Parton Distribution Functions, α_s and Heavy-Quark Masses for LHC Run II

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Based on work done in collaboration with:

- *Parton Distribution Functions, α_s and Heavy-Quark Masses for LHC Run II*
  S. Alekhin, J. Blümlein, S. M. and R. Plačakytė `arXiv:1701.05838`

- *A Critical Appraisal and Evaluation of Modern PDFs*
  A. Accardi, S. Alekhin, J. Blümlein, M.V. Garzelli, K. Lipka, W. Melnitchouk, S. M., J.F. Owens, R. Plačakytė, E. Reya, N. Sato, A. Vogt and O. Zenaiev `arXiv:1603.08906`

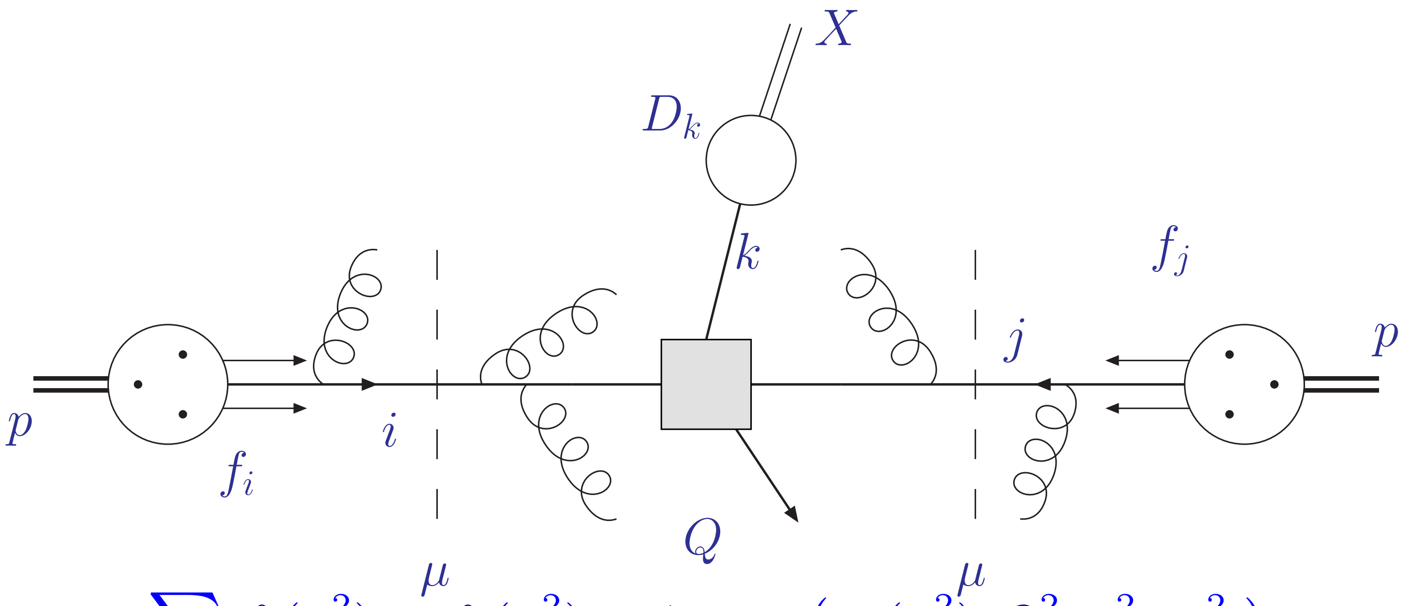
- *Iso-spin asymmetry of quark distributions and implications for single top-quark production at the LHC*
  S. Alekhin, J. Blümlein, S. M. and R. Plačakytė `arXiv:1508.07923`

- *Determination of Strange Sea Quark Distributions from Fixed-target and Collider Data*
  S. Alekhin, J. Blümlein, L. Caminada, K. Lipka, K. Lohwasser, S. M., R. Petti, and R. Plačakytė `arXiv:1404.6469`

- Many more papers of ABM and friends . . .
  `2008 - ...`

QCD factorization

QCD factorization



$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Factorization at scale μ
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij\to k}$ for parton types i, j and hadronic final state X
- Parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Parton content of the proton

The ABMP16 PDF fit

Data sets considered in ABMP16 analysis

- Analysis of world data for deep-inelastic scattering and fixed-target data for Drell-Yan process
  - inclusive DIS data HERA, BCDMS, NMC, SLAC $(NDP = 2155)$
  - DIS heavy-quark production data
    HERA, CCFR, Chorus, NOMAD, NuTeV $(NDP = 313)$
  - Drell-Yan data (fixed target) E-605, E-866 $(NDP = 158)$
  - Tevatron & LHC data for $W^{\pm}$- and Z-boson production
    D0, ATLAS, CMS, LHCb $(NDP = 172)$
  - Top-quark production D0, ATLAS, CMS, LHCb $(NDP = 36)$

Iterative cycle of PDF fits

i) check of compatibility of new data set with available world data

ii) study of potential constraints due to addition of new data set to fit

iii) perform high precision measurement of parton distributions, strong coupling $\alpha_s(M_Z)$ and heavy quark masses

Theory considerations in PDF fits

Theory considerations in ABMP16

- Strictly NNLO QCD for determination of PDFs and α_s
- Consistent scheme for treatment of heavy quarks
  - $\overline{\mathrm{MS}}$-scheme for quark masses and α_s
  - fixed-flavor number scheme for $n_f = 3, 4, 5$
- Consistent theory description for consistent data sets
  - low scale DIS data with account of higher twist
- Full account of error correlations

Interplay with perturbation theory

- Accuracy of determination driven by precision of theory predictions
- PDF parameters, α_s, m_c, m_b and m_t sensitive to
  - radiative corrections at higher orders
  - chosen scheme (e.g. ($\overline{MS}$ scheme)
  - renormalization and factorization scales μ_R, μ_F
  - $\ldots$

Benchmark measurements

DIS

- Structure functions for neutral and charged current known to $\mathcal{O}(\alpha_s^3)$
  - F_2, F_3, known N^{3}LO, F_L known NNLO S.M, Vermaseren, Vogt ‘04–‘08
- Heavy-quark structure functions
  - asymptotic NNLO terms at large $Q^2 \gg m^2$ Bierenbaum, Blümlein, Klein ‘09; Behring, Bierenbaum, Blümlein, De Freitas, Klein, Wissbrock ‘14
  - approximate NNLO expressions for neutral and charged current Lo Presti, Kawamura, S.M., Vogt ‘12, Blümlein, A. Hasselhuhn, and T. Pfoh ‘14
- Dijet production in DIS at NNLO Currie, Gehrmann, Niehues ‘16

LHC

- $W^\pm$- and Z-boson production at NNLO Hamberg, van Neerven, Matsuura ‘91; Harlander, Kilgore ’02
- Hadro-production of top-quark pairs Czakon, Fiedler, Mitov ‘13
- Single top-quark production (t-channel) Brucherseifer, Caola, Melnikov ‘14
- $Z + 1$jet Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello ‘15; Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan ‘15
- Hadroproduction of jets
  - all partonic channels; leading color only Currie, Glover, Pires ‘16

ABMP16 PDF ansatz

- PDFs parameterization at scale $\mu_0 = 3\mathrm{GeV}$ in scheme with $n_f = 3$
  Alekhin, Blümlein, S.M., Placakyte '17
  - ansatz for valence-/sea-quarks, gluon

$$
\begin{aligned}
xq_v(x,\mu_0^2) &= \frac{2\delta_{qu}+\delta_{qd}}{N_q^v}(1-x)^{b_q} x^{a_q P_{qv}(x)} \\
xq_s(x,\mu_0^2) &= x\bar{q}_s(x,\mu_0^2) = A_{qs}(1-x)^{b_{qs}} x^{a_{qs} P_{qs}(x)} \\
xg(x,\mu_0^2) &= A_g(1-x)^{b_g} x^{a_g\, P_g(x)}
\end{aligned}
$$

  - strange quark is taken in charge-symmetric form
  - function $P_p(x)$

$$
P_p(x) = (1+\gamma_{-1,p}\ln x)\left(1+\gamma_{1,p}x+\gamma_{2,p}x^2+\gamma_{3,p}x^3\right) ,
$$

  - 29 parameters in fit including $\alpha_s^{(n_f=3)}(\mu_0 = 3$ GeV), m_c, m_b and m_t
  - simultaneous fit of higher twist parameters (twist-4)
- Ansatz provides sufficient flexibility; no additional terms required to improve the quality of fit

Quality of fit

Statistical tests

- Goodness-of-fit estimator
  - χ^2 values compared to number of data points (typically a few thousand in global fit)

Covariance matrix

- Positive-definite covariance matrix
  - correlations for fit parameters of ABMP16 PDFs

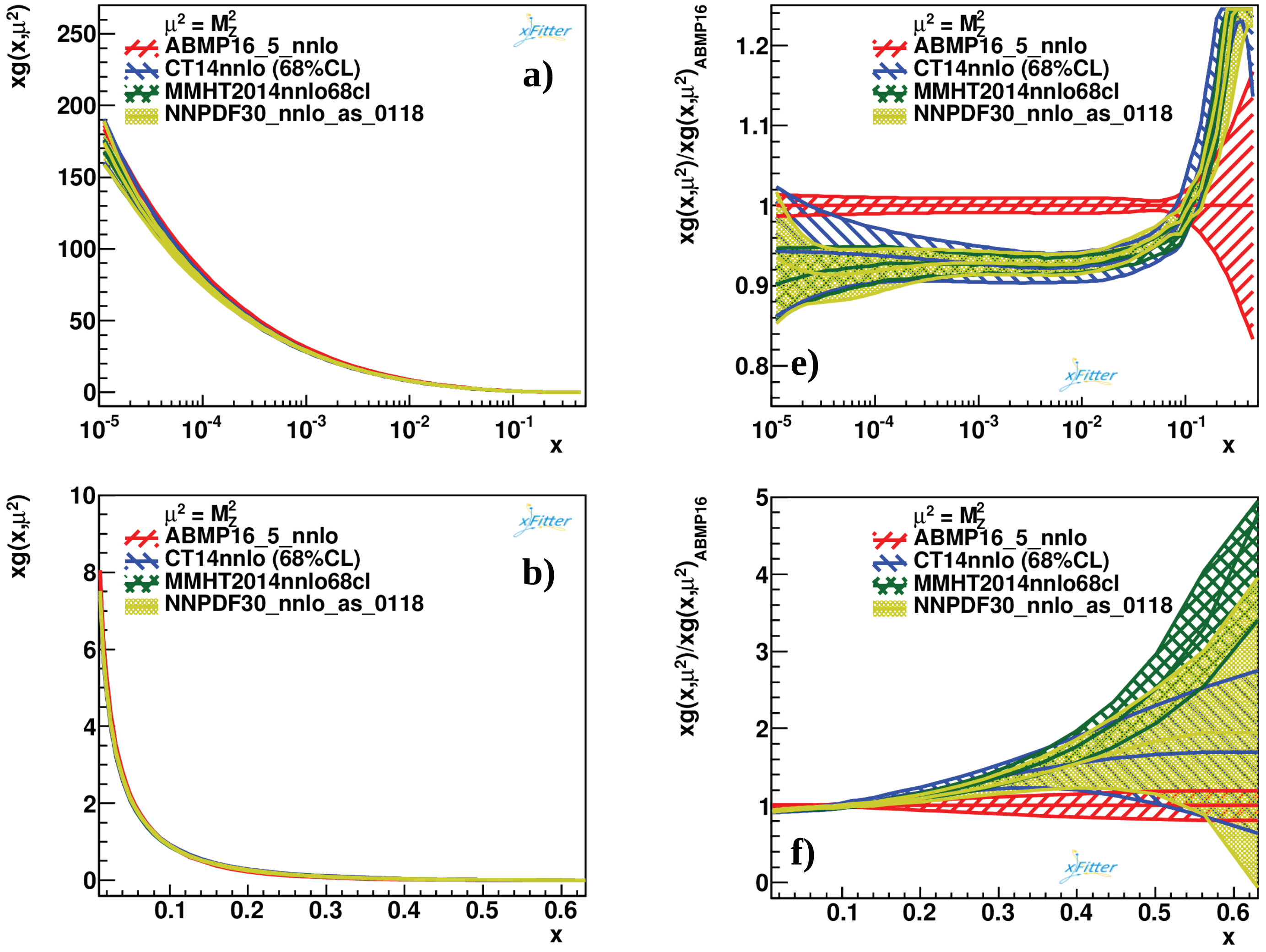
	a_u	b_u	$\gamma_{1,u}$	$\gamma_{2,u}$	$\gamma_{3,u}$	a_d	b_d	$\gamma_{1,d}$	$\gamma_{2,d}$	$\gamma_{3,d}$
a_u	1.0	0.7617	0.9372	- 0.5078	0.4839	0.4069	0.3591	0.4344	- 0.3475	0.0001
b_u	0.7617	1.0	0.6124	- 0.1533	- 0.0346	0.3596	0.2958	0.3748	- 0.2748	0.0001
$\gamma_{1,u}$	0.9372	0.6124	1.0	- 0.7526	0.7154	0.2231	0.2441	0.2812	- 0.2606	0.0001
$\gamma_{2,u}$	- 0.5078	- 0.1533	- 0.7526	1.0	- 0.9409	0.2779	0.2276	0.2266	- 0.1860	0.0
$\gamma_{3,u}$	0.4839	- 0.0346	0.7154	- 0.9409	1.0	- 0.1738	- 0.1829	- 0.1327	0.1488	0.0
a_d	0.4069	0.3596	0.2231	0.2779	- 0.1738	1.0	0.7209	0.9697	- 0.6529	0.0001
b_d	0.3591	0.2958	0.2441	0.2276	- 0.1829	0.7209	1.0	0.7681	- 0.9786	- 0.0001
$\gamma_{1,d}$	0.4344	0.3748	0.2812	0.2266	- 0.1327	0.9697	0.7681	1.0	- 0.7454	0.0002
$\gamma_{2,d}$	- 0.3475	- 0.2748	- 0.2606	- 0.1860	0.1488	- 0.6529	- 0.9786	- 0.7454	1.0	- 0.0002
$\gamma_{3,d}$	0.0001	0.0001	0.0001	0.0	0.0	0.0001	- 0.0001	0.0002	- 0.0002	1.0
a_{us}	- 0.0683	- 0.0081	- 0.2094	0.3881	- 0.3206	0.2266	0.1502	0.2000	- 0.1293	0.0
b_{us}	- 0.3508	- 0.3089	- 0.3462	0.0906	- 0.0537	- 0.1045	- 0.2000	- 0.2241	0.2798	0.0
$\gamma_{-1,us}$	0.2296	0.1387	0.3367	- 0.4043	0.3474	- 0.1171	- 0.1127	- 0.0810	0.0767	0.0
$\gamma_{1,us}$	- 0.4853	- 0.4119	- 0.3844	- 0.0365	0.0064	- 0.4380	- 0.3592	- 0.4957	0.3771	- 0.0001
A_{us}	0.0506	0.0807	- 0.0949	0.3198	- 0.2560	0.2527	0.1648	0.2350	- 0.1509	0.0
a_{ds}	- 0.0759	- 0.0443	- 0.0951	0.0263	- 0.0382	- 0.2565	- 0.2541	- 0.2666	0.2380	0.0
b_{bs}	0.0452	- 0.0197	0.0345	- 0.0589	0.0683	- 0.2084	0.0190	- 0.1841	- 0.0522	0.0
$\gamma_{1,ds}$	- 0.0492	- 0.0809	0.0101	- 0.1791	0.1309	- 0.5576	- 0.2029	- 0.4584	0.0946	0.0
A_{ds}	- 0.1980	- 0.1262	- 0.2349	0.1526	- 0.1428	- 0.1113	- 0.2167	- 0.1739	0.2407	0.0
a_{ss}	- 0.2034	- 0.1285	- 0.2362	0.2328	- 0.2080	0.0960	0.1596	0.0661	- 0.1054	0.0
b_{ss}	- 0.1186	- 0.0480	- 0.1532	0.1549	- 0.1536	0.0486	0.1508	0.0267	- 0.1161	0.0
A_{ss}	- 0.1013	- 0.0411	- 0.1458	0.1802	- 0.1625	0.1216	0.1678	0.0924	- 0.1196	0.0
a_g	0.0046	- 0.0374	0.1109	- 0.1934	0.1653	- 0.0288	- 0.0122	0.0053	0.0059	0.0
b_g	0.2662	0.3141	0.1579	- 0.0050	- 0.0207	0.0973	0.0870	0.0646	- 0.0666	0.0
$\gamma_{1,g}$	0.2008	0.2274	0.0706	0.0876	- 0.0835	0.0919	0.0574	0.0493	- 0.0364	0.0
$\alpha_s^{(n_f=3)}(\mu_0)$	0.1083	- 0.0607	0.0848	- 0.0250	0.0765	0.0763	- 0.0306	0.0725	0.0243	0.0
$m_c(m_c)$	- 0.0006	0.0170	- 0.0104	0.0206	- 0.0201	- 0.0123	- 0.0161	- 0.0114	0.0108	0.0
$m_b(m_b)$	0.0661	0.0554	0.0605	- 0.0367	0.0287	- 0.0116	0.0029	- 0.0074	- 0.0051	0.0
$m_t(m_t)$	- 0.1339	- 0.2170	- 0.0816	0.0081	0.0250	- 0.0616	- 0.0813	- 0.0491	0.0736	0.0

	a_{us}	b_{us}	$\gamma_{-1,us}$	$\gamma_{1,us}$	A_{us}	a_{ds}	b_{bs}	$\gamma_{1,ds}$	A_{ds}	a_{ss}
a_u	- 0.0683	- 0.3508	0.2296	- 0.4853	0.0506	- 0.0759	0.0452	- 0.0492	- 0.1980	- 0.2034
b_u	- 0.0081	- 0.3089	0.1387	- 0.4119	0.0807	- 0.0443	- 0.0197	- 0.0809	- 0.1262	- 0.1285
$\gamma_{1,u}$	- 0.2094	- 0.3462	0.3367	- 0.3844	- 0.0949	- 0.0951	0.0345	0.0101	- 0.2349	- 0.2362
$\gamma_{2,u}$	0.3881	0.0906	- 0.4043	- 0.0365	0.3198	0.0263	- 0.0589	- 0.1791	0.1526	0.2328
$\gamma_{3,u}$	- 0.3206	- 0.0537	0.3474	0.0064	- 0.2560	- 0.0382	0.0683	0.1309	- 0.1428	- 0.2080
a_d	0.2266	- 0.1045	- 0.1171	- 0.4380	0.2527	- 0.2565	- 0.2084	- 0.5576	- 0.1113	0.0960
b_d	0.1502	- 0.2000	- 0.1127	- 0.3592	0.1648	- 0.2541	0.0190	- 0.2029	- 0.2167	0.1596
$\gamma_{1,d}$	0.2000	- 0.2241	- 0.0810	- 0.4957	0.2350	- 0.2666	- 0.1841	- 0.4584	- 0.1739	0.0661
$\gamma_{2,d}$	- 0.1293	0.2798	0.0767	0.3771	- 0.1509	0.2380	- 0.0522	0.0946	0.2407	- 0.1054
$\gamma_{3,d}$	0.0	0.0	0.0	- 0.0001	0.0	0.0	0.0	0.0	0.0	0.0
a_{us}	1.0	- 0.3156	- 0.8947	- 0.5310	0.9719	0.2849	0.0241	- 0.0470	0.2983	0.4131
b_{us}	- 0.3156	1.0	0.1372	0.8258	- 0.3995	0.0467	- 0.0221	- 0.1190	0.1856	0.0291
$\gamma_{-1,us}$	- 0.8947	0.1372	1.0	0.2611	- 0.7829	- 0.1695	0.0156	0.0501	- 0.2117	- 0.7191
$\gamma_{1,us}$	- 0.5310	0.8258	0.2611	1.0	- 0.6479	0.0086	0.0076	0.1460	0.0781	- 0.0010
A_{us}	0.9719	- 0.3995	- 0.7829	- 0.6479	1.0	0.2983	0.0515	- 0.0404	0.3055	0.2811
a_{ds}	0.2849	0.0467	- 0.1695	0.0086	0.2983	1.0	- 0.1608	0.0719	0.9152	- 0.2941
b_{bs}	0.0241	- 0.0221	0.0156	0.0076	0.0515	- 0.1608	1.0	0.7834	- 0.3022	- 0.0390
$\gamma_{1,ds}$	- 0.0470	- 0.1190	0.0501	0.1460	- 0.0404	0.0719	0.7834	1.0	- 0.1838	- 0.1373
A_{ds}	0.2983	0.1856	- 0.2117	0.0781	0.3055	0.9152	- 0.3022	- 0.1838	1.0	- 0.1833
a_{ss}	0.4131	0.0291	- 0.7191	- 0.0010	0.2811	- 0.2941	- 0.0390	- 0.1373	- 0.1833	1.0
b_{ss}	0.2197	0.0643	- 0.4479	0.1286	0.1193	- 0.1579	- 0.0260	0.0169	- 0.0896	0.6522
A_{ss}	0.3627	0.0261	- 0.6319	0.0102	0.2412	- 0.2688	- 0.0180	- 0.0960	- 0.1797	0.9280
a_g	- 0.2570	0.0001	0.2196	0.0039	- 0.2493	- 0.2190	- 0.0454	- 0.1031	- 0.2571	0.0626
b_g	- 0.1419	0.1266	0.0694	0.2648	- 0.1715	- 0.0515	0.0917	0.2130	- 0.0469	- 0.0092
$\gamma_{1,g}$	- 0.0241	0.0332	- 0.0226	0.1296	- 0.0489	- 0.0137	0.0503	0.1409	0.0022	- 0.0279
$\alpha_s^{(n_f=3)}(\mu_0)$	0.0954	- 0.2866	- 0.0341	- 0.3493	0.1110	- 0.0604	- 0.1265	- 0.1811	- 0.1330	- 0.0841
$m_c(m_c)$	0.0704	- 0.0093	- 0.0033	- 0.0462	0.1182	0.0849	0.0547	0.0413	0.1193	- 0.0728
$m_b(m_b)$	- 0.0183	- 0.0132	0.0044	0.0209	- 0.0298	- 0.0006	0.0332	0.0695	- 0.0432	- 0.0159
$m_t(m_t)$	0.0641	- 0.1841	- 0.0408	- 0.2635	0.0755	- 0.0573	- 0.1067	- 0.2003	- 0.0869	0.0169

	b_{ss}	A_{ss}	a_g	b_g	$\gamma_{1,g}$	$\alpha_s^{(n_f=3)}(\mu_0)$	$m_c(m_c)$	$m_b(m_b)$	$m_t(m_t)$
a_u	- 0.1186	- 0.1013	0.0046	0.2662	0.2008	0.1083	- 0.0006	0.0661	- 0.1339
b_u	- 0.0480	- 0.0411	- 0.0374	0.3141	0.2274	- 0.0607	0.0170	0.0554	- 0.2170
$\gamma_{1,u}$	- 0.1532	- 0.1458	0.1109	0.1579	0.0706	0.0848	- 0.0104	0.0605	- 0.0816
$\gamma_{2,u}$	0.1549	0.1802	- 0.1934	- 0.0050	0.0876	- 0.0250	0.0206	- 0.0367	0.0081
$\gamma_{3,u}$	- 0.1536	- 0.1625	0.1653	- 0.0207	- 0.0835	0.0765	- 0.0201	0.0287	0.0250
a_d	0.0486	0.1216	- 0.0288	0.0973	0.0919	0.0763	- 0.0123	- 0.0116	- 0.0616
b_d	0.1508	0.1678	- 0.0122	0.0870	0.0574	- 0.0306	- 0.0161	0.0029	- 0.0813
$\gamma_{1,d}$	0.0267	0.0924	0.0053	0.0646	0.0493	0.0725	- 0.0114	- 0.0074	- 0.0491
$\gamma_{2,d}$	- 0.1161	- 0.1196	0.0059	- 0.0666	- 0.0364	0.0243	0.0108	- 0.0051	0.0736
$\gamma_{3,d}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
a_{us}	0.2197	0.3627	- 0.2570	- 0.1419	- 0.0241	0.0954	0.0704	- 0.0183	0.0641
b_{us}	0.0643	0.0261	0.0001	0.1266	0.0332	- 0.2866	- 0.0093	- 0.0132	- 0.1841
$\gamma_{-1,us}$	- 0.4479	- 0.6319	0.2197	0.0694	- 0.0226	- 0.0341	- 0.0034	0.0044	- 0.0408
$\gamma_{1,us}$	0.1286	0.0102	0.0039	0.2648	0.1296	- 0.3493	- 0.0462	0.0209	- 0.2635
A_{us}	0.1193	0.2412	- 0.2493	- 0.1715	- 0.0489	0.1110	0.1182	- 0.0298	0.0755
a_{ds}	- 0.1579	- 0.2688	- 0.2190	- 0.0515	- 0.0137	- 0.0604	0.0849	- 0.0006	- 0.0573
b_{bs}	- 0.0260	- 0.0180	- 0.0454	0.0917	0.0503	- 0.1265	0.0547	0.0332	- 0.1067
$\gamma_{1,ds}$	0.0169	- 0.0960	- 0.1031	0.2130	0.1409	- 0.1811	0.0413	0.0695	- 0.2003
A_{ds}	- 0.0896	- 0.1797	- 0.2571	- 0.0469	0.0022	- 0.1330	0.1193	- 0.0432	- 0.0869
a_{ss}	0.6522	0.9280	0.0626	- 0.0092	- 0.0279	- 0.0841	- 0.0728	- 0.0159	0.0169
b_{ss}	1.0	0.6427	- 0.0179	0.1967	0.1164	- 0.2390	- 0.0965	0.0169	- 0.1675
A_{ss}	0.6427	1.0	- 0.0211	0.1403	0.0997	- 0.1385	0.0216	0.0072	- 0.1109
a_g	- 0.0179	- 0.0211	1.0	- 0.5279	- 0.8046	0.1838	- 0.2829	0.0076	0.3310
b_g	0.1967	0.1403	- 0.5279	1.0	0.8837	- 0.5124	0.1438	0.1255	- 0.7275
$\gamma_{1,g}$	0.1164	0.0997	- 0.8046	0.8837	1.0	- 0.2511	0.1829	0.0814	- 0.5180
$\alpha_s^{(n_f=3)}(\mu_0)$	- 0.2390	- 0.1385	0.1838	- 0.5124	- 0.2511	1.0	- 0.1048	0.0423	0.6924
$m_c(m_c)$	- 0.0965	0.0216	- 0.2829	0.1438	0.1829	- 0.1048	1.0	0.0328	- 0.1577
$m_b(m_b)$	0.0169	0.0072	0.0076	0.1255	0.0814	0.0423	0.0328	1.0	- 0.0900
$m_t(m_t)$	- 0.1675	- 0.1109	0.3310	- 0.7275	- 0.5180	0.6924	- 0.1577	- 0.0900	1.0

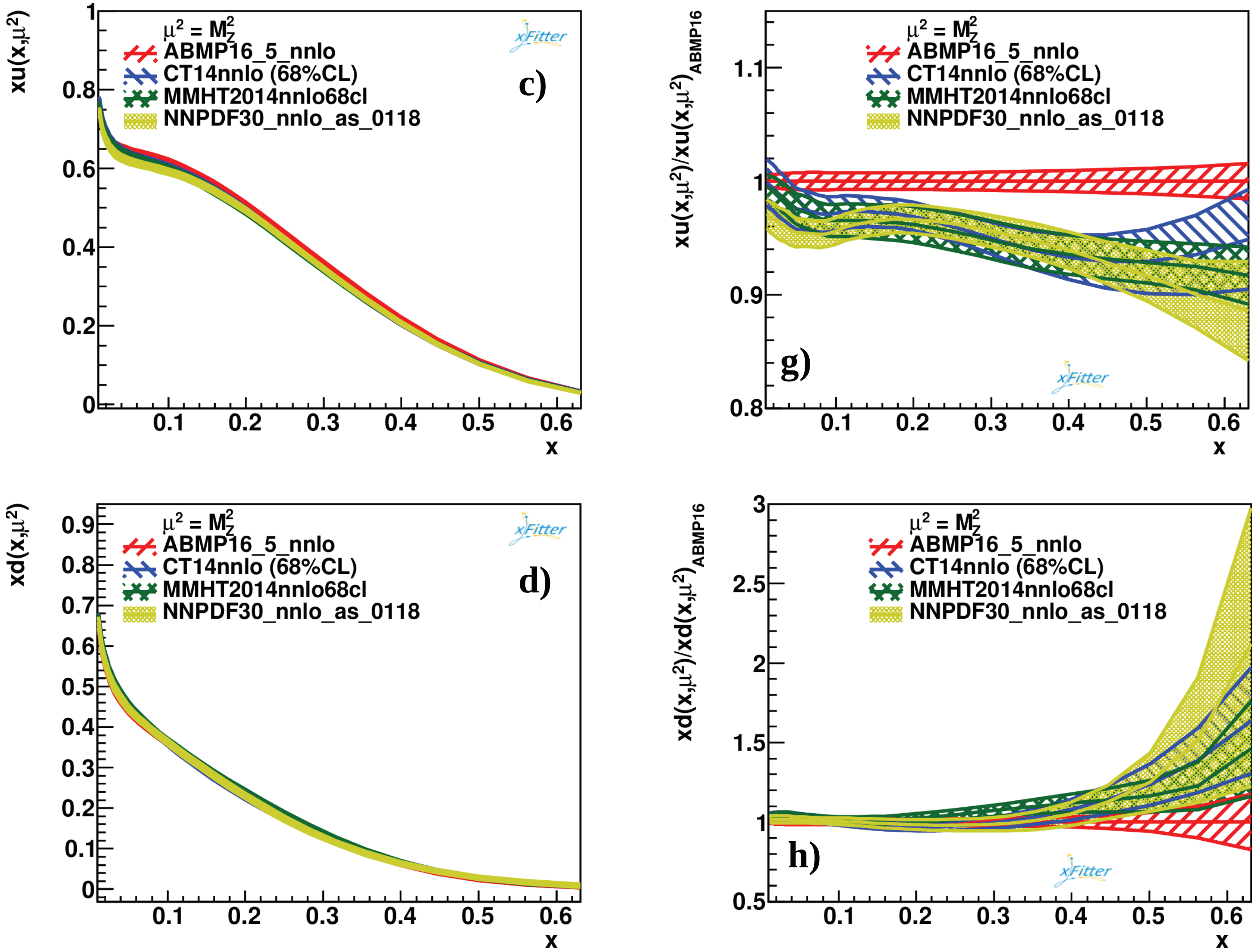
Results for parton distributions (I)

- PDFs with 1σ uncertainty bands; compare ABMP16, CT14, MMHT14 NNPDF3.0
- Gluon $g(x)$



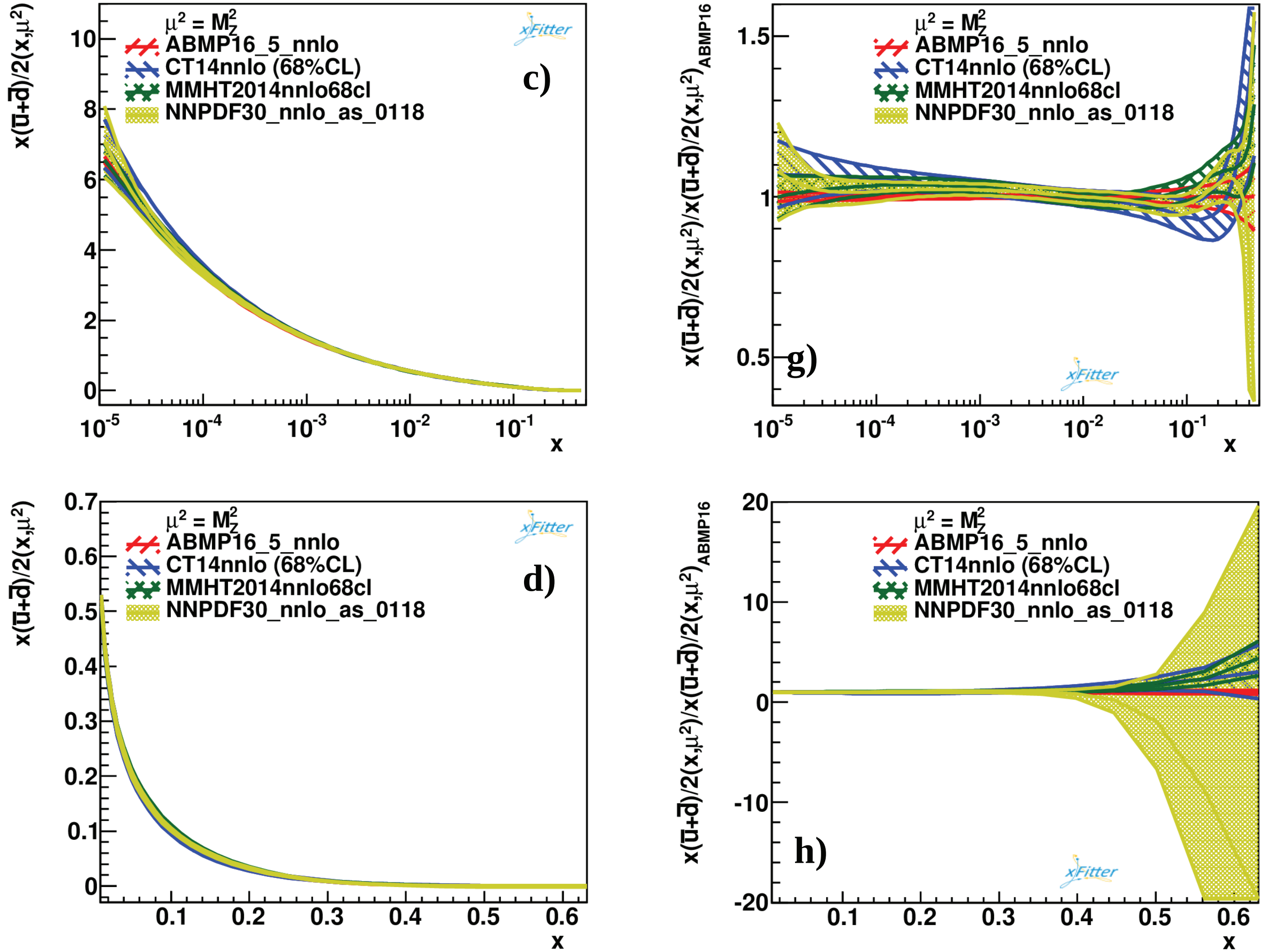
Results for parton distributions (II)

- PDFs with 1σ uncertainty bands; compare ABMP16, CT14, MMHT14 NNPDF3.0
- Light valence quarks $u(x)$, $d(x)$



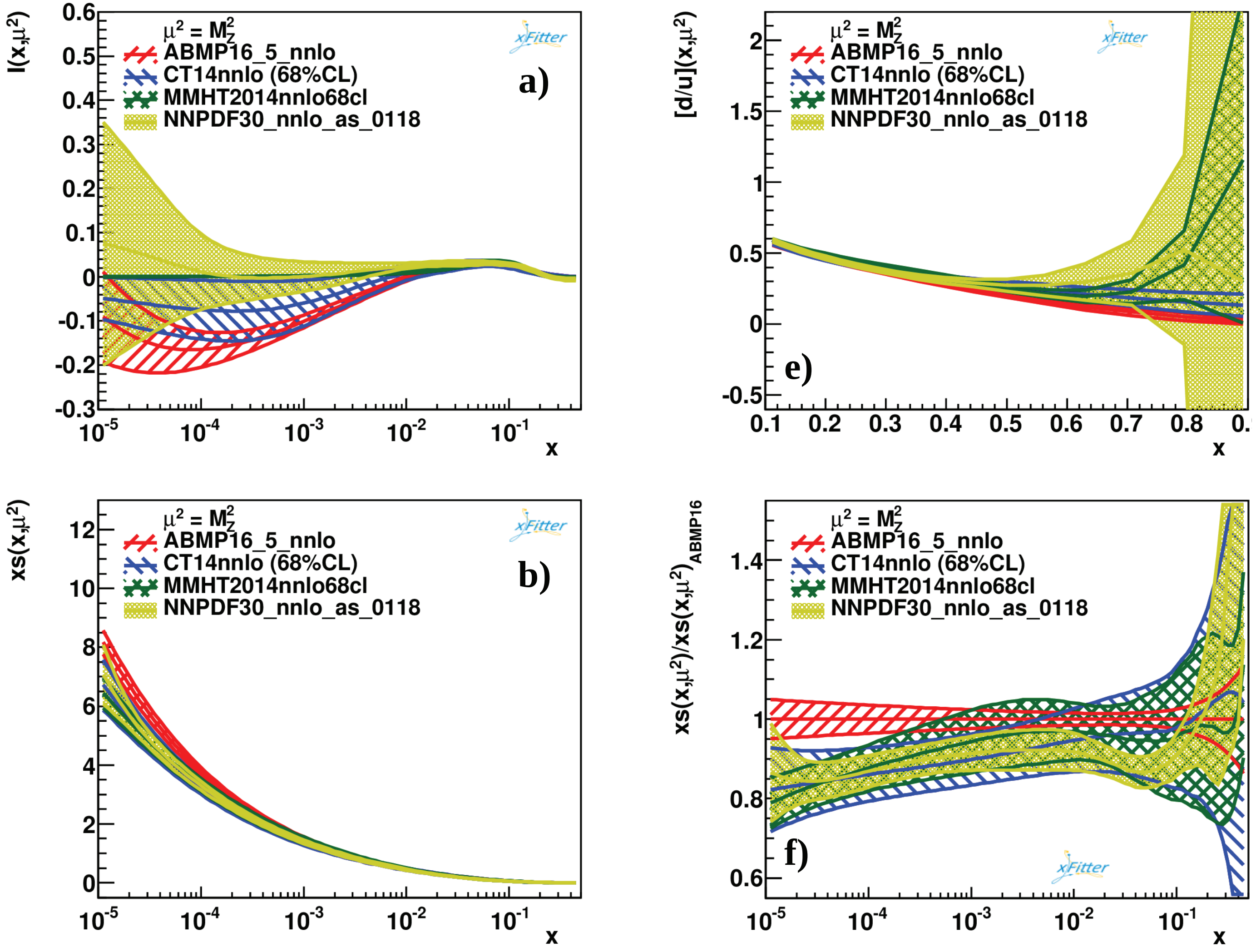
Results for parton distributions (III)

- PDFs with 1σ uncertainty bands; compare ABMP16, CT14, MMHT14 NNPDF3.0
- Sea quarks $\bar{u}(x) + \bar{d}(x)$



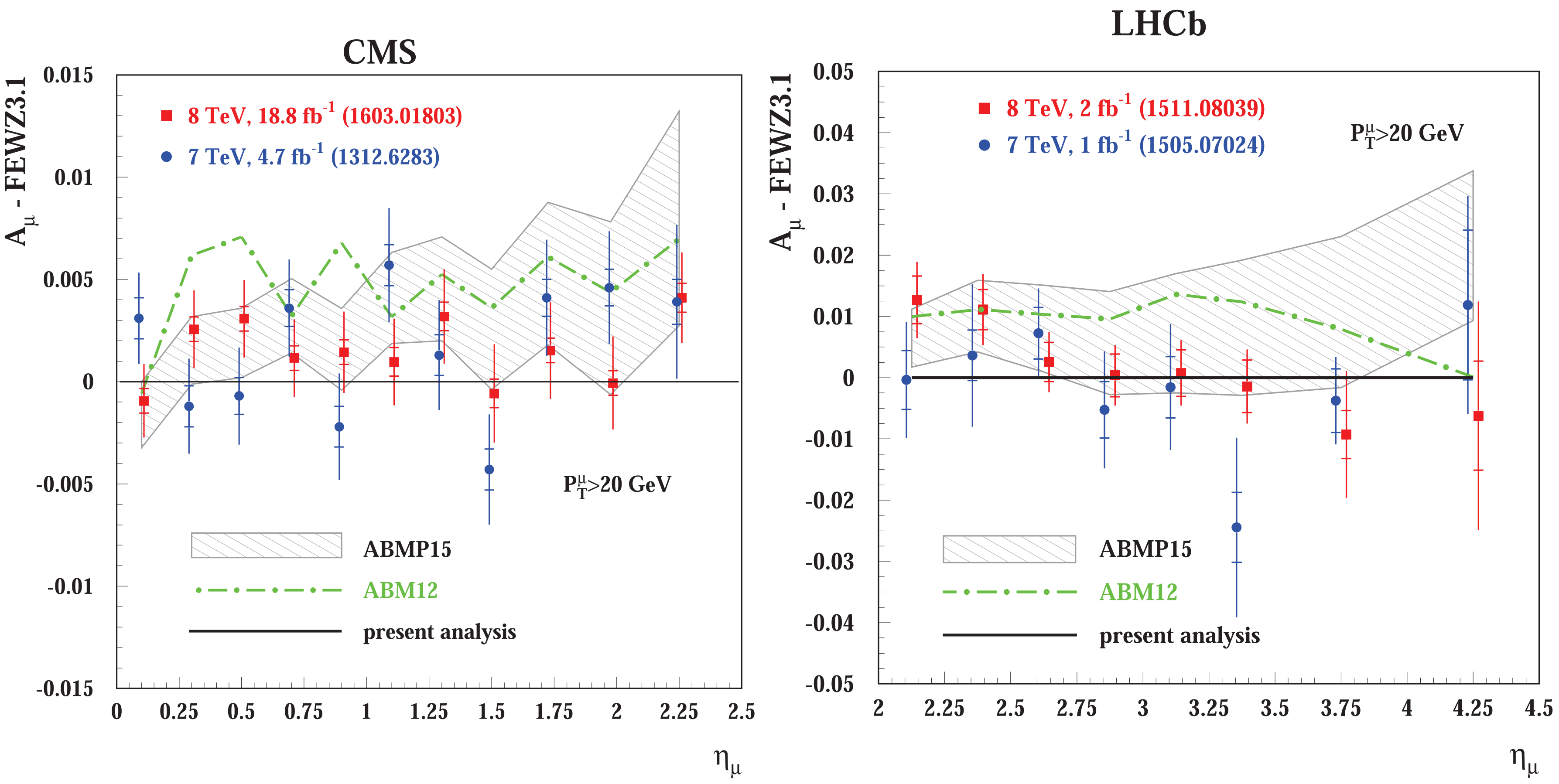
Results for parton distributions (IV)

- PDFs with 1σ uncertainty bands; compare ABMP16, CT14, MMHT14 NNPDF3.0
- Iso-spin asymmetry $x(\bar{d}(x) - \bar{u}(x))$; ratio $d(x)/u(x)$; strange $s(x)$



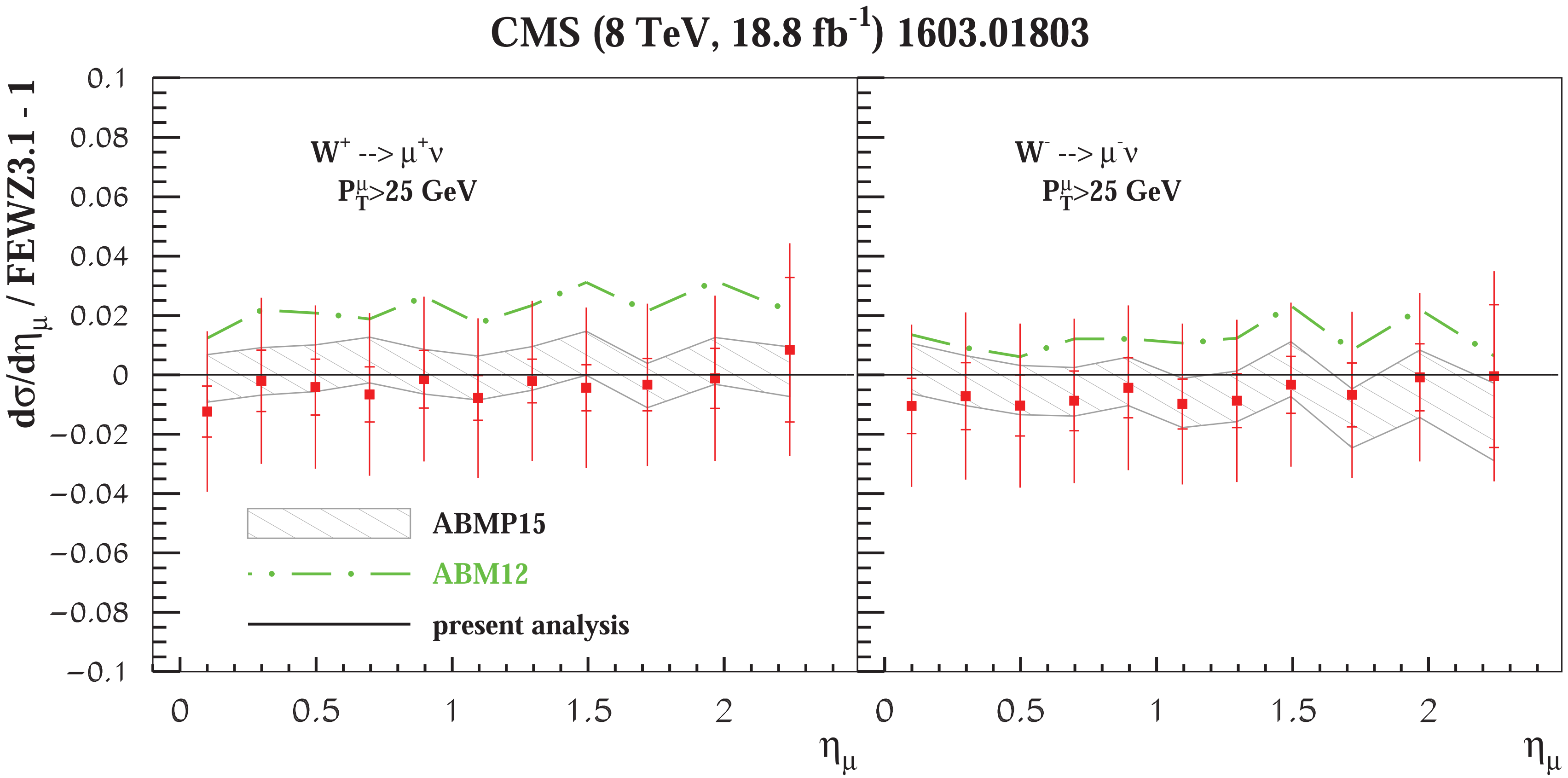
$W^{\pm}$- and Z-boson production

Muon charge asymmetry from LHC



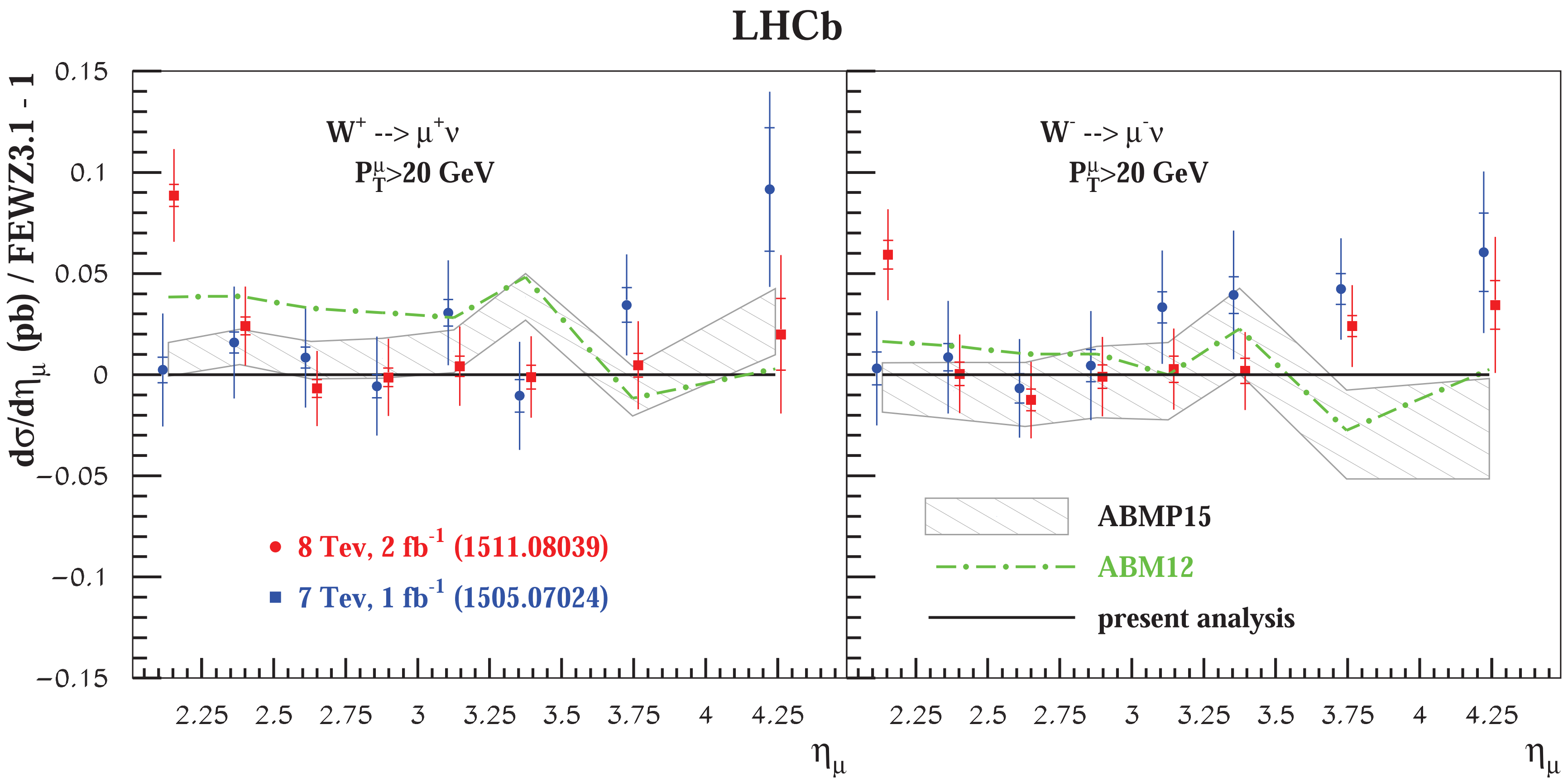
- CMS and LHCb data for $pp \to W^\pm + X \to \mu^\pm \nu + X$ at $\sqrt{s} = 7$ TeV and $\sqrt{s} = 8$ TeV
  - comparison of ABM12, ABMP15 and ABMP16 fits
- Problematic data point at $\eta_\mu = 3.375$ for $\sqrt{s} = 7$ TeV in LHCb data are omitted in fit

$W^{\pm}$-boson production from LHC (I)



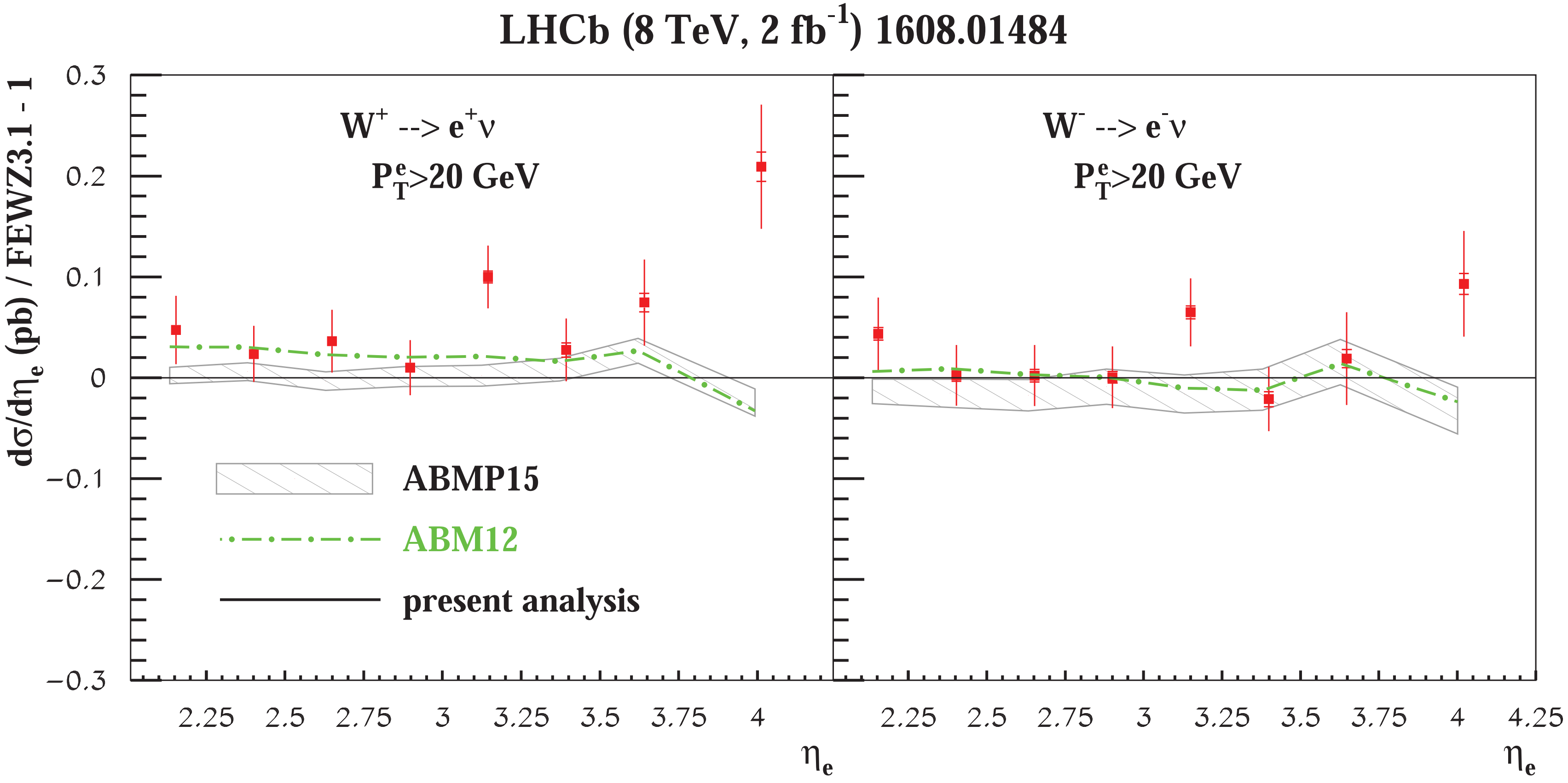
- CMS data on cross section of inclusive $W^{\pm}$-boson production at $\sqrt{s} = 8$ TeV
  - channel $W^{\pm} \to \mu^{\pm}\nu$

$W^{\pm}$-boson production from LHC (II)



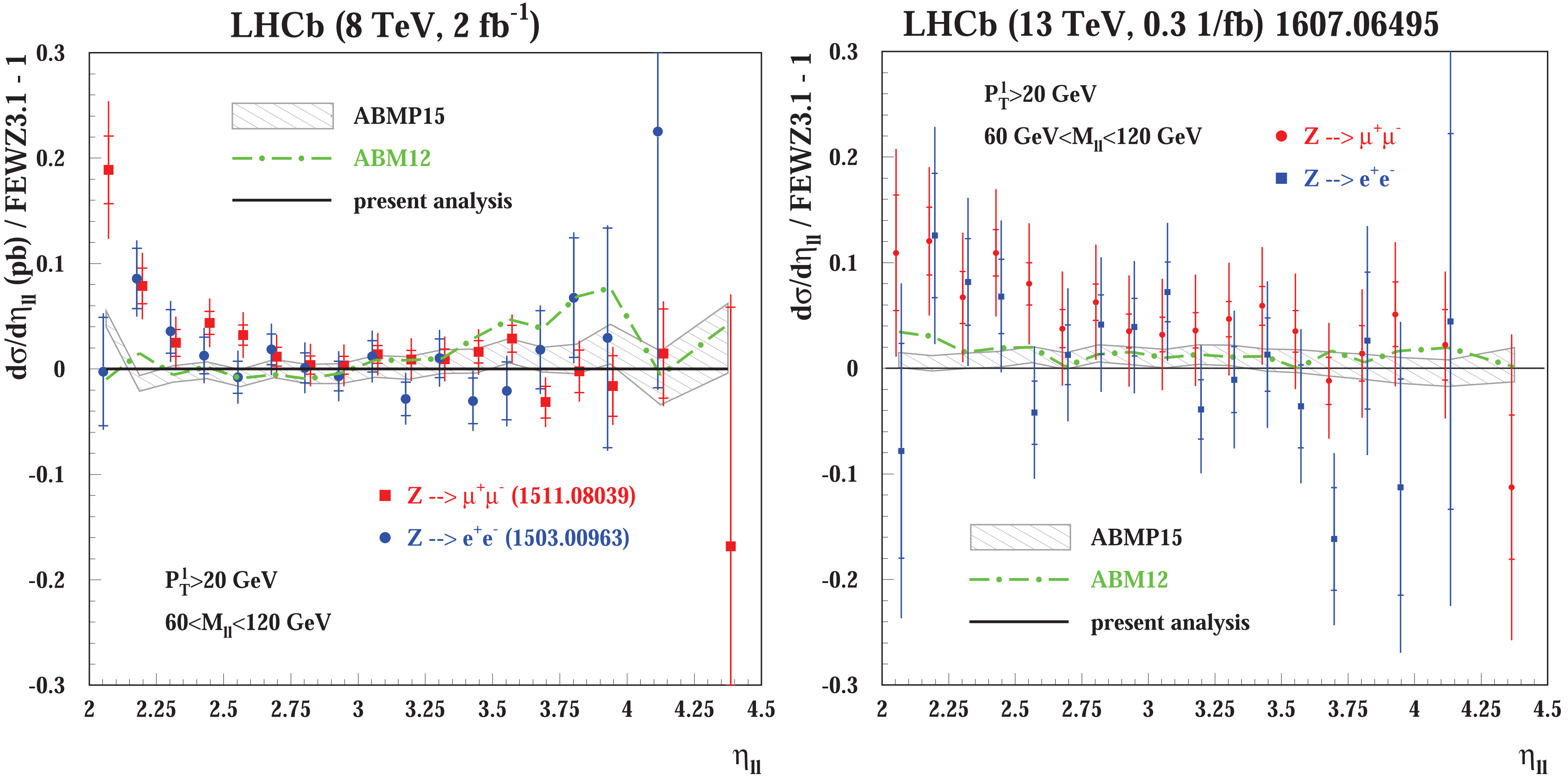
- LHCb data on cross section of inclusive $W^{\pm}$-boson production at $\sqrt{s} = 7$ TeV and $\sqrt{s} = 8$ TeV
  - channel $W^{\pm} \to \mu^{\pm}\nu$
- Points at $\eta_{\mu} = 2.125$ for $\sqrt{s} = 8$ TeV are not used in fit

$W^\pm$-boson production from LHC (III)



- LHCb data on cross section of inclusive $W^\pm$-boson production at $\sqrt{s} = 8$ TeV
    - channel $W^\pm \to e^\pm \nu$

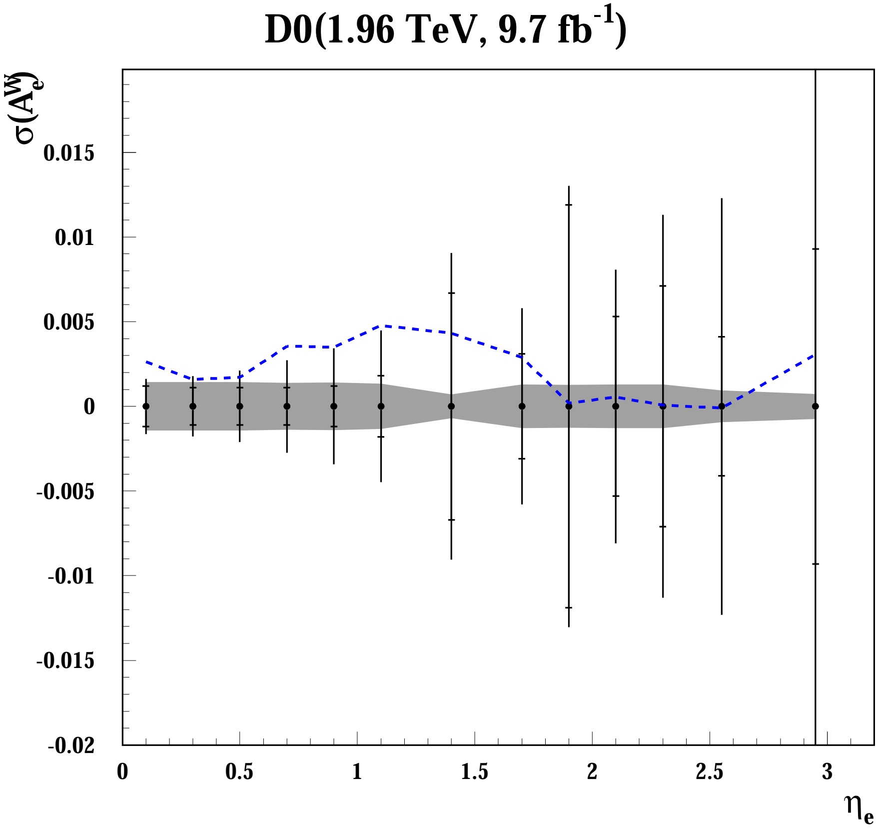
Z-boson production from LHC



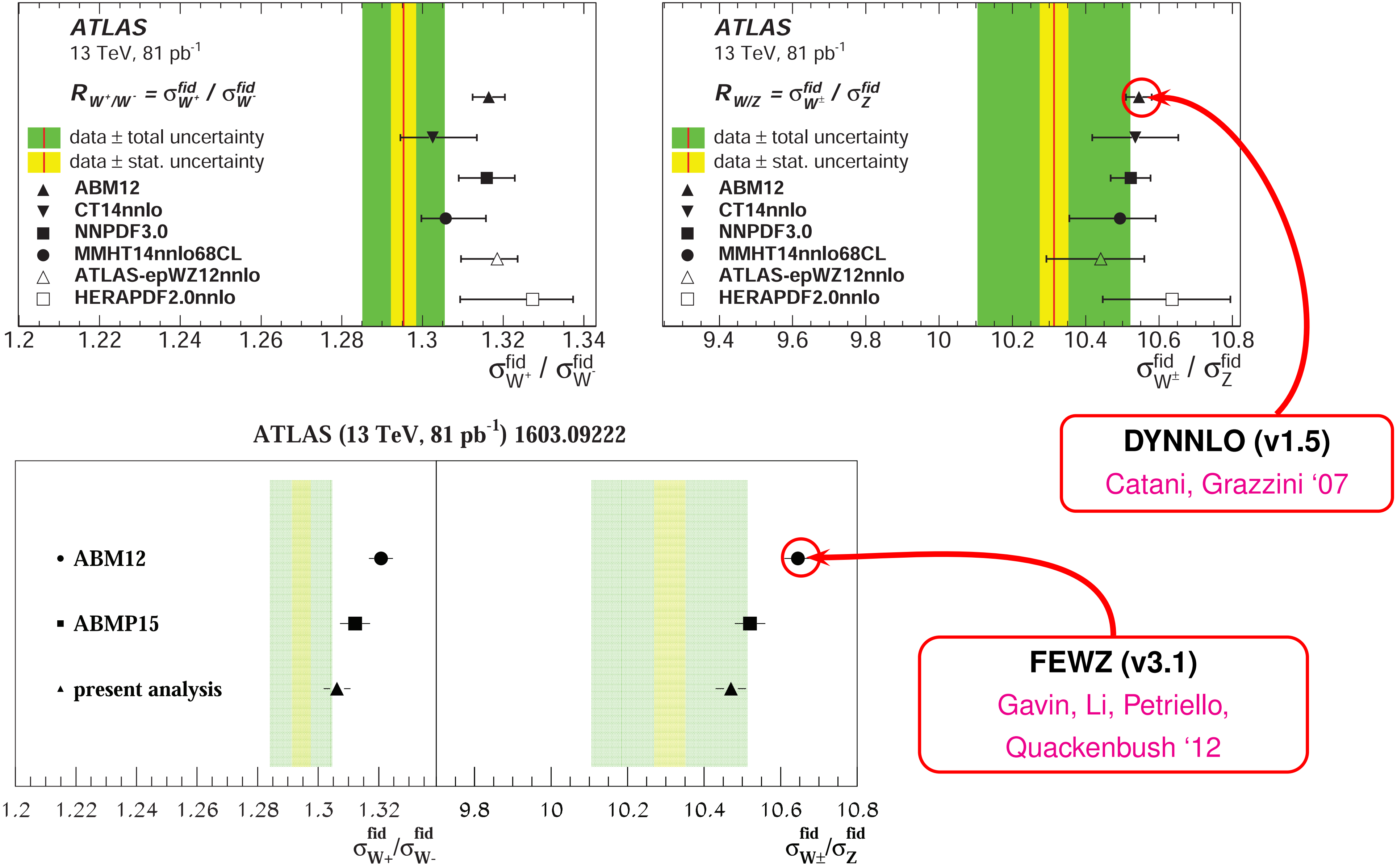
- LHCb data for $pp \to Z + X \to l\bar{l}$ at $\sqrt{s} = 8$ TeV and $\sqrt{s} = 13$ TeV
  - channels $Z \to e^+e^-$ and $Z \to \mu^+\mu^-$

Theory issues (I)

- Data on electron asymmetry with high precision at central rapidities D0
- NNLO corrections in coefficient functions not uniform in η_e (dashed curve)
- Numerical accuracy at NNLO (shaded area) obtained with FEWZ (v3.1)
- Accuracy of $\mathcal{O}(1\ \mathrm{ppm})$ to meet uncertainties in experimental data requires $\mathcal{O}(10^4 \mathrm{h})$ of running FEWZ (v3.1) at NNLO



Theory issues (II)



DYNNLO (v1.5)

Catani, Grazzini ‘07

FEWZ (v3.1)

Gavin, Li, Petriello, Quackenbush ‘12

- Differences at NNLO between DYNNLO and FEWZ up to $\mathcal{O}(1\%)$ or more

Strong coupling constant

Strong coupling constant (1992)

	$\alpha_s(\mathrm{M_Z^2})$
R_τ	$0.117 \begin{smallmatrix} +\ 0.010 \\ -\ 0.016 \end{smallmatrix}$
DIS	0.112 ± 0.007
Υ Decays	0.110 ± 0.010
$R_{e^+e^-}(s < 62\text{GeV})$	0.140 ± 0.020
$p\bar{p} \to W + jets$	0.121 ± 0.024
$\Gamma(Z \to \text{hadrons})/\Gamma(Z \to l\bar{l})$	0.132 ± 0.012
Jets at LEP	0.122 ± 0.009
Average	0.118 ± 0.007

G. Altarelli (1992)
in ***QCD - 20 Years Later,***
CERN-TH-6623-92

Essential facts

- World average 1992 $\alpha_s(M_Z) = 0.118 \pm 0.007$
- Central value at NLO QCD
  - still right, but for very different reasons
- Error at NLO QCD
  - now down to $\sim 0.0050 - 0.0040$ (theory scale uncertainty)

Strong coupling constant (2017)

Measurements at NNLO

- Values of $\alpha_s(M_Z)$ at NNLO from PDF fits

SY	0.1166 ± 0.013	F_2^{ep}	Santiago, Yndurain ‘01
	0.1153 ± 0.063	$xF_3^{\nu N}$ (heavy nucl.)	
A02	0.1143 ± 0.013	DIS	Alekhin ‘01
MRST03	0.1153 ± 0.0020		Martin, Roberts, Stirling, Thorne ‘03
BBG	$0.1134 \, {}^{+\,0.0019}_{-\,0.0021}$	valence analysis, NNLO	Blümlein, Böttcher, Guffanti ‘06
GRS	0.112	valence analysis, NNLO	Glück, Reya, Schuck ‘06
A06	0.1128 ± 0.015		Alekhin ‘06
JR08	0.1128 ± 0.0010	dynamical approach	Jimenez-Delgado, Reya ‘08
	0.1162 ± 0.0006	including NLO jets	
ABKM09	0.1135 ± 0.0014	HQ: FFNS $n_f = 3$	Alekhin, Blümlein, Klein, S.M. ‘09
	0.1129 ± 0.0014	HQ: BSMN	
MSTW	0.1171 ± 0.0014		Martin, Stirling, Thorne, Watt ‘09
Thorne	0.1136	[DIS+DY, HT*] (2013)	Thorne ‘13
ABM11$_J$	$0.1134 \ldots 0.1149 \ \pm 0.0012$	Tevatron jets (NLO) incl.	Alekhin, Blümlein, S.M. ‘11
NN21	0.1173 ± 0.0007	(+ heavy nucl.)	NNPDF ‘11
ABM12	0.1133 ± 0.0011		Alekhin, Blümlein, S.M. ‘13
	0.1132 ± 0.0011	(without jets)	
CT10	0.1140	(without jets)	Gao et al. ‘13
CT14	$0.1150 \, {}^{+\,0.0060}_{-\,0.0040}$	$\Delta\chi^2 > 1$ (+ heavy nucl.)	Dulat et al. ‘15
MMHT	0.1172 ± 0.0013	(+ heavy nucl.)	Martin, Motylinski, Harland-Lang, Thorne ‘15

Strong coupling constant (2017)

Other measurements of α_s at NNLO

- Values of $\alpha_s(M_Z)$ at NNLO from measurements at colliders

3-jet rate	0.1175 ± 0.0025	Dissertori et al. 2009	arXiv:0910.4283
e^+e^- thrust	$0.1131 \, {}^{+\,0.0028}_{-\,0.0022}$	Gehrmann et al.	arXiv:1210.6945
e^+e^- thrust	0.1140 ± 0.0015	Abbate et al.	arXiv:1204.5746
C-parameter	0.1123 ± 0.0013	Hoang et al.	arXiv:1501.04111
CMS	0.1151 ± 0.0033	$t\bar{t}$	arXiv:1307.1907
NLO Jets ATLAS	$0.111 \, {}^{+\,0.0017}_{-\,0.0007}$		arXiv:1312.5694
NLO Jets CMS	0.1148 ± 0.0055		arXiv:1312.5694

PDG average

- Some tension with the PDG average at NNLO

  PDG (Bethke, Dissertori, Salam) ‘16

$$\alpha_s(M_Z) = 0.1181 \pm 0.0013$$

  - PDG value driven by lattice determinations (and low scale τ-data)

Differences in α_s determinations

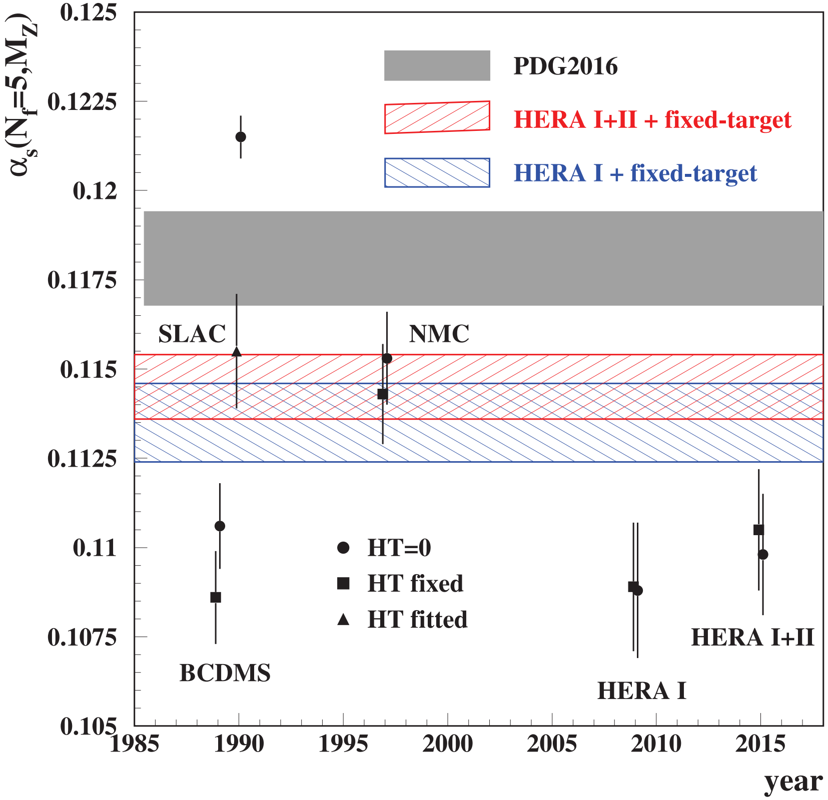
Why α_s values from MSTW, MMHT and NNPDF are large

- Differences result from different physics models and analysis procedures
- Fits of DIS data
  - target mass corrections (powers of nucleon mass M_N^2/Q^2)
  - higher twist $F_2^{\rm ht} = F_2 + ht^{(4)}(x)/Q^2 + ht^{(6)}(x)/Q^4 + \dots$
  - correlation of errors among different data sets

	α_s	NNLO	target mass corr.	higher twist	error correl.
ABM12	0.1132 ± 0.0011	**yes**	**yes**	**yes**	**yes**
NNPDF21	0.1173 ± 0.0007	**(yes)**	**yes**	**no**	**yes**
MSTW	0.1171 ± 0.0014	**(yes)**	**no**	**no**	**no**
MMHT	0.1172 ± 0.0013	**(yes)**	**no**	**no**	–

- Effects for differences are understood
  - variants of ABM with no higher twist etc. reproduce larger α_s values
    Alekhin, Blümlein, S.M. '11

World DIS data and value of α_s

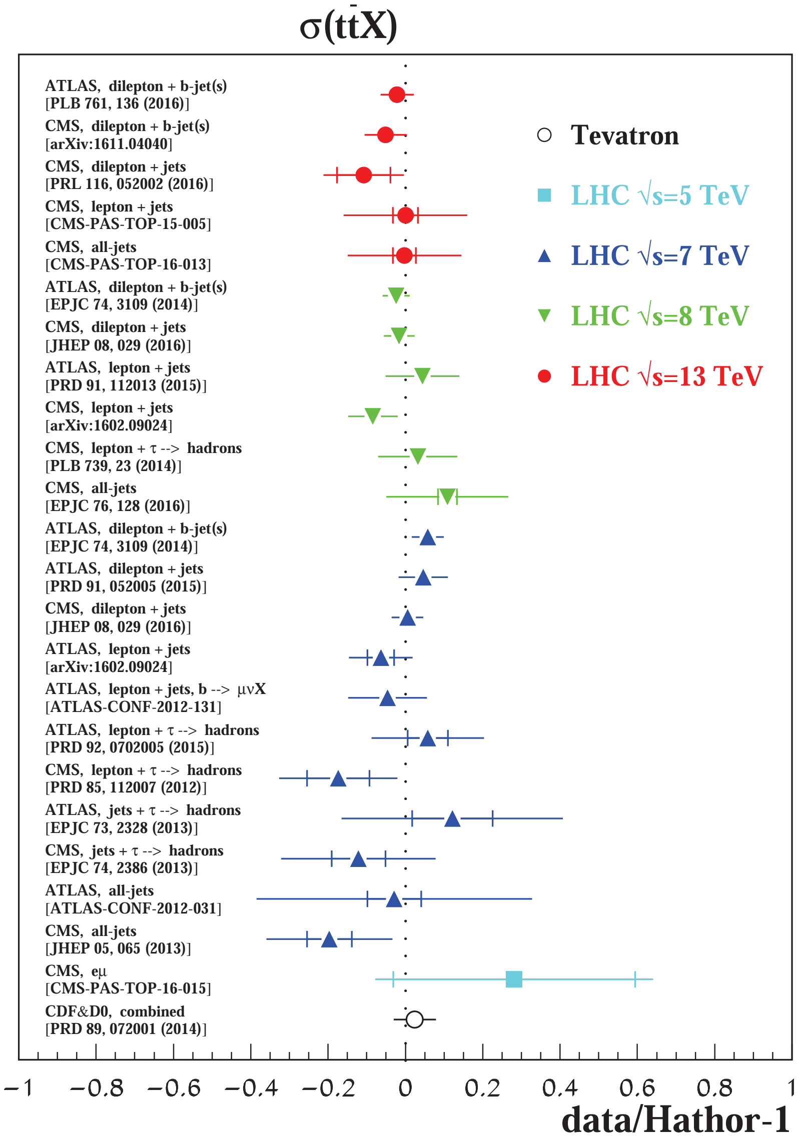


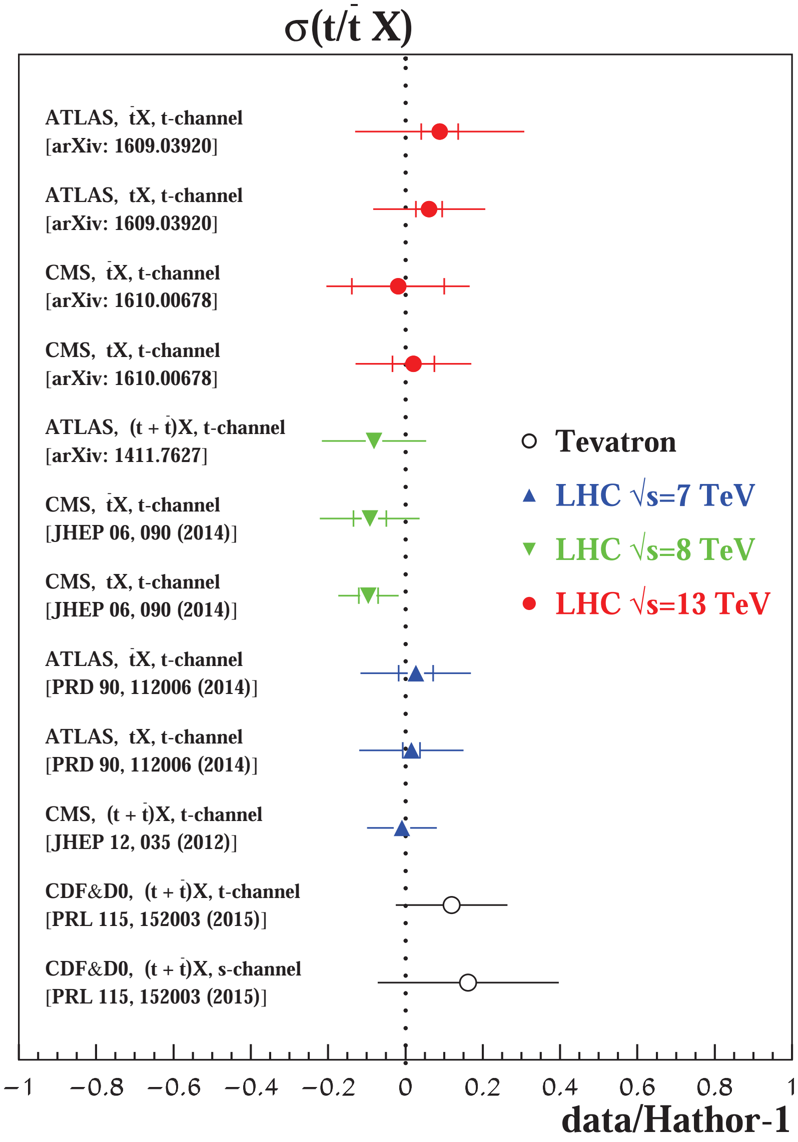
- Value of $\alpha_{(}M_Z)$ is lower than PDG average
  - value of $\alpha_{(}M_Z)$ is pulled up by SLAC and NMC and pulled down by BCDMS and HERA data
- Only $\alpha_{(}M_Z)$ preferred by SLAC data is compatible with PDG average (provided higher twist terms are accounted for)
- Update of the α_s determination with combined data HERA I+II
  - value of $\alpha_{(}M_Z)$ increases by 1σ

Heavy-quark masses

Data on top-quark cross sections

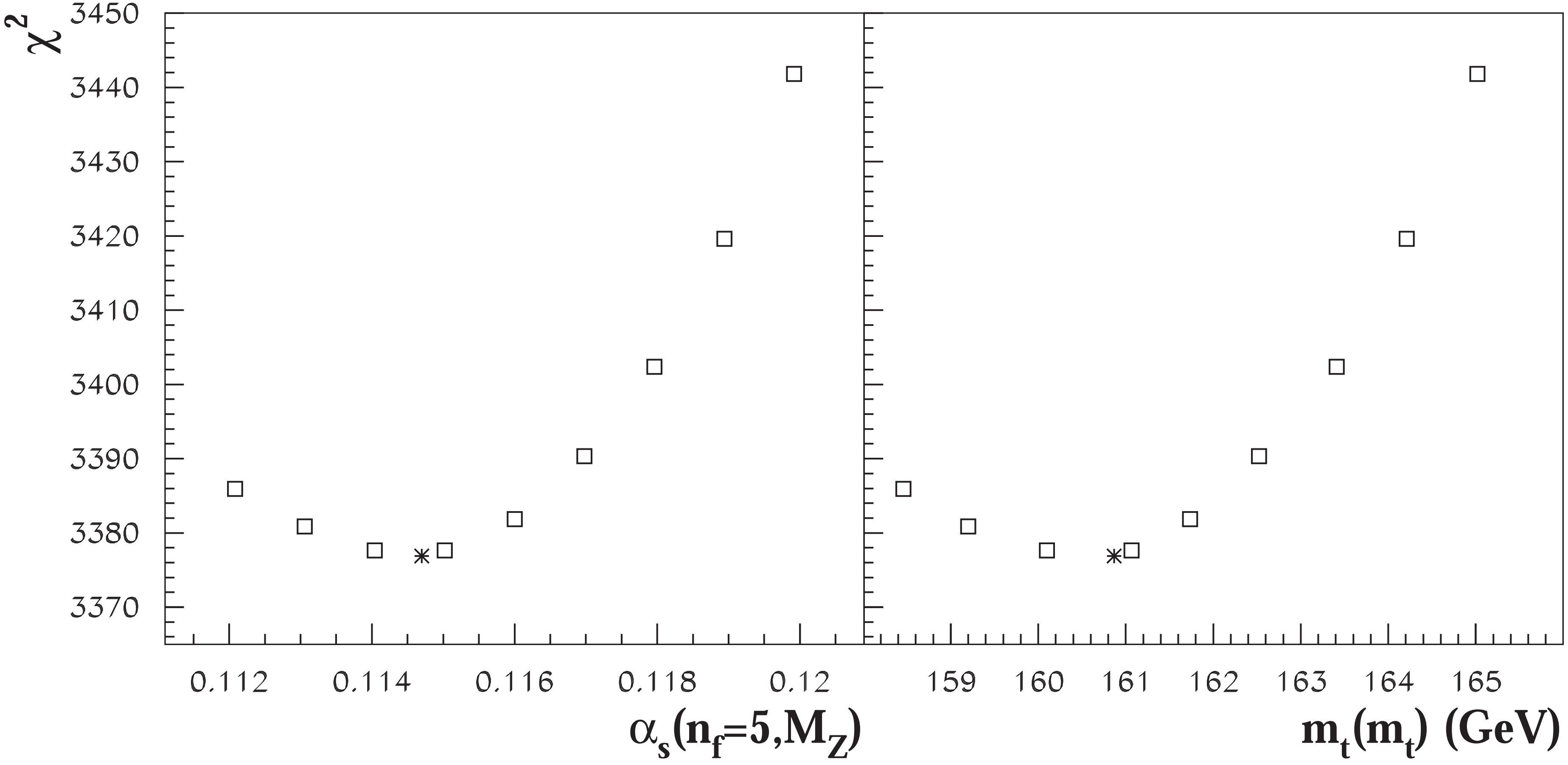
- Pulls for $t\bar{t}$- and single-t inclusive cross sections


σ(tt̄X)


σ(t/t̄ X)

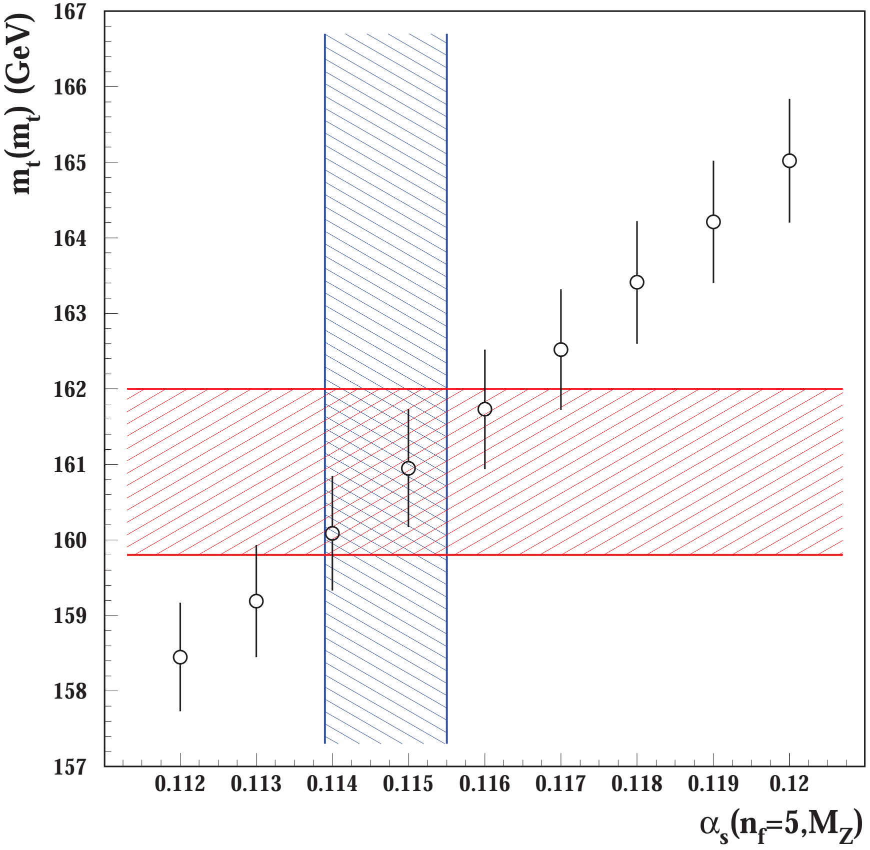
Fit quality

- Goodness-of-fit estimator χ^2 for extracted $\alpha_s(M_Z)$ and $m_t(m_t)$ values
  - χ^2 of global fit with $NDP = 2834$
  - data on top-quark production with $NDP = 36$ D0, ATLAS, CMS, LHCb



Correlations

- Cross section for $t\bar{t}$-production with parametric dependence
  $\sigma_{t\bar{t}} \sim \alpha_s^2\, m_t^2\, g(x) \otimes g(x)$
- Correlations between gluon PDF $g(x)$, $\alpha_s(M_Z)$ and $m_t(m_t)$
  - PDFs and $\alpha_s(M_Z)$ already well constrained by global fit

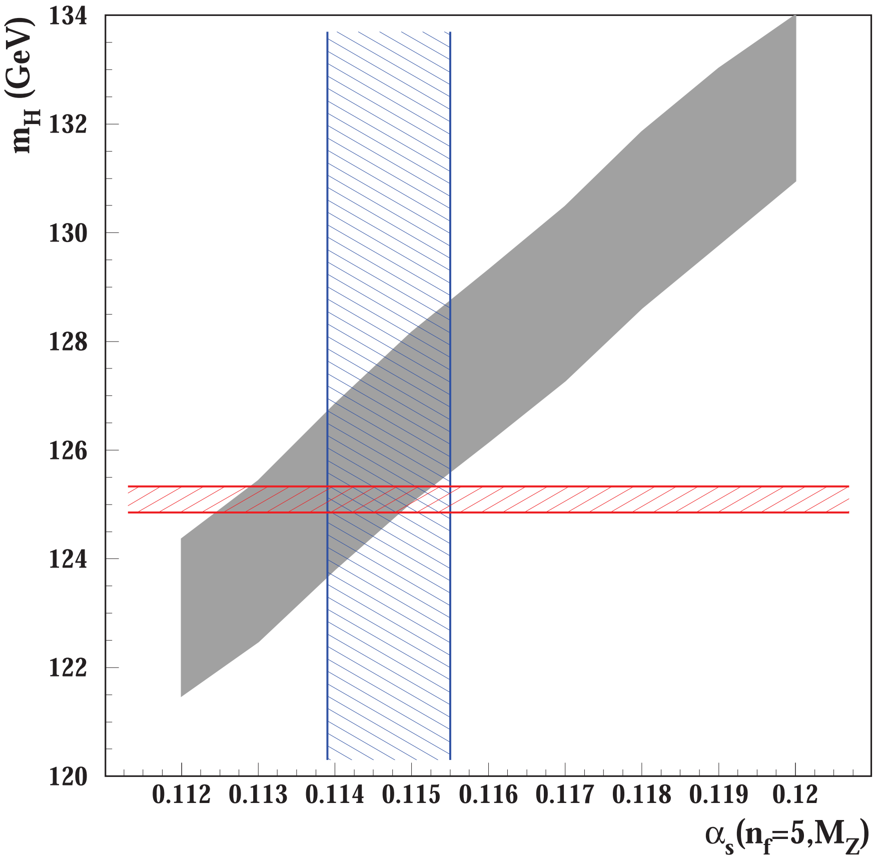


Implications on electroweak vacuum

Implications on electroweak vacuum

- Condition of absolute stability of electroweak vacuum at Planck scale M_{Planck} requires Higgs self-coupling $\lambda(\mu_r) \geq 0$
  - correlation between Higgs mass m_H, m_t and $\alpha_s(M_Z)$ at $\mu = M_{\text{Planck}}$

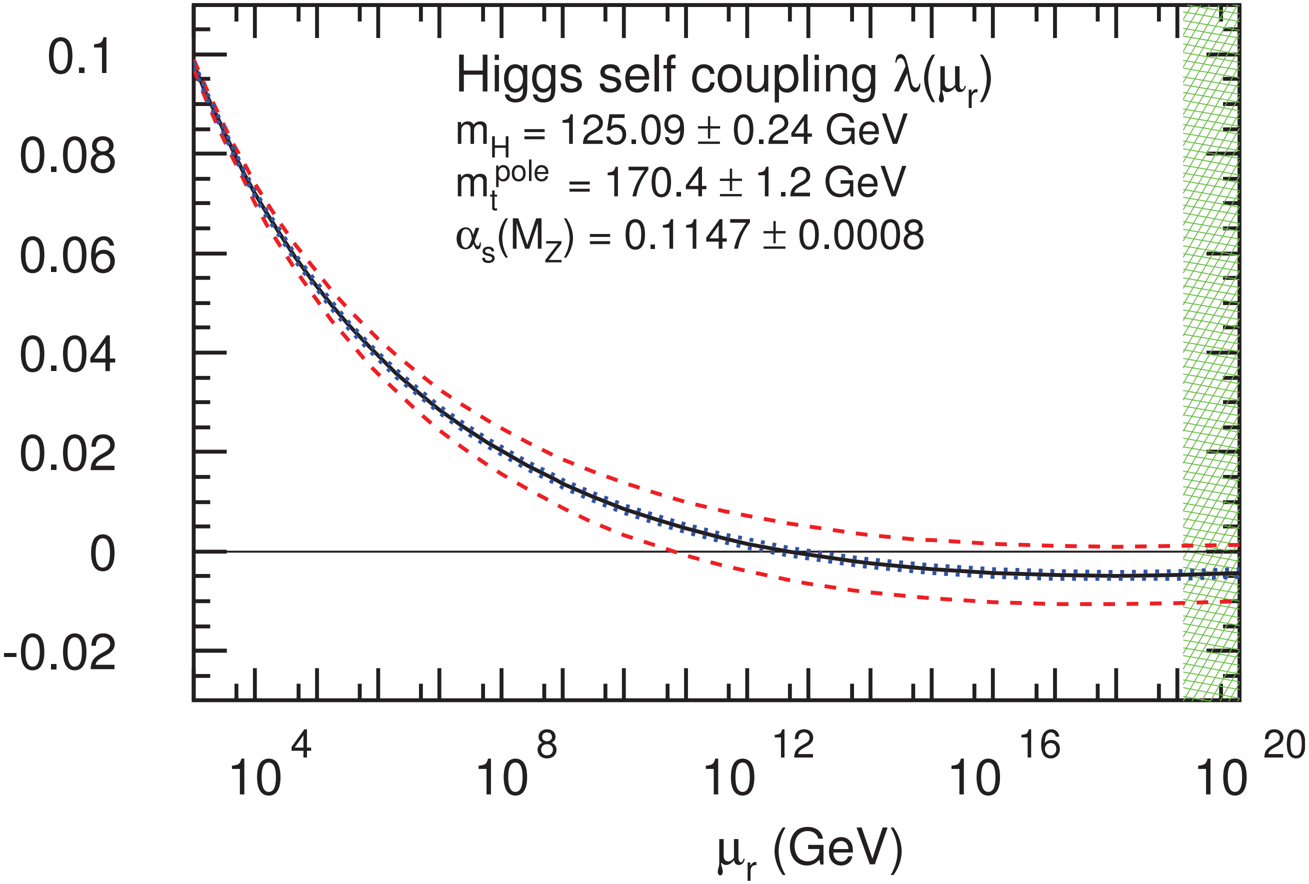
$$m_H \;\geq\; 129.6 + 2.0 \times \left(m_t^{\text{pole}} - 173.34 \text{ GeV} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 0.3 \;\; \text{GeV}$$



- NNLO analyses

  Bezrukov, Kalmykov, Kniehl, Shaposhnikov ‘12; Degrassi et al. ‘12; Buttazzo et al. ‘13; Bednyakov, Kniehl, Pikelner, Veretin ‘15

Higgs self-coupling



- Renormalization group evolution of λ with uncertainties in m_H, m_t and α_s up to $\mu_r = M_{\text{Planck}}$ (using program mr Kniehl, Pikelner, Veretin '16)
  - top-quark mass least precise parameter
- $\lambda(\mu_r = M_{\text{Planck}}) \simeq 0$ implies "fate of universe" may not be fatal, after all

Where to from here?

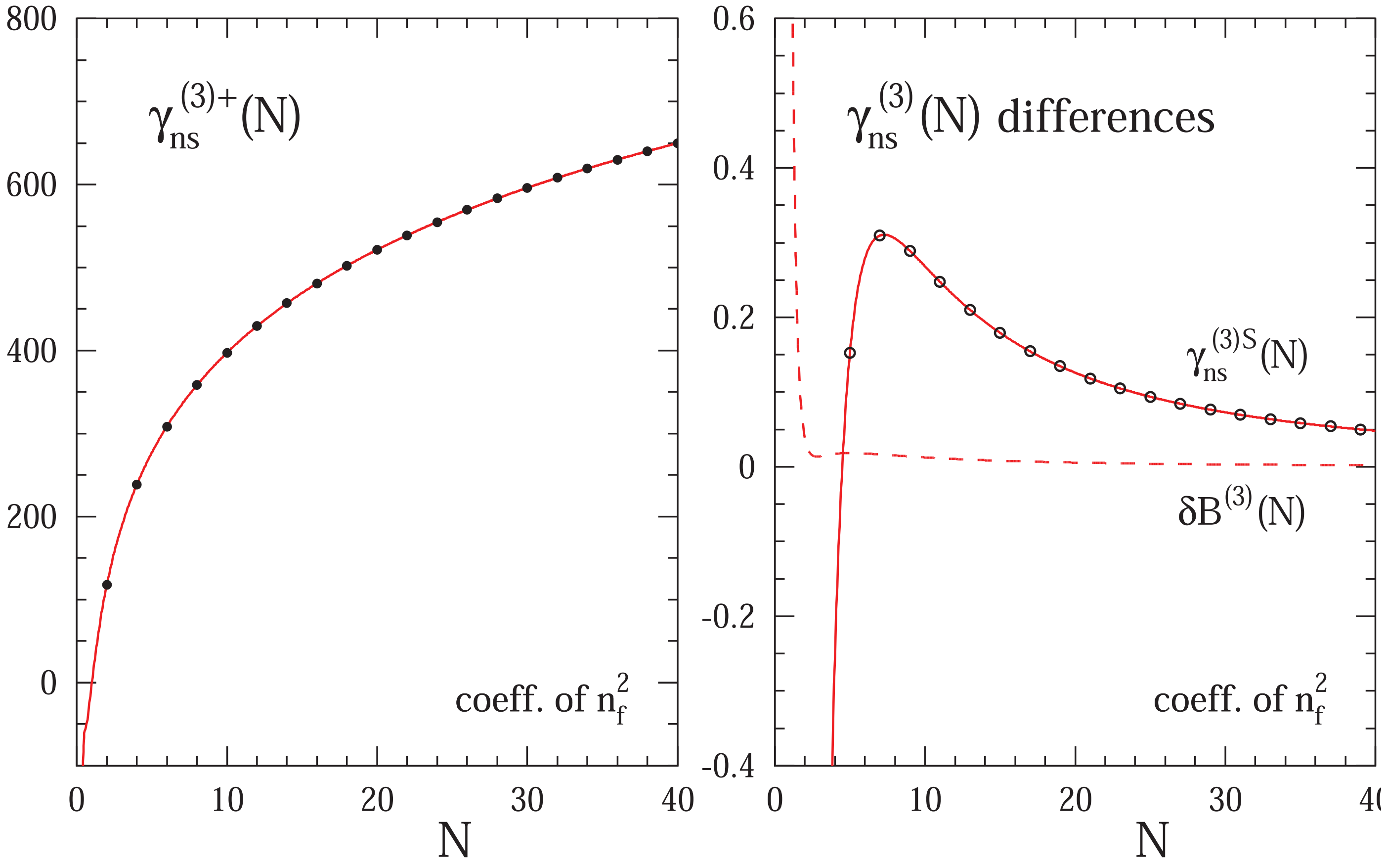
Next-to-next-to-next-to-leading order

The N^{3}LO era

- Increase of precision at next-to-next-to-next-to-leading order
  - further reduction of dependence on renormalization and factorization scales μ_R, μ_F
- Known three-loop QCD corrections for hard scattering processes
  - DIS structure functions F_2, F_3 and F_L S.M., Vermaseren, Vogt ‘05–‘08
  - Higgs boson production in gluon-gluon fusion
    Anastasiou, Duhr, Dulat, Herzog, Mistlberger ‘15
  - Higgs boson production in VBF (double DIS approximation)
    Dreyer, Karlberg ‘16
- QCD factorization requires PDFs at N^{3}LO
  - scale evolution with N^{3}LO splitting functions for consistency

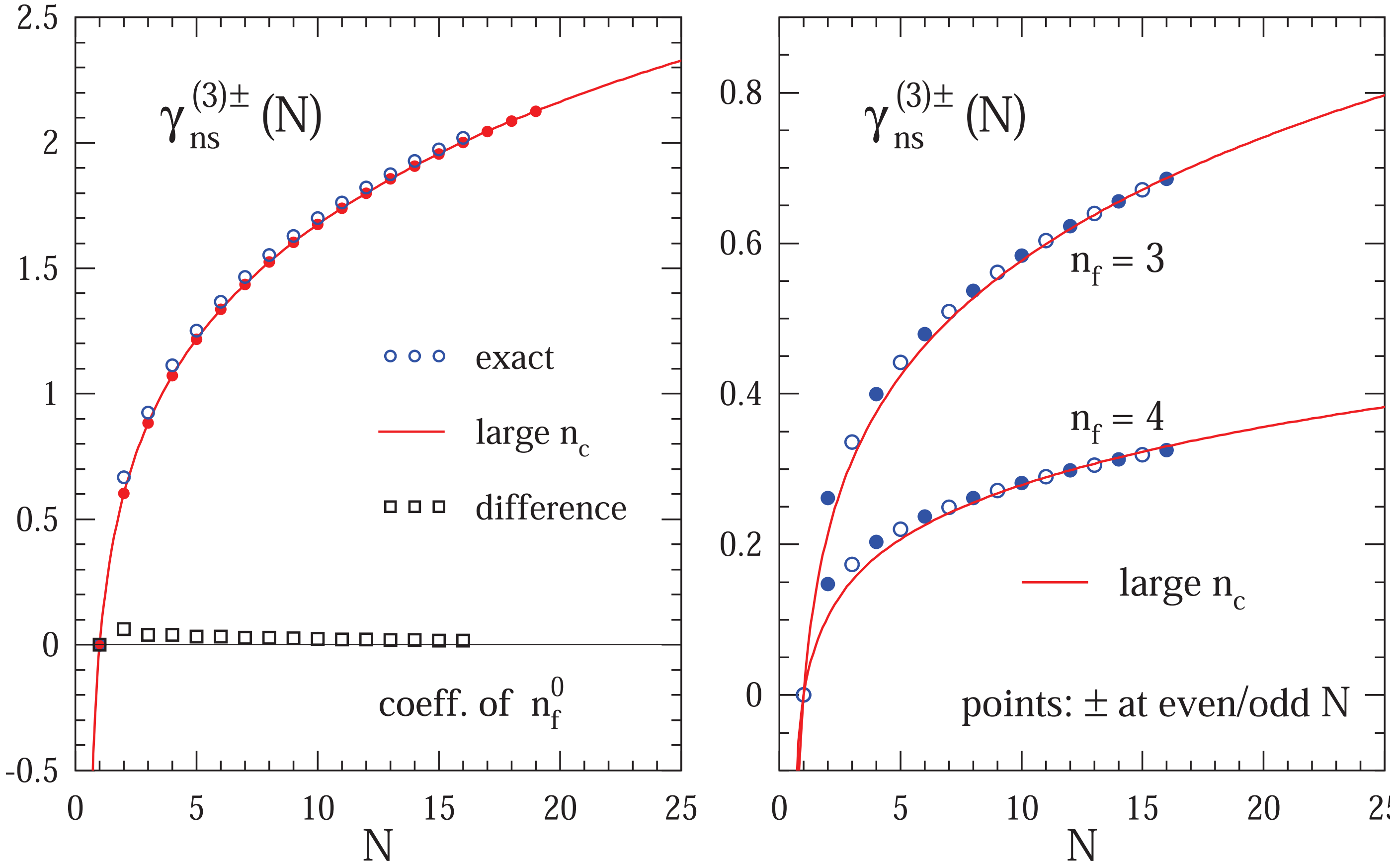
Splitting functions at N^{3}LO

- Non-singlet splitting functions $P_{\rm ns}^{(3)\,\pm}(x)$, anomalous dimension $\gamma_{\rm ns}^{(3)\,\pm}(N)$
- Fermionic contributions $\mathcal{O}(n_f^2)$ known at N^{3}LO Ruijl, Ueda, Vermaseren, Vogt '16
- Computation based on FORCER program in FORM Ruijl, Ueda, Vermaseren '17



Splitting functions at N^{3}LO

- Leading color (large-n_c) non-singlet anomalous dimension $\gamma_{\rm ns}^{(3)\,\pm}(N)$ known at N^{3}LO S.M., Ruijl, Ueda, Vermaseren, Vogt to appear



- All-N result reconstructed from fixed Mellin moments with input for small-/large-x and solution of Diophantine equations

Cusp anomalous dimension

- Cusp anomalous dimension $A(\alpha_s)$ emerges in large-N limit

$$\gamma_{\rm ns}^{(n-1)}(N) \;=\; A_n \left(\ln N + \gamma_e\right) - B_n + C_n \,\frac{\ln N + \gamma_e}{N} - D_n + \mathcal{O}\left(N^{-2} \ln^{\ell} N\right)$$

- Leading color result and fermionic contributions known at four loops

$$
\begin{aligned}
A_4 = {} & \\
& C_F n_c^3 \left(\frac{84278}{81} - \frac{88832}{81}\,\zeta_2 + \frac{20992}{27}\,\zeta_3 + 1804\,\zeta_4 - \frac{352}{3}\,\zeta_2\zeta_3\right. \\
& \qquad \left. -352\,\zeta_5 - 32\,\zeta_3^2 - 876\,\zeta_6\right) \\
& + C_F n_c^2 n_f \left(-\frac{39883}{81} + \frac{26692}{81}\,\zeta_2 - \frac{16252}{27}\,\zeta_3 - \frac{440}{3}\,\zeta_4 + \frac{256}{3}\,\zeta_2\zeta_3 + 224\,\zeta_5\right) \\
& + \mathcal{O}\left(C_F n_c n_f^0\right) \;+\; \mathcal{O}\left(C_F n_c^0 n_f\right) \;+\; C_F C_A n_f^2 \left(\frac{923}{81} - \frac{608}{81}\,\zeta_2 + \frac{2240}{27}\,\zeta_3 - \frac{112}{3}\,\zeta_4\right) \\
& + C_F^2 n_f^2 \left(\frac{2392}{81} - \frac{640}{9}\,\zeta_3 + 32\,\zeta_4\right) - C_F n_f^3 \left(\frac{32}{81} - \frac{64}{27}\,\zeta_3\right)
\end{aligned}
$$

- Other calculations: n_f^3: Gracey '94, Beneke, Braun '95; $C_F^2 n_f^2$: Grozin '16; large-n_c and n_f^2: (Henn), Lee, Smirnov^2, Steinhauser '16, 17.

Summary

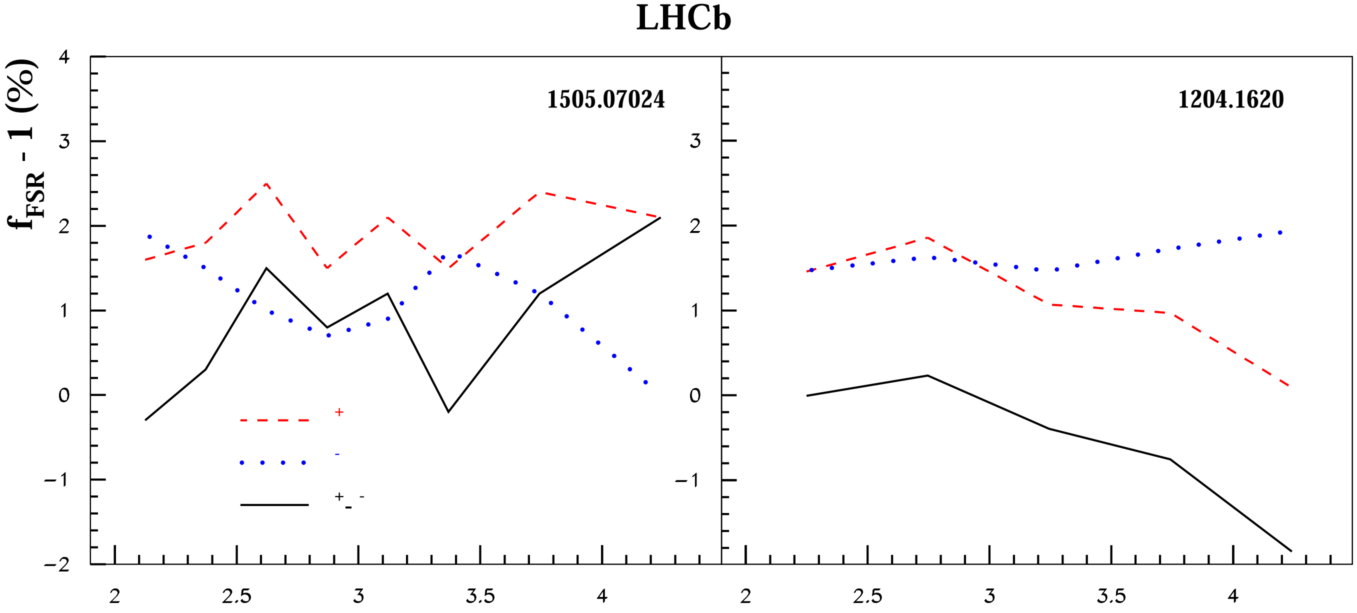
- Precision determination of non-perturbative parameters is essential
  - parton content of proton (PDFs), strong coupling constant $\alpha_s(M_Z)$, quark masses m_c, m_b, m_t
  - correlations are important and need to be taken into account
- LHC data for $W^{\pm}$- and Z-boson production provides valuable information on light flavor PDFs u, d and s over wide range of x
- Values of $\alpha_s(M_Z)$ at NNLO from measurements at colliders lower than world average
  - $\alpha_s(M_Z) = 0.118$ at NNLO not preferred by data
  - data analysis with fixed value of $\alpha_s(M_Z)$ lacks correlation with parameters of PDF fits
- Experimental precision of $\lesssim 1\%$ makes theoretical predictions at NNLO in QCD mandatory
  - Efforts towards at N^{3}LO are under way

Back-up

Theory issues

Final-state-radiation effects

- QED corrections in $W^{\pm}$- and Z-boson decays applied to data of LHCb
  - left: FSR effects from mean of simulations with Herwig++ and Pythia8 with anomalous irregularity at $\eta_\mu = 3.375$
  - right: earlier analysis of LHCb with smooth FSR corrections from PHOTOS Monte Carlo Golonka, Was '05



- Dropping problematic data points at $\eta_\mu = 3.375$ reduces χ^2 value by some 10 units