Combined threshold and transverse momentum resummation for pt distributions at NNLL

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based on Muselli, Forte, Ridolfi, 2017
arXiv:1701.01464

LoopFestXVI, 31 May 2017

Outline

- Resummations of p_{T} distributions
- Combined Resummation
- Preliminary Resummed Results for the Higgs pt distributions
- Conclusion

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Transverse Momentum Distributions: Kinematics

- Inclusive Kinematics: $Q^2 = M^2$, $\hat{\tau} = \frac{M^2}{\hat{s}}$, $\hat{s} = x_1 x_2 S$, $\tau = \frac{M^2}{S}$

$$\sigma\left(\tau, M^2\right) = \sum_{i,j} \int_{\tau}^{1} dx_1 \int_{\frac{\tau}{x_1}}^{1} dx_2\, f_i\left(x_1, \mu_{\mathrm{F}}^2\right) f_j\left(x_2, \mu_{\mathrm{F}}^2\right) \hat{\sigma}_{ij}\left(\hat{\tau}, \alpha_s\left(\mu_{\mathrm{R}}^2\right), \mu_{\mathrm{F}}^2\right) \quad (1)$$

- Differential Kinematics: $Q^2 = \left(\sqrt{M^2 + p_{\mathrm{T}}^2} + p_{\mathrm{T}}\right)^2$, $\tau' = \frac{Q^2}{S}$, $\xi_p = \frac{p_{\mathrm{T}}^2}{M^2}$

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$$f_i\left(x_1, \mu_{\mathrm{F}}^2\right) f_j\left(x_2, \mu_{\mathrm{F}}^2\right) \frac{d\bar{\sigma}}{d\xi_p}\left(\hat{\tau}, \xi_p, \alpha_s\left(\mu_{\mathrm{R}}^2\right), \mu_{\mathrm{F}}^2\right) \qquad (2)$$

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Attention!

$\tau \neq \tau'$

Transverse Momentum Distributions: Kinematics

- Mellin Space
  - Inclusive Kinematics: Mellin with respect to τ

$$\sigma\left(N, M^2\right) = \sum_{i,j} \mathcal{L}_{ij}\left(N+1, \mu_{\mathrm{F}}^2\right) \sigma_{ij}\left(N, \alpha_s\left(\mu_{\mathrm{R}}^2\right), \mu_{\mathrm{F}}^2\right)$$

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- Integration of p_{T}
  - Momentum space
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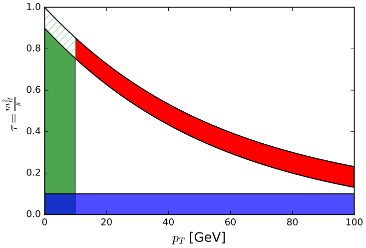
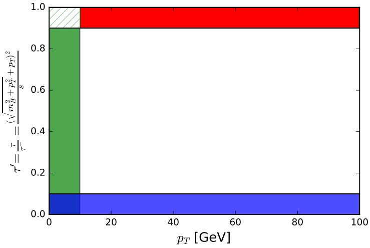
Which regions do we want to resum?

Inclusive



Differential



- High Energy Limit: $\tau \sim \tau' \ll 1$
- Threshold Limit: $\tau \sim \tau_{\max} = \frac{\sqrt{M^2+p_{\mathrm{T}}^2}-p_{\mathrm{T}}}{M^2}, \tau' \sim 1$
- Small-p_{T}: $p_{\mathrm{T}} \lesssim m, \xi_p \ll 0$
- High Energy and Small-p_{T}
- Threshold and Small-p_{T}

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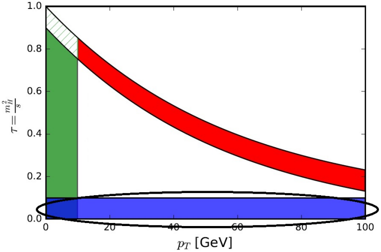

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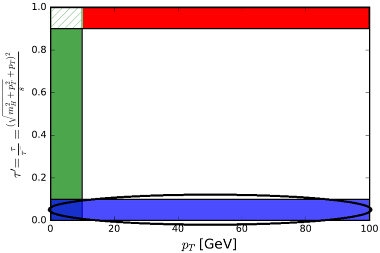

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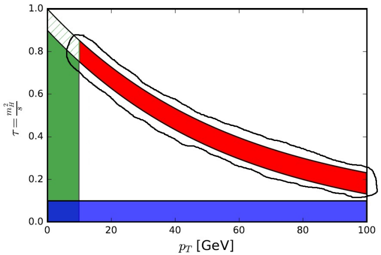

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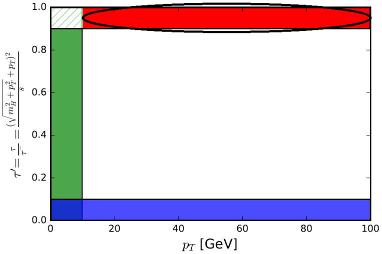

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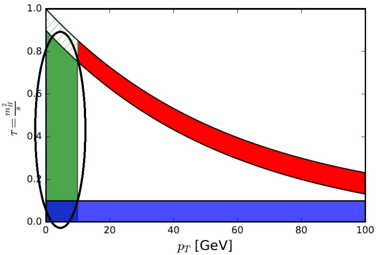

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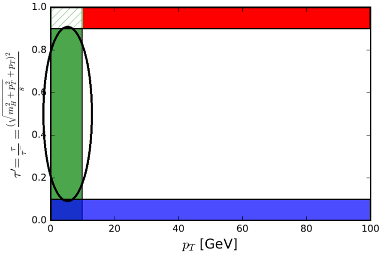

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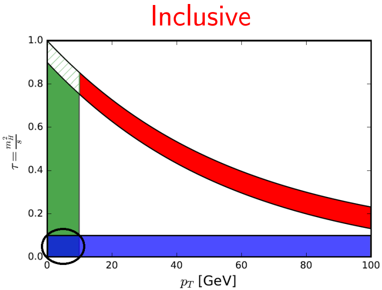

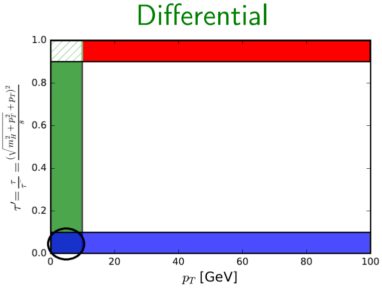

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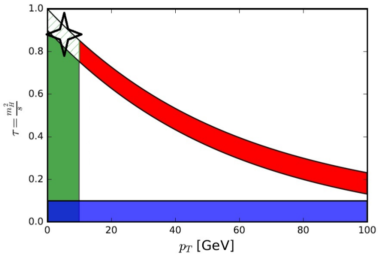

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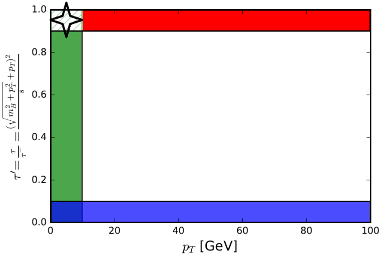

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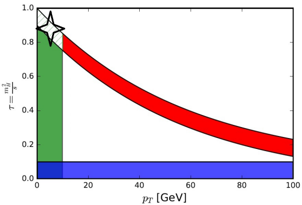

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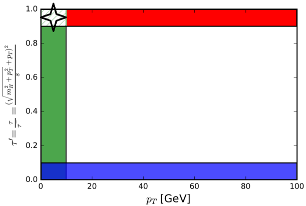

- Threshold and Small-p_{T}

$$\xi_p \sim (1 - \hat{\tau})^2$$

Standard p_{T} resummation (CSS) requires

$$\xi_p \ll (1 - \hat{\tau})^2$$

Objective

We want to construct a resummation valid at threshold for all the p_{T}. To reach this objective we combine in a single formula

1. The threshold resummation performed at fixed p_{T}.
2. The resummation of small-p_{T} contribution.

Doing this properly, it must happen that

3. Under integration over p_{T}, our result must reproduce all threshold contributions in the total cross section

We have to slightly change small-p_{T} resummation definition...

... in order to deal with the region when $\xi_p \sim (1-\hat{\tau})^2$

$\Rightarrow$ Consistent small-p_{T} Resummation (Joint)

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Previous Results in Literature

- Small-p_{T} Resummation known in *b*-space formalism up to NNLL

  (Collins, Soper, Sterman; Bozzi, Catani, De Florian, Grazzini, '05)

  and now in direct momentum space up to N^{3}LL

  (Bizon, Monni, Re, Rottoli, Torrielli, '17)

- Threshold Resummation for fixed-p_{T} up to NLL

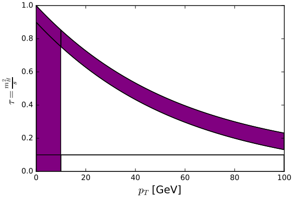
  (De Florian, Kulesza, Vogelsang, '05)

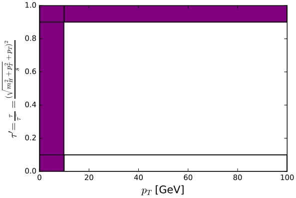
- Joint Resummation at NLL (Kulesza, Sterman, Vogelsang, '04)
- SCET factorization analysis in the collinear and soft limit

  (Lustermans, Waalewijn, Zeune, '16)

- First phenomenological study of a possible extension of Joint Resummation at NNLL for Drell-Yan (Marzani, Theeuwes, '16)

Our Solution





Combined Resummation

$$\frac{d\sigma_{ij}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(\mu_{\rm R}^2\right),\mu_{\rm F}^2\right) = \left(1 - T\left(N,\xi_p\right)\right)\frac{d\hat{\sigma}_{ij}^{\rm tr'}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(\mu_{\rm R}^2\right),\mu_{\rm F}^2\right) \quad (3)$$

$$+ T\left(N,\xi_p\right)\frac{d\hat{\sigma}_{ij}^{\rm fixed}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(\mu_{\rm R}^2\right),\mu_{\rm F}^2\right) \quad (4)$$

$$T\left(N,\xi_p\right) = \mathcal{O}\left(\xi_p\right) \qquad \xi_p \to 0 \text{ at fixed } N, \quad (5)$$

$$T\left(N,\xi_p\right) = \mathcal{O}\left(1\right) \qquad N \to \infty \text{ at fixed } \xi_p, \quad (6)$$

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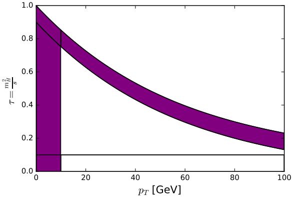
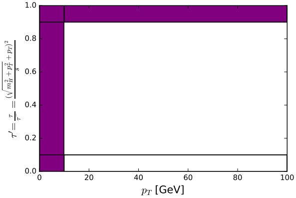




Combined Resummation

$$\frac{d\sigma_{ij}}{d\xi_p}\left(N, \xi_p, \alpha_s\left(\mu_{\mathrm{R}}^2\right), \mu_{\mathrm{F}}^2\right) = \left(1 - T\left(N, \xi_p\right)\right) \frac{d\hat{\sigma}_{ij}^{\mathrm{tr}'}}{d\xi_p}\left(N, \xi_p, \alpha_s\left(\mu_{\mathrm{R}}^2\right), \mu_{\mathrm{F}}^2\right) \quad (3)$$

$$+ T\left(N, \xi_p\right) \frac{d\hat{\sigma}_{ij}^{\mathrm{fixed}}}{d\xi_p}\left(N, \xi_p, \alpha_s\left(\mu_{\mathrm{R}}^2\right), \mu_{\mathrm{F}}^2\right) \quad (4)$$

$$T\left(N, \xi_p\right) = \mathcal{O}\left(\xi_p\right) \qquad \xi_p \to 0 \text{ at fixed } N, \quad (5)$$

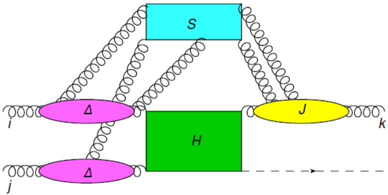
$$T\left(N, \xi_p\right) = \mathcal{O}\left(1\right) \qquad N \to \infty \text{ at fixed } \xi_p, \quad (6)$$

Threshold Resummation at fixed p_T: $\frac{d\hat{\sigma}_{ij}^{\text{fixed}}}{d\xi_p}$

(De Florian, Kulesza, Vogelsang, '05))

$$\frac{d\hat{\sigma}_{ij}^{\text{fixed}}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(Q^2\right),Q^2\right)=\sigma_0\,C_{0,ij}\left(N,\xi_p\right)g_{0,ij}\left(\xi_p,\alpha_s\right)\exp\left[G\left(N,\alpha_s\right)\right]\exp\left[S\left(N,\xi_p,\alpha_s\right)\right]$$

$$G\left(N,\alpha_s\right)=\Delta_i\left(N,\alpha_s\right)+\Delta_j\left(N,\alpha_s\right)+J_k\left(N,\alpha_s\right)$$



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$$\Delta_i\left(N,\alpha_s\right)=\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\int_{Q^2}^{Q^2(1-z)^2}\frac{dq^2}{q^2}A_i^{\text{th}}\left(\alpha_s\left(q^2\right)\right) \tag{7}$$

$$J_k\left(N,\alpha_s\right)=\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\int_{Q^2(1-z)^2}^{Q^2(1-z)}\frac{dq^2}{q^2}A_k^{\text{th}}\left(\alpha_s\left(q^2\right)\right)+B_k^{\text{th}}\left(\alpha_s\left(Q^2\left(1-z\right)\right)\right) \tag{8}$$

$$S\left(N,\xi_p\right)=-\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}A_k^{\text{th}}\left(\alpha_s\left(Q^2\left(1-z\right)^2\right)\right)\ln\frac{\left(\sqrt{1+\xi_p}+\sqrt{\xi_p}\right)^2}{\xi_p} \tag{9}$$

with A^{th} the cusp anomalous dimension.

Threshold Resummation at fixed $p_{\rm T}$: $\frac{d\hat{\sigma}_{ij}^{\rm fixed}}{d\xi_p}$

$$\frac{d\hat{\sigma}_{ij}^{\rm fixed}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(Q^2\right),Q^2\right)=\sigma_0\,C_{0,ij}\left(N,\xi_p\right)g_{0,ij}\left(\xi_p,\alpha_s\right)\exp\left[G\left(N,\alpha_s\right)\right]\exp\left[S\left(N,\xi_p,\alpha_s\right)\right]$$

Problem!

At small-$p_{\rm T}$:

$$\frac{d\hat{\sigma}_{ij}^{\rm fixed}}{d\xi_p}\sim\alpha_s^n\frac{\ln^n\xi_p}{\xi_p}\ln^{n-1}N \qquad (7)$$

while fixed order calculations and small-$p_{\rm T}$ resummation predict

$$\frac{d\hat{\sigma}_{ij}}{d\xi_p}\sim\alpha_s^n\frac{\ln^{n-1}\xi_p}{\xi_p}\ln N \qquad (8)$$

Soft behaviour completely wrong at small-$p_{\rm T}$ since new soft configurations arise, previously suppressed by the finite value of $p_{\rm T}$.

Threshold Resummation at fixed p_{T}: $\frac{d\hat{\sigma}_{ij}^{\mathrm{fixed}}}{d\xi_p}$

Problem!

Soft behaviour completely wrong at small-p_{T} since new soft configurations arise, previously suppressed by the finite value of p_{T}.

Another Problem!

Fixed-order calculations and threshold Resummation at fixed p_{T} at large N:

$$\frac{d\hat{\sigma}_{ij}^{\mathrm{fixed}}}{d\xi_p} \sim \alpha_s^n \frac{1}{\sqrt{N}} \ln^{2n-1} N \tag{7}$$

while CSS small-p_{T} resummation at large N scales as

$$\frac{d\hat{\sigma}_{ij}^{\mathrm{CSS}}}{d\xi_p} \sim \alpha_s^n \ln N \tag{8}$$

At large-p_{T}, CSS resummation shows a not-physical logarithmic behaviour at large N

Small-p_{T} limit: phase space analysis

At small-p_{T}, phase-space for n emissions factorizes in Mellin-Fourier space:

$$d\Phi_{n+1}\left(p_1, p_2; p, k_1, \ldots, k_n\right) = M^{2n} \frac{8\pi^3}{\left[4(2\pi)^2\right]^{n+1}} d\xi_p \int db^2 \, J_0\left(b p_{\mathrm{T}}\right)$$
$$J_0\left(b k_{\mathrm{T}_1}\right) \frac{d\xi_1 dz_1}{\sqrt{\left(1-z_1\right)^2 - 4\xi_1}} \ldots J_0\left(b k_{\mathrm{T}_n}\right) \frac{d\xi_n dz_n}{\sqrt{\left(1-z_n\right)^2 - 4\xi_n}}$$
$$\delta\left(\hat{\tau} - z_1 \ldots z_n\right) + \mathcal{O}\left(\frac{1}{b}\right). \tag{9}$$

Now standard p_{T} resummation (CSS) considers $\xi_i \ll \left(1-z_i\right)^2$ and rewrites the square-root as

$$\frac{1}{\sqrt{\left(1-z\right)^2 - 4\xi}} \to \left(\frac{1}{1-z}\right)_+ - \frac{1}{2}\delta\left(1-z\right)\ln\xi \tag{10}$$

By taking this limit:

- We destroy the large-N behaviour at fixed-$p_{\rm T}$

$$\mathcal{M}\left[\frac{1}{\sqrt{(1-z)^2-4\xi}}\right] \sim \frac{1}{\sqrt{N}} \qquad \mathcal{M}\left[\left(\frac{1}{1-z}\right)_+\right] \sim \ln N \qquad (11)$$

- This approximation ruins when $(1-z)^2 \sim \xi$, which in Mellin-Fourier space means

$$\mathcal{F}\mathcal{M}\left[\frac{1}{\sqrt{(1-z)^2-4\xi}}\right] = \frac{2}{b^2}\left(1-\frac{4N^2}{b^2}+\frac{16N^4}{b^4}+\dots\right), \qquad (12)$$

we are missing terms suppressed by powers of b but enhanced with the same powers of N.

- Integral over ξ can not be right since

$$\int_0^{\frac{(1-z)^2}{4}} d\xi \, \frac{1}{\sqrt{(1-z)^2-4\xi}} = \frac{(1-z)}{4}\left(1+\frac{1}{4}+\frac{1}{8}+\dots\right) \qquad (13)$$

after integration all the terms in the expansion are of the same order.

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Consistent Small-$p_{\rm T}$ Resummation: $\frac{d\hat{\sigma}_{ij}^{\rm joint}}{d\xi_p}$

CSS Small-$p_{\rm T}$ Resummation: $b \to \infty$ at fixed N

In this way, it misses terms at $N \to \infty$ which are of the same order of b.

Our proposal: Consistent Small-$p_{\rm T}$ Resummation: $b \to \infty$ at fixed $\frac{N}{b}$

- Now our resummation is accurate at small-$p_{\rm T}$ for all the energy, even at threshold.
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Consistent Small-p_{T} Resummation: $\frac{d\hat{\sigma}_{ij}^{\mathrm{tr}'}}{d\xi_p}$

We end up with a complicated expression...

$$\frac{d\hat{\sigma}_{ij}^{\mathrm{tr}'}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(M^2\right),M^2\right)=\sigma_0\int_0^\infty db\,\frac{b}{2}J_0\left(bM\sqrt{\xi_p}\right)\left(\sqrt{1+\xi_p}-\sqrt{\xi_p}\right)^{-2N}$$
$$\mathcal{H}_{ij}\left(N,\alpha_s\left(M^2\right)\right)\exp\left[\int_0^\infty d\xi\left(\sqrt{1+\xi}-\sqrt{\xi}\right)^{2N}J_0\left(b\sqrt{\xi}\right)\left(\frac{\mathcal{B}\left(N,\alpha_s\left(M^2\xi\right)\right)}{\xi}\right)_+^{p_{\mathrm{T}}}+\mathcal{O}\left(\frac{1}{b}\right)\right]$$
$$\exp\left[\int_0^\infty d\xi\left(\sqrt{1+\xi}-\sqrt{\xi}\right)^{2N}J_0\left(b\sqrt{\xi}\right)\int_0^1 dz\,z^{N-1}\right.$$
$$\left(\left(\frac{2A^{p_{\mathrm{T}}}\left(\alpha_s\left(M^2\xi\right)\right)}{\xi}\right)_+^{p_{\mathrm{T}}}\left(\frac{1}{\sqrt{(1-z)\left(1-\left(\sqrt{1+\xi}-\sqrt{\xi}\right)^4 z\right)}}\right)_+^z\right.$$
$$+\,\delta\left(1-z\right)\frac{1}{2\left(\sqrt{1+\xi}-\sqrt{\xi}\right)^2}$$
$$\left.\left.\left(2A^{p_{\mathrm{T}}}\left(\alpha_s\left(M^2\xi\right)\right)\frac{\ln\left(1+\xi\right)}{\xi}-\left(\frac{2A^{p_{\mathrm{T}}}\left(\alpha_s\left(M^2\xi\right)\right)\ln\xi}{\xi}\right)_+^{p_{\mathrm{T}}}\right)\right)+\mathcal{O}\left(\frac{1}{b}\right)\right]. \tag{14}$$

Consistent Small-$p_{\rm T}$ Resummation: $\frac{d\hat{\sigma}_{ij}^{\rm tr'}}{d\xi_p}$

We end up with a complicated expression...
but by solving the integrals at the exponent and by taking the limit we are interested in, it can be simplified in a rather compact formula:

$$\frac{d\sigma^{\rm tr'}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(M^2\right),M^2\right)=\sigma_0 H\left(\alpha_s\left(M^2\right)\right)\int_0^\infty db\,\frac{b}{2}J_0\left(b\sqrt{\xi_p}\right)\left(\sqrt{1+\xi_p}-\sqrt{\xi_p}\right)^{-2N}$$
$$\exp\left[S\left(\chi,N,\alpha_s\left(M^2\right)\right)\right]\sum_{ij}C_i\left(N,\alpha_s\left(\frac{1}{\chi}\right)\right)C_j\left(N,\alpha_s\left(\frac{1}{\chi}\right)\right)f_i\left(N,\frac{1}{\chi}\right)f_j\left(N,\frac{1}{\chi}\right) \quad (14)$$

$$S\left(\chi,\alpha_s\right)=\frac{1}{\alpha_s}g_1\left(\chi\right)+g_2\left(\chi,N\right)+\alpha_s\,g_3\left(\chi,N\right) \quad (15)$$

$$\chi=\bar{N}^2+\frac{b^2}{b_0^2}\qquad \bar{N}=Ne^{\gamma_E}\qquad b_0=2e^{-\gamma_E} \quad (16)$$

$$g_1(\chi) = \frac{A_c^{p_{\mathrm{T}},(1)}}{\beta_0^2}\left(\lambda_\chi + \ln\left(1-\lambda_\chi\right)\right) \tag{17}$$

$$\begin{aligned} g_2(\chi) = {} & \frac{A_c^{p_{\mathrm{T}},(1)}\beta_1}{\beta_0^3}\left[\frac{\lambda_\chi + \ln\left(1-\lambda_\chi\right)}{1-\lambda_\chi} + \frac{1}{2}\ln\left(1-\lambda_\chi\right)^2\right] \\ & - \frac{A_c^{p_{\mathrm{T}},(2)}}{\beta_0^2}\frac{\lambda_\chi + \left(1-\lambda_\chi\right)\ln\left(1-\lambda_\chi\right)}{1-\lambda_\chi} + \frac{B_c^{p_{\mathrm{T}},(1)}}{\beta_0}\ln\left(1-\lambda_\chi\right) \end{aligned} \tag{18}$$

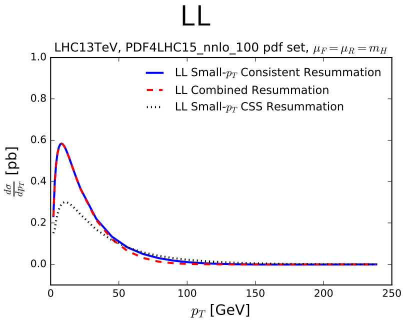
$$\begin{aligned} g_3(\chi) = {} & \frac{A_c^{p_{\mathrm{T}},(1)}\beta_1}{2\beta_0^4}\left[\frac{\lambda_\chi + \ln\left(1-\lambda_\chi\right)}{\left(1-\lambda_\chi\right)^2}\left(\lambda_\chi + \left(1-2\lambda_\chi\right)\ln\left(1-\lambda_\chi\right)\right)\right] + A_c^{p_{\mathrm{T}},(1)}\mathrm{Li}_2\left(\frac{\mathrm{N}^2}{\chi}\right) \\ & + \frac{A_c^{p_{\mathrm{T}},(1)}\beta_2}{\beta_0^3}\left[\frac{\left(2-3\lambda_\chi\right)\lambda_\chi}{2\left(1-\lambda_\chi\right)^2} + \frac{\left(1-2\lambda_\chi\right)\ln\left(1-\lambda_\chi\right)}{\left(1-\lambda_\chi\right)^2}\right] + \frac{B_c^{p_{\mathrm{T}},(1)}\beta_1}{\beta_0}\frac{\lambda_\chi + \ln\left(1-\lambda_\chi\right)}{1-\lambda_\chi} \\ & - \frac{A_c^{p_{\mathrm{T}},(3)}}{2\beta_0^2}\frac{\lambda_\chi^2}{\left(1-\lambda_\chi\right)^2} - \frac{B_c^{p_{\mathrm{T}},(2)}}{\beta_0}\frac{\lambda_\chi}{1-\lambda_\chi} + A_c^{p_{\mathrm{T}},(1)}\frac{\lambda_N}{1-\lambda_N}\mathrm{Li}_2\left(\frac{\mathrm{N}^2}{\chi}\right) \end{aligned} \tag{19}$$

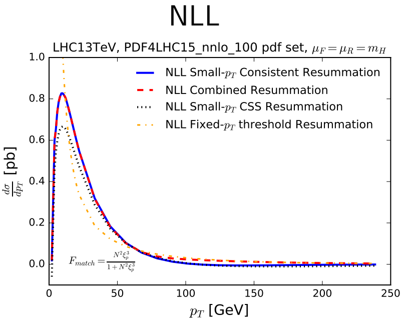
$$\lambda_\chi = \alpha_s\beta_0 \ln\chi \qquad \lambda_N = \alpha_s\beta_0\ln\bar{N}^2 \tag{20}$$

- $\chi = \frac{b^2}{b_0^2}$ CSS Small-p_{T} resummation
- $\chi = \bar{N}^2$ Threshold Resummation total cross section

Phenomenological Results: Higgs Boson Production

Resummed Component... no matching with fixed order





PRELIMINARY (Forte, Muselli, Ridolfi in preparation)

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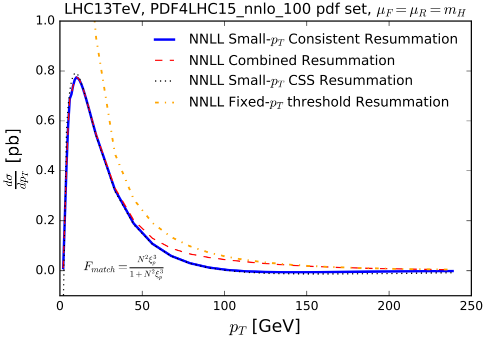
NNLL



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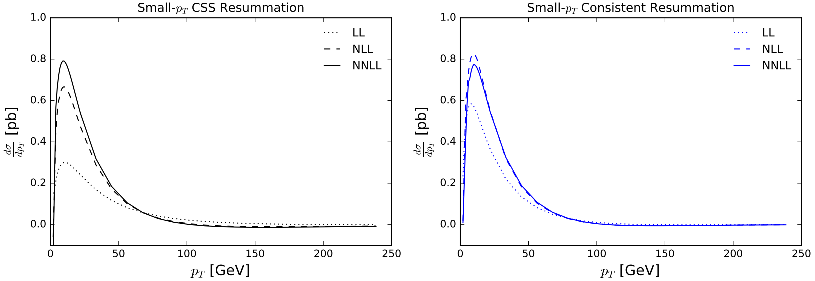
Resummed Component... no matching with fixed order

CSS | Consistent



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- Difference between Consistent and CSS approaches becomes smaller and smaller increasing the logarithmic order at small-p_T.
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- Any corrections not at small-p_T of a joint expression is completely unreliable, since it has been constructed on a factorized expression at small-p_T.
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  No need to change Logs of b to turn off resummed component at large p_T as necessary in CSS approach

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Conclusion

- We construct a combined cross section to resum properly soft and collinear regions of a generic p_{T} distribution.
- This combined expression is formed by two ingredients: a modified version of the small-p_{T} resummation and a threshold resummation valid at fixed value of p_{T}.
- Our modified version of the small-p_{T} resummation resums all leading contributions in Mellin-Fourier space at large b with fixed ratio $\frac{N}{b}$, rather than fixed N.
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- Our modified version of the small-p_{T} resummation resums all leading contributions in Mellin-Fourier space at large b with fixed ratio $\frac{N}{b}$, rather than fixed N.
- With this modified definition the large-N limit of the integral over p_{T} of our combined expression naturally coincides with the threshold resummation of inclusive cross section.
  Thus with this construction we obtained the so-called Joint resummation for free.
- In the case of Higgs boson production, results at NNLL show small effect at small-p_{T} but better convergence.

Outlook

- We are going to conclude the phenomenological analysis on Higgs and Drell-Yan p_{T} distributions.
- We want to join this combined expression with the known high-energy resummation for transverse momentum distribution to construct a complete resummed formula for the partonic distribution.
- Thanks to these tools, in addition to construct resummed and matched results, we are going to be able to properly approximate unknown higher order coefficients of the transverse momentum distribution as already done for inclusive cross section

(akin Ball, Bonvini, Forte, Marzani, Ridolfi, '13; Muselli, Bonvini, Forte, Marzani, Ridolfi, '15)

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That's all Folks!

Backup Slides

Prescriptions in Mellin-Fourier space

On N complex plane $\ln \chi$ owns a very particular branch cut, on the imaginary axis, in the range

$$\Big(- i\infty, -i\frac{b}{2}\Big] \cup \Big[i\frac{b}{2}, i\infty \Big) \tag{22}$$

Its inverse Mellin-Fourier transform exists since any $N_0 > 0$ is a convergence abscissa.

But it is very numerical unstable since we can not bend the path to increase convergence without hitting the brunch cut.

However, even if we could be able to compute the inverse order by order, the inverse of the whole series does not exist due to Landau pole problem and we should always need a prescription.

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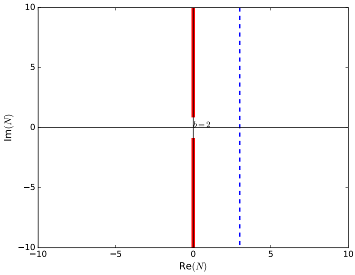
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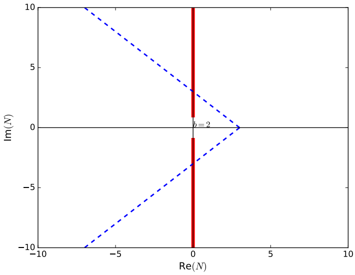
Im(N)
Re(N)
b = 2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Minimal Presciption vs Borel Prescription

The most common prescription in resummation performed in conjugate space is the Minimal Presciption

(Catani, Mangano, Nason, Trentadue, '96 ; Laenen, Sterman, Vogelsang, '00)

Owing to MP...

We need to move integration paths to leave all the singularity on the left, except the Landau branch cut which must remain on the right.

However...

...such an integration path does not exist for our resummation.
This means that MP in this form can not be apply directly to Consistent Resummation

Minimal Presciption vs Borel Prescription

So we can use a different prescription. For example, the Borel Prescription

(Abbate, Forte, Ridolfi, '07; Bonvini, Forte, Ridolfi, '08)

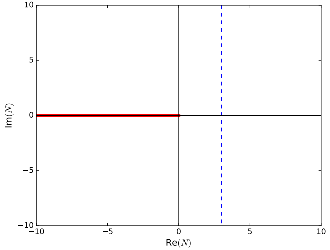
Owing to BP...

We need to integrate the series term by term and to sum the series à la Borel. The Borel inversion integral is then computed with a proper cutoff.

In our case...

...this prescription can be applied and moreover, by computing the inverse Fourier transform analytically term by term, we are able to remove the cut on the imaginary axis, improving in this way also the numerical efficiency of the Mellin inverse

$$\int_0^\infty db\, \frac{b}{2} J_0\left(b\sqrt{\xi_p}\right) \ln^k\left(\bar{N}^2 + \frac{b^2}{b_0^2}\right) = \frac{\partial^k}{\partial \epsilon^k}\left[\frac{2\xi_p^{\frac{1}{2}+\epsilon 2} N^{1+\epsilon} \exp\left(2\gamma_E\right) K_{1+\epsilon}\left(2\sqrt{\xi_p} N\right)}{\Gamma\left(-\epsilon\right)}\right]_{\epsilon\to 0} \quad (23)$$



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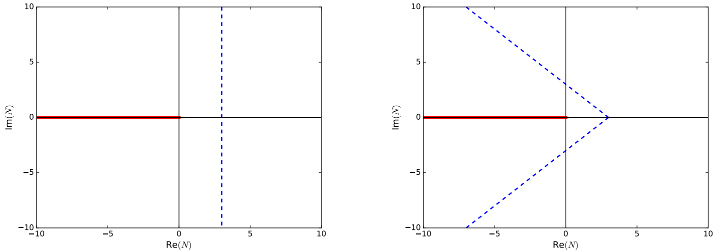

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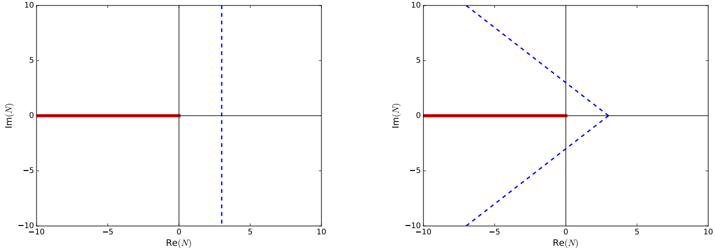

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Commutating limits: the high energy case

- High energy limit for transverse momentum distributions: $\tau' \to 0$ at fixed p_{T}

$$\frac{d\sigma_{gg}^{\mathrm{h.e}}}{d\xi_p}(N,b) = \sigma_0 R\left(\gamma\left(\frac{\alpha_s}{N}\right)\right)^2 \frac{\Gamma\left(1+\gamma\left(\frac{\alpha_s}{N}\right)\right)^2}{\Gamma\left(2-\gamma\left(\frac{\alpha_s}{N}\right)\right)^2}\left(1-2\gamma\left(\frac{\alpha_s}{N}\right)+2\gamma\left(\frac{\alpha_s}{N}\right)^2\right) e^{-\gamma\left(\frac{\alpha_s}{N}\right)\ln\frac{b^2M^2}{4}} \quad (23)$$

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(Forte et al, arXiv:1511.05561)

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TR-HE joint resummation $\Rightarrow$ (Marzani, arXiv:1511.06039v3)

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$$\frac{d\sigma_{ij}^{\mathrm{th}}}{d\xi_p}(N,\xi_p) = \sigma_0 C_0(N,\xi_p) \exp[G(N)] \exp[S(N,\xi_p)] \approx \frac{1}{\sqrt{N}} \ln^k N \qquad (26)$$

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- After integration over p_{T}, we found very strict relations between Treshold resummation for inclusive cross section and transverse momentum resummation anomalous dimensions. In particular:

$$A_g^{\mathrm{p_T},(1)} = A_g^{\mathrm{th},(1)} \tag{29a}$$

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$$A_g^{\mathrm{p_T},(3)} = A_g^{\mathrm{th},(3)} - \beta_0 D_g^{\mathrm{p_T},(2)} \tag{29c}$$

$$D_g^{\mathrm{p_T},(2)} = D_g^{\mathrm{th},(2)} - 4A_g^{\mathrm{p_T},(1)}\zeta_2\beta_0 \tag{29d}$$

$$H_{gg}^{\mathrm{p_T},(1)} = H_{gg}^{\mathrm{th},(1)} - A_g^{\mathrm{p_T},(1)}\zeta_2 \tag{29e}$$

- All these relations are fulfilled by known coefficients.
- Eq. (29c) is the known collinear anomaly relation.
- This is (I think) the first independent derivation of the collinear anomaly in standard QCD.

Collinear Anomaly

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