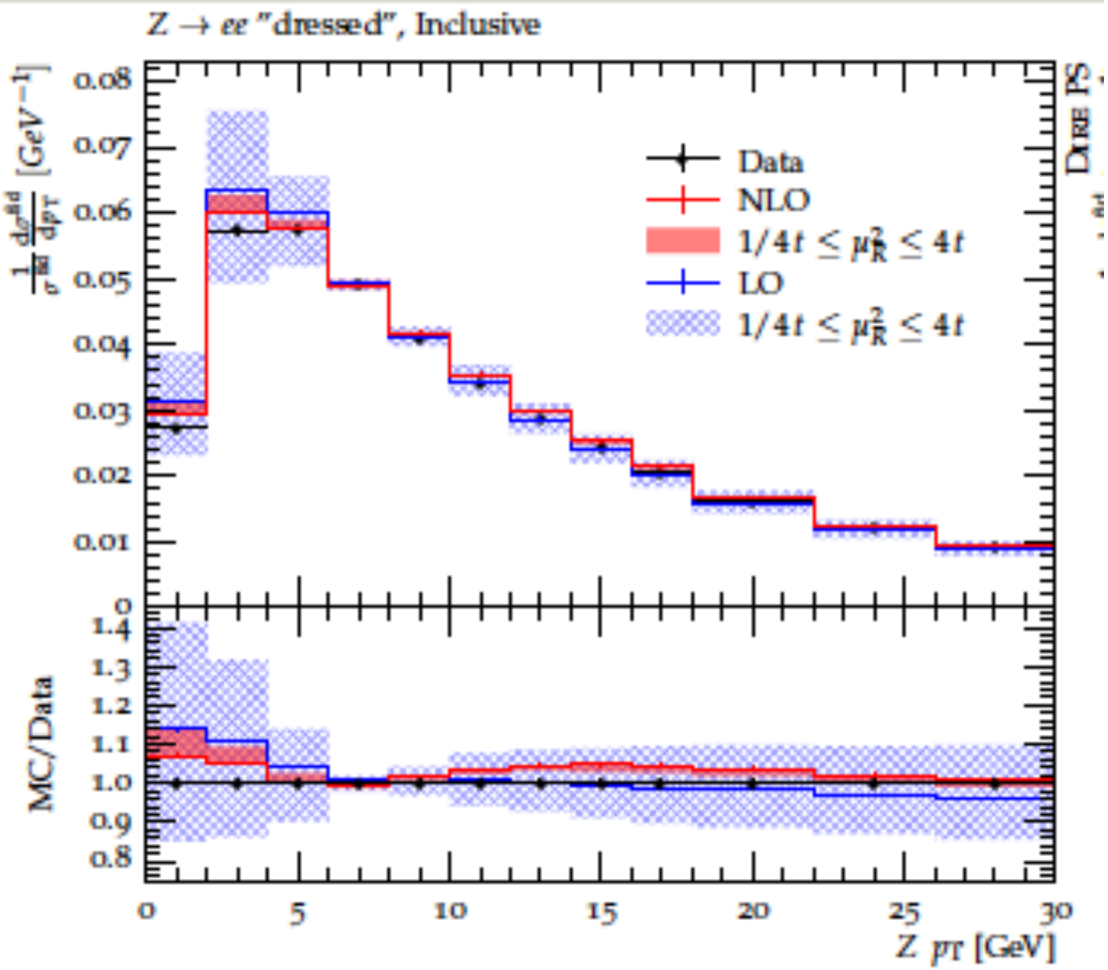
Pro: On-the-fly numerical recalculation of known NLO results.

Con: Full-fledged implementation requires recalculating loops.

⇒ For now, use a simpler scheme as baseline implementation.

From talks of Prestel and Hoche: NLO Parton Showers

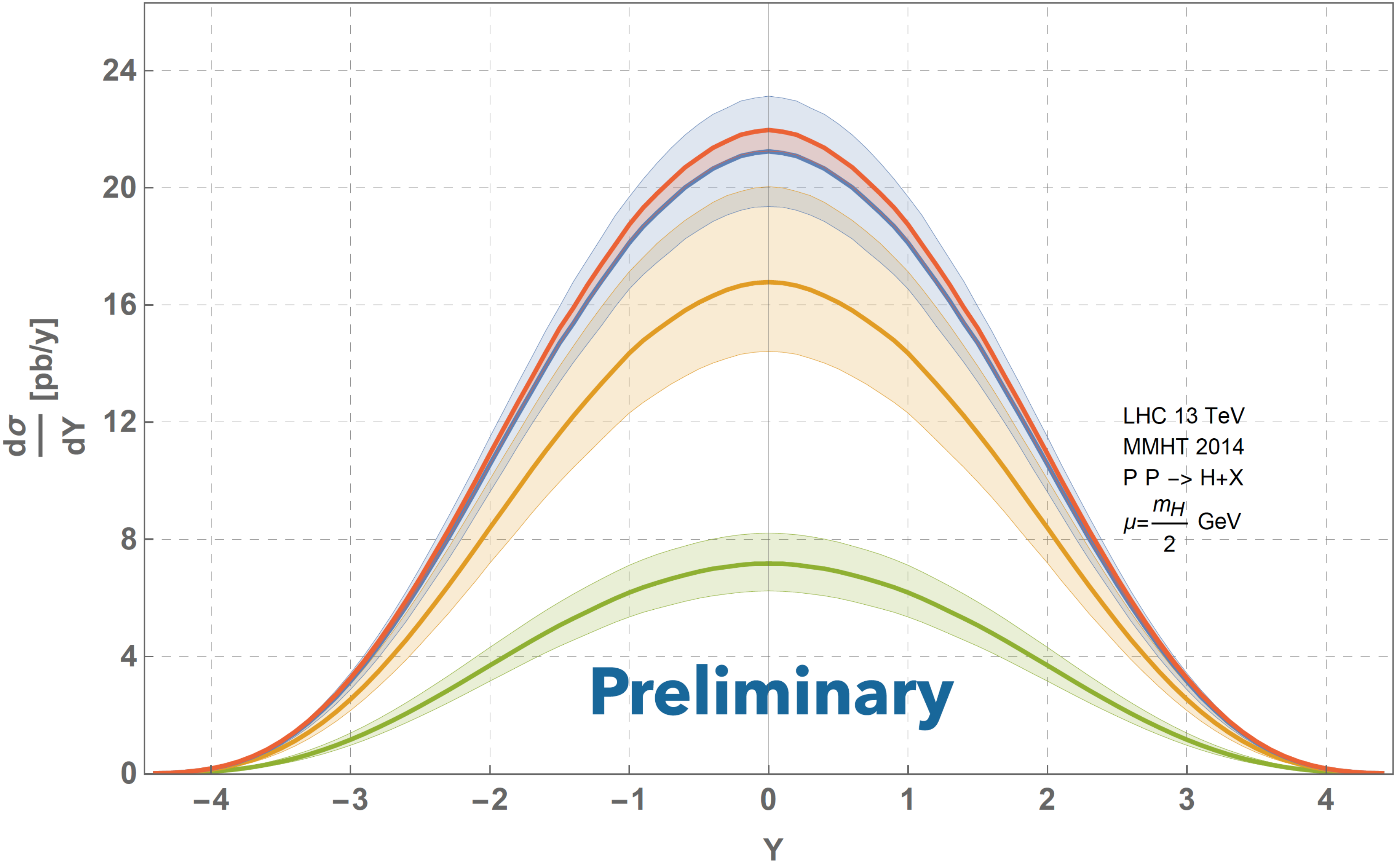
Dire



NLO parton showering can be done!

N^{3}LO Teaser From Dulat

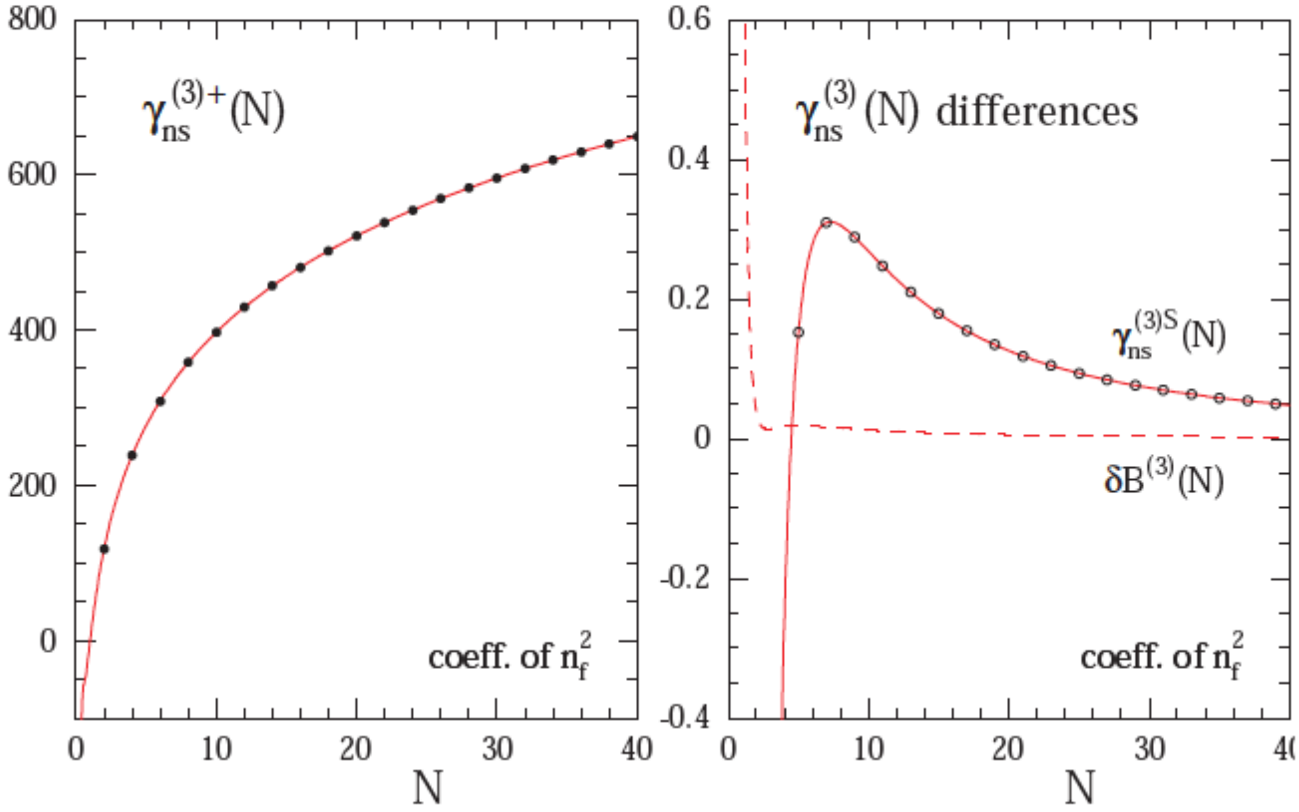
SV RAPIDITIY DISTRIBUTION @ N3LO



Splitting functions at N^3LO

Talk from Moch

- Non-singlet splitting functions $P_{\rm ns}^{(3)\,\pm}(x)$, anomalous dimension $\gamma_{\rm ns}^{(3)\,\pm}(N)$
- Fermionic contributions $\mathcal{O}(n_f^2)$ known at N^{3}LO Ruijl, Ueda, Vermaseren, Vogt ‘16
- Computation based on FORCER program in FORM Ruijl, Ueda, Vermaseren ‘17



Relation to Formal Issues

No shortage of technical firepower!

I want to show you how the advances are playing a crucial role not only in phenomenology, but in formal questions.

IBP Multiloop Technology

Basic technology for multiloop integrals: integration by parts

Chetyrkin and Tkachov

$$\int \prod_i d^D\ell_i \frac{\partial}{\partial \ell_j^\mu}\left(\frac{v_j^\mu}{\prod_k D_k}\right) = 0$$

Laporta alogithm: make the system big enough and you can solve, in terms of master integrals.

Basic tools: AIR, FIRE5, REDUZE2, LiteRed…

Anastasiou; A. Smirnov; von Manteuffel; Lee

Awesome tools, even at 4 loops!

But systems get out of control. Always have harder problems to solve!

Is there a better way? Yes! Syzygy, alegbraic geometry, unitarity compatibility…

Talks from Ita, Zeng, Schabinger, Zhang

New or Improved Loop Integration Technologies

Many talk on new ideas and advances:

- IBPs and differential equations without doubled propagators.
  Mao Zeng
- Asymptotic expansion of Feynman integrals.
  Go Mishima
- Numerical unitarity method in QCD
  Harald Ita
- First two-loop amplitudes with numerical unitarity method
  Ben Page
- Azurite: a package to determine master integrals via computational algebraic geometry.
  Yang Zhang
- Numerical approach to multi-scale multi-loop integrals
  Zhao Li
- Baikov-Lee representations of cut Feynman integrals
  Robert Schabinger
- Finite fields for linear equations.
  Andreas von Mantuefel and Robert Schabinger

From Ita’s talk

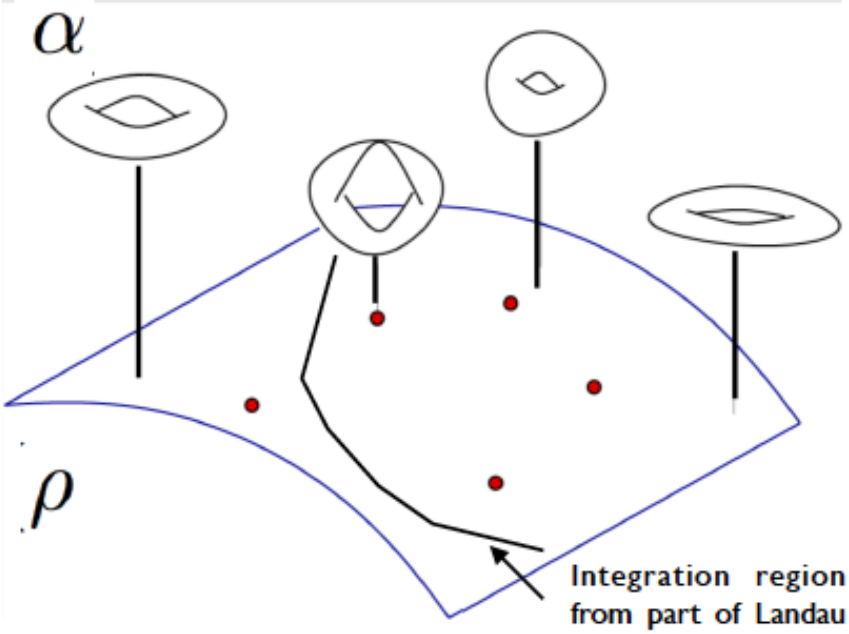
Hidden Geometry

Algebraic geometry

Coordinate change exposes fiber structure:

$$\mathcal{I}[t] = \int \frac{[d\rho]}{\rho^0 \cdots \tilde{\rho}^{(\tilde{N}-1)}} \times t(\rho,\alpha)\,\mu(\rho,\alpha)[d\alpha]$$

See also: ‘The Analytic S-Matrix’, Eden, Landshoff, Olive, Polkinhorne; Baikov; HI; Zhang Larsen



Integration region from part of Landau equations

Functional dependence on internal spaces important for full integral.

One loop example: internal spaces are spheres; all non-constant harmonic functions integrate to zero = IBP relations.

Geometry comes with natural structures:

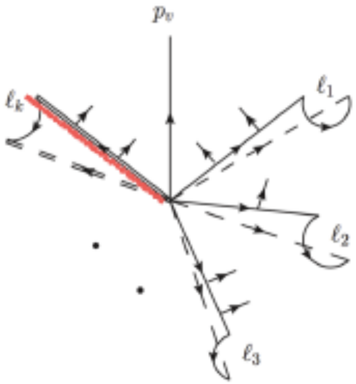
- Function ring => irreducible numerators; tangent vectors => IBP relations
- Cohomology => master integrals; moduli spaces & connections => differential equations

From Ita's talk

Solve IBP Reduction?

IBP-generating vectors:

- Rotation/scaling/translation generators for each rung, consistent with momentum conservation at vertices.
- Can we write down all solutions and solve integral reduction?



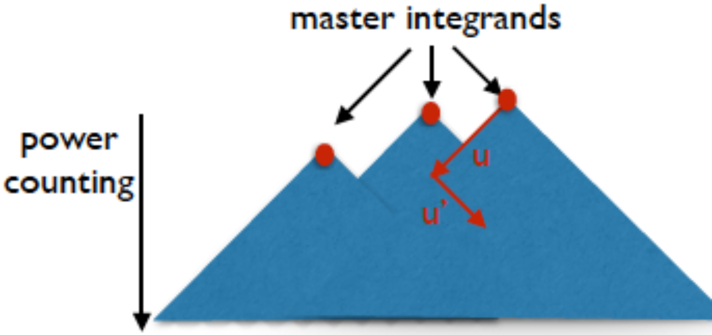
Geometric structures:

- Lie-algebra & representation theory:

$$[u_a, u_b] = f^c_{ab}(\ell, p_i) u_c$$

- Numerators are representations space.
- 'Highers weight' representation are master integrals

Numerator polynomials:



There is structure to be exploited

FROM YANG ZHANG'S TALK

Azurite

Georgoudis, Larsen, YZ 1612.04252

A ZURich-bred InTEgral-determination method

Syzygy → IBPs within one sector → Master integrals

$$0 = \int dz_1 \ldots dz_k \sum_{i=1}^{k} \partial_i \left(a_i(z) G^{-D/2} \right)$$

If the IBP vector satisfies,

$$\sum_{i=1}^{k} a_i(z) \frac{\partial G}{\partial z_i} + b(z) G = 0$$

the resulting IBP has a simple form (without dimension shift)

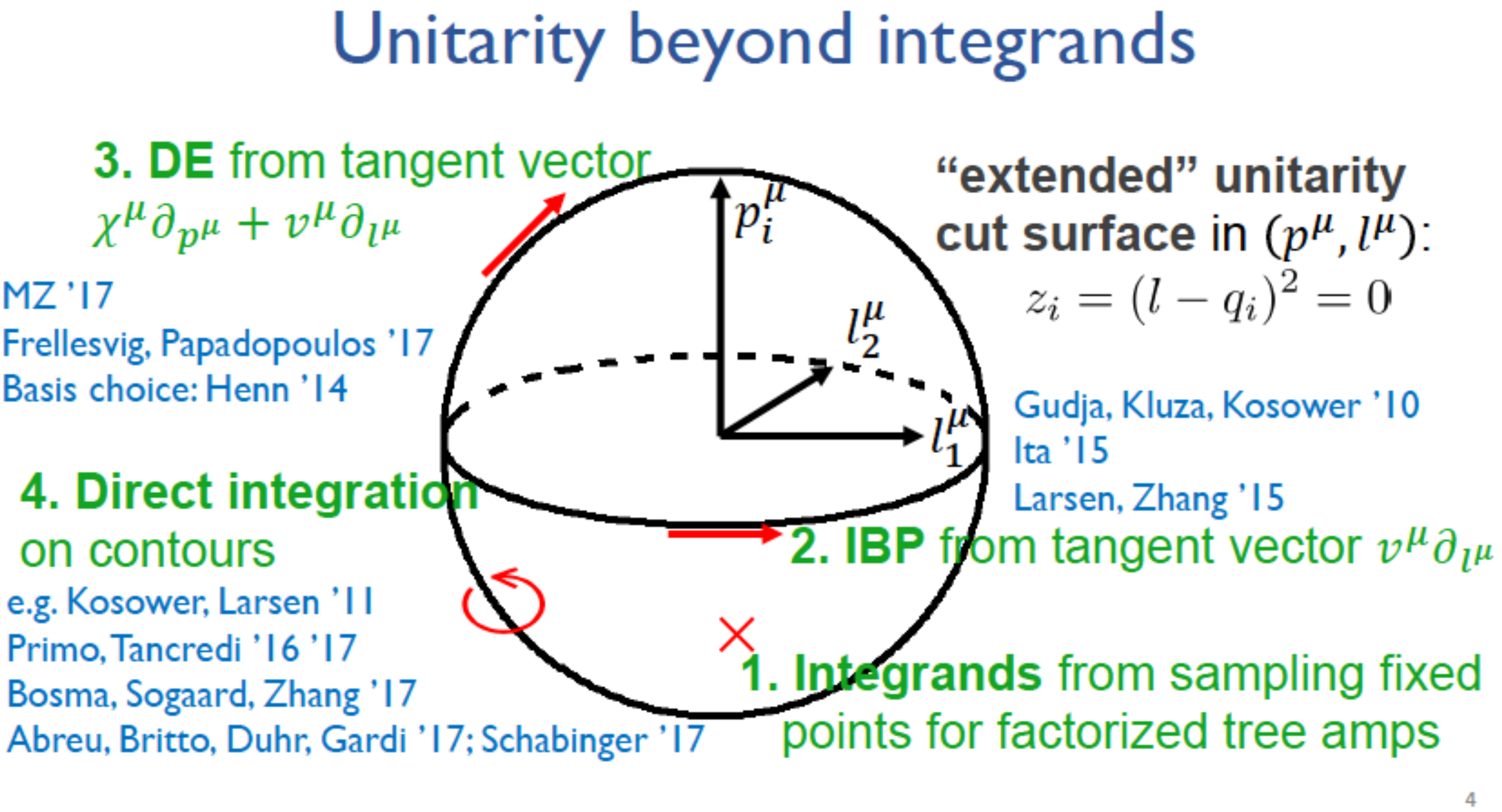
Bases on

Syzygy for IBPs: Gluza, Kjada, Kosower 1009.0472
IBP with arbitrary cuts: Ita 1510.05626, Larsen, YZ 1511.01071

See Ita, Page and Zeng's talks for more applications on unitarity, integral reduction and differential equations

Algebraic geometry to find master integrals.

From Mao's Zeng's talk



Unitarity beyond integrands

3. DE from tangent vector

$\chi^\mu \partial_{p^\mu} + v^\mu \partial_{l^\mu}$

MZ '17
Frellesvig, Papadopoulos '17
Basis choice: Henn '14

"extended" unitarity
cut surface in (p^μ, l^μ):

$$z_i = (l - q_i)^2 = 0$$

Gudja, Kluza, Kosower '10
Ita '15
Larsen, Zhang '15

4. Direct integration
on contours

e.g. Kosower, Larsen '11
Primo, Tancredi '16 '17
Bosma, Sogaard, Zhang '17
Abreu, Britto, Duhr, Gardi '17; Schabinger '17

2. IBP from tangent vector $v^\mu \partial_{l^\mu}$

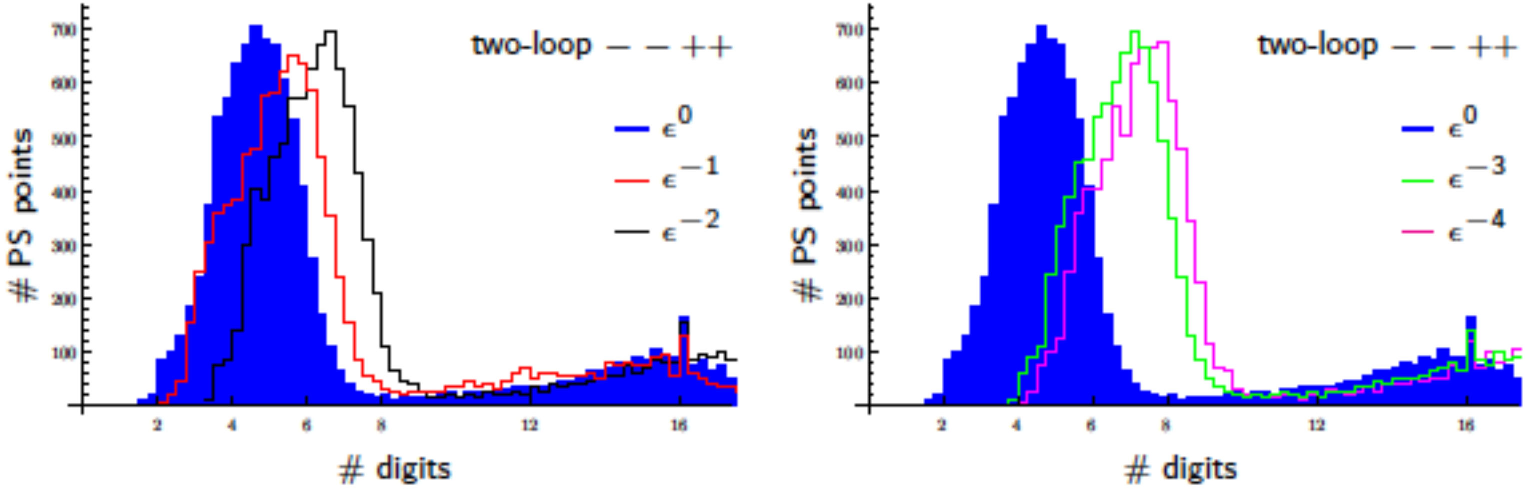
1. Integrands from sampling fixed points for factorized tree amps

4

Analytic understanding.

Numerical Stability of MMPP Amplitude

Talk from B. Page



- Comparison to analytics over 10000 phase space points.
- Rescue system based on accuracy of universal $\frac{1}{\epsilon}$ pole.

Proves new technology works in real situations

Formal Topic: Solve $N = 4$ sYM Theory

See talk from Volovich

The key goal is to "solve" planar $N = 4$ sYM theory.

- Connection to AdS/CFT and Maldacena conjecture.
- Connection to integrability.
- Bootstrap program.

How is QCD Connected to *N* = 4 sYM?

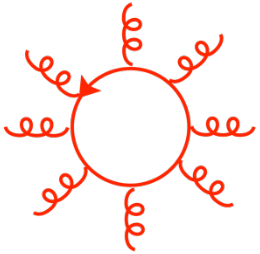
N = 4 sYM plays central role in AdS/CFT and string theory

Connection to QCD is simple.
At *any* loop order to get *N* = 4 sYM from QCD:

- Replace quarks with 1 adjoint fermion.
- Place all states in *D* = 10.
- Put all loop momenta in *D* = 4-2ε.

Dimensional reduction of *D* = 10, *N* = 1 susy is N=4 sYM

If you have a QCD computation, *N* = 4 sYM is essentially free! *N* = 4 sYM is lot simpler.



From Volovich talk

N=4 Yang-Mills Amplitudes

- Despite recent advances, relatively few scattering amplitudes in N=4 Yang-Mills are available in the literature.
- 6-point MHV and NMHV up to 5-loops [Caron-Huot, Dixon, McLeon, Von Hippel 2016]
- All 2-loop MHV [Caron-Huot 2011]
- 7-point 2-loop NMHV [Caron-Huot, He 2011]
- 7-point 3-loop MHV symbol [Drummond, Papathanasiou, Spradlin 2014]
- 7-point 4-loop MHV and 7-point 3-loop NMHV symbol [Dixon, Drummond, Harrington, McLeod, Papathanasiou, Spradlin 2016]

Some Work Related to $\mathcal{N} = 4$ sYM

See talk from Volovich

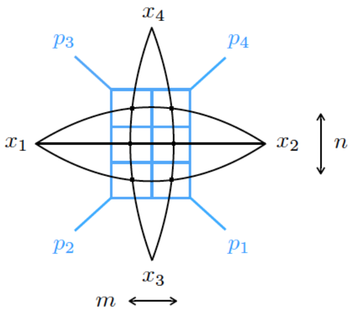
1) **Symbols and polylogs.** Goncharov, Spradlin, Vergu, Volovich

2) **Landau singularities.** Dennen, Prlina, Spradlin, Stanojevic, Volovich

3) **Bootstrap: Abolish integrands.**
Caron-Huot, Dixon, McLeod, Matt von Hippel; Dixon, Drummond, Harrington, McLeod, Papathanasio, Spradlin; Li, Neill, Zhu

4) **Simpler differential equations for multiloop integrals.**
Henn; Henn, Smirnov and Smirnov

5) **Fishnet integrals at any loop order.** Basso and Dixon



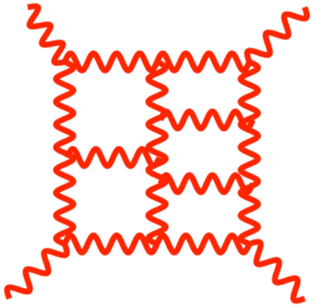
Dual conformal integrals have uniform transcendental weight. Use such integrals as basis in QCD calculations.

Obvious cross-talk between QCD and $\mathcal{N} = 4$ amplitudes

Example: Formal Theory Problem Where We Need Improved Loop Integration

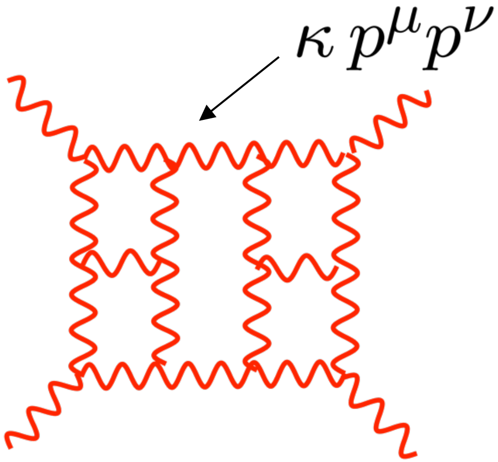
What is the UV behavior of gravity theories?

For major progress we need multi-loop advances of the type discussed at LoopFest!



UV Behavior of Gravity?

$\kappa = \sqrt{32\pi G_N}$ ⟵ Dimensionful coupling



Gravity: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots \kappa p_j^\mu p_j^\nu \cdots}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots g p_j^\nu \cdots}{\text{propagators}}$$

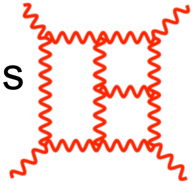
- **Extra powers of loop momenta in numerator means integrals are badly behaved in the UV and must diverge at some loop order.**
- **Much more sophisticated power counting in supersymmetric theories but this is basic idea.**

- **With more supersymmetry expect better UV properties.**
- **Need to worry about "hidden cancellations".**
- ***N* = 8 supergravity best theory to study.**

Feynman Diagrams for Gravity

Suppose we want to check UV properties of gravity theories Using Feynman diagrams:

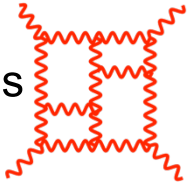
3 loops



~10^{20} TERMS

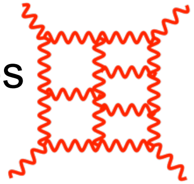
No surprise it has never been calculated via Feynman diagrams.

4 loops



~10^{26} TERMS

5 loops



~10^{31} TERMS

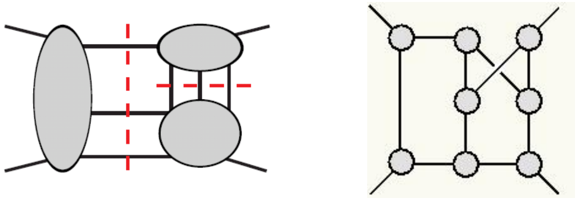
More terms than atoms in your brain!

- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

Need a better approach.

Basic Tools for Attacking the Problem

We use following tools for computing scattering amplitudes and studying their UV properties:



- **Generalized unitarity method.**

  ZB, Dixon, Dunbar, Kosower
  ZB, Carrasco, Johansson, Kosower

- **Duality between color and kinematics. Gravity scattering amplitudes directly from gauge-theory ones. Double copy.**

  ZB, Carrasco and Johansson (BCJ)

- **Advanced loop-integration technology.**

  Chetyrkin, Kataev and Tkachov; Laporta; A.V. Smirnov; V. A. Smirnov; Vladimirov; Marcus, Sagnotti; Czakon; Laporta; Kosower; Ita; Larsen and Zhang; Zeng, etc

Last item is directly connected to LoopFest.

How is Gravity Connected to Gauge Theory

Consider QCD five-gluon tree amplitude:

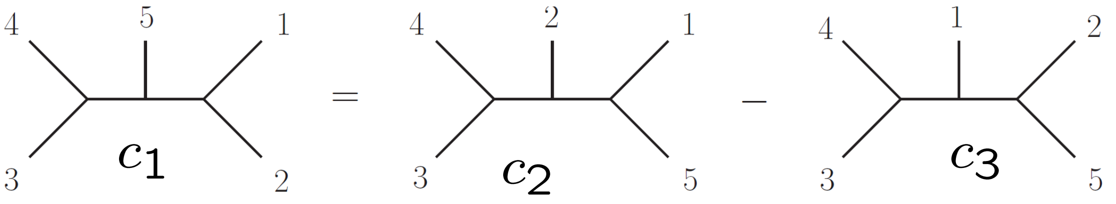
ZB, Carrasco, Johansson (BCJ)

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i\, n_i}{\prod_{\alpha_i} p_i^2}$$

color factor

kinematic numerator factor

Feynman propagators



$$c_1 = f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2} \qquad c_2 = f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5} \qquad c_3 = f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$n_i \sim k_4 \cdot k_5\, k_2 \cdot \varepsilon_1\, \varepsilon_2 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

Duality between color and kinematics:

$$\boxed{c_1 + c_2 + c_3 = 0 \;\Leftrightarrow\; n_1 + n_2 + n_3 = 0}$$

At tree level we can always find a rearrangement so color and kinematics satisfy the *same* algebraic constraint equations.

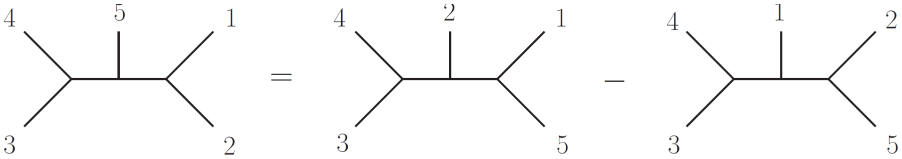
Progress on unraveling relations.

BCJ, Bjerrum-Bohr, Feng, Damgaard, Vanhove, ; Mafra, Stieberger, Schlotterer;
Tye and Zhang; Feng, Huang, Jia; Chen, Du, Feng; Du, Feng, Fu; Naculich, Nastase, Schnitzer
O'Connell and Montiero; Bjerrum-Bohr, Damgaard, O'Connell and Montiero; O'Connell, Montiero, White, etc.

How is Gravity Connected to Gauge Theory

Duality between color and kinematics:

ZB, Carrasco, Johansson (BCJ)



$$c_1 + c_2 + c_3 = 0 \Leftrightarrow n_1 + n_2 + n_3 = 0$$

$$\mathcal{A}_n^{\text{tree}} = \sum_i \frac{c_i \tilde{n}_i}{\prod_{\alpha_i} p^2_{\alpha_i}} \qquad \mathcal{M}_n^{\text{tree}} = \sum_i \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$

gauge theory $c_i \to n_i$ **gravity**

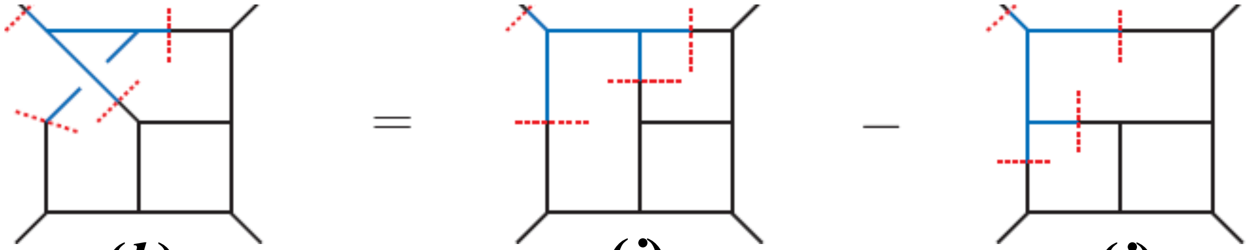
$N = 8$ sugra: ($N = 4$ sYM) × ($N = 4$ sYM)

There is now a whole zoology of theories that can be obtained via "double copy" procedure.

Anastasiou, Bornsten, Duff; Duff, Hughs, Nagy; Johansson and Ochirov; Carrasco, Chiodaroli, Günaydin and Roiban; ZB, Davies, Dennen, Huang and Nohle; Nohle; Chiodaroli, Günaydin, Johansson, Roiban. A. Anastasiou, L. Borsten, M.J. Duff, M.J. Hughes, Marrani, Nagy, Zoccali; Cachazo, He, Yuan; Chen Du, Broedel, Schlotterer and Stieberger; Carrasco, Mafra, Schlotterer;

How is Gravity Connected to Gauge Theory?

ZB, Carrasco, Johansson (BCJ)



(*k*) (*i*) (*j*)

color factor

$$c_k = c_i - c_j$$

$$n_k = n_i - n_j$$

kinematic numerator

If you have a set of duality satisfying numerators.

Conjecture:

gauge theory ⟶ gravity theory

simply take

color factor ⟶ kinematic numerator

$$c_k \longrightarrow n_k$$

Gravity loop integrands follow from gauge theory! The nontrivial part is to find kinematic numerators where duality holds. Double copy is easy to prove.

Supergravity: Ultraviolet Divergence Status

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Björnsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Bossard, Howe, Stelle, Vanhove, etc

In recent years renewed effort to understand UV of supergravity

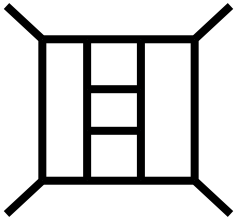
Key point: *all* supersymmetry cancellations are exposed.

Poor UV behavior, unless new types of cancellations between diagrams exist that are "not consequences of supersymmetry in any conventional sense". Bjornsson and Green

Consensus agreement from all methods

Want to check this.

- **$N = 8$ sugra should diverge at 5 loops in $D = 24/5$.** ?
- **$N = 8$ sugra should diverge at 7 loops in $D = 4$.** ?
- **$N = 4$ sugra should diverge at 3 loops in $D = 4$.** ✗
- **$N = 5$ sugra should diverge at 4 loops in $D = 4$.** ✗



New types of cancellations *do* exist: "enhanced cancellations".

ZB, Davies, Dennen

Enhanced UV Cancellations

ZB, Davies, Dennen

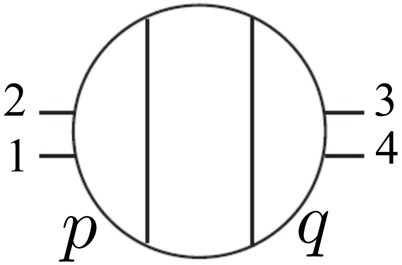
Suppose diagrams in *all* possible Lorentz covariant representations are UV divergent, but amplitude is well behaved.

- **By definition this is an enhanced cancellation.**
- **Not the way nonabelian gauge theory works.**

already log divergent

***N* = 4 sugra: pure YM x *N* = 4 sYM**

***N* = 4 sugra**



$$n_i \sim s^3 t A_4^{\text{tree}} \, (p\cdot q)^2 \, \varepsilon_1\cdot p \, \varepsilon_2\cdot p \, \varepsilon_3\cdot q \, \varepsilon_4\cdot q + \ldots$$

This diagram is log divergent
Amplitude is UV finite.

- **3 loop UV finiteness of *N* = 4 supergravity proves existence of “enhanced cancellation” in supergravity theories.**
- **No known standard symmetry explanation.**

$N = 5$ Supergravity at Four Loops

ZB, Davies and Dennen

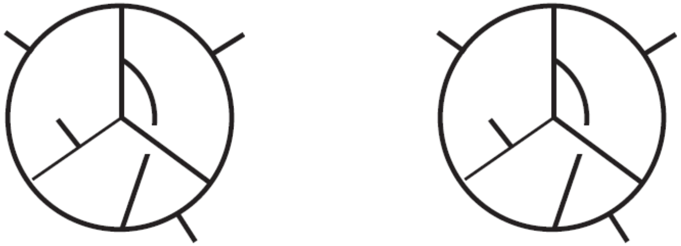
We calculated four-loop divergence in $N = 5$ supergravity.

Industrial strength software needed: FIRE5 and special purpose C++

$N = 5$ sugra: ($N = 4$ sYM) x ($N = 1$ sYM)

Crucial help from (Smirnov)^2

$N = 4$ sYM **$N = 1$ sYM**



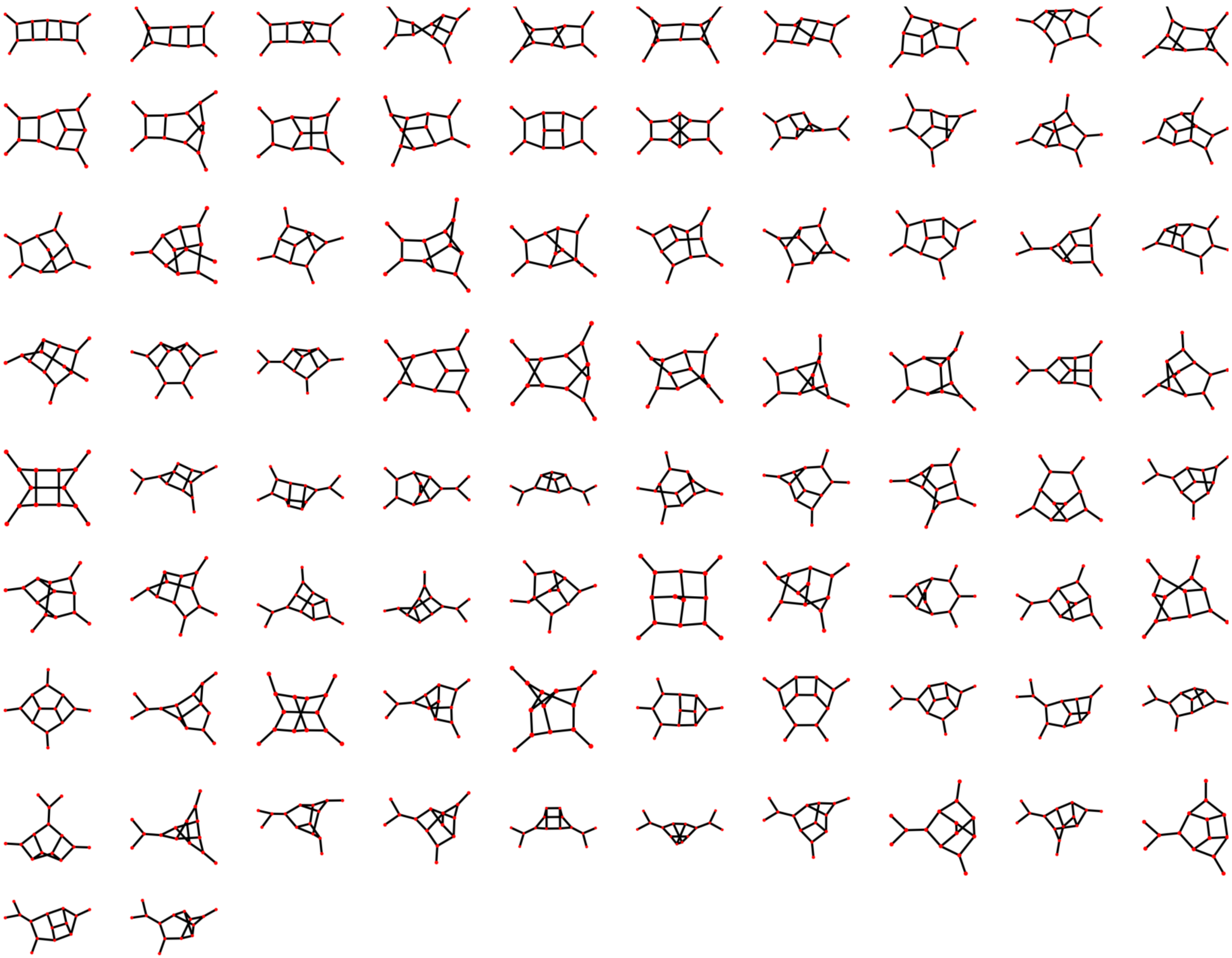
Diagrams necessarily UV divergent.

$N = 5$ supergravity has no divergence at four loops.

Nontrivial example of an “enhanced cancellation”.

82 nonvanishing numerators in BCJ representation

ZB, Carrasco, Dixon, Johansson, Roiban ($\mathcal{N} = 4$ sYM)



$\mathcal{N} = 5$ supergravity at Four Loops

Special purpose C++ and FIRE5

ZB, Davies and Dennen

graphs	(divergence) $\times\, u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 st A^{\mathrm{tree}}(\frac{\kappa}{2})^{10})$
1–30	$\frac{1}{\epsilon^4}\Big[\frac{7358585}{7962624}s^2+\frac{2561447}{2654208}st-\frac{872683}{1990656}t^2\Big]+\frac{1}{\epsilon^3}\Big[\frac{75972559}{35389440}s^2+\frac{240984061}{26542080}st+\frac{1302037}{1310720}t^2\Big]$ $+\frac{1}{\epsilon^2}\Big[\zeta_3\,(-\frac{369234283}{11059200}s^2-\frac{257792411}{4915200}st-\frac{101847769}{14745600}t^2)+\zeta_2\,(\frac{7358585}{3981312}s^2+\frac{2561447}{1327104}st-\frac{872683}{995328}t^2)$ $-\mathrm{S2}\,(\frac{1223621}{49152}s^2+\frac{46816475}{442368}st+\frac{2639903}{221184}t^2)+\frac{206093335871}{11466178560}s^2+\frac{320983191023}{3822059520}st+\frac{53309416589}{2866544640}t^2\Big]$ $+\frac{1}{\epsilon}\Big[\zeta_5\,(-\frac{84777347}{368640}s^2+\frac{382194721}{1474560}st+\frac{417476581}{1474560}t^2)-\zeta_4\,(\frac{3062401}{2457600}s^2+\frac{3881051}{3276800}st-\frac{112081813}{29491200}t^2)$ $+\zeta_3\,(\frac{28162691399797}{53747712000}s^2+\frac{19354492750651}{35831808000}st-\frac{22092683352811}{107495424000}t^2)-\zeta_2\,(\frac{70861961}{17694720}s^2+\frac{227180689}{13271040}st$ $+\frac{105727243}{53084160}t^2)+\mathrm{T1ep}\,(-\frac{1223621}{663552}s^2-\frac{46816475}{5971968}st-\frac{2639903}{2985984}t^2)-\mathrm{S2}\,(\frac{11916028151}{5898240}s^2$ $+\frac{72637733971}{13271040}st+\frac{17223563447}{53084160}t^2)+\mathrm{D6}\,(-\frac{9001177}{552960}s^2-\frac{264491}{10240}st-\frac{2610157}{552960}t^2)$ $+\frac{110945914744727}{1146617856000}s^2+\frac{16989492195991}{127401984000}st-\frac{21362122998269}{573308928000}t^2\Big]$
31–60	$\frac{1}{\epsilon^4}\Big[-\frac{5502451}{2654208}s^2-\frac{3675877}{884736}st+\frac{11269}{497664}t^2\Big]+\frac{1}{\epsilon^3}\Big[\frac{38102993}{26542080}s^2-\frac{291607201}{106168320}st-\frac{565798829}{318504960}t^2\Big]$ $+\frac{1}{\epsilon^2}\Big[\zeta_3\,(\frac{108955183}{2211840}s^2+\frac{653019571}{8847360}st+\frac{9453043}{1769472}t^2)+\zeta_2\,(-\frac{5502451}{1327104}s^2-\frac{3675877}{442368}st+\frac{11269}{248832}t^2)$ $+\mathrm{S2}\,(\frac{16797481}{1327104}s^2+\frac{1172969}{16384}st+\frac{978427}{82944}t^2)-\frac{304243754383}{19110297600}s^2-\frac{2032063711381}{19110297600}st-\frac{257798086613}{7166361600}t^2\Big]$ $+\frac{1}{\epsilon}\Big[\zeta_5\,(\frac{33327659}{122880}s^2+\frac{13276219}{24576}st+\frac{22251887}{184320}t^2)+\zeta_4\,(\frac{12299887}{1474560}s^2+\frac{258056147}{5898240}st+\frac{46913759}{5898240}t^2)$ $+\zeta_3\,(-\frac{26846001990157}{42998169600}s^2-\frac{337106527201}{265420800}st-\frac{5298324906787}{42998169600}t^2)+\zeta_2\,(\frac{282283789}{39813120}s^2+\frac{975199319}{53084160}st$ $+\frac{60394451}{159252480}t^2)+\mathrm{T1ep}\,(\frac{16797481}{17915904}s^2+\frac{1172969}{221184}st+\frac{978427}{1119744}t^2)+\mathrm{S2}\,(\frac{10516980893}{4976640}s^2$ $+\frac{389045625329}{53084160}st+\frac{216032337589}{159252480}t^2)+\mathrm{D6}\,(\frac{503413}{23040}s^2+\frac{12342607}{552960}st+\frac{3661}{184320}t^2)$ $-\frac{166777358259461}{1146617856000}s^2-\frac{565137511429117}{1146617856000}st-\frac{21629055712141}{191102976000}t^2\Big]$
61–82	$\frac{1}{\epsilon^4}\Big[\frac{285899}{248832}s^2+\frac{1058273}{331776}st+\frac{275869}{663552}t^2\Big]+\frac{1}{\epsilon^3}\Big[-\frac{380329649}{106168320}s^2-\frac{74703227}{11796480}st+\frac{124701919}{159252480}t^2\Big]$ $+\frac{1}{\epsilon^2}\Big[\zeta_3\,(-\frac{1371419}{86400}s^2-\frac{236241539}{11059200}st+\frac{4326077}{2764800}t^2)+\zeta_2\,(\frac{285899}{124416}s^2+\frac{1058273}{165888}st+\frac{275869}{331776}t^2)$ $+\mathrm{S2}\,(\frac{8120143}{663552}s^2+\frac{1893289}{55296}st+\frac{92293}{663552}t^2)-\frac{58867708103}{28665446400}s^2+\frac{71191292711}{3185049600}st+\frac{83016363427}{4777574400}t^2\Big]$ $+\frac{1}{\epsilon}\Big[\zeta_5\,(-\frac{1520563}{36864}s^2-\frac{1178767861}{1474560}st-\frac{595491677}{1474560}t^2)-\zeta_4\,(\frac{6539029}{921600}s^2+\frac{313837819}{7372800}st+\frac{21665663}{1843200}t^2)$ $+\zeta_3\,(\frac{20790944575597}{214990848000}s^2+\frac{6505876281371}{8957952000}st+\frac{70676991239557}{214990848000}t^2)+\zeta_2\,(-\frac{491377507}{159252480}s^2-\frac{66476563}{53084160}st$ $+\frac{128393639}{79626240}t^2)+\mathrm{T1ep}\,(\frac{8120143}{8957952}s^2+\frac{1893289}{746496}st+\frac{92293}{8957952}t^2)+\mathrm{S2}\,(-\frac{14810628499}{159252480}s^2$ $-\frac{19698937889}{10616832}st-\frac{10272602953}{9953280}t^2)+\mathrm{D6}\,(-\frac{616147}{110592}s^2+\frac{1939907}{552960}st+\frac{1299587}{276480}t^2)$ $+\frac{9307894793789}{191102976000}s^2+\frac{206124003456599}{573308928000}st+\frac{21562322533673}{143327232000}t^2\Big]$

graphs	(divergence) $\times\, u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 st A^{\mathrm{tree}}(\frac{\kappa}{2})^{10})$
1–30	$\frac{1}{\epsilon^4}\Big[\frac{1052159}{995328}s^2+\frac{509789}{331776}st-\frac{121001}{497664}t^2\Big]+\frac{1}{\epsilon^3}\Big[\frac{9042569}{1474560}s^2+\frac{34360945}{1327104}st+\frac{73518401}{13271040}t^2\Big]$ $+\frac{1}{\epsilon^2}\Big[\zeta_3\,(-\frac{11443919}{2764800}s^2+\frac{32520079}{552960}st+\frac{5836531}{230400}t^2)+\zeta_2\,(\frac{1052159}{497664}s^2+\frac{509789}{165888}st-\frac{121001}{248832}t^2)$ $-\mathrm{S2}\,(\frac{637991}{6144}s^2+\frac{10978729}{27648}st+\frac{5080825}{55296}t^2)+(\frac{270806866183}{7166361600}s^2+\frac{89848068067}{597196800}st+\frac{218093645149}{7166361600}t^2)\Big]$ $+\frac{1}{\epsilon}\Big[\zeta_5\,(\frac{100843}{360}s^2+\frac{17118043}{30720}st-\frac{30266471}{92160}t^2)+\zeta_4\,(\frac{11435323}{614400}s^2+\frac{232002227}{1843200}st+\frac{22211783}{460800}t^2)$ $+\zeta_3\,(\frac{223300432349}{3359232000}s^2-\frac{178732984847}{716636160}st+\frac{951659436383}{53747712000}t^2)$ $-\zeta_2\,(\frac{5492357}{245760}s^2+\frac{53468887}{663552}st+\frac{129714599}{6635520}t^2)+\mathrm{T1ep}\,(-\frac{637991}{82944}s^2-\frac{10978729}{373248}st-\frac{5080825}{746496}t^2)$ $+\mathrm{S2}\,(-\frac{5700088747}{3686400}s^2-\frac{69470348491}{16588800}st-\frac{713512871}{6635520}t^2)+\mathrm{D6}\,(-\frac{357421}{43200}s^2-\frac{2891743}{230400}st-\frac{470219}{138240}t^2)$ $-\frac{3571506237341}{28665446400}s^2-\frac{1611591325291}{5971968000}st+\frac{2301084608777}{143327232000}t^2\Big]$
31–60	$\frac{1}{\epsilon^4}\Big[-\frac{150715}{82944}s^2-\frac{668333}{221184}st-\frac{7213}{1990656}t^2\Big]+\frac{1}{\epsilon^3}\Big[-\frac{68021833}{13271040}s^2-\frac{36852103}{1327104}st-\frac{298377299}{39813120}t^2\Big]$ $+\frac{1}{\epsilon^2}\Big[\zeta_3\,(-\frac{36448033}{2764800}s^2-\frac{455889533}{2764800}st-\frac{82059281}{1382400}t^2)+\zeta_2\,(-\frac{150715}{41472}s^2-\frac{668333}{110592}st-\frac{7213}{995328}t^2)$ $+\mathrm{S2}\,(\frac{13910839}{165888}s^2+\frac{1340033}{4096}st+\frac{26303855}{331776}t^2)-\frac{68286245653}{2388787200}s^2-\frac{20649690431}{119439360}st-\frac{351701043553}{7166361600}t^2\Big]$ $+\frac{1}{\epsilon}\Big[\zeta_5\,(-\frac{2362679}{9216}s^2-\frac{178668311}{92160}st-\frac{1268313}{10240}t^2)+\zeta_4\,(-\frac{124344121}{1843200}s^2-\frac{491722333}{1843200}st-\frac{68141309}{921600}t^2)$ $-\zeta_3\,(\frac{630084012997}{53747712000}s^2-\frac{1250670277213}{663552000}st-\frac{6913218302303}{13436928000}t^2)$ $+\zeta_2\,(\frac{352368061}{19906560}s^2+\frac{35509679}{663552}st+\frac{227699801}{19906560}t^2)+\mathrm{T1ep}\,(\frac{13910839}{2239488}s^2+\frac{1340033}{55296}st+\frac{26303855}{4478976}t^2)$ $+\mathrm{S2}\,(\frac{188312318729}{99532800}s^2+\frac{110749829741}{16588800}st+\frac{5056299197}{3981312}t^2)+\mathrm{D6}\,(\frac{1220779}{76800}s^2+\frac{44791}{6912}st-\frac{1159831}{230400}t^2)$ $+\frac{2755666297013}{28665446400}s^2+\frac{5622513975899}{35831808000}st-\frac{196197363193}{1769472000}t^2\Big]$
61–82	$\frac{1}{\epsilon^4}\Big[\frac{756421}{995328}s^2+\frac{985421}{663552}st+\frac{163739}{663552}t^2\Big]+\frac{1}{\epsilon^3}\Big[-\frac{1670161}{1658880}s^2+\frac{415193}{221184}st+\frac{4863881}{2488320}t^2\Big]$ $+\frac{1}{\epsilon^2}\Big[\zeta_3\,(\frac{110861}{6400}s^2+\frac{16293841}{153600}st+\frac{9408019}{276480}t^2)+\zeta_2\,(\frac{756421}{497664}s^2+\frac{985421}{331776}st+\frac{163739}{331776}t^2)$ $+\mathrm{S2}\,(\frac{1657459}{82944}s^2+\frac{7734025}{110592}st+\frac{4181095}{331776}t^2)-\frac{8243516153}{895795200}s^2+\frac{558349337}{24883200}st+\frac{11133949867}{597196800}t^2\Big]$ $+\frac{1}{\epsilon}\Big[\zeta_5\,(-\frac{1094509}{46080}s^2+\frac{63657091}{46080}st+\frac{5210161}{11520}t^2)+\zeta_4\,(\frac{11254769}{230400}s^2+\frac{129860053}{921600}st+\frac{23717743}{921600}t^2)$ $-\zeta_3\,(\frac{2745647960587}{53747712000}s^2+\frac{3654260151947}{2239488000}st+\frac{5720906529119}{10749542400}t^2)$ $+\zeta_2\,(\frac{11564107}{2488320}s^2+\frac{2244901}{82944}st+\frac{40360999}{4976640}t^2)+\mathrm{T1ep}\,(\frac{1657459}{1119744}s^2+\frac{7734025}{1492992}st+\frac{4181095}{4478976}t^2)$ $+\mathrm{S2}\,(-\frac{420043}{1215}s^2-\frac{825589625}{331776}st-\frac{5785239343}{4976640}t^2)+\mathrm{D6}\,(-\frac{210731}{27648}s^2+\frac{4196129}{691200}st+\frac{1457647}{172800}t^2)$ $+\frac{33976742047}{1194393600}s^2+\frac{4046536311847}{35831808000}st+\frac{212357840779}{2239488000}t^2\Big]$

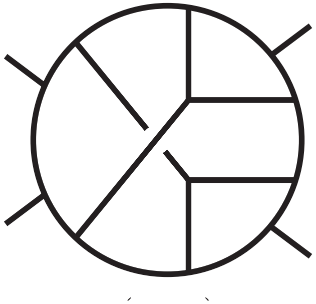
Adds up to zero: no divergence. Enhanced cancellations!
No standard (super)symmetry explanation exists.

Need Better Loop Integration Methods

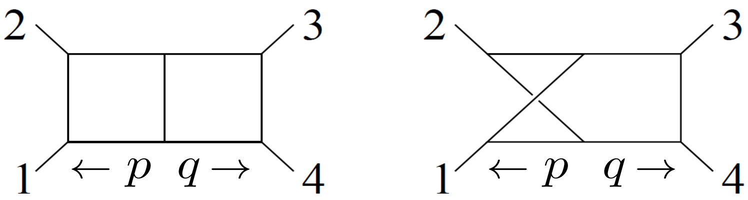
Enhanced cancellations:

- Standard supersymmety powercounting arguments fail.
- Cancellations visible only after integration. Not in integrand.
- Supergravity friends want to help, but no supersymmetry angle available. Kind of frustrating.

At present there is only one technique available:
Do full calculation including integration to extract UV.



Multiloop Enhanced Cancellations



ZB, Enciso, Parra-Martinez, Zeng (2017)

Conjecture: At large loop momentum enhanced cancellations follow from Lorentz symmetry and SL(*L*) relabeling symmetry.

Lorentz symmetry

$$0 = \int \Big(\prod_{a=1}^{L} d^D \ell_a\Big) \sum_{a=1}^{L} \Big(\ell_{a\mu} \frac{\partial}{\partial \ell_a^\nu} - \ell_{a\nu} \frac{\partial}{\partial \ell_a^\mu}\Big) \frac{\mathcal{N}(\ell_i)}{\prod_j \ell_j^2}$$

SL(*L*) relabeling symmetry

$$0 = \int \Big(\prod_{a=1}^{L} d^D \ell_a\Big) \sum_{a=1}^{L} \frac{\partial}{\partial \ell_a^\nu} \frac{\omega_{ab} \ell_{b\mu} \mathcal{N}(\ell_i)}{\prod_j \ell_j^2}$$

L loops

Symmetries generate a generic set of identities between integrals.

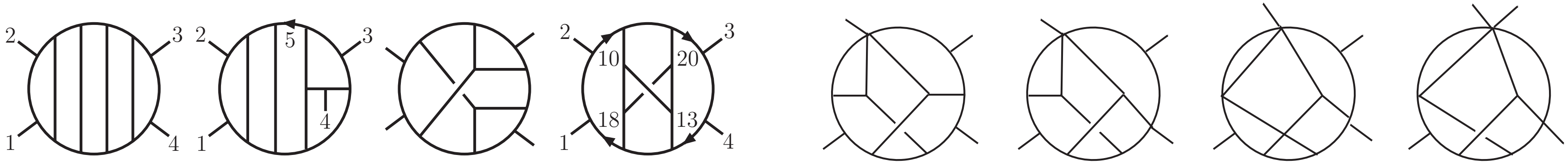
Understanding structures of IBPs is crucial

5 Loop $N = 8$ Supergravity

Finally, after considerable effort we have constructed five-loop integrand. Modified double copy.

ZB, Carrasco, Chen, Johansson, Roiban (2017)

We have the $N = 8$ five-loop four point integrand!



16K nonvanishing diagrams.

We need to extract UV divergence (or lack thereof) from this:

Similar five-loop QCD beta function, except: **See Zoller's talk**

- **Nastier tensor integrals—rank 16.**
- **$D = 24/5$ instead of $D = 4$.**

We are currently setting up an ibp program.

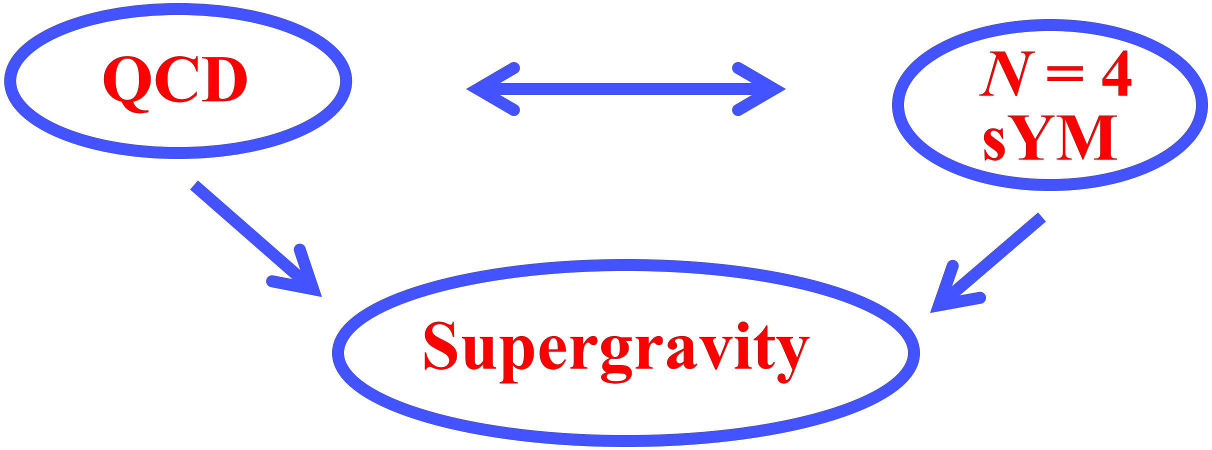
ZB, Carrasco, Chen, Johansson, Roiban, Zeng

Need high tech ibp: See talks from Ita, Page, Zhang and Zeng.

Numerical approaches for checking. See talks from Borowka, Li

Take Home Message

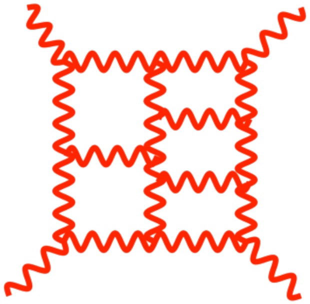
The formal and collider phenomenology communities can learn from each other. Feeds into supergravity.



QCD

$N = 4$ sYM

Supergravity



Our ability to understand UV of supergravity relies crucially on loop advances.

Keep up the great work! Pheno not only reason.

LoopFest XVII

second half of July 2018

Michigan State University

Let's thank the organizers for this great conference:

Local Committee: Radja Boughezal, Andrea Isgro, Ulrich Schubert and Hongxi Xing

Advisors: Sally Dawson, Lance Dixon, Frank Petriello, Laura Reina and Doreen Wackeroth

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LOOPFEST XVI

Radiative Corrections for the LHC and Future Colliders

May 31 - June 2, 2017
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