Resummation of transverse observables at N3LL

P. F. Monni
CERN

[Monni, Re, Torrielli, Phys.Rev.Lett. 116 (2016) no.24, 242001]
[Bizon, Monni, Re, Rottoli, Torrielli, 1705.09127]

LoopFest XVI - Argonne National Laboratory

Transverse observables at colliders

- Transverse observables in colour-singlet production offer a clean experimental and theoretical environment for precision physics:

$$V(\{\tilde{p}\}, k) \equiv V(k) = d_\ell \, g_\ell(\phi) \left(\frac{k_t}{M}\right)^a$$

    - SM measurements (e.g. W, Z, photon,…): parton distributions, strong coupling, W mass, …
    - BSM measurements/constraints (e.g. Higgs): light/heavy NP, Yukawa couplings,…
- Of this class, the family of "inclusive" observables probes directly the kinematics of the colour singlet:

$$V(\{\tilde{p}\}, k_1, \ldots, k_n) = V(\{\tilde{p}\}, k_1 + \cdots + k_n)$$

    - sensitive to non-perturbative effects (hadronisation, intrinsic kt) only through transverse recoil
    - very little/no sensitivity to multi-parton interactions
    - measured precisely at experiments
- Experimental uncertainty is already at the % level (or less) in some cases (e.g. Z production)
    - theory should aim at this level of accuracy for some differential distributions

Fixed order and resummed PT

- Fixed-order predictions for $H/W/Z/\gamma + \mathrm{jet}$ available through NNLO in QCD - residual perturbative uncertainty in the range ~ 5%-10% depending on the process

  [Boughezal et al. 1504.07922, Boughezal et al. 1505.03893, Chen et al. 1607.08817]
  [Boughezal et al. 1504.02131]
  [Gehrmann-De Ridder et al. 1507.02850, Boughezal et al. 1512.01291]
  [Campbell et al. 1612.04333]

- In regions of the differential spectra in which the radiation is restrained to be emitted in the soft and/or collinear edges of its phase space, the PT series for the cross section has a logarithmic divergence: all-order calculation necessary to cancel the IRC poles. Effects propagate away from the singularity -> resummation necessary for a good control of this region

- Logarithmic counting commonly defined at the level of the logarithm of the integrated cross section

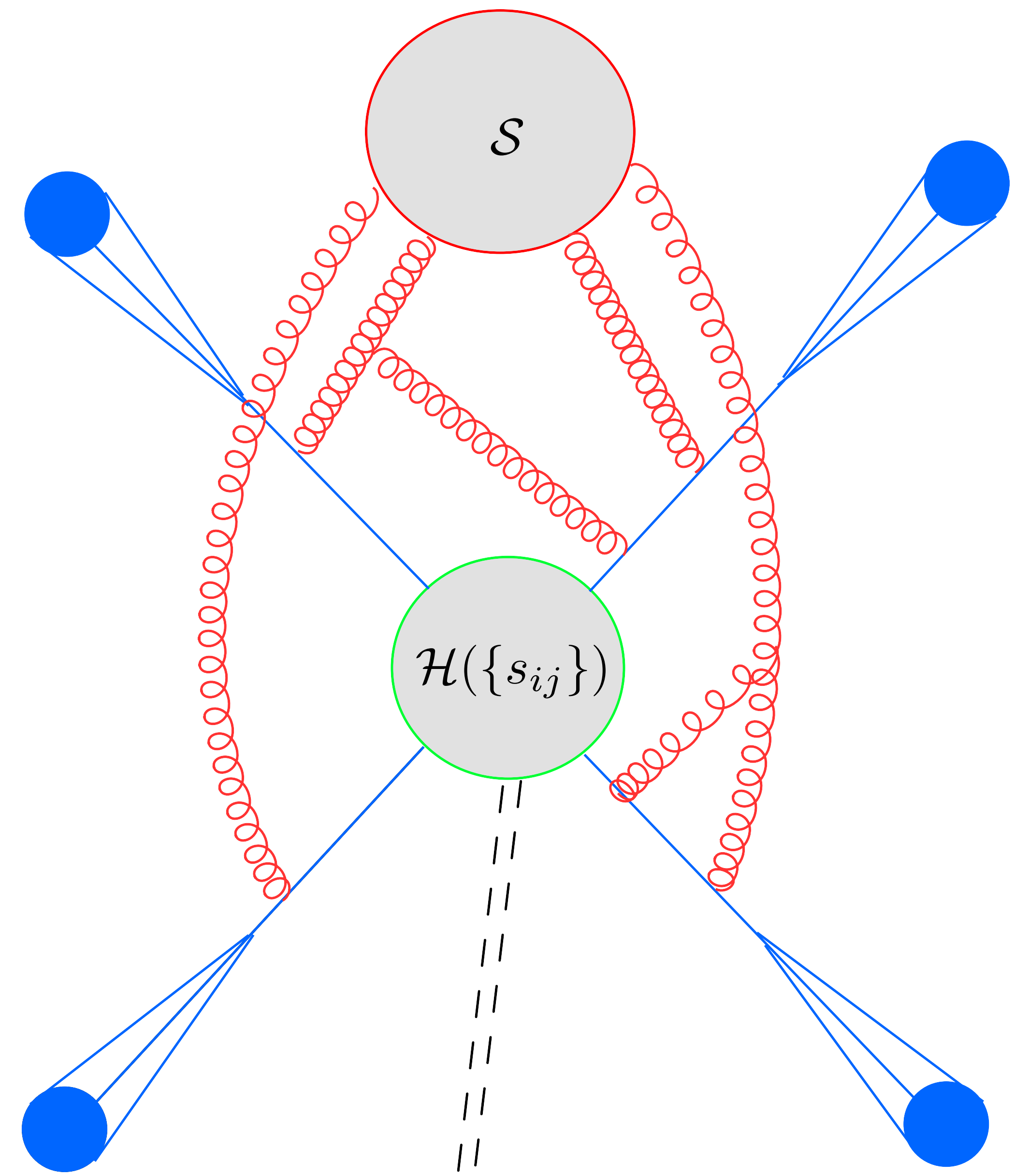
$$\frac{\Sigma(v)}{\sigma_{\mathrm{Born}}} = \int_0^v \frac{1}{\sigma_{\mathrm{Born}}} \frac{d\sigma}{dv'} dv' \sim e^{\overbrace{\alpha_s^n L^{n+1}}^{\mathrm{LL}} + \overbrace{\alpha_s^n L^n}^{\mathrm{NLL}} + \overbrace{\alpha_s^n L^{n-1}}^{\mathrm{NNLL}} + \dots}$$

- In the large logarithms region $L \sim \frac{1}{\alpha_s}$: NLL~LO; NNLL~NLO; N3LL~NNLO,…

  strict LL not even sensible for some observables
  (multiple emission effects important)

Factorisation of amplitudes in the IRC

soft wide − angle : $\alpha_s^n L^m \;(m \leq n)$



soft − collinear : $\alpha_s^n L^m \;(m \leq 2n)$

colourless system

hard − collinear : $\alpha_s^n L^m \;(m \leq n)$

- Consider a IRC observable $V = V(\{\tilde{p}\}, k_1, ..., k_n) \leq 1$ in the Born-like limit $V \to 0$
- In this limit radiative corrections are described exclusively by virtual corrections, and collinear and/or soft real emissions (singular limit) — QCD squared **amplitudes factorise** in these regimes w.r.t. the Born, up to regular corrections
- Different observables are sensitive to different singular modes which determine the logarithmic structure of the perturbative expansion (e.g. (non) global, hard-collinear logarithms, …)

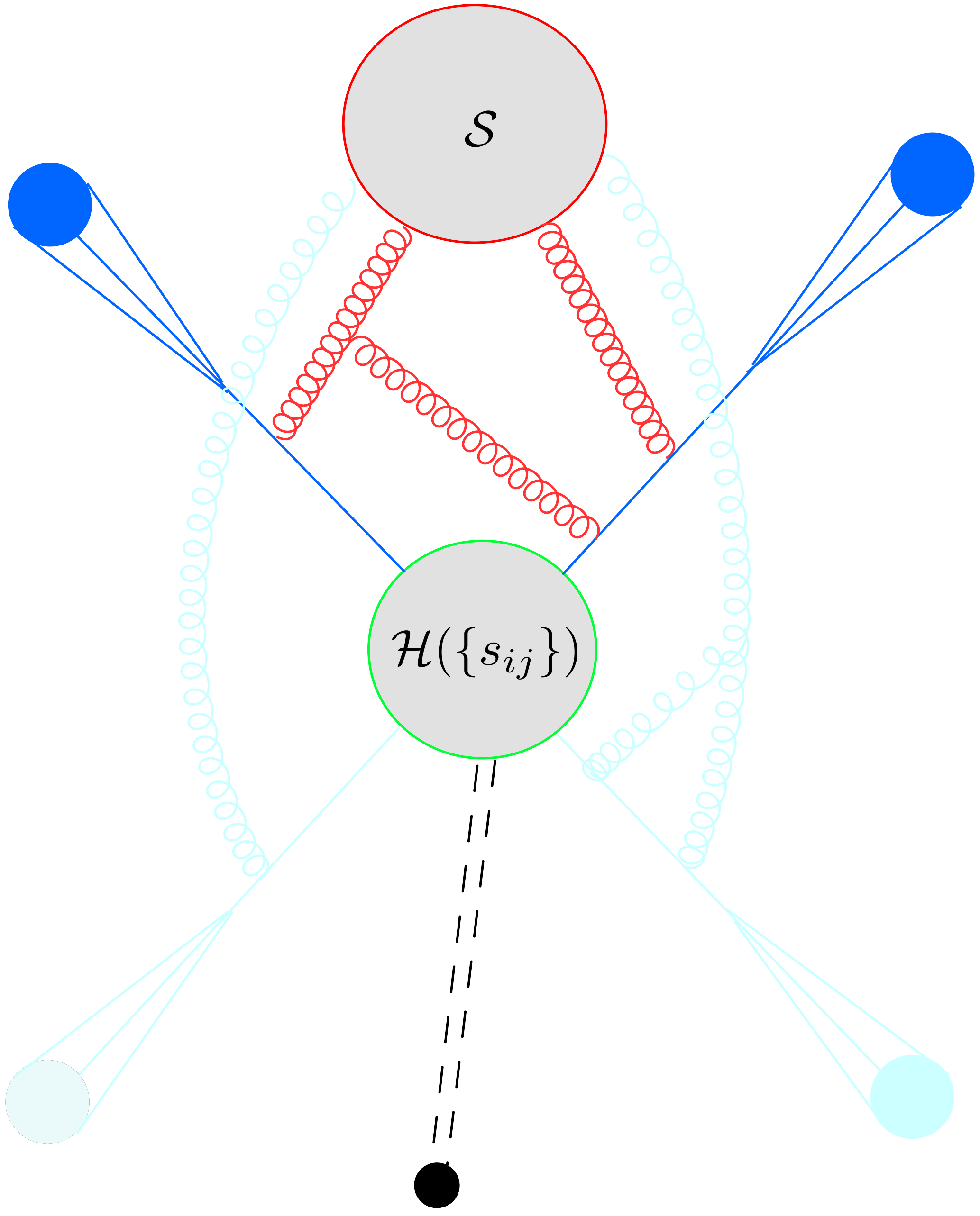
Collinear factorisation breaking due to exchange of Glauber modes found at high perturbative orders (additional double logarithms)

[Forshaw, Kyrieleis, and Seymour ’06-’09]
[Catani, de Florian, and Rodrigo ’12]
[Forshaw, Seymour, and Siodmok ’12]
[Gaunt ‘14]
[Angeles-Martinez, Forshaw, and Seymour ’15-‘16]
[Zeng ‘15]
[Rothstein, Stewart ‘16]

Two-emitter processes

- The strong angular separation between different modes ensures they evolve independently at late times after the collision

- The structure of the coherent soft radiation at large angles (interference between emitters) gets increasingly complex with the number of emitting legs

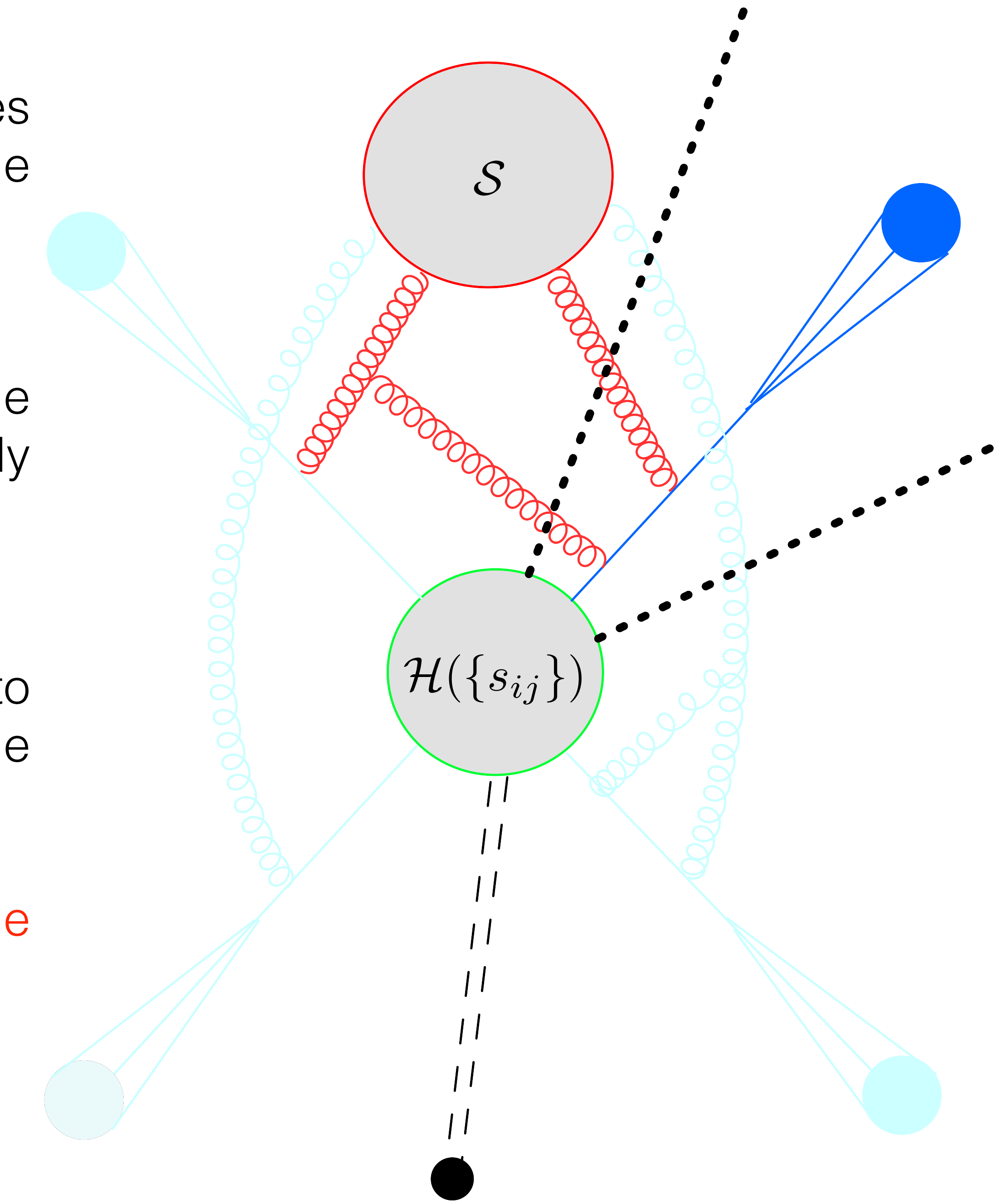


Non-Global observables

- The strong angular separation between different modes ensures they evolve independently at late times after the collision

- The structure of the coherent soft radiation at large angles (interference between emitters) gets increasingly complex with the number of emitting legs

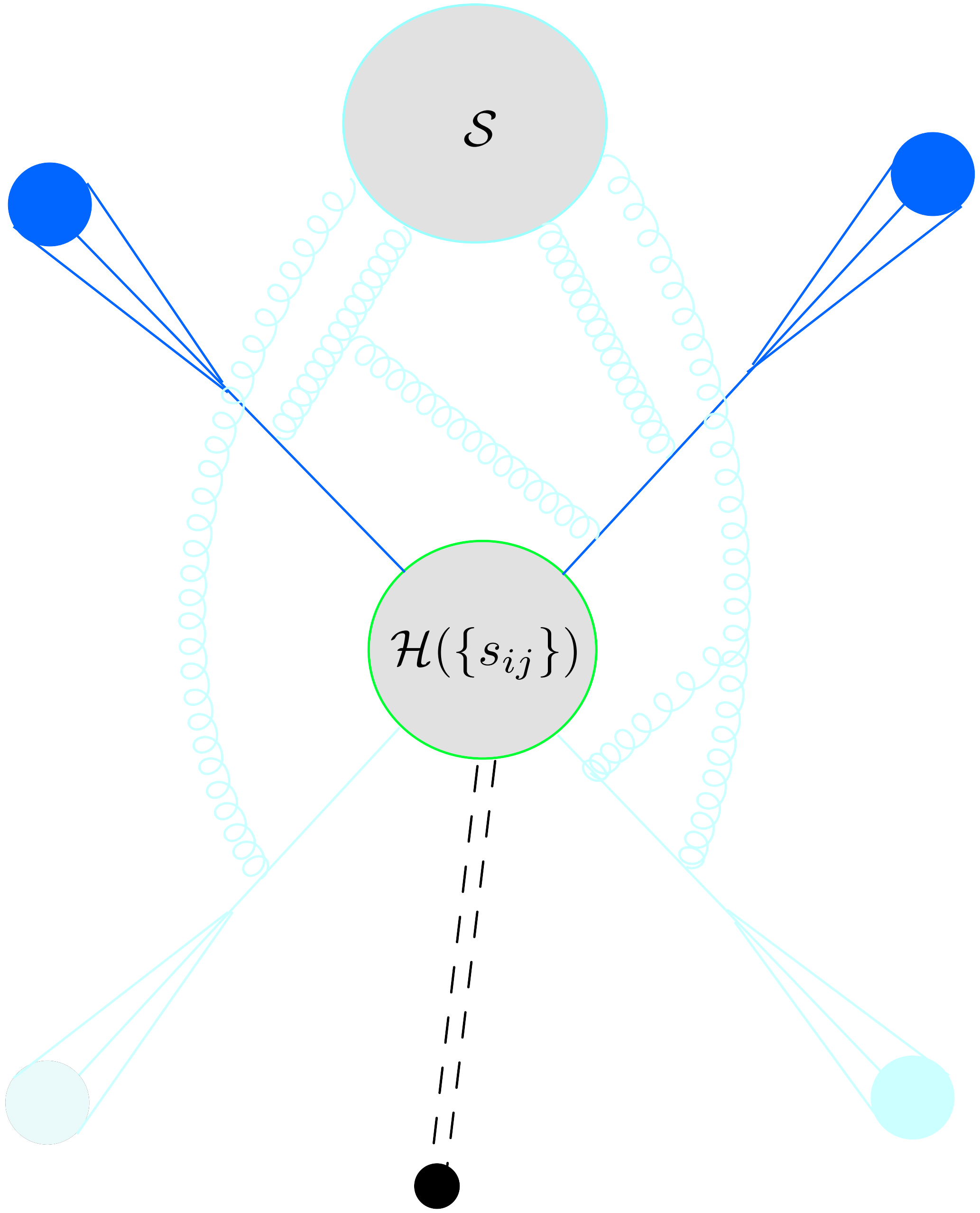
- For non-global observables one is always sensitive to the full evolution of the soft radiation outside of the resolved phase-space region
  - Both soft and collinear modes are present in the general case
  - Collinear modes are absent for some observables



[Dasgupta, Salam ’01; Banfi, Marchesini, Smye ’02]
[Caron-Huot ’15-‘16; Larkoski, Moult, Neill ’15; Becher, Neubert, Rothen, Shao ’15-‘16]

Two-emitter processes

- The strong angular separation between different modes ensures they evolve independently at late times after the collision
- The structure of the coherent soft radiation at large angles (interference between emitters) gets increasingly complex with the number of emitting legs
- For continuously global observables in processes with two emitters, colour coherence forces the effect of soft modes exchanged with large angles to vanish
  - Only collinear (soft/hard) modes effectively remain
  - Soft modes can be absent in specific cases



What (exclusive observables) can we resum ?

- The all-order treatment for an observable $V(\{\tilde{p}\}, k_1, \ldots, k_n)$ that satisfies the IRC-safety criteria relies on the concept of factorisation …
  - … of the QCD amplitudes in the IRC limits
  - … of the observable. This implies that hard and singular IRC modes are not mixed when radiative corrections are considered
- The latter is often interpreted as the existence of factorised formula for the cross section in some conjugate space.
- It is however important to translate the separation of singular modes from the hard ones into scaling requirements for the observable for a systematic solution
  - e.g. rIRC safety: scaling conditions for the observable in the presence of radiation. It allows one to define a logarithmic hierarchy for the squared amplitudes at all perturbative orders. Resummation can be formulated systematically [Banfi, Salam, Zanderighi '04]

    —> A. Banfi's talk
  - Exceptions exist: e.g. rIRC unsafe observables (JADE, Geneva algorithms,…), Sudakov-safe observables. No general structure beyond LL for these is known yet

Zeros away from the Sudakov limit

Probability of emitting the hardest parton v1 = v(k1)

Probability of secondary radiation given the first emission, and the observable's value v

$$\frac{\Sigma(v)}{\sigma_{\rm Born}} = \int_0^v \frac{1}{\sigma_{\rm Born}} \frac{d\sigma}{dv'} dv' = \frac{1}{\sigma_{\rm Born}} \int dv_1 P(v_1) P(v|v_1), \qquad P(v_1) \sim \alpha_s \frac{e^{-R(v_1)}}{v_1} \ln \frac{1}{v_1}$$

- Commonly $v_i/v = \mathcal{O}(1)$ and one can expand about $v_i \sim v$

$$\frac{\Sigma(v)}{\sigma_{\rm Born}} \simeq \frac{P(v)}{\sigma_{\rm Born}} \int \frac{dv_1}{v_1} \left(\frac{v_1}{v}\right)^{R'(v)} P(v|v_1) = e^{-R(v)} \mathcal{F}(v), \qquad R'(v) = dR(v)/d\ln(1/v)$$

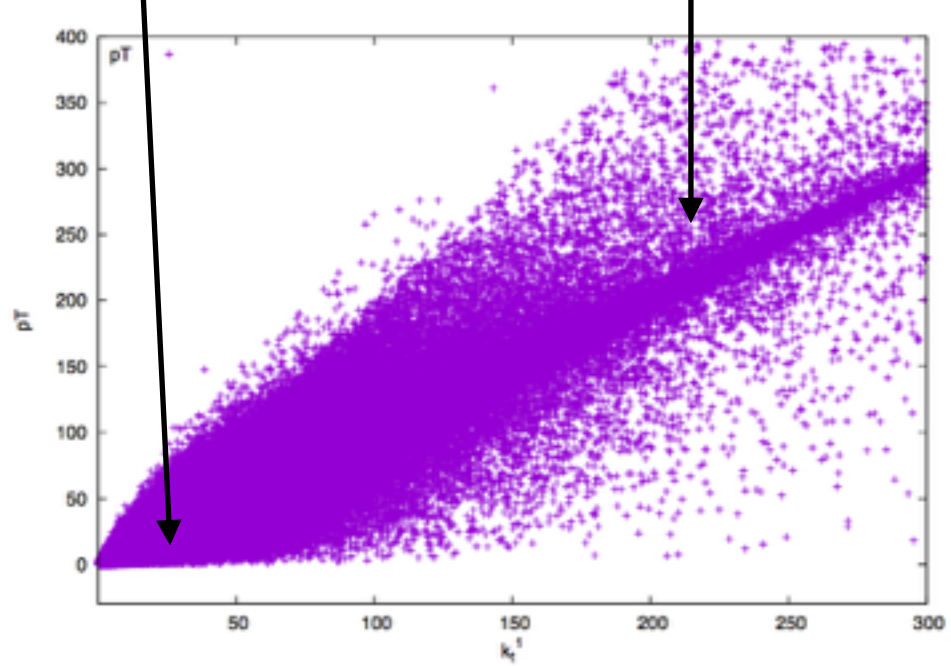
- Answer may diverge if $P(v|v_1) \neq 0$ for $v_1 \gg v$. Two mechanisms compete:

e.g. transverse momentum of the final-state system

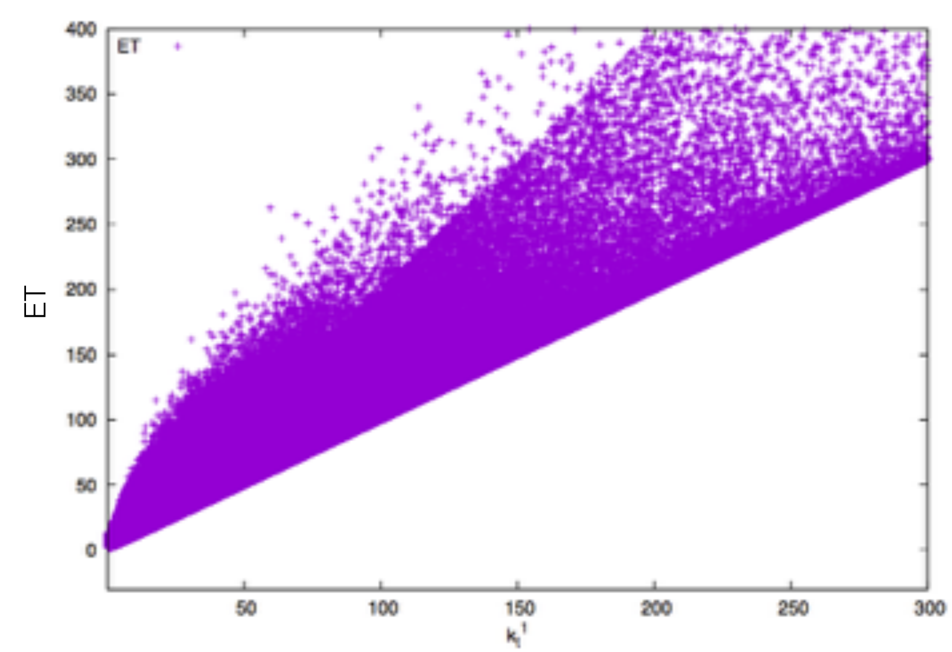
Transverse Momentum: $P(v|v_1) \sim v_1^{-2}$ for $v_1 \gg v$

many emissions : $k_{t1} \gg p_T$ few emissions : $k_{t1} \sim p_T$

At some value of kt1 a transition takes place and the second mechanism becomes more likely way to get pT->0: power-like suppression



Transverse Energy: $P(v|v_1) = 0$ for $v_1 \gg v$
single (Sudakov) suppression mechanism for all values of kt1: $k_{t1} \leq E_T$



b-space formulation

- For inclusive observables the vectorial nature of the cancellations can be handled via a Fourier transform: [Parisi, Petronzio ’78; Collins, Soper, Sterman ’85]

$$\delta^{(2)}\left(\vec{p}_t - \left(\vec{k}_{t1} + \cdots + \vec{k}_{t(n+1)}\right)\right) = \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^{n+1} e^{i\vec{b}\cdot\vec{k}_{ti}}$$

Coefficient functions (and form factor) known to O(as^2) [Catani, Grazzini ’11] [Catani et al. ‘12] [Gehrmann, Luebbert, Yang ‘14]

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \int b\, db\, p_t J_0(p_t b)\, \mathbf{f}^T(b_0/b) \mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b)) H_{\rm CSS}(M) \mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b)) \mathbf{f}(b_0/b)$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\int_0^M \frac{dk_t}{k_t} \mathbf{R}'_{{\rm CSS},\ell}(k_t)\, \Theta(k_t - \frac{b_0}{b})\right\}.$$

Anomalous dimensions known to N3LL (except for four-loop cusp) [Davies, Stirling ‘84] [De Florian, Grazzini ’01] [Becher, Neubert ‘10] [Li, Zhu ’16] [Vladimirov ’16]

$$R_{\rm CSS}(b) = \sum_{\ell=1}^{2}\int_{b_0/b}^{M} \frac{dk_t}{k_t} R'_{{\rm CSS},\ell}(k_t) = \sum_{\ell=1}^{2}\int_{b_0/b}^{M} \frac{dk_t}{k_t}\left(A_{{\rm CSS},\ell}(\alpha_s(k_t)) \ln\frac{M^2}{k_t^2} + B_{{\rm CSS},\ell}(\alpha_s(k_t))\right)$$

- Any solution in momentum space (e.g. expanding about $b \sim 1/p_t$) at any order beyond LL cannot be simultaneously free of both subleading terms (in terms of $\ln(M/p_t)$) and spurious singularities (as above) [Frixione, Nason, Ridolfi ‘98]
  - the logarithmic counting at small pt is not sensible: the correct power-suppressed scaling is lost if only logarithms are retained
- Why bother so much about a single observable ?
  - What we learn here will have a broader application range, it can be generalised beyond the simple inclusive-observables case

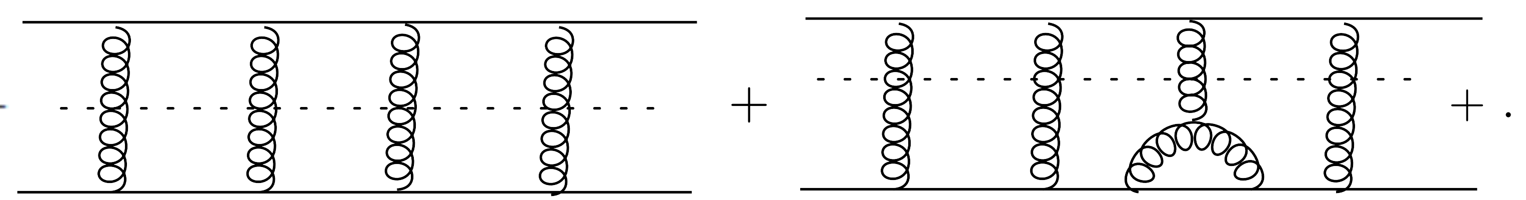
Momentum-space formulation

- Pure virtual corrections (massless form factor) exponentiate at all orders in momentum space: need to devise a way to cancel poles against *all-order* real corrections

- rIRC safety suggests to decompose the squared amplitude in terms of n-particle-correlated blocks:

e.g. n soft partons case (analogous considerations for hard-collinear)

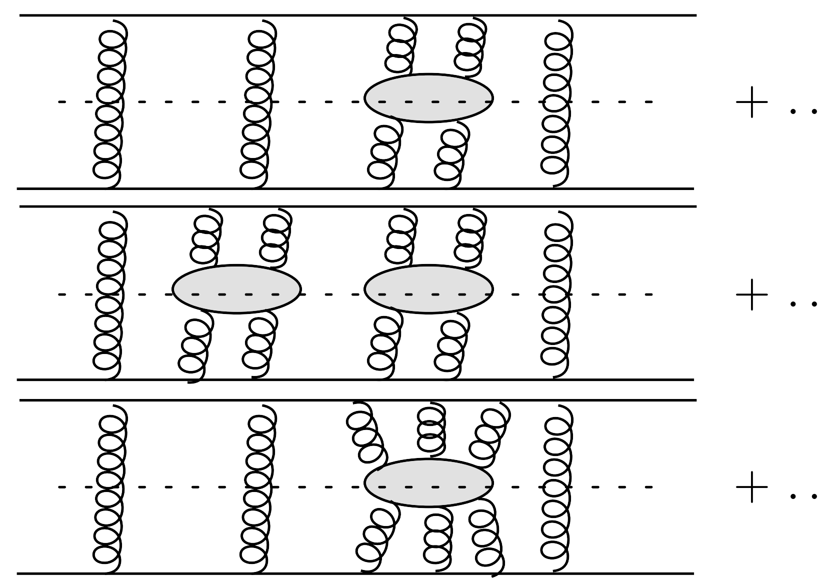
$$|M(\tilde{p}_1,\tilde{p}_2,k_1,\dots,k_n)|^2 = |M_B(\tilde{p}_1,\tilde{p}_2)|^2 \Bigg\{ \left(\frac{1}{n!}\prod_{i=1}^{n}|M(k_i)|^2\right) +$$



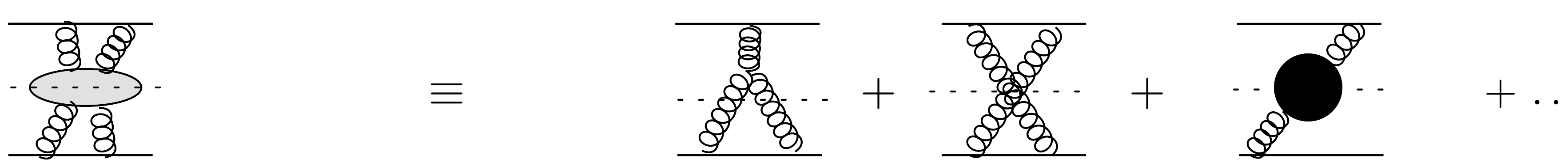
$$\Bigg[\sum_{a>b}\frac{1}{(n-2)!}\left(\prod_{\substack{i=1\\ i\neq a,b}}^{n}|M(k_i)|^2\right)\left|\tilde{M}(k_a,k_b)\right|^2 +$$

$$\sum_{a>b}\sum_{\substack{c>d\\ c,d\neq a,b}}\frac{1}{(n-4)!2!}\left(\prod_{\substack{i=1\\ i\neq a,b,c,d}}^{n}|M(k_i)|^2\right)\left|\tilde{M}(k_a,k_b)\right|^2\left|\tilde{M}(k_c,k_d)\right|^2 + \dots\Bigg]$$

$$+\left[\sum_{a>b>c}\frac{1}{(n-3)!}\left(\prod_{\substack{i=1\\ i\neq a,b,c}}^{n}|M(k_i)|^2\right)\left|\tilde{M}(k_a,k_b,k_c)\right|^2 + \dots\right] + \dots\Bigg\},$$



2-particle-correlated (i.e. 2 real emissions) squared amplitude defined in terms of cut webs



Momentum-space formulation

- For inclusive transverse observables we can recast the squared amplitude as follows

$$V(\{\tilde{p}\}, k_1, \ldots, k_n) = V(\{\tilde{p}\}, k_1 + \cdots + k_n) \qquad V(\{\tilde{p}\}, k) \equiv V(k) = d_\ell \, g_\ell(\phi) \left(\frac{k_t}{M}\right)^a$$

$$|M(\tilde{p}_1, \tilde{p}_2, k_1, \ldots, k_n)|^2 = |M_B(\tilde{p}_1, \tilde{p}_2)|^2$$
$$\times \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(|M(k_i)|^2 + \int [dk_a][dk_b] |\tilde{M}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) \right.\right.$$
$$\left.\left. + \int [dk_a][dk_b][dk_c] |\tilde{M}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \ldots \right) \right\}$$

Corrections to this picture starting appear at NNLL for non-inclusive observables (see e.g. A. Banfi's talk)

- Introduce a resolution scale ϵk_{t1}^a: owing to rIRC safety, radiation with $k_{ti}^a < \epsilon k_{t1}^a$ is neglected in the observable can be integrated inclusively to cancel the divergences of the virtuals
    - This results in an exponential factor $\propto e^{-R(\epsilon k_{t1}^a)}$

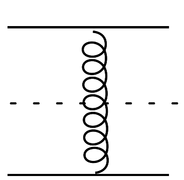
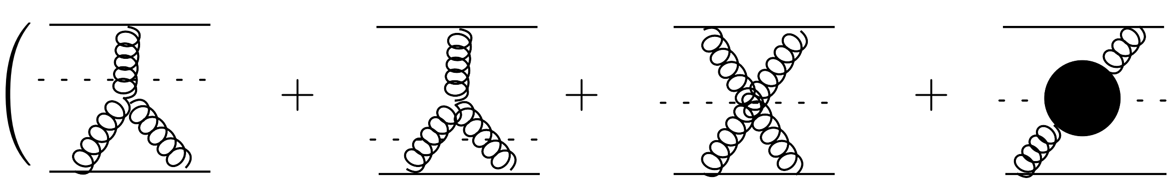
Product of all Sudakov factors that fill the gap between emissions

$$e^{-R(\epsilon k_{t1}^a)} = e^{-R(k_{t1}^a) - R'(k_{t1}^a)\ln(1/\epsilon) - \frac{1}{2}R''(k_{t1}^a)\ln^2(1/\epsilon) \ldots}$$

ϵ dependence cancels against the resolved real corrections

- Remaining (resolved) emissions are treated exclusively: can be parametrised as a Sudakov unintegrated in kt and azimuthal angle

e.g. soft case (analogous counting for hard-collinear emissions)

$R'(k_{ti}^a) \sim \frac{\alpha_s}{2\pi}$  $+ \frac{\alpha_s^2}{(2\pi)^2}\Big($  $\Big) + \ldots$

$\mathcal{O}(\alpha_s L)$ $\qquad$ $\mathcal{O}(\alpha_s^2 L^2)$

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$$|M(\tilde p_1,\tilde p_2,k_1,\ldots,k_n)|^2=|M_B(\tilde p_1,\tilde p_2)|^2$$
$$\times\frac{1}{n!}\left\{\prod_{i=1}^{n}\left(|M(k_i)|^2+\int[dk_a][dk_b]|\tilde M(k_a,k_b)|^2\delta^{(2)}(\vec k_{ta}+\vec k_{tb}-\vec k_{ti})\delta(Y_{ab}-Y_i)\right.\right.$$
$$\left.\left.+\int[dk_a][dk_b][dk_c]|\tilde M(k_a,k_b,k_c)|^2\delta^{(2)}(\vec k_{ta}+\vec k_{tb}+\vec k_{tc}-\vec k_{ti})\delta(Y_{abc}-Y_i)+\ldots\right)\right\}$$

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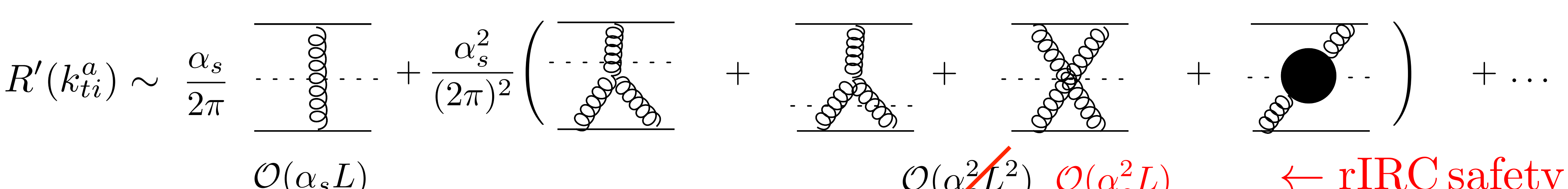


Momentum-space resummation

- Hard-collinear emissions off initial-state legs require some care in the treatment of kinematics. Final result reads

$$\frac{d\Sigma(v)}{dp_t d\Phi_B} = \int_{\mathcal{C}_1}\frac{dN_1}{2\pi i}\int_{\mathcal{C}_2}\frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0)\frac{d\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v)}{dp_t}\mathbf{f}_{N_2}(\mu_0)$$

e.g. for all inclusive observables with a=1 (pt, phi*,…):

DGLAP anomalous dimensions

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))H(\mu_R)\mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right]\int_0^M \frac{dk_{t1}}{k_{t1}}\int_0^{2\pi}\frac{d\phi_1}{2\pi}$$
$$\times e^{-\mathbf{R}(\epsilon k_{t1})}\exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$
$$\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$
$$\times\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_{\epsilon}^{1}\frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$
$$\times\Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

RGE evolution of coeff. functions

- Formulation equivalent to b-space result, up to a scheme change. Using the delta representation one finds

$$\frac{d\Sigma(v)}{dp_t d\Phi_B} = \int_{\mathcal{C}_1}\frac{dN_1}{2\pi i}\int_{\mathcal{C}_2}\frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0)\frac{d\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v)}{dp_t}\mathbf{f}_{N_2}(\mu_0) =$$

$$(1 - J_0(bk_t)) \simeq \Theta(k_t - \frac{b_0}{b}) + \frac{\zeta_3}{12}\frac{\partial^3}{\partial\ln(Mb/b_0)^3}\Theta(k_t - \frac{b_0}{b}) + \ldots$$

$$= \sum_{c_1,c_2}\frac{d|M_B|^2_{c_1c_2}}{d\Phi_B}\int b\,db\,p_t J_0(p_t b)\,\mathbf{f}^T(b_0/b)\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b))H(M)\mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b))\mathbf{f}(b_0/b)$$
$$\times\exp\left\{-\sum_{\ell=1}^{2}\int_0^M\frac{dk_t}{k_t}\mathbf{R}'_\ell(k_t)\,(1 - J_0(bk_t))\right\}.$$

Momentum-space resummation

- Since $k_{ti}/k_{t1} = \zeta_i = \mathcal{O}(1)$ we can expand (although unnecessary) the integrands about $k_{ti} \sim k_{t1}$ to the desired accuracy for a more efficient evaluation

- At N3LL, only two resolved, hard-collinear emissions are relevant: Mellin inversion is analytic

$$\int d\mathcal{Z}[\{R', k_i\}] G(\{\tilde{p}\}, \{k_i\}) = \epsilon^{R'(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(k_{t1}) G(\{\tilde{p}\}, k_1, \dots, k_{n+1})$$

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}_{\mathrm{N^3LL}}(k_{t1})\right) \int d\mathcal{Z}[\{R', k_i\}] \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})\right)$$

$$+ \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z}[\{R', k_i\}] \int_0^1 \frac{d\zeta_s}{\zeta_s} \frac{d\phi_s}{2\pi} \left\{ \left(R'(k_{t1}) \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) \right) \right.$$

$$\times \left(R''(k_{t1}) \ln\frac{1}{\zeta_s} + \frac{1}{2} R'''(k_{t1}) \ln^2\frac{1}{\zeta_s} \right) - R'(k_{t1}) \left(\partial_L \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - 2\frac{\beta_0}{\pi} \alpha_s^2(k_{t1}) \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \ln\frac{1}{\zeta_s} \right)$$

$$\left. + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \right\} \left\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_s)\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})\right) \right\}$$

$$+ \frac{1}{2} \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z}[\{R', k_i\}] \int_0^1 \frac{d\zeta_{s1}}{\zeta_{s1}} \frac{d\phi_{s1}}{2\pi} \int_0^1 \frac{d\zeta_{s2}}{\zeta_{s2}} \frac{d\phi_{s2}}{2\pi} R'(k_{t1})$$

$$\times \left\{ \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \left(R''(k_{t1})\right)^2 \ln\frac{1}{\zeta_{s1}} \ln\frac{1}{\zeta_{s2}} - \partial_L \mathcal{L}_{\mathrm{NLL}}(k_{t1}) R''(k_{t1}) \left(\ln\frac{1}{\zeta_{s1}} + \ln\frac{1}{\zeta_{s2}} \right) \right.$$

$$\left. + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \right\}$$

$$\times \left\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_{s1}, k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_{s1})\right) - \right.$$

$$\left. \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})\right) \right\} + \mathcal{O}\left(\alpha_s^n \ln^{2n-6}\frac{1}{v}\right)$$

- This formula can be evaluated by means of fast Monte-Carlo methods

- Coefficient functions and hard-virtual corrections absorbed into the parton luminosity

- Valid for all inclusive observables with a=1. A similar formula holds for any a>0

$$V(\{\tilde{p}\}, k) \equiv V(k) = d_\ell \, g_\ell(\phi) \left(\frac{k_t}{M}\right)^a$$

$$V(\{\tilde{p}\}, k_1, \dots, k_n) = V(\{\tilde{p}\}, k_1 + \dots + k_n)$$

Some remarks

- In the region where $k_{t1} \sim p_t$, the above formula controls all logarithmic terms $\ln(M/p_t)$ up to N3LL. In addition, it contains subleading logarithmic terms that *cannot be neglected*
  - This can be seen by expanding $k_{t1} \sim p_t$ and evaluating the integral over the first emission analytically. No singularity is present in this region as cancellations occur at much smaller values of pt

- In the region dominated by azimuthal cancellations, the kinematics is correctly reproduced and the above formula yields the power-like scaling for the differential cross section

e.g. NLL formula

$$\frac{d^2\Sigma(v)}{d^2\vec{p}_t d\Phi_B} = \sigma^{(0)}(\Phi_B)\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi} e^{-R(k_{t1})}R'(k_{t1})\int d\mathcal{Z}[\{R', k_i\}]\delta^{(2)}\left(\vec{p}_t - \left(\vec{k}_{t1} + \cdots + \vec{k}_{t(n+1)}\right)\right)$$

$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} \simeq 4\sigma^{(0)}(\Phi_B)\,p_t \int_{\Lambda_{\rm QCD}}^{M} \frac{dk_{t1}}{k_{t1}^3} e^{-R(k_{t1})} \simeq 2\sigma^{(0)}(\Phi_B) p_t \left(\frac{\Lambda_{\rm QCD}^2}{M^2}\right)^{\frac{16}{25}\ln\frac{41}{16}}$$

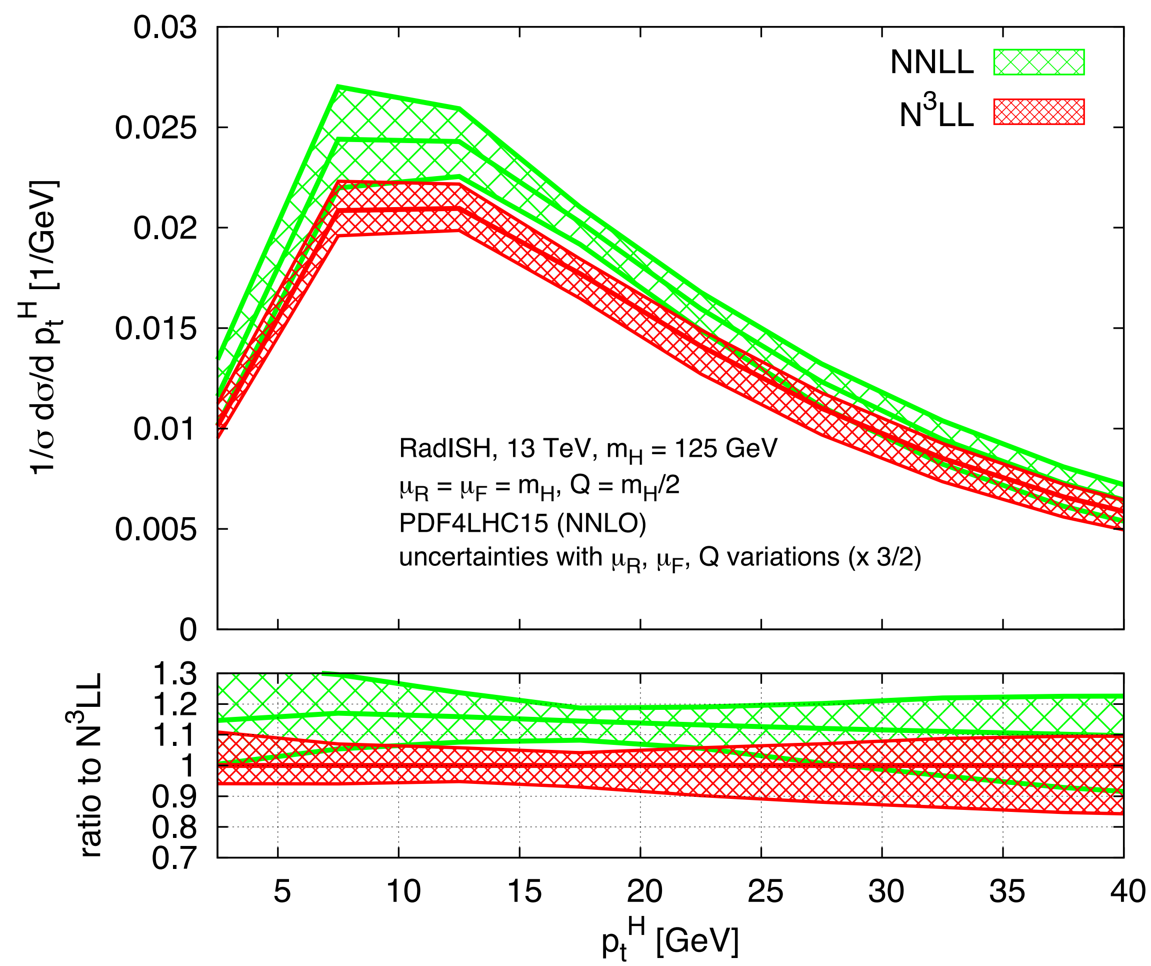
  - Higher-order logarithmic corrections modify the value of the intercept in a perturbative way according to the scaling

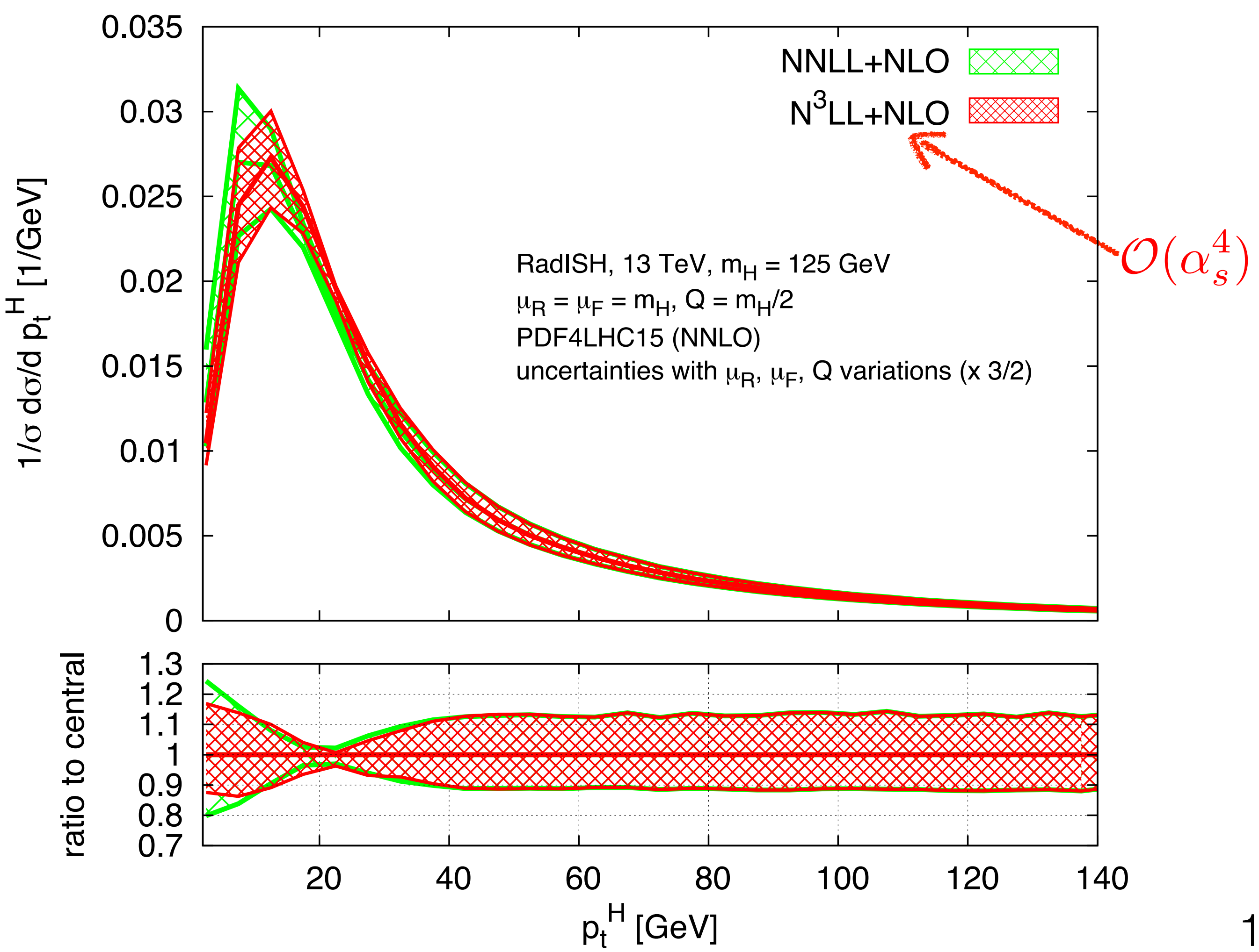
$$L \sim \frac{1}{\alpha_s}$$

Matching to fixed order: Higgs

- N3LL/NNLL corrections amount to 10%-15%, partly due to the constant change in normalisation from the two-loop coefficient functions

- Residual scale uncertainty O(10%)





- At the matched level, the (N3LL+NLO)/(NNLL+NLO) correction is O(10%) near the peak of the spectrum, and somewhat larger at small pt. Perturbative uncertainty nearly halved below 10 GeV

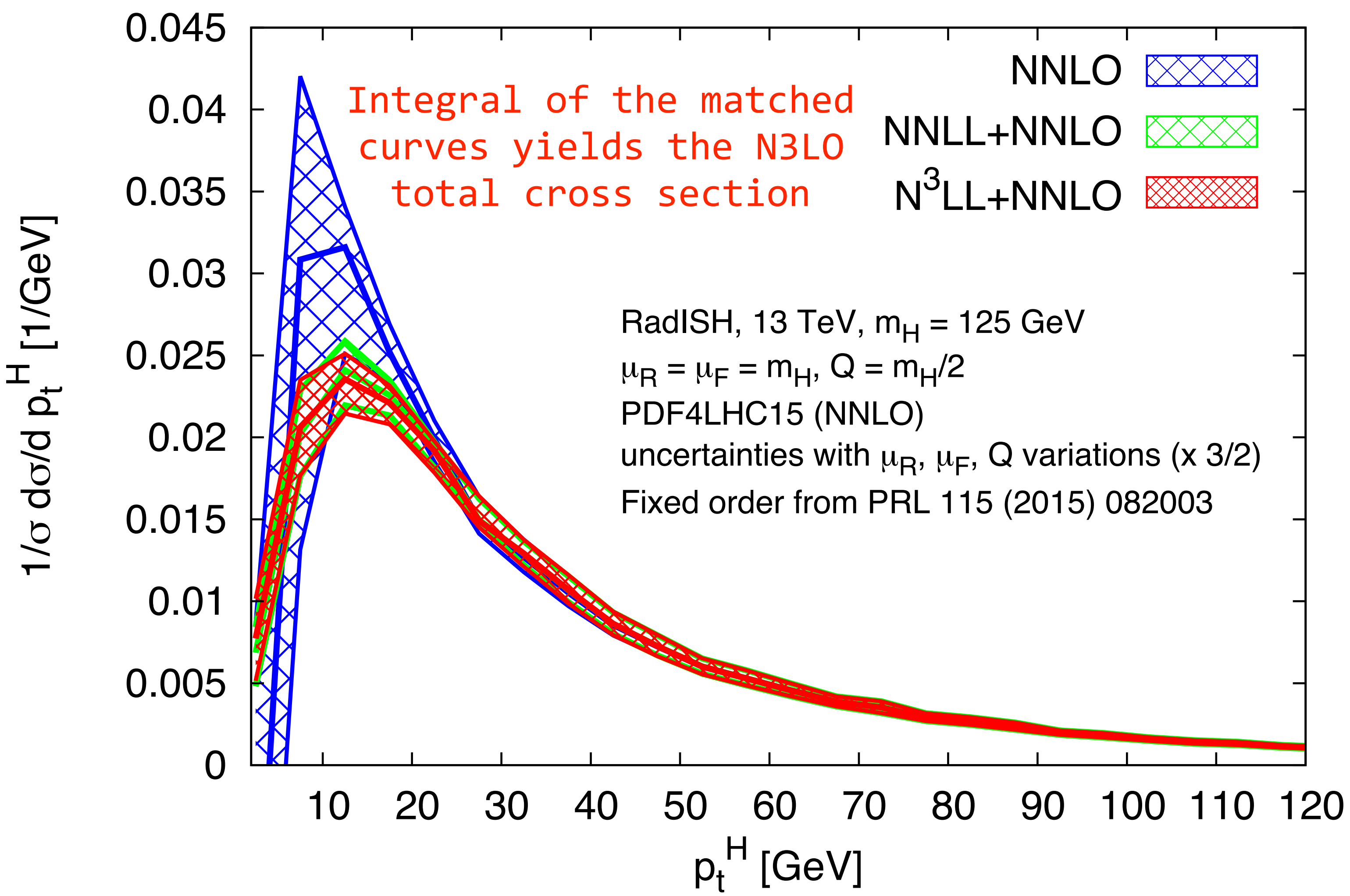
Matching to fixed order: Higgs

- Constant terms at N3LO are recovered through the matching to fixed order via a multiplicative scheme.

- The matching to NNLO compensates part of the difference. In comparison to NNLL+NNLO, the N3LL corrections amount to a few percent around the peak, and get more sizeable (~10%) below 10 GeV

- Moderate reduction in the perturbative uncertainty observed, more stable NNLO distributions on the making

- Quark-mass corrections necessary to improve further, non-perturbative corrections still subleading



Total N3LO XS from
[Anastasiou, Duhr, Dulat, Herzog, Mistlberger ’15]
NNLO distribution obtained in:
[Boughezal, Caola, Melnikov, Petriello, Schulze ‘15; Caola, Melnikov, Schulze ’15;
Boughezal, Focke, Giele, Liu, Petriello ’15; Chen, Cruz-Martinez, Gehrmann, Glover,Jaquier ‘16]

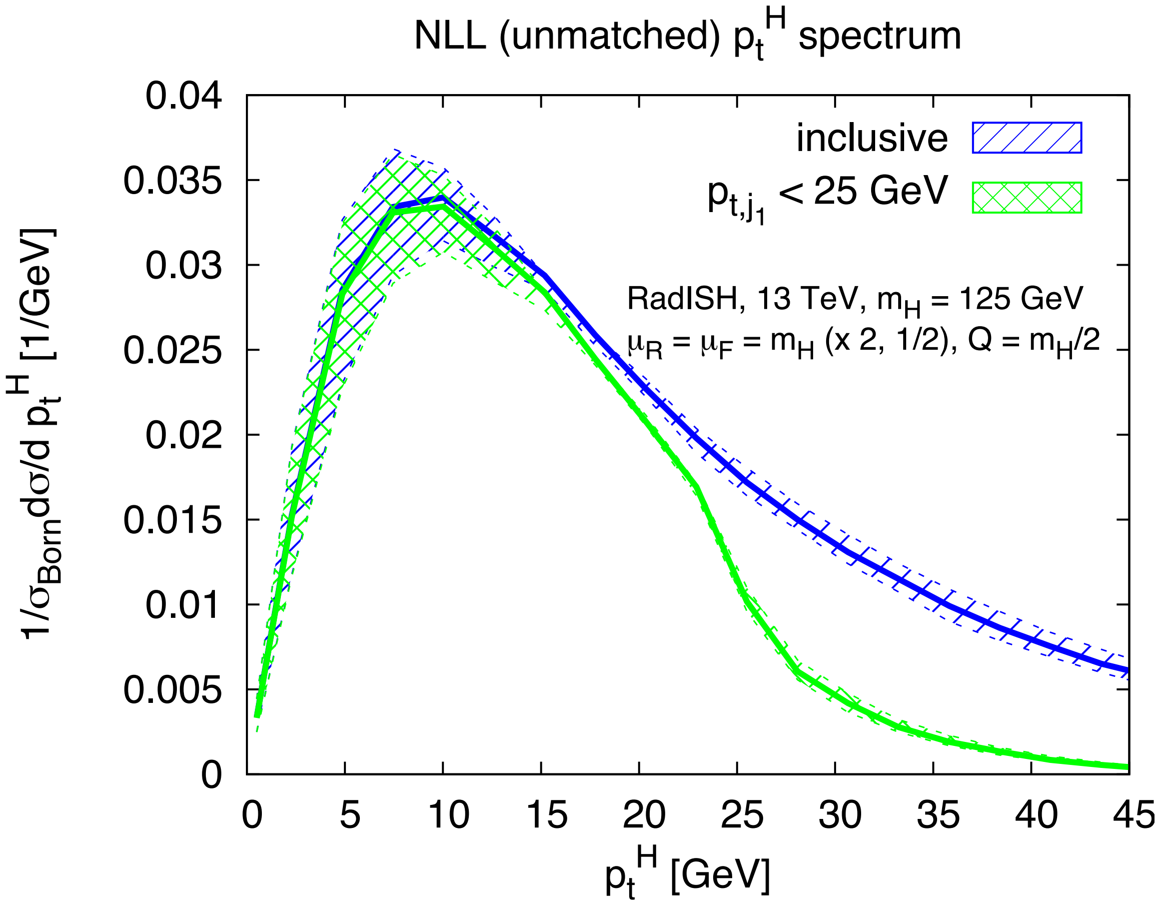
Generalisation to other observables

- Extension to non-inclusive observables at N3LL:
  - Although the resummed formula obtained here is valid for inclusive observables, the Sudakov radiator is universal for all observables which feature the same scaling for a single, soft-collinear emission [Banfi, McAslan, Monni, Zanderighi ’16]

$$V_{\rm sc}(\{\tilde{p}\},k) = \left(\frac{k_t}{M}\right)^a$$

  - The resolved real-emission corrections in the general case can be computed by extending the ARES approach beyond NNLL. This is not trivial, but it can be done systematically (the case reported here is an example)

- Multi-differential cross sections:
  - Not being fully inclusive in the radiation allows us to have more exclusive cuts. The logarithmic accuracy can be spoiled and it must be understood case-by-case
  - This makes it possible to compute more exclusive cross sections with higher logarithmic order



Comparison to literature for pT

See also talk by Varun Vaidya on Friday

- An alternative approach to momentum-space resummation for the transverse-momentum distribution was proposed in [Ebert, Tackmann 1611.08610]

All of the terms in the infinite sum compensate for the exponential suppression

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\,\mathrm{d}\vec{q}_T} = \sigma_0\, H(Q,\mu_H)\frac{1}{2\pi q_T}\frac{\mathrm{d}}{\mathrm{d}q_T}\int_{|\vec{p}_T|\le q_T}\mathrm{d}^2\vec{p}_T$$
$$\times \exp\left[\int_{\mu_H}^{\mu_T}\frac{\mathrm{d}\mu'}{\mu'}\gamma_H(Q,\mu')\right]\int \mathrm{d}^2\vec{k}_a\,\mathrm{d}^2\vec{k}_b\,\mathrm{d}^2\vec{k}_s\,\delta(\vec{p}_T-\vec{k}_a-\vec{k}_b-\vec{k}_s)\int \mathrm{d}^2\vec{k}_s'$$
$$\times\left[\delta(\vec{k}_s-\vec{k}_s') + \sum_{n=1}^{\infty}\prod_{i=1}^{n}\int_{k_{i-1}|_+}^{\nu_{i-1}}\frac{\mathrm{d}\nu_i}{\nu_i}\int \mathrm{d}^2\vec{k}_i\,\gamma_\nu(\vec{k}_{i-1}-\vec{k}_i,\mu_T)\,\delta\Big(\vec{k}_s-\vec{k}_s'-\sum_i \vec{k}_i\Big)\right]$$
$$\times B_a(\omega_a,\vec{k}_a,\mu_T,\nu_a)\,B_b(\omega_b,\vec{k}_b,\mu_T,\nu_b)\,S(\vec{k}_s',\mu_T,k_s'|_+)\,. \qquad (6.4)$$

e.g. at LL:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\,\mathrm{d}\vec{q}_T} = \sigma_0\frac{1}{2\pi q_T}\frac{\mathrm{d}}{\mathrm{d}q_T}\theta(q_T)\,f_a(\omega_a,q_T)\,f_b(\omega_b,q_T)\exp\left[-\frac{\Gamma_{\rm cusp}}{2}\ln^2\frac{Q^2}{q_T^2}\right]$$
$$\times\left[1-2\Gamma_{\rm cusp}^2\zeta_3\ln\frac{Q^2}{q_T^2}+\Gamma_{\rm cusp}^3\left(\frac{2\zeta_3}{3}\ln^3\frac{Q^2}{q_T^2}+6\zeta_5\ln\frac{Q^2}{q_T^2}\right)\right.$$
$$\left.+\Gamma_{\rm cusp}^4\left(-4\zeta_5\ln^3\frac{Q^2}{q_T^2}+10\zeta_3^2\ln^2\frac{Q^2}{q_T^2}-30\zeta_7\ln\frac{Q^2}{q_T^2}\right)+\mathcal{O}(\Gamma_{\rm cusp}^5)\right]$$

Note that eq. (6.4) could be the starting point for a numerical evaluation. Although the infinite number of convolutions cannot be calculated in closed form, one could for example evaluate the result iteratively and truncate the sum in eq. (6.4) once a desired numerical accuracy is reached.

- Similar structure obtained by expanding $b \sim 1/p_t$ in the b-space formulation and retaining all terms in the Taylor series.
- Quantitative comparisons to our formula at higher orders under consideration

Conclusions

- I presented a method to obtain all-order predictions for inclusive, transverse observables up to N3LL:
  - it is formally equivalent to the standard b-space formulation in the known cases (modulo the treatment of non-perturbative corrections and Landau pole)
  - it can be implemented efficiently on a computer code
  - the universality of the Sudakov radiator allows for an extension to more general transverse observables. Resolved real corrections can be formulated systematically if needed - practical implementation left
  - We implement the method in the computer code `RadISH` that can process any colour singlet with arbitrary cuts in the Born phase space. Public release soon
- Phenomenological application: Higgs pt spectrum
  - matching to NLO: N3LL corrections are of O(10%) near the peak and below. Uncertainty reduction below the peak of the distributions
  - matching to NNLO: N3LL corrections are a few percent at the peak, and get larger at smaller pt. Moderate reduction of the perturbative uncertainty, higher-statistics runs for the fixed order ongoing. Final uncertainty at the ~10% across the whole spectrum