Proposed general structure for parton showers

Davison E. Soper
University of Oregon

work with Zoltan Nagy, DESY

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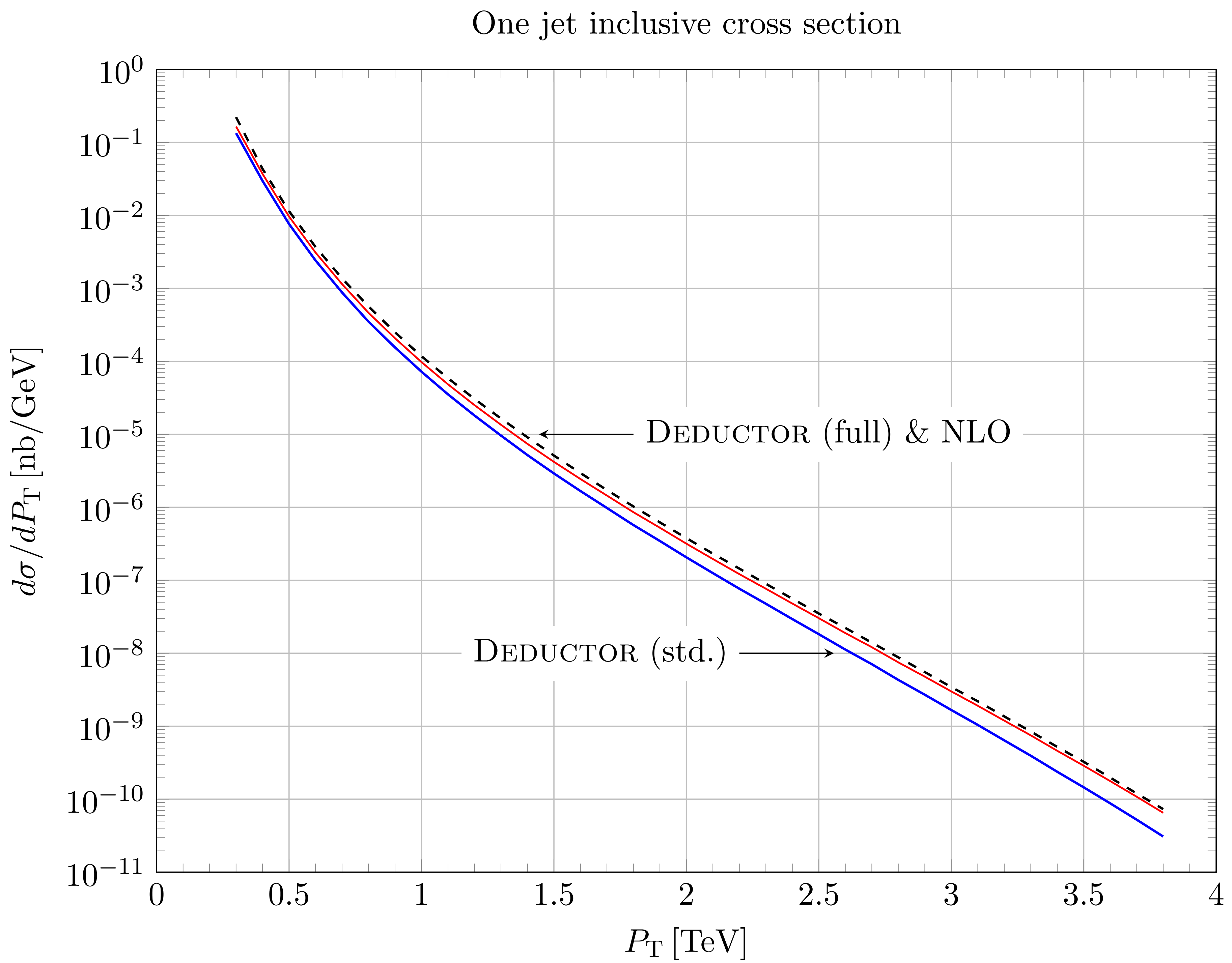
Prequel

- Zoltan Nagy (DESY) and I have a parton shower event generator, DEDUCTOR.
    - Hadronization and underlying event not included.
    - Also, matching to NLO not included in the code.
- DEDUCTOR now includes summation of threshold logs.

One jet cross section $d\sigma/dP_{\mathrm{T}}$



Motivation for this talk

- Perturbative calculations at order α_s^k can be systematically improved by working to higher order.

- But what about parton showers?

- Are they just “QCD inspired” or do they fit into a scheme that can by systematically improved?

The statistical space

- Consider production of a Higgs boson plus QCD partons.

- Describe momenta and flavors for m final state partons with

$$\{p,f\}_m = \{\eta_{\rm a}, a, \eta_{\rm b}, b, p_{\rm H}, p_1, f_1, \cdots, p_m, f_m\}$$

- Use amplitudes $\big|M(\{p,f\}_m)\big\rangle$.
- This is a vector in spin and color space.
- Expand in color$\otimes$spin basis vectors $\big|\{s,c\}_m\big\rangle$.
- Also need conjugate amplitudes $\big\langle M(\{p,f\}_m)\big|$.
- Expand these in basis vectors $\big\langle\{s',c'\}_m\big|$.

- Describe the statistical state of an ensemble of simulations of the scattering.

- The spins and colors are quantum.

- So we need quantum statistical mechanics.

- Use the density operator

$$\rho(\{p,f\}_m) = \sum_{\{s,s',c,c'\}_m} \rho(\{p,f,s,s',c,c'\}_m)\big|\{s,c\}_m\big\rangle\big\langle\{s',c'\}_m\big|$$

$$\rho(\{p,f\}_m) = \sum_{\{s,s',c,c'\}_m} \rho(\{p,f,s,s',c,c'\}_m)\big|\{s,c\}_m\big\rangle\big\langle\{s',c'\}_m\big|$$

- Think of this as a function named ρ.
- The space of such functions is a linear vector space.
- Call this the statistical space.
- Call a vector $\big|\rho\big)$.
- Everything happens in the statistical space.

$$\rho(\{p,f\}_m) = \sum_{\{s,s',c,c'\}_m} \rho(\{p,f,s,s',c,c'\}_m)\big|\{s,c\}_m\big\rangle\big\langle\{s',c'\}_m\big|$$

- Basis vectors $\big|\{p,f,s,s',c,c'\}_m\big)$:

$$\big(\{p,f,s,s',c,c'\}_m\big|\rho\big) = \rho(\{p,f,s,s',c,c'\}_m)$$

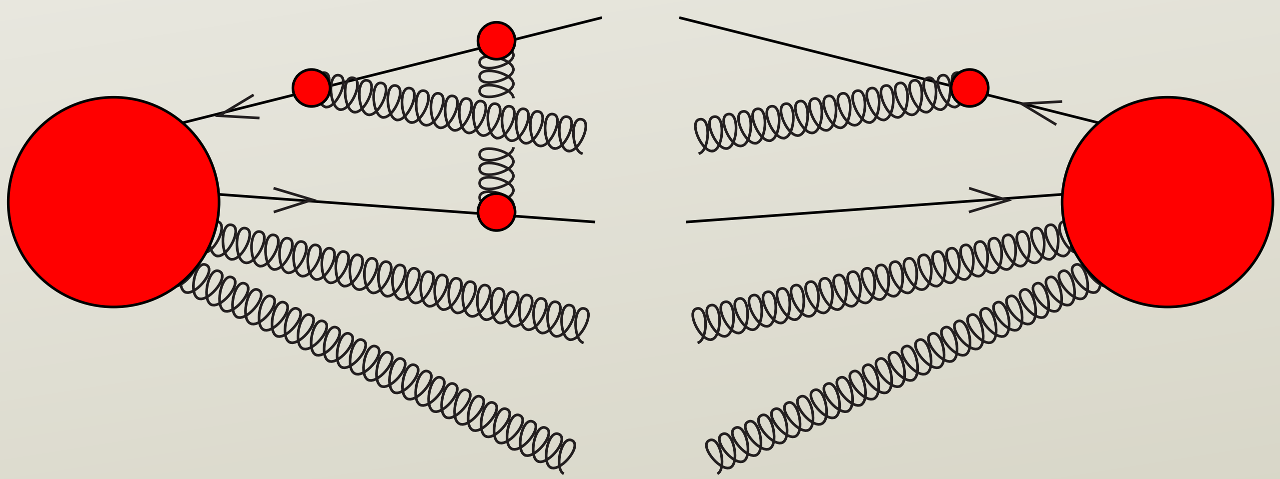
- Inclusive measurement

$$\big(1\big|\rho\big) = \sum_m \frac{1}{m!}\int[d\{p\}_m] \sum_{\{f\}_m} \sum_{\{s,s',c,c'\}_m} \big\langle\{s',c'\}_m\big|\{s,c\}_m\big\rangle \times \big(\{p,f,s,s',c,c'\}_m\big|\rho\big)$$

The infrared sensitive operator

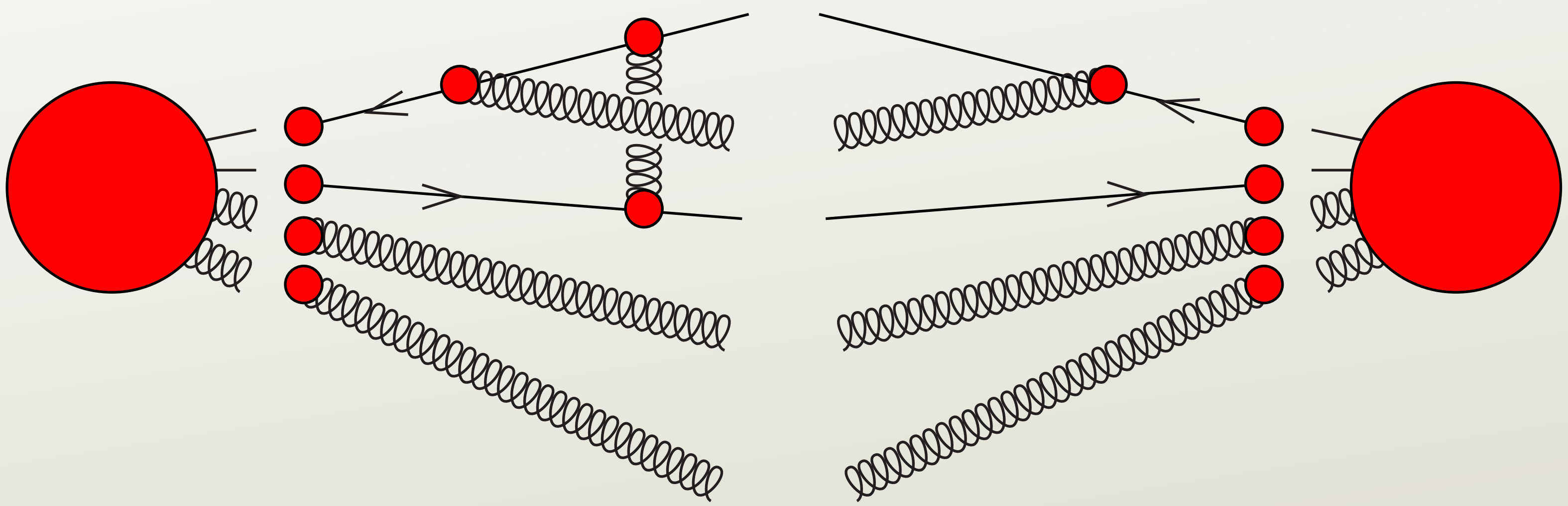
$$\mathcal{D}(\mu^2)$$

- Amplitudes have singularities when partons are soft or collinear.
- They have divergences $1/\epsilon$ from loops.



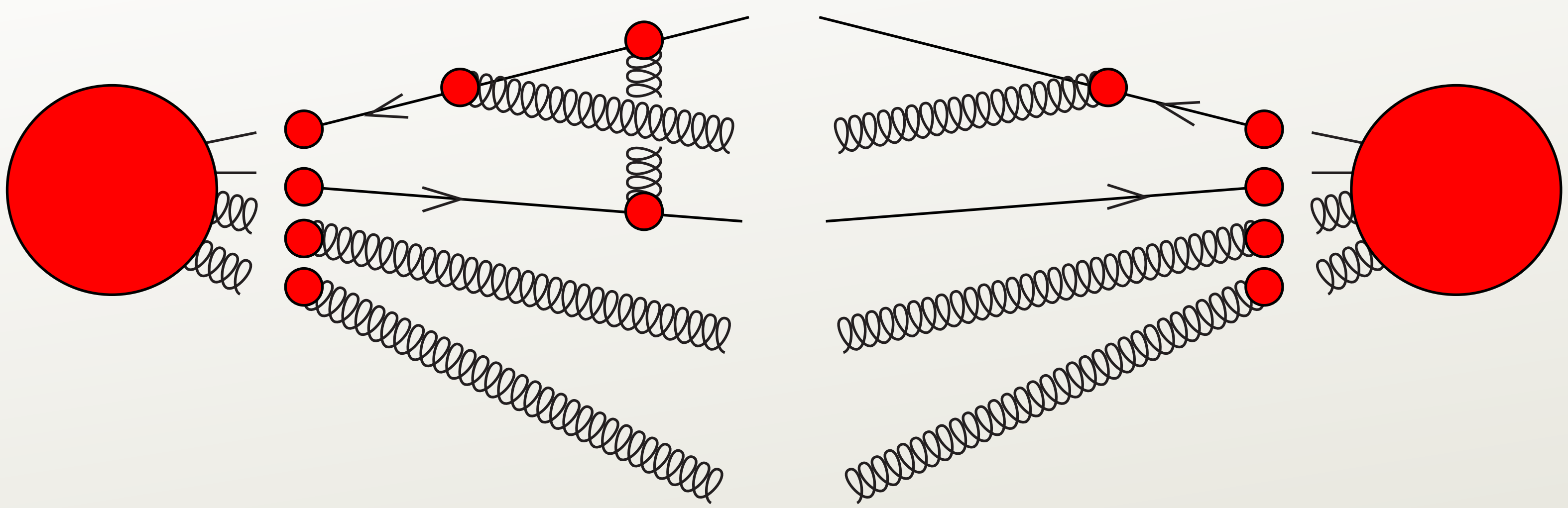
- We want to describe the singularity structure.
- Consider that everything inside the red subamplitudes is hard.

- Approximate the momenta coming from the hard part as fixed and on shell.



- This gives us an operator $\mathcal{D}(\mu^2)$.

$$
\begin{aligned}
&\big(\{\hat{p},\hat{f},\hat{s},\hat{s}',\hat{c},\hat{c}'\}_{m+n}\big|\rho(\mu^2)\big) \\
&\qquad\sim \frac{1}{m!}\int[d\{p\}_m]\sum_{\{f\}_m}\sum_{\{s,s',c,c'\}_m} \\
&\qquad\qquad\times\big(\{\hat{p},\hat{f},\hat{s},\hat{s}',\hat{c},\hat{c}'\}_{m+n}\big|\mathcal{D}(\mu^2)\big|\{p,f,s,s',c,c'\}_m\big) \\
&\qquad\qquad\times\big(\{p,f,s,s',c,c'\}_m\big|\rho_{\rm hard}(\mu^2)\big)
\end{aligned}
$$



- In $\mathcal{D}(\mu^2)$ we need an ultraviolet cutoff μ_s^2.

- *e.g.* $\Lambda^2 < \mu_\mathrm{s}^2$.

- We set $\mu_\mathrm{s}^2 = \mu^2$.

- We also need a momentum mapping.

- We assume that $\mathcal{D}(\mu^2)$ is available and investigate what to do with it.

Standard perturbation theory

cancel initial state poles

“Add back” the subtractions

Subtractions

$$\sigma[J] = \big(1\big|\big[\mathcal{F}_{\overline{\mathrm{MS}}}(\mu_{\mathrm{H}}^2) \circ \mathcal{Z}_F(\mu_{\mathrm{H}}^2)\big]\, \mathcal{D}(\mu_{\mathrm{H}}^2)\, \mathcal{D}^{-1}(\mu_{\mathrm{H}}^2)\, \mathcal{O}_J\big|\rho(\mu_{\mathrm{H}}^2)\big) + \mathcal{O}(\alpha_{\mathrm{s}}^{k+1}) + \mathcal{O}(\mu_{\mathrm{f}}^2/Q[J]^2)$$

Operator to measure desired cross section

Feynman diagrams to α_{s}^k

- Normally $\mathcal{D}^{-1}(\mu_{\mathrm{H}}^2)$ is constructed by hand and $\mathcal{D}(\mu_{\mathrm{H}}^2)$ is its inverse.

- It is useful to rewrite our cross section a bit:

$$\sigma[J] = \big(1\big|\big[\mathcal{F}_{\overline{\rm MS}}(\mu_{\rm H}^2) \circ \mathcal{Z}_F(\mu_{\rm H}^2)\big]\, \mathcal{D}(\mu_{\rm H}^2)\, \mathcal{D}^{-1}(\mu_{\rm H}^2)\, \mathcal{O}_J\big|\rho(\mu_{\rm H}^2)\big)$$

becomes

$$\sigma[J] = \big(1\big|\big[\mathcal{F}_{\overline{\rm MS}}(\mu_{\rm H}^2) \circ \mathcal{Z}_F(\mu_{\rm H}^2)\big]\, \mathcal{O}_J\, \mathcal{D}(\mu_{\rm H}^2)\big|\rho_{\rm H}\big)$$

where

$$\big|\rho_{\rm H}\big) = \mathcal{D}^{-1}(\mu_{\rm H}^2)\big|\rho(\mu_{\rm H}^2)\big)$$

Shower oriented parton distribution functions

- Standard $\overline{\mathrm{MS}}$ parton distribution functions are not quite right for use in a shower.

- *E.g.* if you use k_T for the shower hardness parameter, $\overline{\mathrm{MS}}$ is almost OK, but imposing a UV cut is not the same as subtracting a UV pole.

- Define

$$\mathcal{F}_{\overline{\mathrm{MS}}}(\mu^2) = [\mathcal{F}(\mu^2) \circ \mathcal{K}(\mu^2)]$$

The inclusive infrared finite operator $\mathcal{V}(\mu^2)$

- Define

$$\mathcal{X}(\mu^2) = \left[\mathcal{F}(\mu^2) \circ \mathcal{K}(\mu^2) \circ \mathcal{Z}_F(\mu^2)\right] \mathcal{D}(\mu^2)\, \mathcal{F}^{-1}(\mu^2)$$

- Then $\big(1\big|\mathcal{X}(\mu^2)\big|\{p, f, s, s', c, c'\}_m\big)$ is IR finite.

- Define an operator $\mathcal{V}(\mu^2)$ that leaves $\{p, f\}_m$ unchanged, with

$$\big(1\big|\mathcal{V}(\mu^2) = \big(1\big|\mathcal{X}(\mu^2)$$

- Then $\mathcal{V}(\mu^2)$ is IR finite.

The result

…without the derivation.

Operator to measure desired cross section

parton distribution functions and luminosity factor

$$\sigma[J] = \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)\,\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)\,\mathcal{F}(\mu_\mathrm{H}^2)\big|\rho_\mathrm{H}\big) + \mathcal{O}(\alpha_\mathrm{s}^{k+1}) + \mathcal{O}(\mu_\mathrm{f}^2/Q[J]^2)$$

Feynman diagrams to α_s^k with subtractions

- The most important parts are $\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)$ and $\mathcal{U}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)$.

$$\sigma[J] = \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)\,\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)\,\mathcal{F}(\mu_\mathrm{H}^2)\big|\rho_\mathrm{H}\big) + \mathcal{O}(\alpha_\mathrm{s}^{k+1}) + \mathcal{O}(\mu_\mathrm{f}^2/Q[J]^2)$$

$$\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2) = \mathcal{V}^{-1}(\mu_\mathrm{f}^2)\,\mathcal{V}(\mu_\mathrm{H}^2) = \mathbb{T}\exp\left(\int_{\mu_\mathrm{f}^2}^{\mu_\mathrm{H}^2}\frac{d\mu^2}{\mu^2}\,\mathcal{S}_\mathcal{V}(\mu^2)\right)$$

$$\mathcal{S}_\mathcal{V}(\mu^2) = \mathcal{V}^{-1}(\mu^2)\,\mu^2\frac{d}{d\mu^2}\,\mathcal{V}(\mu^2)$$

- Does not create new partons.
- Provides perturbative corrections to the hard scattering state $\big|\rho_\mathrm{H}\big)$.
- Sums threshold logarithms associated with $\big|\rho_\mathrm{H}\big)$.

$$\sigma[J] = \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)\,\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)\,\mathcal{F}(\mu_\mathrm{H}^2)\big|\rho_\mathrm{H}\big) + \mathcal{O}(\alpha_\mathrm{s}^{k+1}) + \mathcal{O}(\mu_\mathrm{f}^2/Q[J]^2)$$

$$\mathcal{U}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2) = \mathcal{V}(\mu_\mathrm{f}^2)\,\mathcal{X}^{-1}(\mu_\mathrm{f}^2)\,\mathcal{X}(\mu_\mathrm{H}^2)\,\mathcal{V}^{-1}(\mu_\mathrm{H}^2) = \mathbb{T}\exp\left(\int_{\mu_\mathrm{f}^2}^{\mu_\mathrm{H}^2}\frac{d\mu^2}{\mu^2}\,\mathcal{S}(\mu^2)\right)$$

$$\mathcal{S}(\mu^2) = \mathcal{V}(\mu^2)\,\mathcal{F}(\mu^2)\,\mathcal{D}^{-1}(\mu^2)\left[\mu^2\frac{d}{d\mu^2}\mathcal{D}(\mu^2)\right]\mathcal{F}^{-1}(\mu^2)\,\mathcal{V}^{-1}(\mu^2) - \left[\mu^2\frac{d}{d\mu^2}\mathcal{F}(\mu^2)\right]\mathcal{F}^{-1}(\mu^2) - \left[\mu^2\frac{d}{d\mu^2}\mathcal{V}(\mu^2)\right]\mathcal{V}^{-1}(\mu^2)\,.$$

- Creates new partons.
- Preserves probabilities: $\big(1\big|\mathcal{U}(\mu_2^2,\mu_1^2) = \big(1\big|$.

- What about the error estimate?

$$\sigma[J] = \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_{\rm f}^2,\mu_{\rm H}^2)\,\mathcal{U}_{\mathcal{V}}(\mu_{\rm f}^2,\mu_{\rm H}^2)\,\mathcal{F}(\mu_{\rm H}^2)\big|\rho_{\rm H}\big) + \mathcal{O}(\alpha_{\rm s}^{k+1}) + \mathcal{O}(\mu_{\rm f}^2/Q[J]^2)$$

- There is a power suppressed error $\mathcal{O}(\mu_{\rm f}^2/Q[J]^2)$.
- Such an error is part of factorization.
- There is a perturbative error $\mathcal{O}(\alpha_{\rm s}^{k+1})$.
- This is because we calculate only to order $\alpha_{\rm s}^{k}$.
- If $\mathcal{O}_J$ involves different scales, say $\mu_{\rm H}^2$ and $\mu_{\rm L}^2$ we could have $[\alpha_{\rm s}\log^2(\mu_{\rm H}^2/\mu_{\rm L}^2)]^{k+1}$.
- We can hope that the shower sums the most important large logarithms and leaves us with a smaller error.
- Implementations will involve further approximations.

Summary

- Perturbative calculations

$$\sigma[J] = \big(1\big|\big[\mathcal{F}_{\overline{\mathrm{MS}}}(\mu^2) \circ \mathcal{Z}_F(\mu^2)\big]\,\mathcal{D}(\mu^2)\,\mathcal{D}^{-1}(\mu^2)\,\mathcal{O}_J\big|\rho(\mu^2)\big) + \mathcal{O}(\alpha_{\mathrm{s}}^{k+1}) + \mathcal{O}(\mu_{\mathrm{f}}^2/Q[J]^2)$$

can be systematically improved by working to higher order.

- Parton shower calculations

$$\sigma[J] = \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_{\mathrm{f}}^2,\mu_{\mathrm{H}}^2)\,\mathcal{U}_{\mathcal{V}}(\mu_{\mathrm{f}}^2,\mu_{\mathrm{H}}^2)\,\mathcal{F}(\mu_{\mathrm{H}}^2)\big|\rho_{\mathrm{H}}\big) + \mathcal{O}(\alpha_{\mathrm{s}}^{k+1}) + \mathcal{O}(\mu_{\mathrm{f}}^2/Q[J]^2)$$

can be systematically improved by working to higher order.