# NNLL resummation with the ARES method

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# Outline

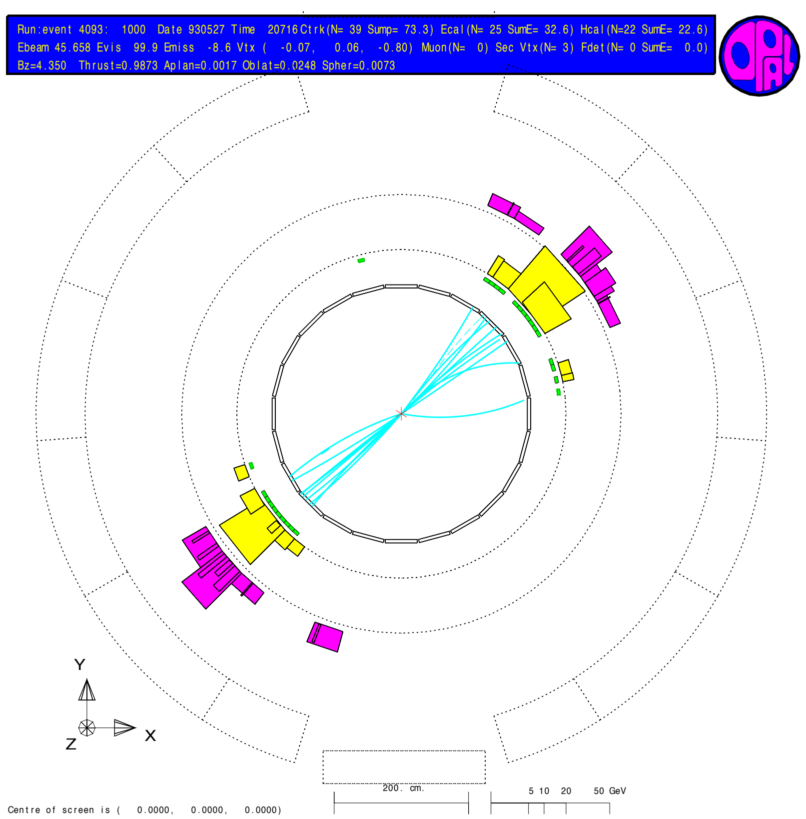
- Resummation of final-state observables
- Observable properties (rIRC safety)
- The ARES method for NNLL resummation
- Applications to $e^+e^-$ annihilation
- Current work in progress and outlook

# Final-state observables

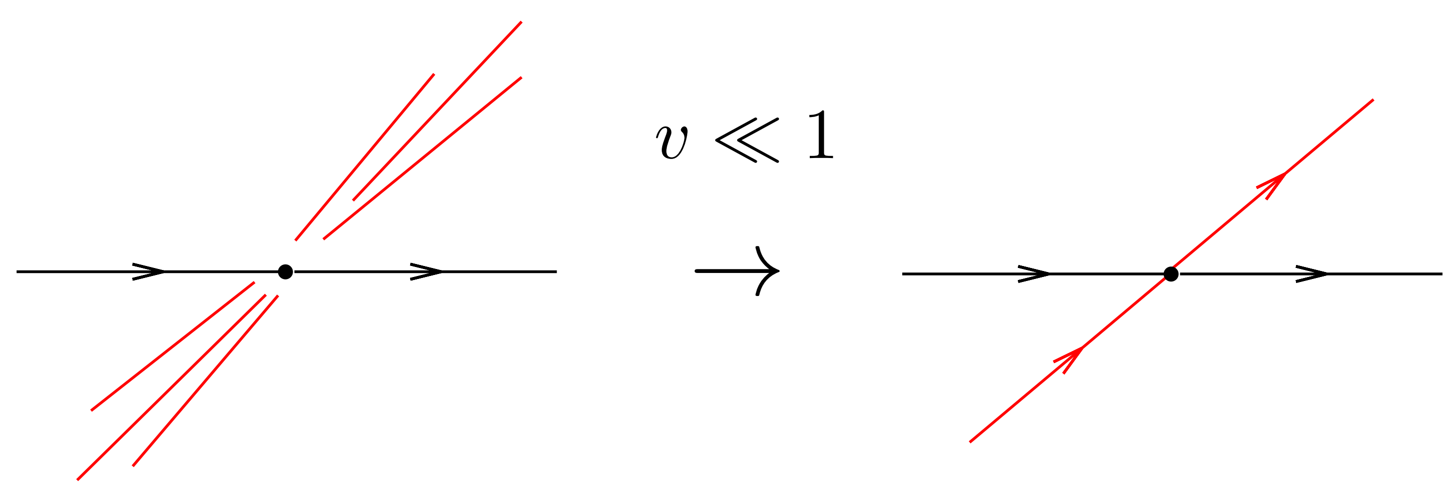
- We consider a generic final-state observable, a function $V(p_1, \ldots, p_n)$ of all possible final-state momenta $p_1, \ldots, p_n$

- Examples: leading jet transverse momentum in Higgs production or thrust in $e^+e^- \to \text{hadrons}$

$$\frac{p_{\mathrm{t,max}}}{m_H} = \max_{j \in \text{jets}} \frac{p_{t,j}}{m_H} \qquad\qquad T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$




$$\Sigma(v) = \mathrm{Prob}[V(p_1, \ldots, p_n) < v]$$



# The narrow-jet limit

- Selecting events close to the Born limit, i.e. $v \ll 1$, produces large logarithms of the resolution variable $v$ due to incomplete real-virtual cancellations

$$\Sigma(v) \simeq 1 - C\,\frac{\alpha_s}{\pi}\ln^2\frac{1}{v} + \ldots$$

LO NLO

**breakdown of perturbation theory!**

# All-order resummation

- All-order resummation of large logarithms $\Rightarrow$ reorganisation of the PT series in the region $\alpha_s L \sim 1$, with $L = \ln(1/v)$

$$\Sigma(v) \simeq \exp\left[\underbrace{L g_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \dots\right]$$

# All-order resummation

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$$\Sigma(v) \simeq e^{\overbrace{Lg_1(\alpha_s L)}^{\text{LL}}} \times \begin{pmatrix} 1 \quad + \quad \alpha_s \quad + \dots \\ \underbrace{G_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s G_3(\alpha_s L)}_{\text{NNLL}} + \dots \end{pmatrix}$$

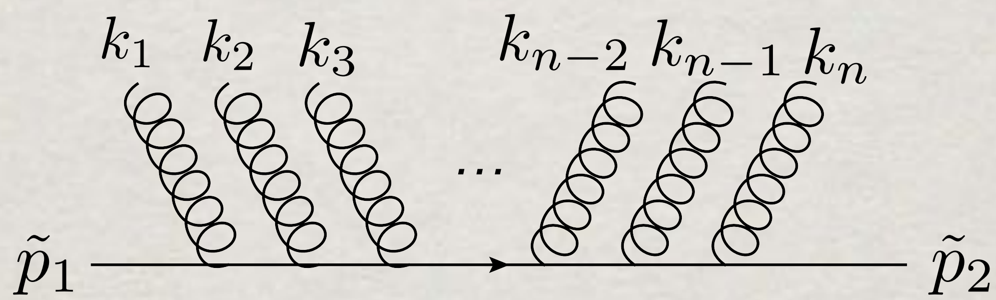
# The ARES method

- NNLL corrections are often sizeable and important for precision physics
- The most important limitation is the analytical treatment of the observable in some (smartly defined) conjugate space
- The Automated Resummer for Event Shapes (ARES) is a semi-numerical approach that:
  - does not rely on analytical properties of the observable
  - is NNLL accurate and extendable to higher orders
  - is fully general for a very broad category of observables (~ all that can be possibly resummed at NNLL accuracy)
  - is flexible and automated (only input: observable’s routine in suitable limits)

# Basic observable properties

- We consider an infrared and collinear (IRC) safe observable normalised as

$$v = V(\{\tilde{p}\}, k_1, \ldots, k_n) \leq 1$$




- In the Born limit, $V(\{\tilde{p}\}) = 0$
- In the limit $v \to 0$, quasi-Born kinematics, all secondary emissions $k_1, \ldots, k_n$ are soft and/or collinear

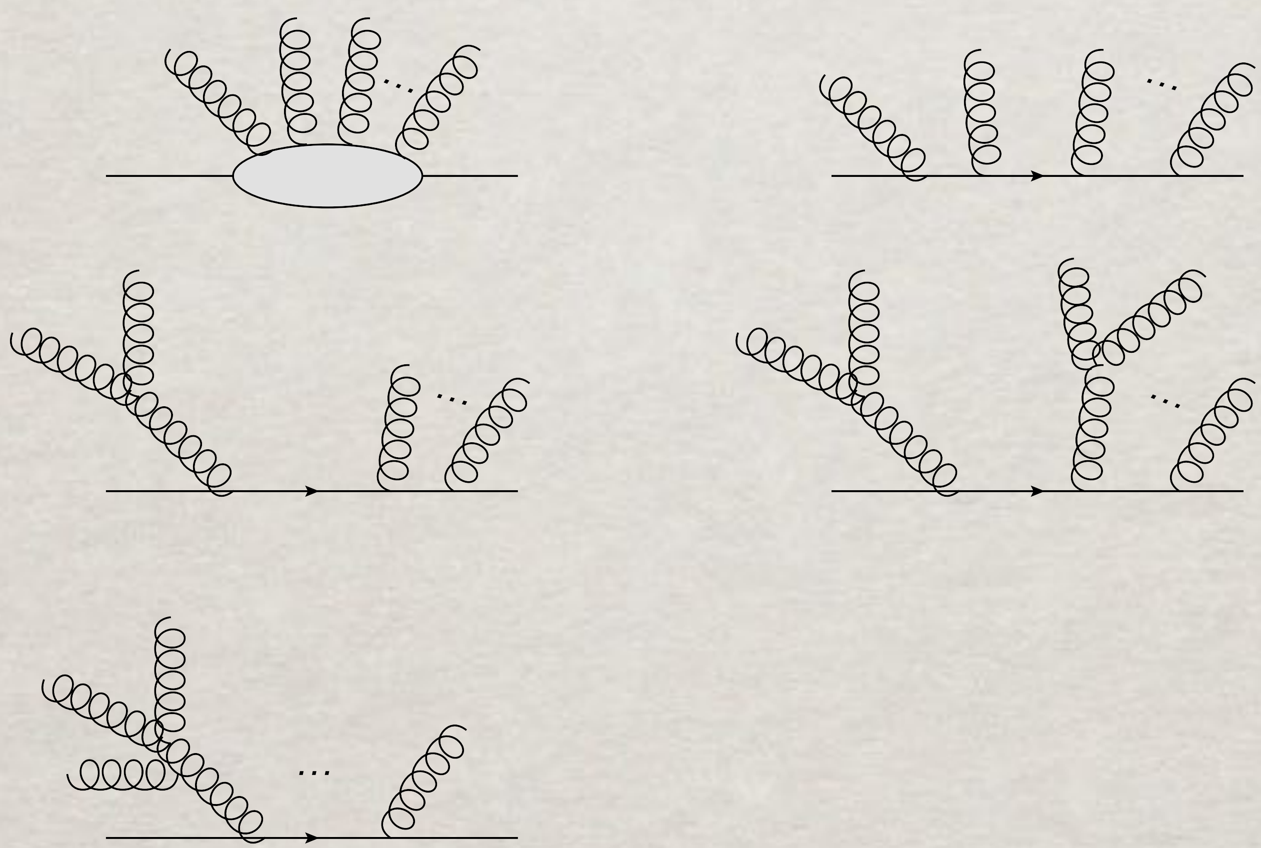
# Recursive IRC safety

- We restrict ourselves to recursive IRC (rIRC) safe observables, for which
  - the observable scaling properties when we make all emissions simultaneously soft-collinear are the same with any number of secondary emissions
  - such scaling properties are unchanged after an infinitely soft emission or a perfect collinear splitting
- Examples of rIRC observables:
  - most global event shapes
  - Durham and Cambridge jet resolution parameters (no JADE and Geneva)
  - transverse momentum of the leading jet in Higgs or vector boson production

# Implications of rIRC safety

- The only emissions that contribute to $\Sigma(v) = \mathrm{Prob}[V(\{\tilde{p}\}, k_1, \ldots, k_n)] < v$ in the limit $v \to 0$, up to powers of $v$, are those for which

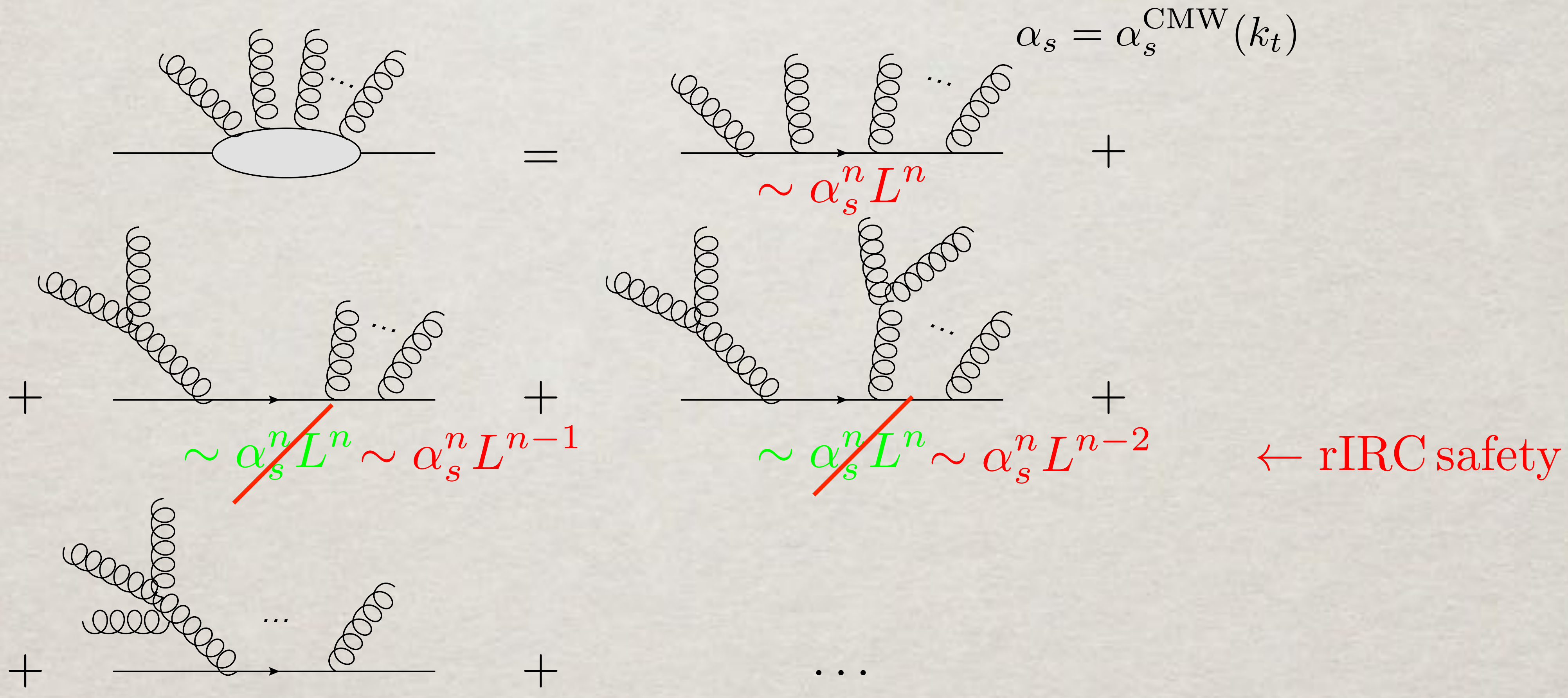
$$V(\{\tilde{p}\}, k_1) \sim V(\{\tilde{p}\}, k_2) \sim \cdots \sim V(\{\tilde{p}\}, k_n) \sim V(\{\tilde{p}\}, k_1, \ldots, k_n) \sim v$$



- This, together with QCD coherence, is enough to establish the relative importance of soft-collinear contributions at all logarithmic orders

# Relevant emissions

- At NLL the only relevant emissions are soft and collinear gluons widely separated in angle (rapidity separation $\sim \ln(1/v)$ )



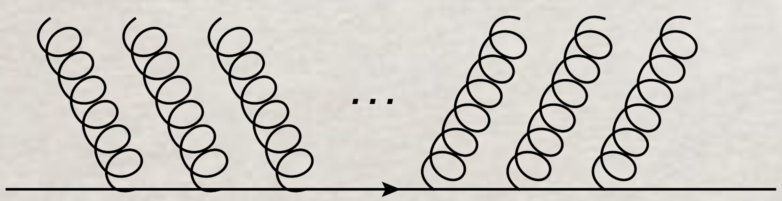

- Any other emission gives a contribution of relative order $\alpha_s$

# NLL resummation

- Unresolved emissions and virtual corrections result in a double-logarithmic Sudakov exponent, the radiator [Banfi Salam Zanderighi '05]

$$\Sigma(v) = e^{-R_{\mathrm{NLL}}(v)} \mathcal{F}_{\mathrm{NLL}}(v)$$

- The effect of multiple soft and collinear gluons widely separated in angle is encoded in the single-logarithmic function $\mathcal{F}_{\mathrm{NLL}}(v)$



Measure defined by the soft-collinear ensemble

$$= \int d\mathcal{Z}[\{R'_{\ell_i}, k_i\}]$$

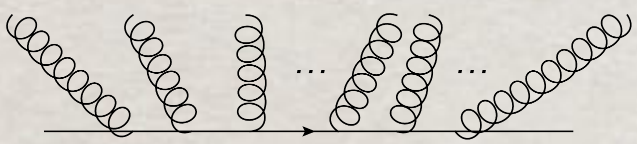
$$\mathcal{F}_{\mathrm{NLL}}(v) = \left\langle \Theta\left(1 - \lim_{v\to 0} \frac{V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\}, \{k_i\})}{v}\right)\right\rangle$$

- The function $V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\}, \{k_i\})$ is the observable for soft and collinear emissions widely separated in angle. For jet rates, this is different from $V_{\mathrm{sc}}(\{\tilde{p}\}, \{k_i\})$, the observable in the soft-collinear limit

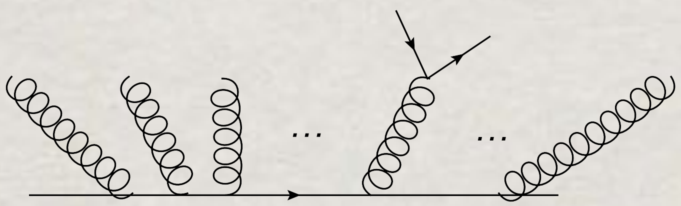
# NNLL resummation

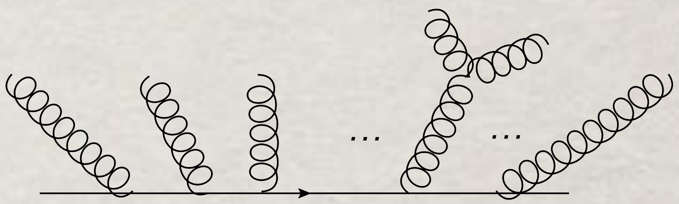
- The NNLL radiator, encoding the cancellation of real and virtual corrections, is known only for observables that scale like the jet mass or jet broadening

$$\Sigma(v) = e^{-R_{\rm NNLL}(v)} \left[\mathcal{F}_{\rm NLL}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\rm NNLL}(v)\right]$$



Clustering
(jet algorithms only)

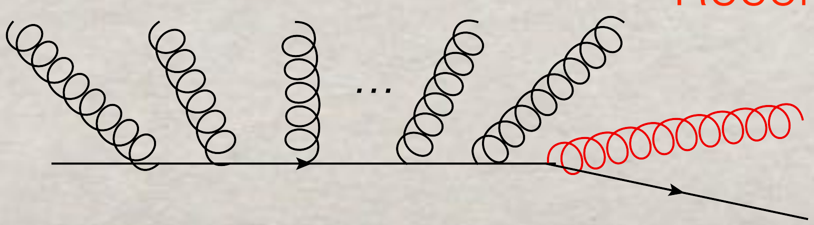




Correlated emission

$$\delta\mathcal{F}_{\rm NNLL} = \delta\mathcal{F}_{\rm clust} + \delta\mathcal{F}_{\rm correl} + \delta\mathcal{F}_{\rm hc} + \delta\mathcal{F}_{\rm rec} + \delta\mathcal{F}_{\rm wa} + \delta\mathcal{F}_{\rm sc}$$

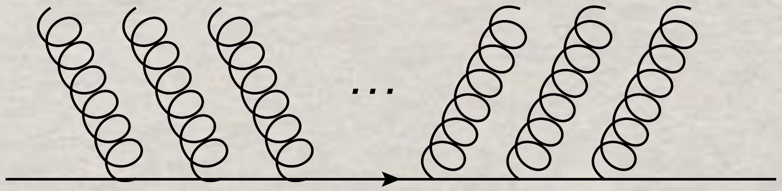
Hard collinear
Recoil



Running coupling
Rapidity
(jet algorithms)

Soft wide-angle

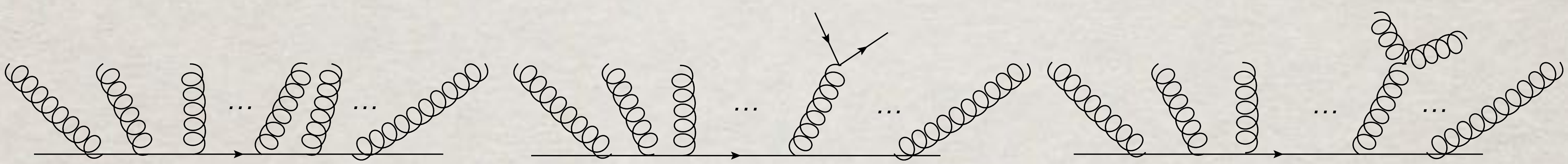
Soft-collinear
(event shapes)



# NNLL resummation

- All NNLL corrections can be written in terms of finite integrals in four dimensions

$$\Sigma(v) = e^{-R_{\mathrm{NNLL}}(v)}\left[\mathcal{F}_{\mathrm{NLL}}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(v)\right]$$



Clustering
(jet algorithms only)

Correlated emission

$$\delta\mathcal{F}_{\mathrm{NNLL}} = \delta\mathcal{F}_{\mathrm{clust}} + \delta\mathcal{F}_{\mathrm{correl}} + \delta\mathcal{F}_{\mathrm{hc}} + \delta\mathcal{F}_{\mathrm{rec}} + \delta\mathcal{F}_{\mathrm{wa}} + \delta\mathcal{F}_{\mathrm{sc}}$$

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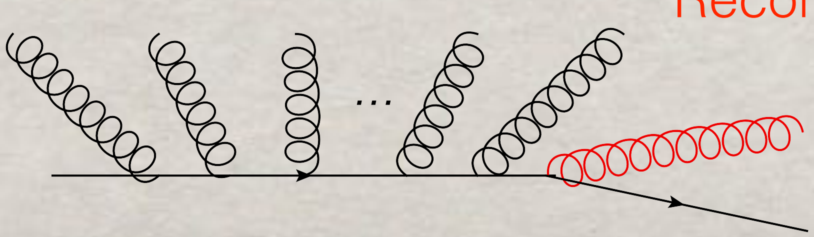
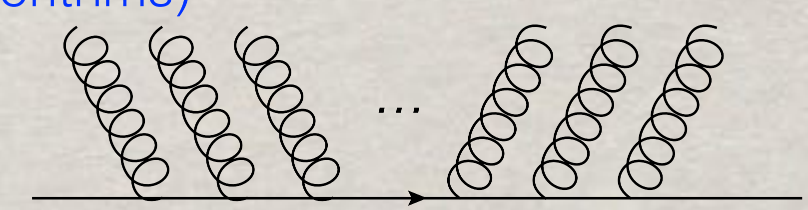


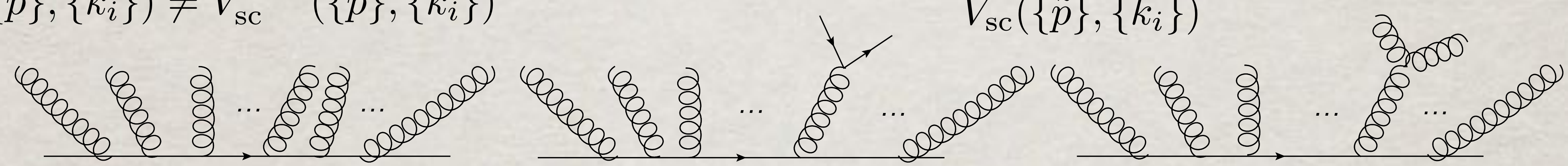


# NNLL resummation

- Every NNLL correction requires to determine an approximate expression for the observable in the relevant kinematic limit

$$\Sigma(v) = e^{-R_{\rm NNLL}(v)}\left[\mathcal{F}_{\rm NLL}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\rm NNLL}(v)\right]$$

$V_{\rm sc}(\{\tilde p\},\{k_i\}) \neq V_{\rm sc}^{\rm NLL}(\{\tilde p\},\{k_i\})$ $V_{\rm sc}(\{\tilde p\},\{k_i\})$



Clustering
(jet algorithms only)

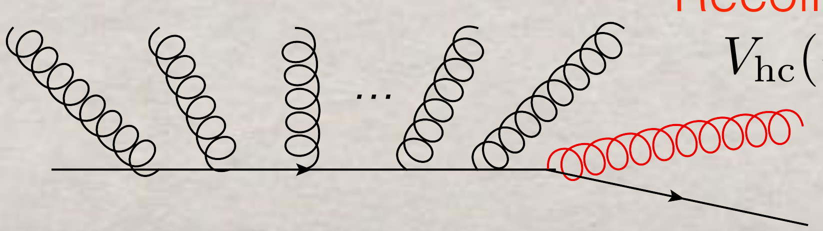
Correlated emission

$$\delta\mathcal{F}_{\rm NNLL} = \delta\mathcal{F}_{\rm clust} + \delta\mathcal{F}_{\rm correl} + \delta\mathcal{F}_{\rm hc} + \delta\mathcal{F}_{\rm rec} + \delta\mathcal{F}_{\rm wa} + \delta\mathcal{F}_{\rm sc}$$

$V_{\rm sc}(\{\tilde p\},\{k_i\})$

Hard collinear
Recoil

$V_{\rm hc}(\{\tilde p\},\{k_i\})$



$V_{\rm sc}^{\rm NLL}(\{\tilde p\},\{k_i\})$

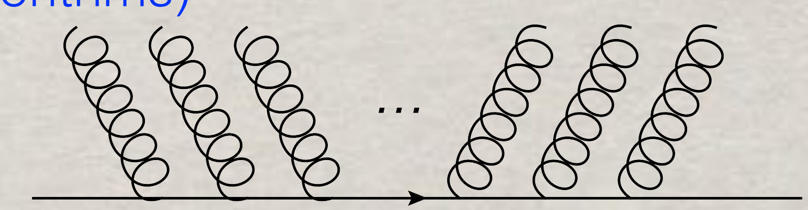
Running coupling
Rapidity
(jet algorithms)

$V_{\rm wa}(\{\tilde p\},\{k_i\})$

Soft wide-angle

$V_{\rm sc}(\{\tilde p\},\{k_i\})$

Soft-collinear
(event shapes)



# CAESAR vs ARES

[Banfi Salam Zanderighi ‘05]

- Establishes the range in which actual emissions can be considered soft and collinear
- Uses the actual observable subroutine and computes its soft-collinear limit numerically
- Requires careful extrapolations to be extended at NNLL

[Banfi McAslan Monni Zanderighi ‘15]

- Generates emissions that are by construction soft and collinear (no energy-momentum conservation)
- Uses analytically determined soft and collinear limits of each observable
- Can be in principle extended to any logarithmic accuracy

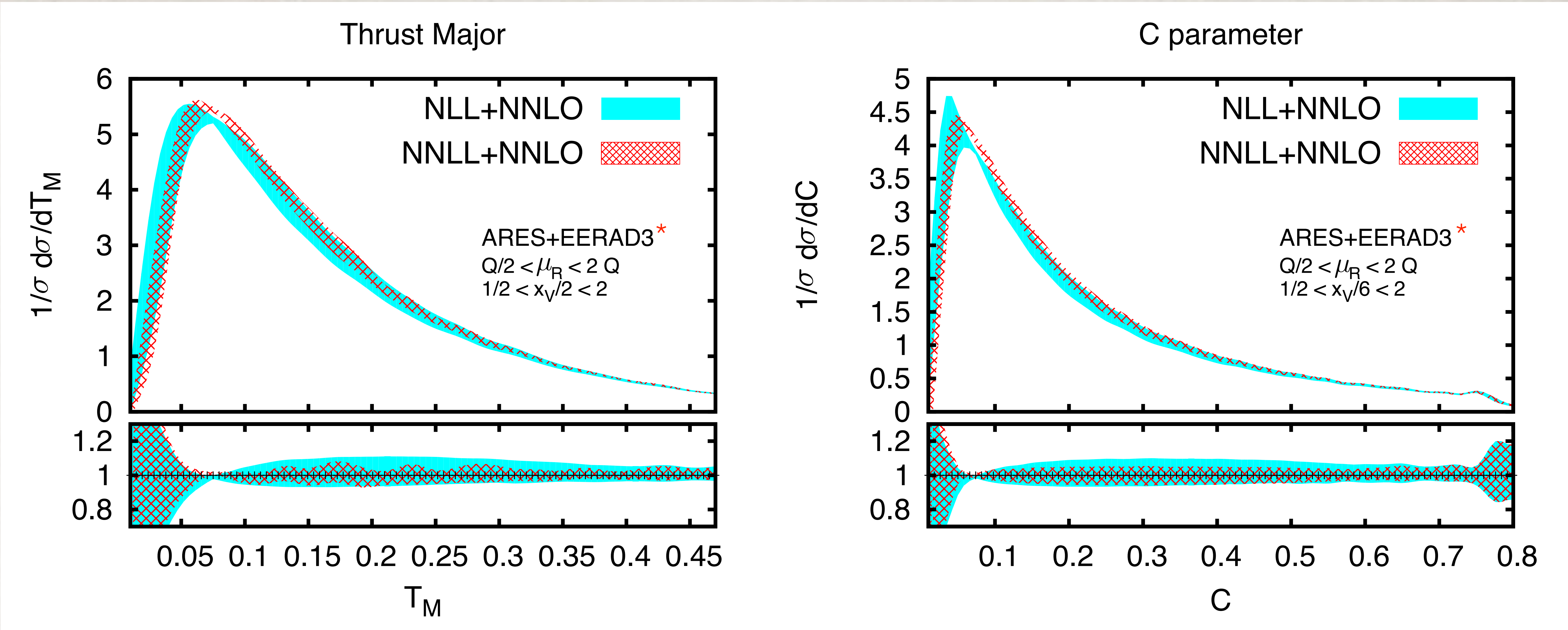
# Event-shape variables

- Event-shapes distributions at NNLL matched to exact NNLO

  [Banfi McAslan Monni Zanderighi '15]

- Reproduced existing results for thrust, heavy-jet mass and broadenings

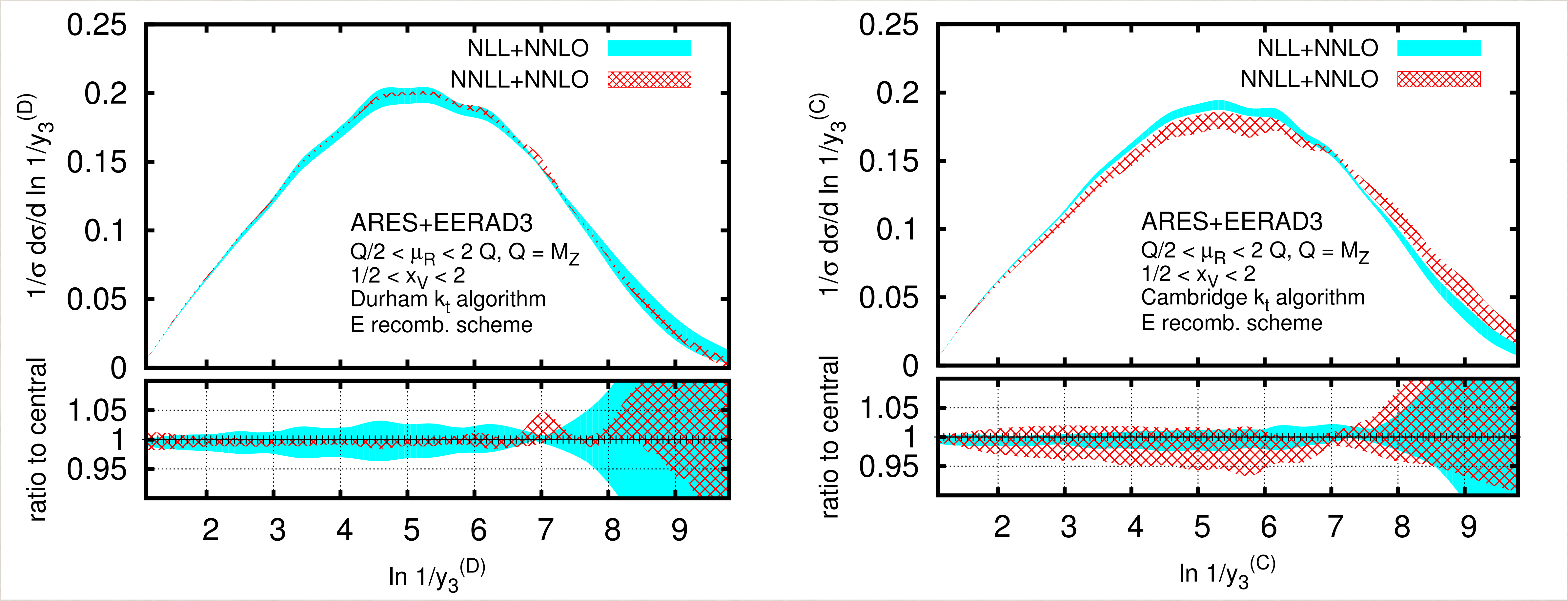
- New results for thrust-major, C-parameter and oblateness




[* Gehrmann-De Ridder Gehrmann Glover Heinrich]

# Two-jet rate

- First-ever NNLL resummation of the two-jet rate for the Durham and Cambridge algorithms [Banfi McAslan Monni Zanderighi '16]




- NNLL resummation of the two-jet rate has been performed also for other rIRC safe jet algorithms (flavour $k_t$, angular-ordered Durham, inclusive $k_t$)

# Two-jet rate

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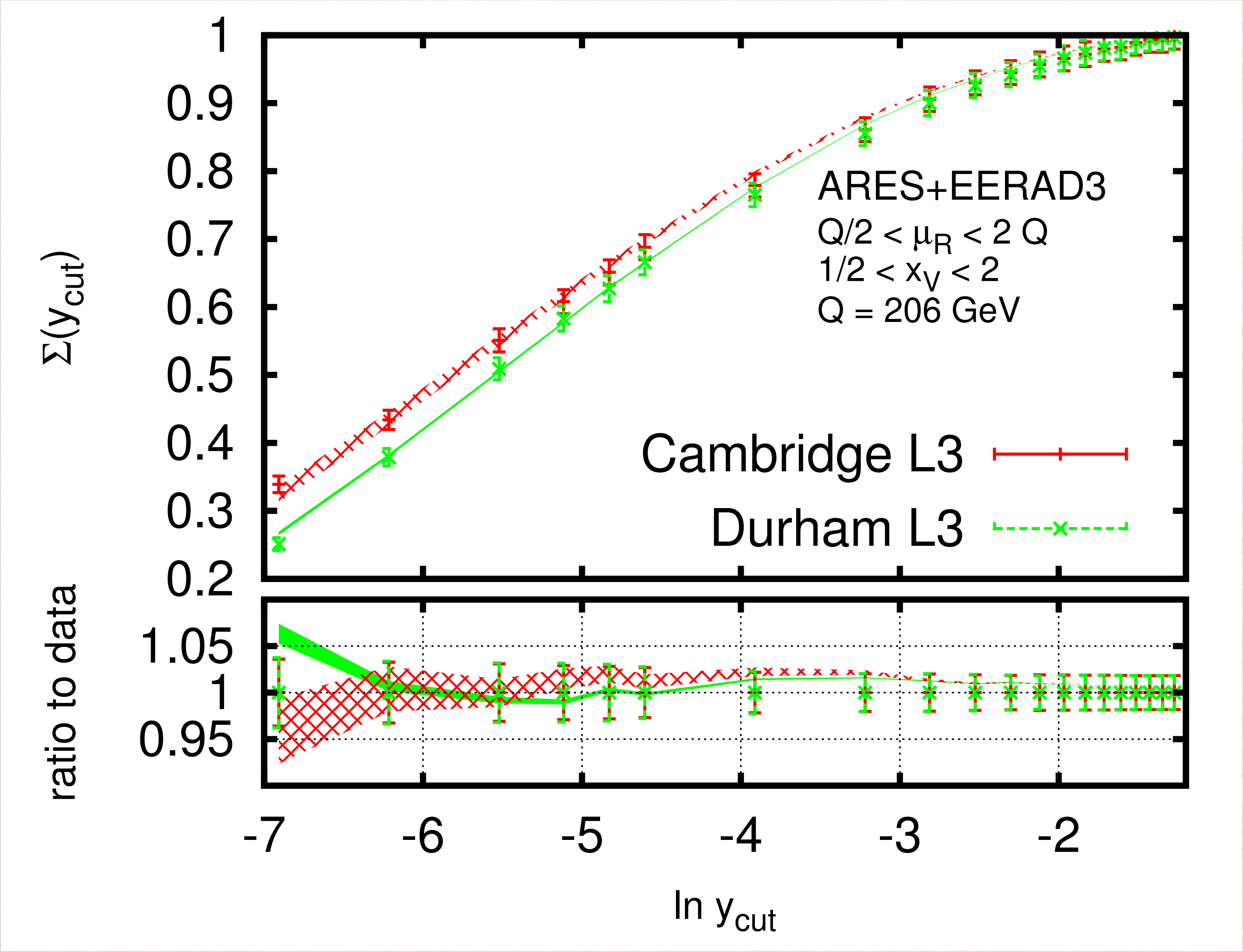


- Good agreement with LEP data $\Rightarrow$ fit of $\alpha_s(M_Z)$ in progress

# Conclusions

- Novel general method for the resummation of any rIRC safe (global) observable in the two-scale regime
    - weak applicability conditions
    - cancellation of poles between real and virtual corrections performed analytically in dimensional regularisation
    - contributions of resolved real emissions formulated in terms of four-dimensional integrals (suitable for Monte Carlo implementation)
- Work in progress
    - New global fit of $\alpha_s$ from $e^+e^-$ event shapes
    - More NNLL resummations in multi-jet events, e.g. hadron collisions
- Ambitious goal: comparison to parton shower event generators to obtain branching algorithms that are NNLL accurate

# Conclusions

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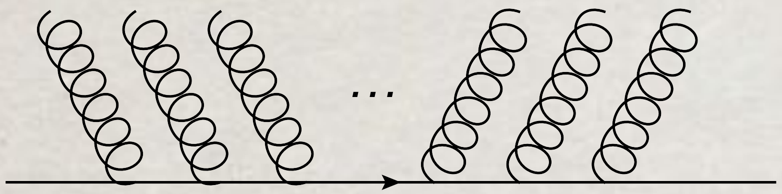
**Thank you for your attention!**

# EXTRA

# Soft-Collinear emissions

- Integration measure for soft and collinear emissions widely separated in angle



Measure defined by the soft-collinear ensemble

$$= \int d\mathcal{Z}[\{R'_{\ell_i}, k_i\}]$$

$$\langle G(\{k_i\})\rangle = \int d\mathcal{Z}[\{R'_{\ell_i}, k_i\}]\, G(\{k_i\})$$

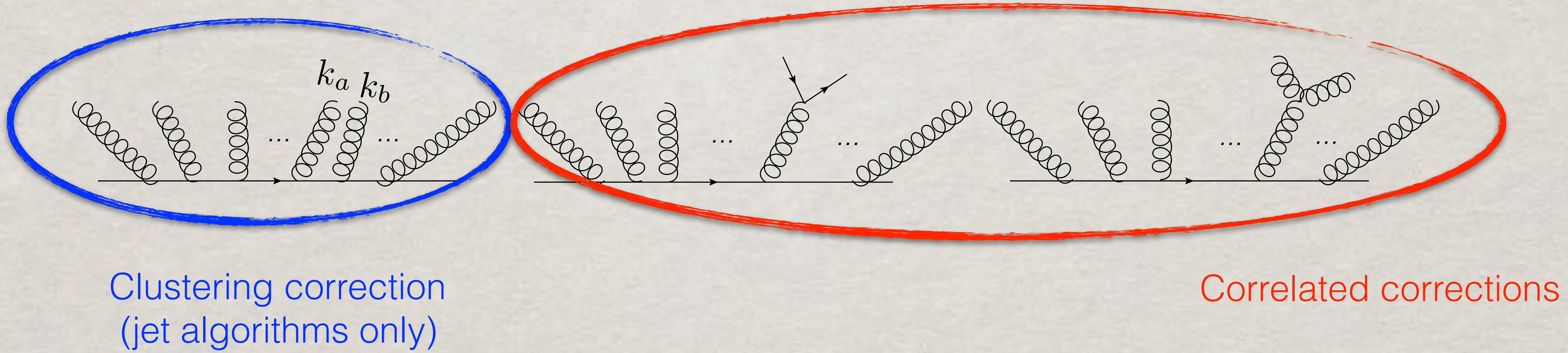
$$= \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \left( \int_{\epsilon}^{\infty} \frac{d\zeta_i}{\zeta_i} \sum_{\ell_i} R'_{\ell_i} \int_0^{2\pi} \frac{d\phi_i^{(\ell_i)}}{2\pi} \int_0^1 d\xi_i^{(\ell_i)} \right) G(k_1, \dots, k_n)$$

- Probability to emit a soft and collinear gluon $k_i$ with $V(\{p\}, k_i) = v\zeta_i$ off hard leg $\ell_i$ with azimuthal angle $\phi_i^{(\ell_i)}$ and rapidity fraction $\xi_i^{(\ell_i)}$

$$dP(\{R'_{\ell_i}\}, k_i) \sim \frac{R'_{\ell_i}}{R'} \frac{d\zeta_i}{\zeta_i} \left( \frac{\zeta_i}{\zeta_{i-1}} \right)^{R'} \Theta\left(\zeta_{i-1} - \zeta_i\right) \Theta\left(\zeta_i - \epsilon\right) \frac{d\phi_i^{(\ell_i)}}{2\pi} d\xi_i$$

# Event-shape vs jet rates

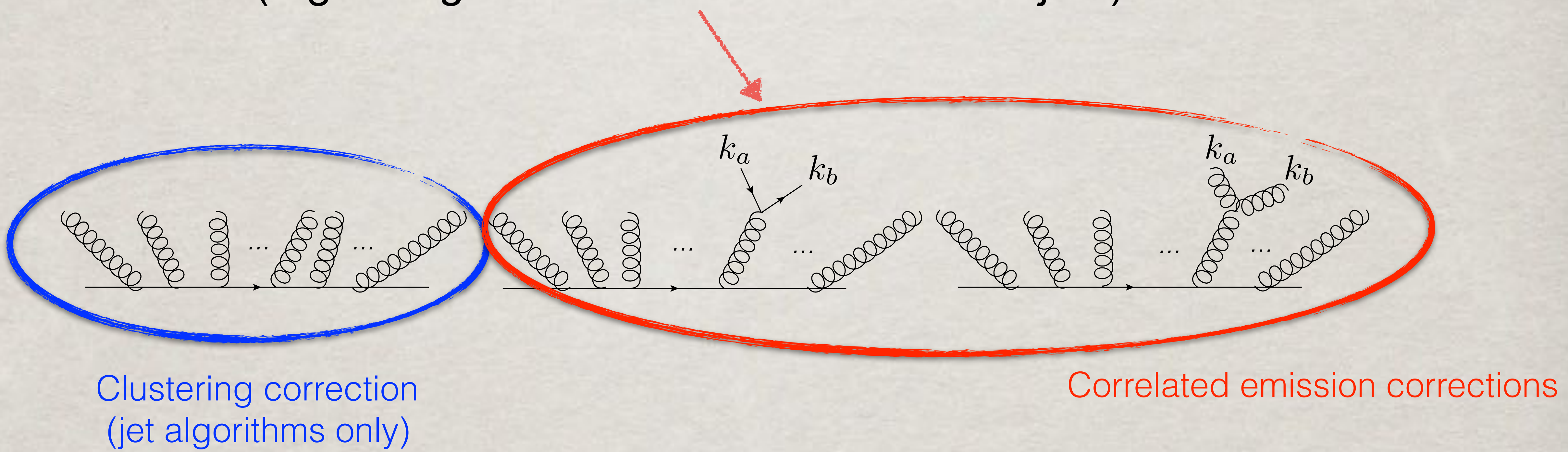
- Event shape variables have the property that, for fixed $V(\{\tilde{p}\}, k_i)$, their value does not depend on emissions' rapidities
- This implies that some NNLL corrections are zero for event shapes
- For instance, two soft and collinear gluons close in rapidity give a NNLL clustering correction only to two-jet rates




$$\delta\mathcal{F}_{\rm clust} = \left\langle \Theta\left(1 - \lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde{p}\}, k_a, k_b, \{k_i\})}{v}\right) - \Theta\left(1 - \lim_{v\to 0}\frac{V_{\rm sc}^{\rm NLL}(\{\tilde{p}\}, k_a, k_b, \{k_i\})}{v}\right)\right\rangle$$

# An example of NNLL correction

- Two soft-collinear partons close in rapidity
- Correlated emission corrections taking into account non-inclusiveness of the observable (e.g. two gluons clustered into different jets)




$$\delta\mathcal{F}_{\rm correl} = \left\langle \Theta\left(1 - \lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde{p}\}, k_a, k_b, \{k_i\})}{v}\right) - \Theta\left(1 - \lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde{p}\}, k_a + k_b, \{k_i\})}{v}\right)\right\rangle$$