Collinear factorization breaking from effective field theory

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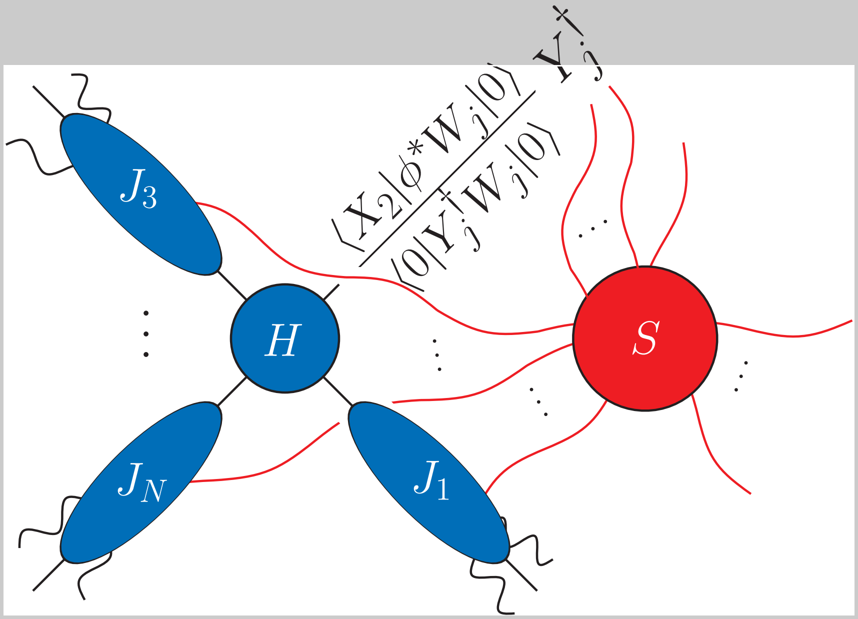
Factorization property of perturbative amplitude

$\langle X_1| \cdots \langle X_N|$: collinear, time-like separated

$\langle X_s|$: soft

$$\langle X_1 \cdots X_N; X_s| \mathcal{O} |0\rangle \cong$$

$$C(S_{ij}) \frac{\langle X_1| \phi^* W_1 |0\rangle}{\langle 0| Y_1^\dagger W_1 |0\rangle} \cdots \frac{\langle X_N| W_N^\dagger \phi |0\rangle}{\langle 0| W_N^\dagger Y_N |0\rangle} \langle X_s| Y_1^\dagger \cdots Y_N |0\rangle$$



Universal behavior of the amplitude in time-like collinear regime :

$$|\mathcal{M}\rangle = \mathbf{Sp} \cdot |\overline{\mathcal{M}}\rangle$$

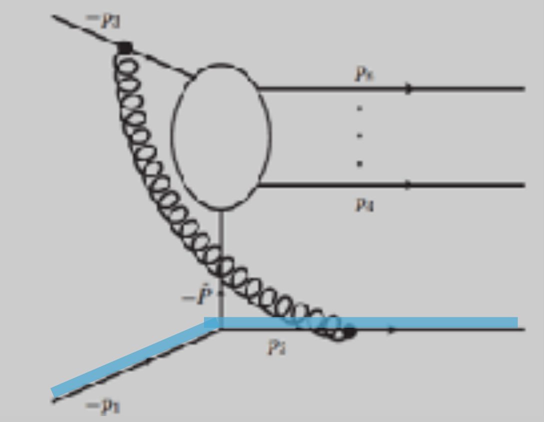
splitting amplitude depends only on collinear physics

Sp no longer universal when the collinear partons are space-like separated.
Factorization breaks down due to the long-range Coulomb/Glauber interaction.

When does factorization fail and have observable consequences?

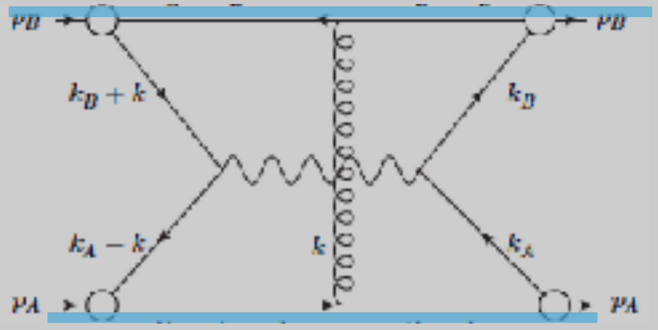
- single splitting: Generalized splitting amplitude (Catani et al) ;

  super-leading- logs in the gaps-between-jets cross-section (Forshaw, Seymour)



- double splitting: spectator-spectator Glauber exchange.

  Inclusive Drell-Yan (Colin-Soper-Sterman); More exclusive observables: spin asymmetry in beam thrust (Zeng) ; MPI sensitive observables (Gaunt);

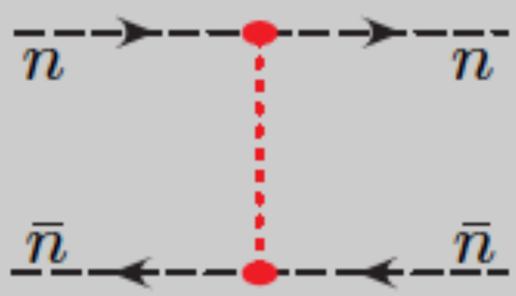


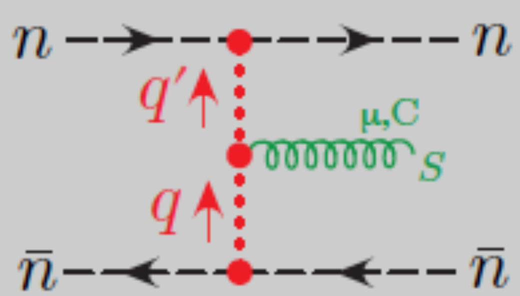
- A new Effective Lagrangian incorporating Glauber modes into the SCET framework (Rothestein, Stewart)

$$\mathcal{L}^{(0)}_{\mathrm{SCET_{II}}} = \Big[\mathcal{L}^{(0)}_{\mathrm{S}}(\psi_{\mathrm{S}}, A_{\mathrm{S}}) + \sum_{n_i} \mathcal{L}^{(0)}_{n_i}(\xi_{n_i}, A_{n_i})\Big] + \mathcal{L}^{\mathrm{II}(0)}_{G}(\{\xi_{n_i}, A_{n_i}\}, \psi_{\mathrm{S}}, A_{\mathrm{S}})$$

$$\mathcal{O}^{\mathrm{QCD}}_{G12} = \Big[\bar{\xi}_1 W_1 \frac{\bar{n}\!\!\!/_2}{2} T^A W_1^\dagger \xi_1\Big] \frac{1}{\mathcal{P}_\perp^2}$$

$$\times \Big[\mathcal{P}^\mu_\perp Y_1^\dagger Y_2 \mathcal{P}_{\perp\mu} - g\mathcal{P}^\perp_\mu \mathcal{B}^\mu_{1S\perp} Y_1^\dagger Y_2 - g Y_1^\dagger Y_2 \mathcal{B}^\mu_{2S\perp}\mathcal{P}^\perp_\mu - g\mathcal{B}^\mu_{1S\perp} Y_1^\dagger Y_2 \mathcal{B}^\mu_{2S\perp} - \frac{ig}{2} n_1^\mu n_2^\nu Y_1 \widetilde{G}^S_{\mu\nu} Y_2\Big]^{AB}$$





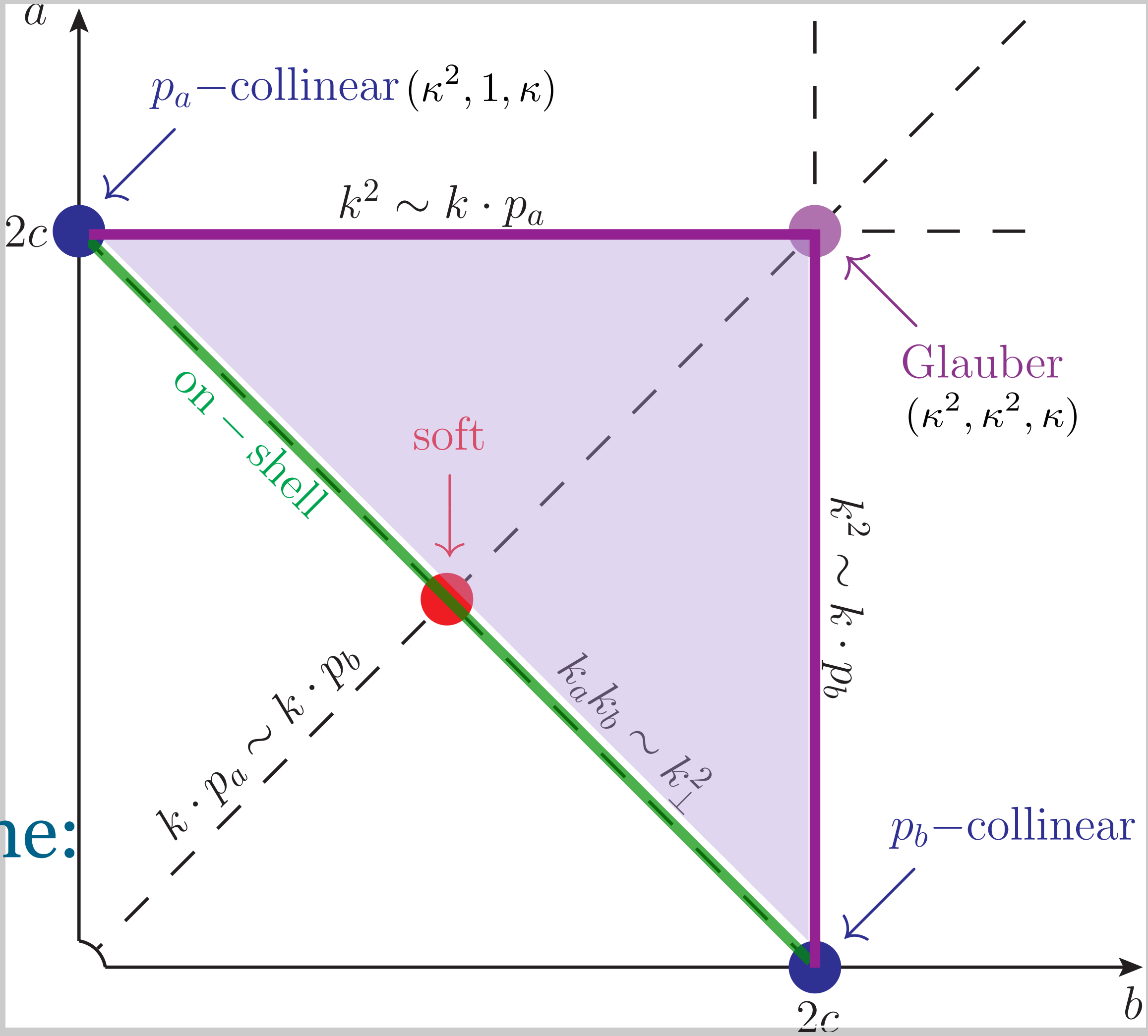
$$\times \frac{1}{\mathcal{P}_\perp^2} \Big[\bar{\xi}_2 W_2 \frac{\bar{n}\!\!\!/_1}{2} T^B W_2^\dagger \xi_2\Big]$$

allows separately calculating factorization breaking terms, advantageous in studying its physical implications.

Isolating the Glauber contribution

$$k^\mu = k_b\, n_a^\mu + k_a\, n_b^\mu + k_\perp^\mu$$

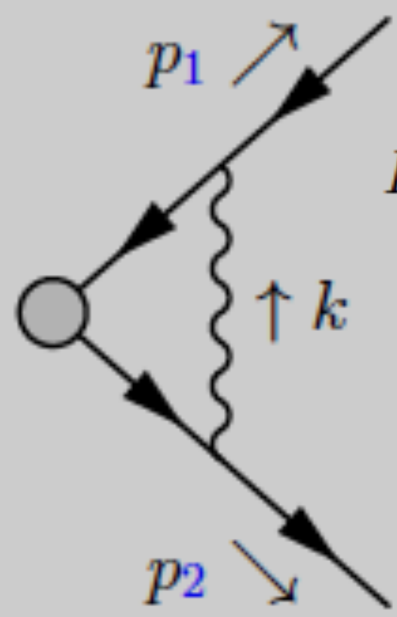
$$k_a = \frac{n_a \cdot k}{n_a \cdot n_b}, \qquad k_b = \frac{n_b \cdot k}{n_a \cdot n_b}$$



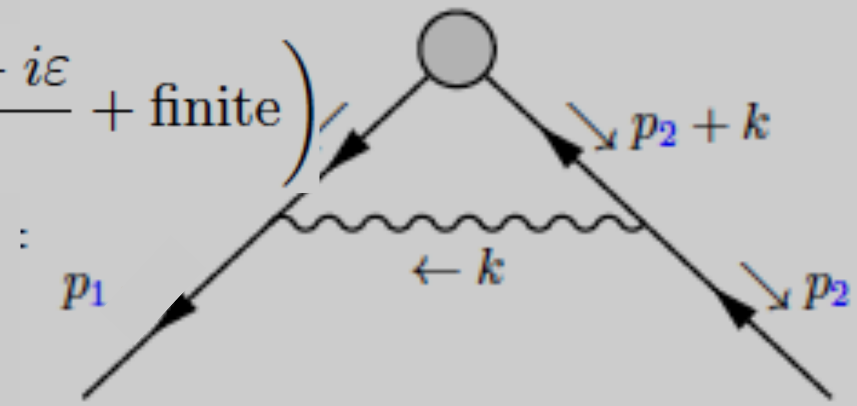
zoom into a particular infrared regime:

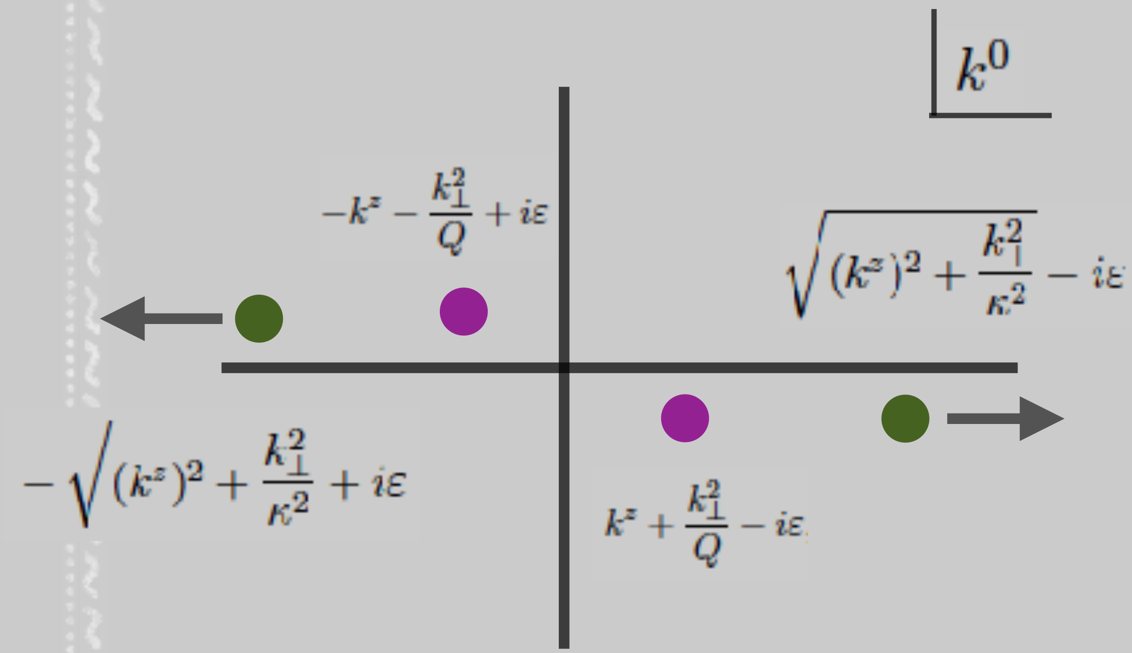
$$k^\mu \to \kappa^b k_b\, n_a^\mu + \kappa^a k_a\, n_b^\mu + \kappa^c k_\perp^\mu$$

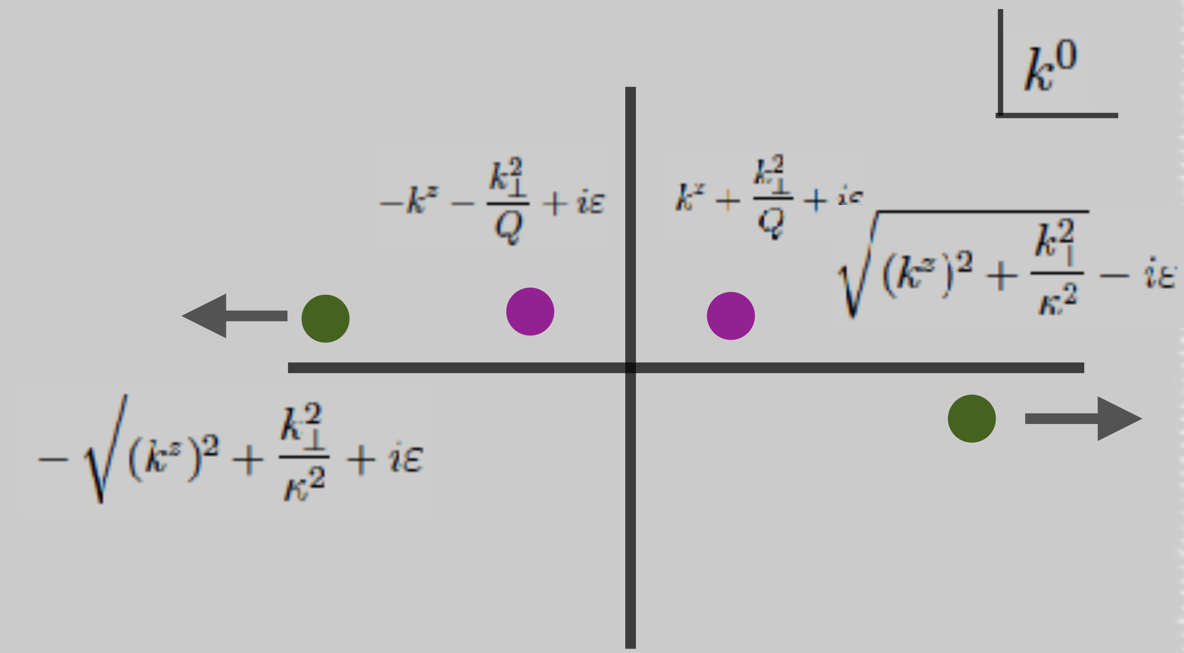
Glauber scaling: take a=b=2, c=1
zooming into an off-shell, soft regime where the validation of eikonal approximation is challenged $\quad p_j \cdot k \sim k^2,\ k^+ k^- \ll k_\perp^2$

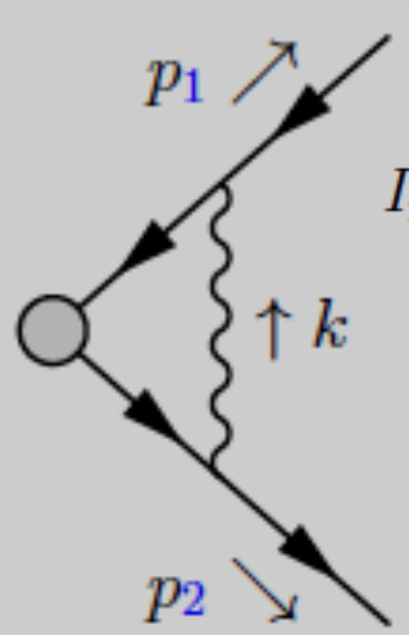


$$I_{\text{full}} = \frac{\alpha_s}{2\pi}\left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\ln\frac{-2p_1\cdot p_2 - i\varepsilon}{\mu^2} + \text{finite}\right)$$

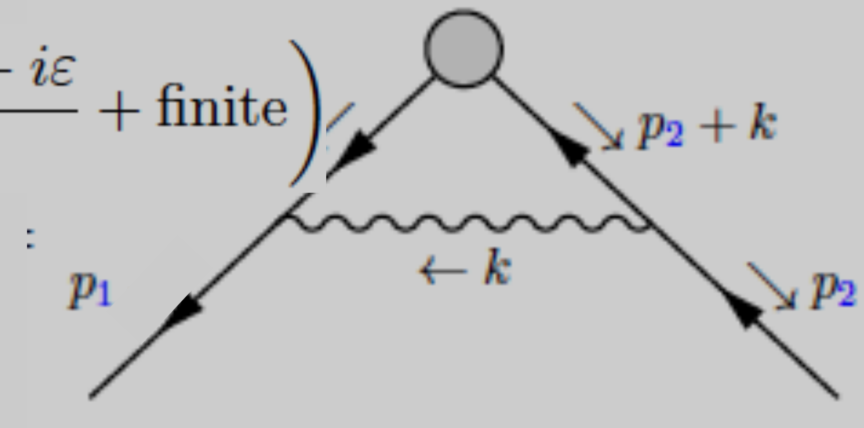




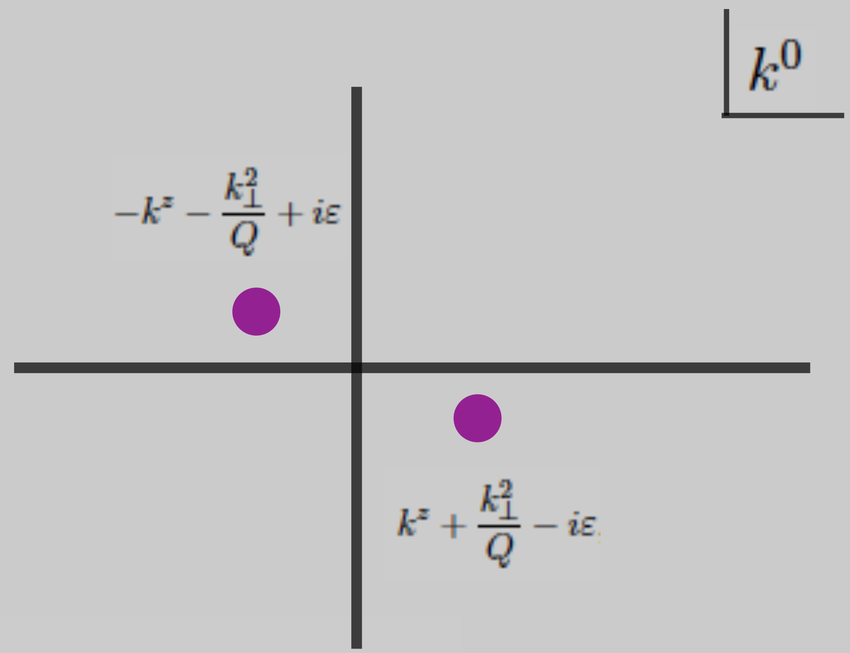


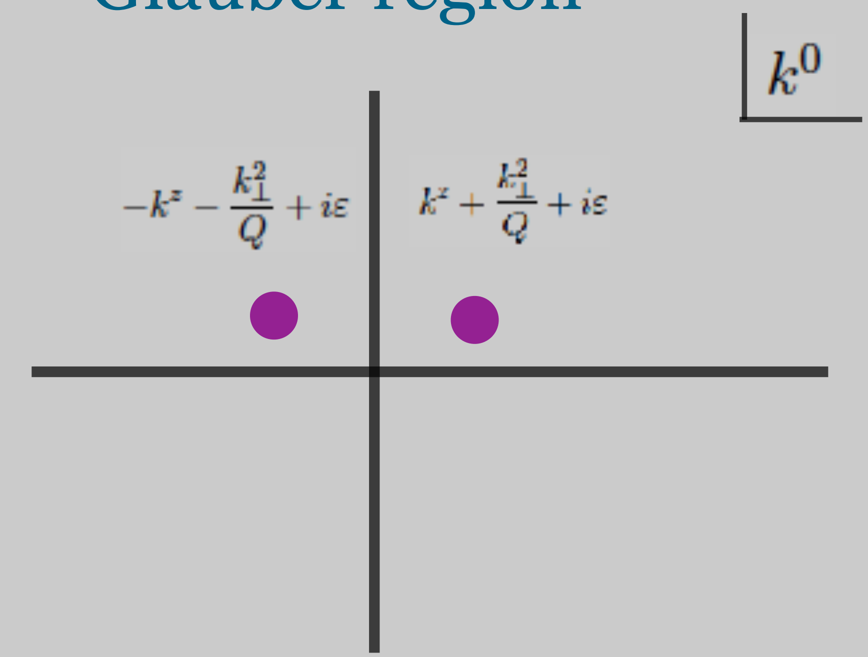


$$I_{\text{full}} = \frac{\alpha_s}{2\pi}\left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\ln\frac{-2p_1\cdot p_2 - i\varepsilon}{\mu^2} + \text{finite}\right)$$



no pinched singularity in Glauber region





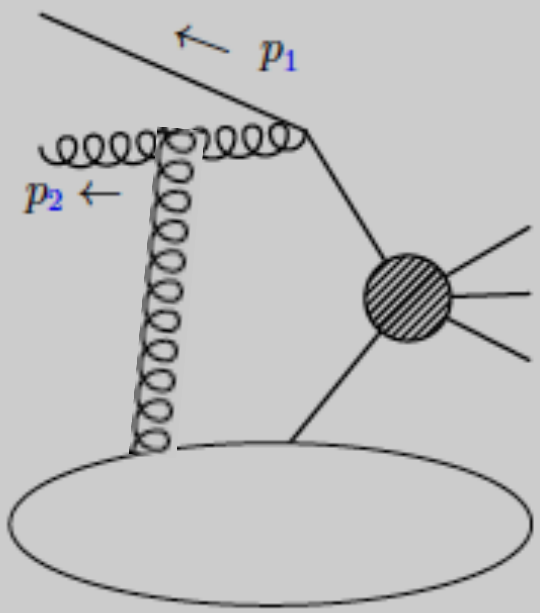
$I(s_{12}) - I(-s_{12}) =$ purely imaginary

- Glauber contribution is non-analytic in external momenta

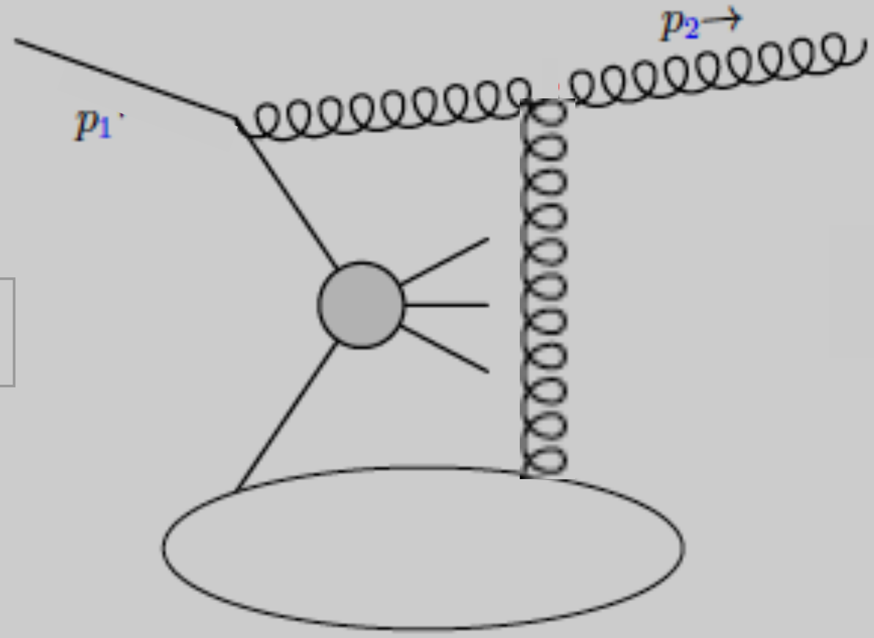
Timelike vs. spacelike splitting

Absorptive part of the soft virtual correction violates collinear factorization

TL



SL



$$\boldsymbol{T}_j \cdot \left[\boldsymbol{T}_1 \, \Theta(s_{j1}) + \boldsymbol{T}_2 \, \Theta(s_{j2}) \right]$$

$$= \boldsymbol{T}_j \cdot \boldsymbol{T}_{\tilde{P}} \, \Theta(s_{j\tilde{P}})$$

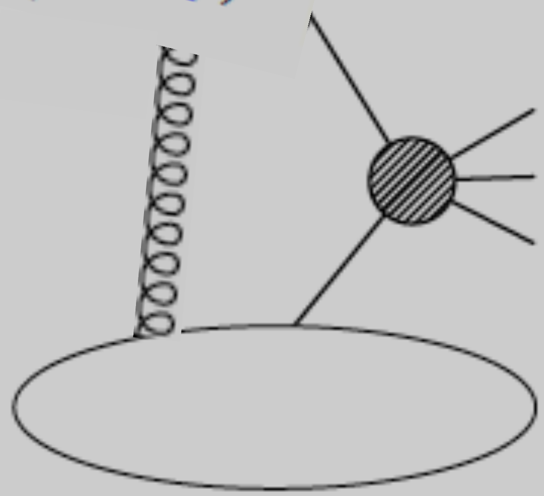
- Strict factorization holds.

- Color coherence breaks down due to contribution from the Glauber regime.

Timelike vs. spacelike splitting

$$\frac{\langle X_j | \bar{\psi} W_j | 0 \rangle^{h'}}{\mathrm{tr}\, \langle 0 | Y_j^\dagger W_j | 0 \rangle} \times (Y_j^\dagger)^{h'h}$$

TL



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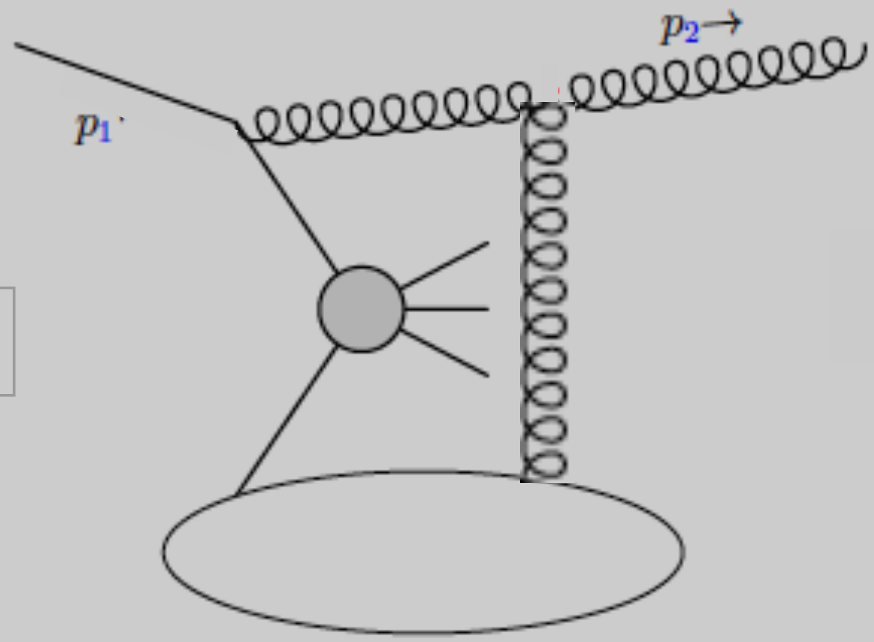


$$\boldsymbol{T}_j \cdot \left[\boldsymbol{T}_1 \, \Theta(s_{j1}) + \boldsymbol{T}_2 \, \Theta(s_{j2}) \right]$$

$$= \boldsymbol{T}_j \cdot \boldsymbol{T}_{\tilde{P}} \, \Theta(s_{j\tilde{P}})$$

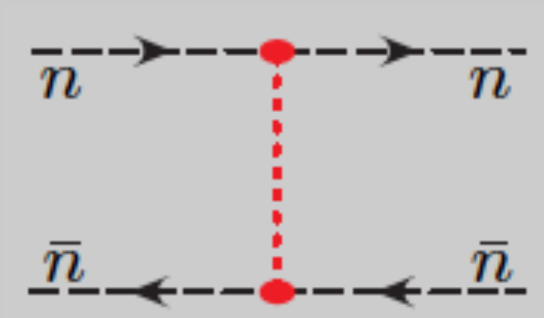
- Strict factorization holds.

- Color coherence breaks down due to contribution from the Glauber regime.

How the SCET approach works

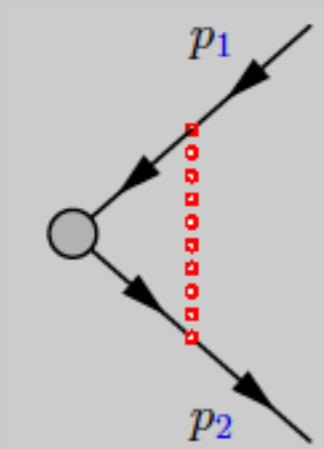
SCET Glauber operator

with the conspiracy of power expansion and rapidity regulator :

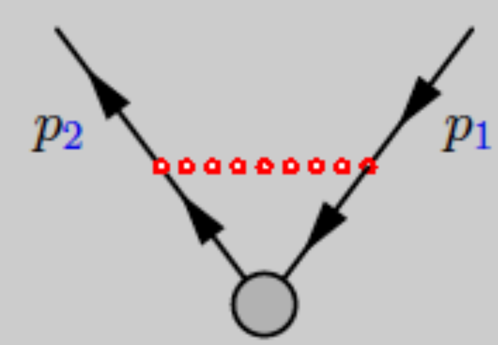


$$\propto |2\mathcal{P}^z|^{-\eta}\frac{1}{\mathcal{P}_\perp^2}$$

- reproduce the non-analytic property of Glauber contribution to the one-loop vertex correction



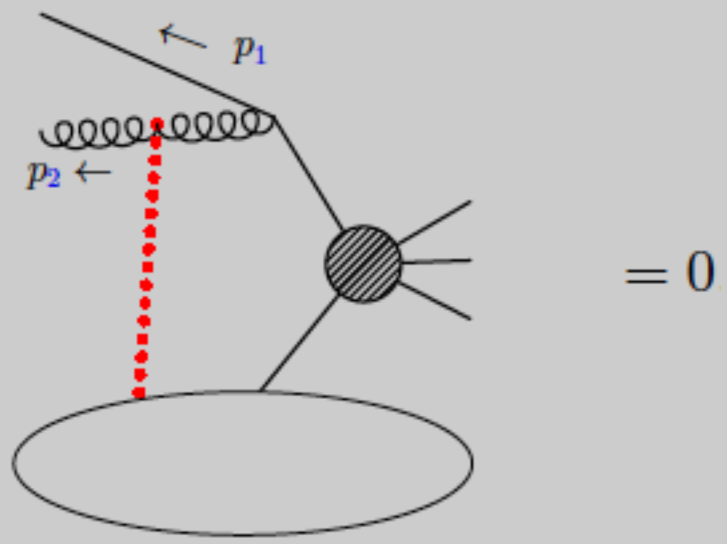
$$= \frac{\alpha_s}{2\pi}\frac{i\pi}{\epsilon_{\rm IR}}$$

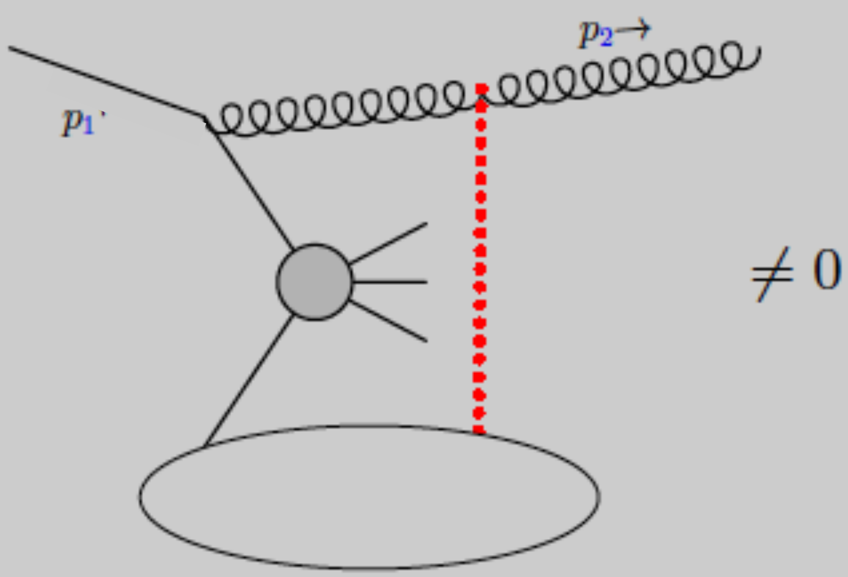


$$= 0$$

The splitting is computed from emissions of an amplitude with (n-m) collinear sectors, rather than by taking collinear limits of n parton amplitudes

- respect collinear factorization in time like regime





Generalized splitting amplitude

$$|\mathcal{M}\rangle \simeq \boldsymbol{Sp}(p_1, \ldots, p_m; \widetilde{P}; \underline{p_{m+1}, \ldots, p_n}) \; |\overline{\mathcal{M}}\rangle$$

depend on color and momenta on the non-collinear partons

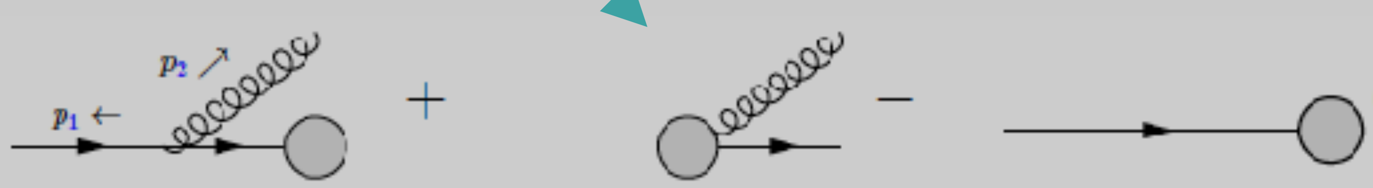
Catani's approach

- Extract from the collinear limit of the n-point formula, compared with the (n-m)-point formula

Reformulation in SCET

- Separately calculate

$$\mathbf{Sp}^{\text{fact}} = \frac{\langle p_2 | \, W_1^\dagger \, \psi \, | p_1 \rangle}{\langle 0 | \, W_1^\dagger \, \psi \, | P \rangle} \qquad + \qquad \mathbf{Sp}^{\text{non-fac.}}$$



diagrams with insertions of Glauber operators

Catani's formula

IR divergences of one-loop splitting amplitude:

$$= \frac{\alpha_S(\mu^2)}{2\pi} \frac{1}{2} \left\{ \left(\frac{1}{\epsilon^2} C_{\widetilde{P}} + \frac{1}{\epsilon} \gamma_{\widetilde{P}} \right) - \sum_{i \in C} \left(\frac{1}{\epsilon^2} C_i + \frac{1}{\epsilon} \gamma_i - \frac{2}{\epsilon} C_i \ln |z_i| \right) \right.$$

$$\left. - \frac{1}{\epsilon} \sum_{\substack{i,\ell \in C \\ i \neq \ell}} \boldsymbol{T}_i \cdot \boldsymbol{T}_\ell \ln \left(\frac{-s_{i\ell} - i0}{|z_i|\,|z_\ell|\,\mu^2} \right) \right\} + \boldsymbol{\Delta}^{(1)}_{mC}(\epsilon)$$

radiative part; real; factorized

absorptive part; purely imaginary

$$\boldsymbol{\Delta}^{(1)}_{mC}(\epsilon) = \frac{\alpha_S(\mu^2)}{2\pi} \frac{i\pi}{\epsilon} \sum_{\substack{i \in C \\ j \in NC}} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, \Theta(-z_i) \, \mathrm{sign}(s_{ij})$$

dependence on non-collinear dynamics

IR divergences of two-loop splitting amplitude:

$$\mathbf{Sp}^2 = \left[\boldsymbol{\Delta}^2_C(\epsilon) + \frac{1}{2}(\mathbf{I}^1_C)^2\right] \cdot \mathbf{Sp}^0 + \frac{1}{\epsilon} \text{ terms} + \text{finite}$$

exponentiation of one-loop result

two-loop non-abelian web; subleading pole

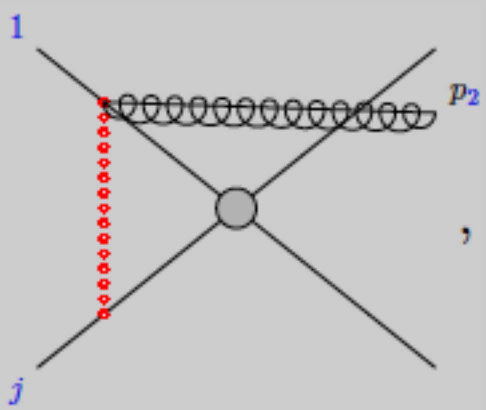
$$[I(\epsilon), \overline{I}(\epsilon)] =$$

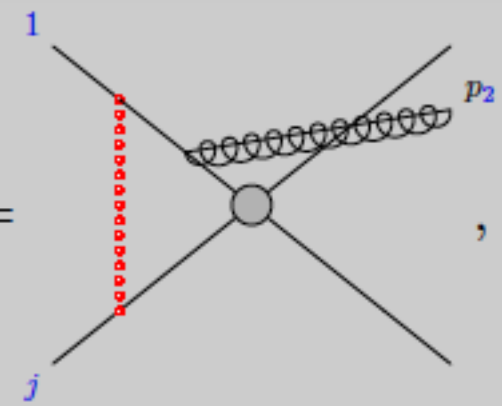
(leading pole; do not exponentiation)

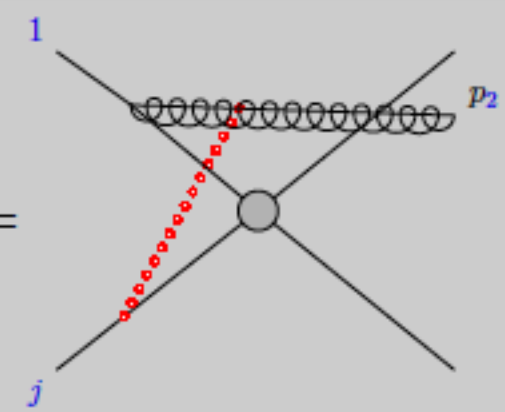
$$\boldsymbol{\Delta}^2_C(\epsilon) = \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{-s_{12} - i\varepsilon}{\mu^2}\right)^{-2\epsilon} \pi f_{abc} \sum_{i=1,2} \sum_{j,k=3}^{n} \mathbf{T}^a_i \mathbf{T}^b_j \mathbf{T}^c_k \Theta(-z_i) \mathrm{sign}(s_{ij}) \Theta(-s_{jk})$$
$$\times \ln\left(-\frac{s_{jP} s_{kP} z_1 z_2}{s_{jk} s_{12}} - i\varepsilon\right) \left[-\frac{1}{2\epsilon^2} + \frac{1}{\epsilon} \ln\left(\frac{-z_i}{1 - z_i}\right)\right]$$

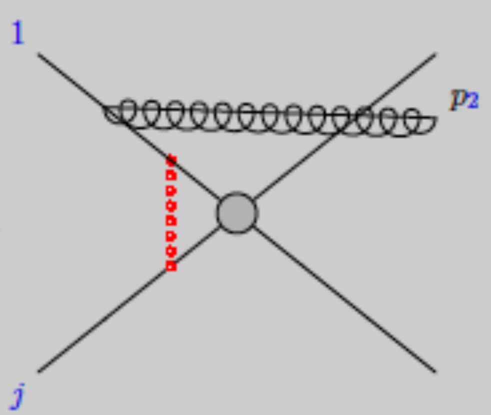
One-loop splitting amplitude in SCET

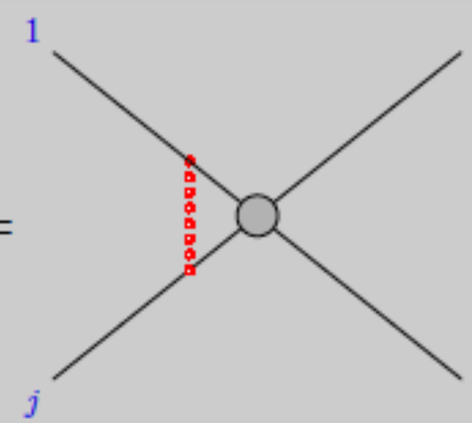
$$\mathbf{Sp}^{1,\text{non-fac.}} = |\mathcal{M}^1\rangle_{\text{Glauber}} - \mathbf{Sp}^0|\overline{\mathcal{M}}^1\rangle_{\text{Glauber}}$$

$I^j_{\text{Gc}} =$ 

, $I^j_{\text{Gd}} =$ 

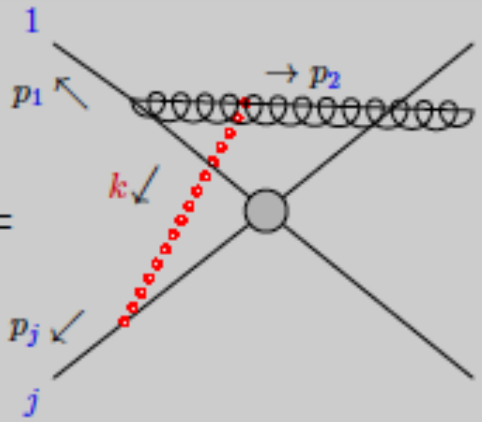
, $I^j_{\text{Ge}} =$ 

$I_{\text{Gb}} =$ 

$- \mathbf{Sp}^{(0)} \cdot \bar{I}_{\text{Gb}} =$ 

non-vanishing only if glauber exchange takes place **after** the splitting : Glauber loop cannot interrupted

spectator-active interaction

$I^{j}_{\text{Ge}} =$ 

$$= (\mathbf{T}_2 \cdot \mathbf{T}_j) \int \frac{d^d k}{(2\pi)^d} \frac{1}{|2k^z|^\eta} \frac{1}{k^+ + \delta_j \mp i\varepsilon} \frac{1}{k^- - \delta_2 + i\varepsilon} \frac{1}{k^- - \delta_1 - i\varepsilon} \frac{N^\mu}{\vec{k}_\perp^2}$$

sign depends on the direction of the non-collinear parton

summing over j:

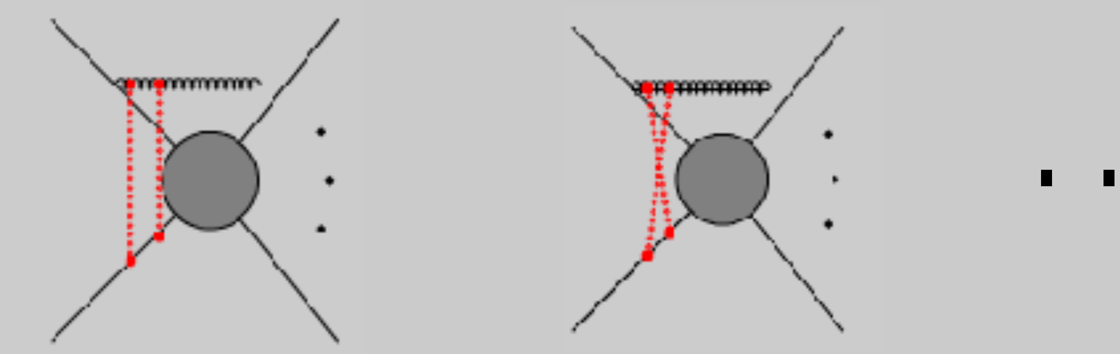
$$\mathbf{Sp}^{1,\text{non-fact}} = \frac{\alpha_s}{2\pi}(4\pi e^{-\gamma_E})^\epsilon (i\pi) \left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-2p_1 \cdot p_2} + \ln\frac{z-1}{z}\right) \left(-\mathbf{T}_2 \cdot \mathbf{T}_3 + \sum_{j>3} \mathbf{T}_2 \cdot \mathbf{T}_j\right) \mathbf{Sp}^0$$

agrees with Catani's formula $\Delta^{(1)}(\epsilon)$

Two-loop splitting amplitude in SCET

$$\mathbf{Sp}^{2,\text{non-fac.}} \cdot |\overline{\mathcal{M}}^0\rangle = |\overline{\mathcal{M}}^2\rangle_{\text{double Glauber}} - \mathbf{Sp}^{1,\text{non-fac.}} \cdot |\overline{\mathcal{M}}^1\rangle_{\text{Glauber loops}} + |\overline{\mathcal{M}}^2\rangle^{\text{non-fac.}}_{\text{Glauber-soft}} + |\overline{\mathcal{M}}^2\rangle^{\text{non-fac.}}_{\text{Glauber-coll.}} - \mathbf{Sp}^{1,\text{non-fac.}} \cdot |\overline{\mathcal{M}}^1\rangle_{n_1-\text{coll. loops}}$$

First line:

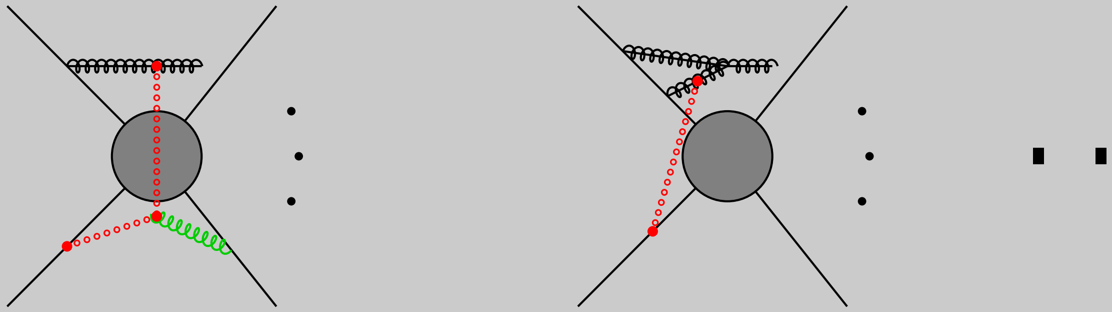


anti-hermitian (real) two-loop factorization violation term (Catani)

exponentiated one-loop

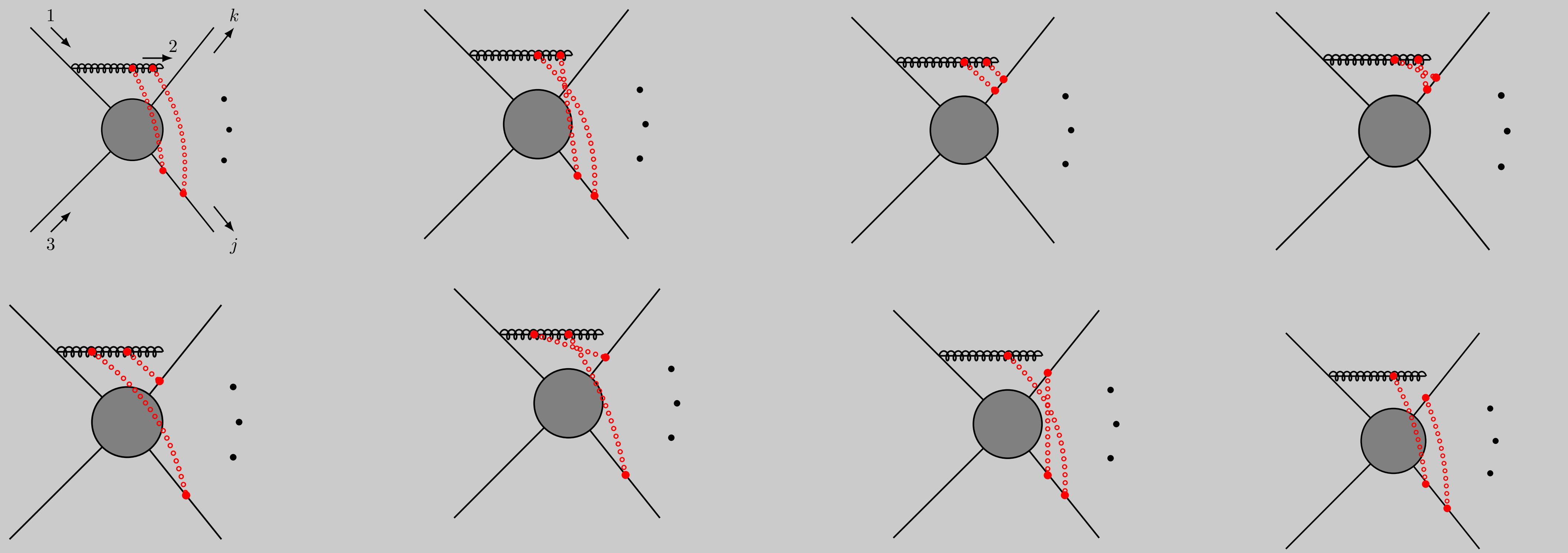
$$\overset{\text{double pole}}{=} \left\{ \frac{1}{2}\left[\frac{\alpha_s}{2\pi}\frac{i\pi}{\epsilon}\left(-\mathbf{T}_2\cdot\mathbf{T}_3 + \sum_{j=4}^{m}\mathbf{T}_2\cdot\mathbf{T}_j\right)\right]^2 - \frac{\alpha_s^2}{4\epsilon^2}\sum_{j=4}^{m} i f_{abc}\,\mathbf{T}_2^a\,\mathbf{T}_3^b\,\mathbf{T}_j^c \right\} \mathbf{Sp}^0\,\overline{\mathcal{M}}^0$$

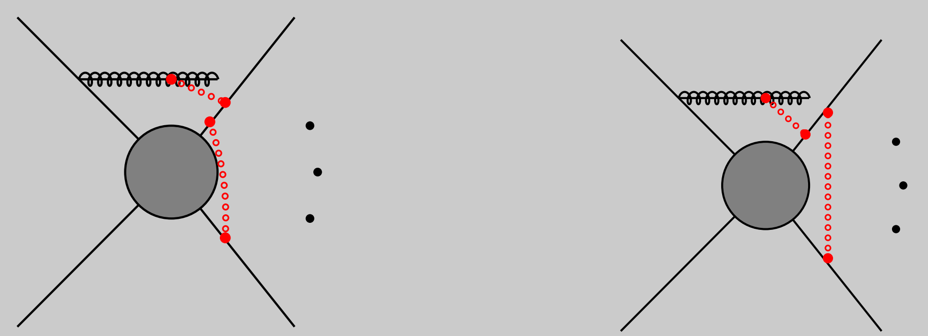
Second line:



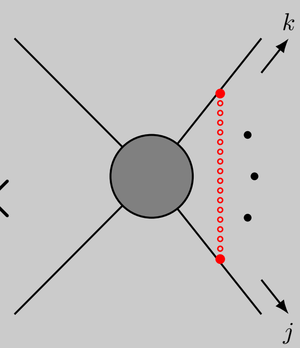
soft/collinear-glauber mixing effects, purely imaginary

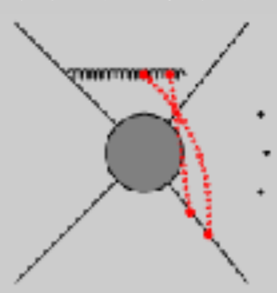
Both Glaubers on outgoing legs:
strict exponentiation of one-loop diagrams



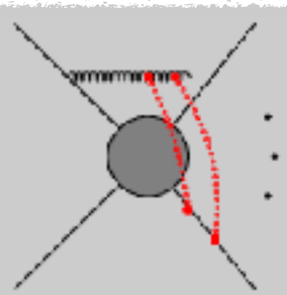


$$- \mathbf{Sp}^{(1),\text{non-fac.}} \times$$



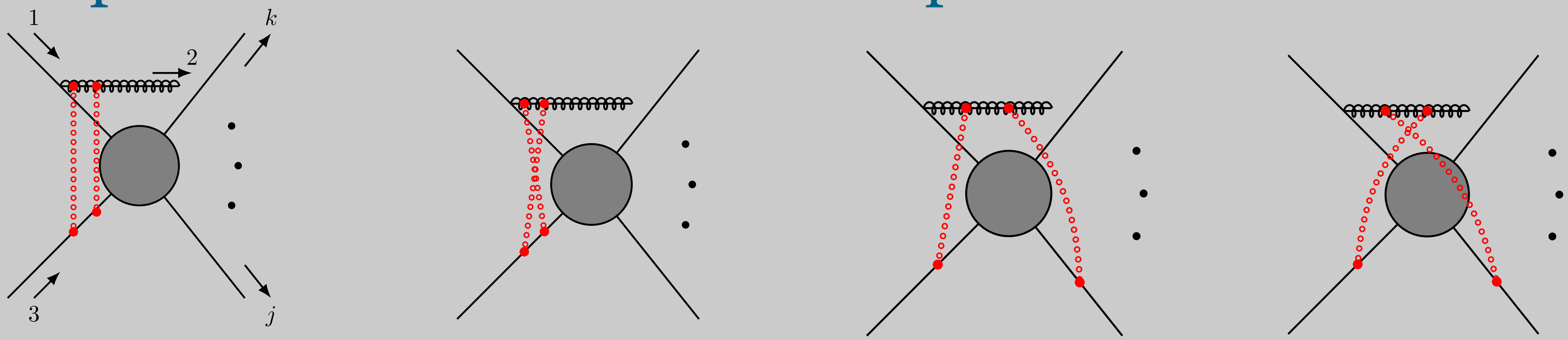


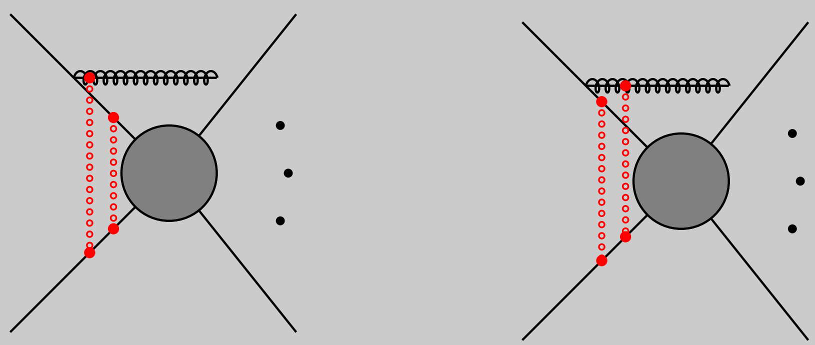
$= 0$



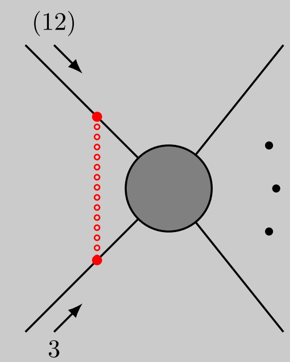
$$\overset{\text{double pole}}{=} \frac{1}{2!} (\mathbf{T}_2 \cdot \mathbf{T}_j)^2\, \mathbf{Sp}^0\, \overline{\mathcal{M}}^0 \left(\frac{\alpha_s}{2\pi}\right)^2 (i\pi)^2\, \frac{1}{\epsilon^2}$$

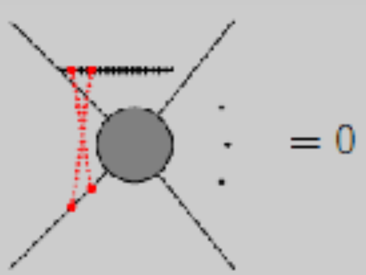
At least one Glauber on an incoming leg: two-loop non-abelian terms do not exponentiate



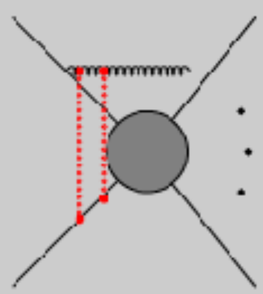


$$- \mathbf{Sp}^{(1),\text{non-fac.}} \times$$

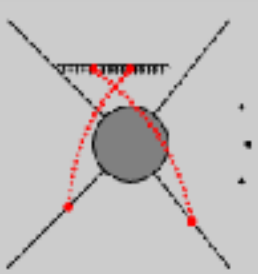




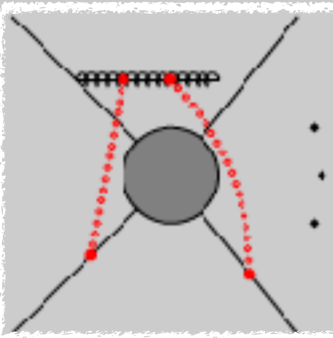
$= 0$



$$\stackrel{\text{double pole}}{=} \frac{1}{2!}\left[(\mathbf{T}_2\cdot\mathbf{T}_3)^2 + C_A\,\mathbf{T}_2\cdot\mathbf{T}_3\right]\,\mathbf{Sp}^0\,\overline{\mathcal{M}}^0\left(\frac{\alpha_s}{2\pi}\right)^2 (i\pi)^2\,\frac{1}{\epsilon^2}$$



$= 0$



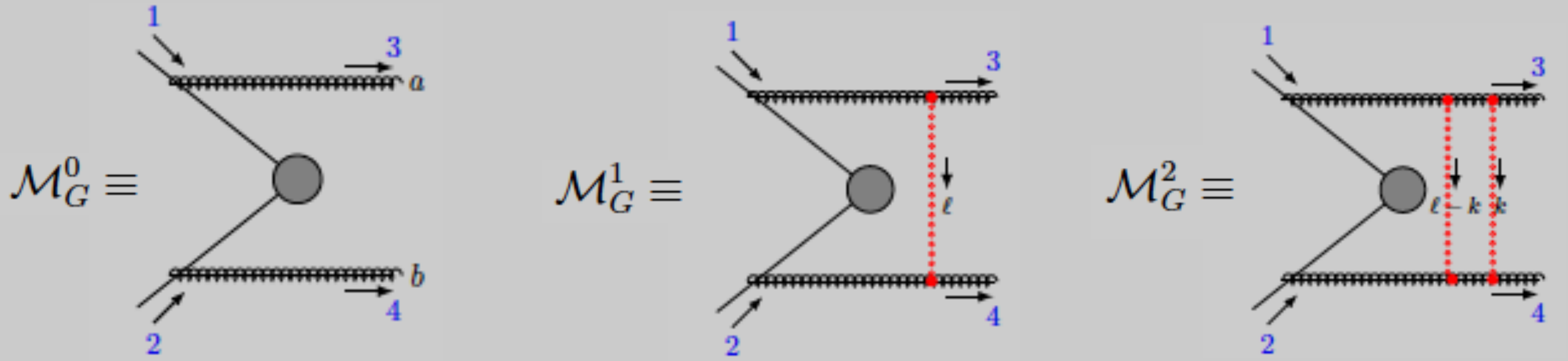
$$\stackrel{\text{double pole}}{=} \left(-\frac{1}{2}(\mathbf{T}_2\cdot\mathbf{T}_j)(\mathbf{T}_2\cdot\mathbf{T}_3) - \frac{i}{2} f_{abc}\,\mathbf{T}_2^a\,\mathbf{T}_3^b\,\mathbf{T}_j^c\right)\mathbf{Sp}^0\,\overline{\mathcal{M}}^0\left(\frac{\alpha_s}{2\pi}\right)^2 (i\pi)^2\,\frac{1}{\epsilon^2}$$

$$\stackrel{\text{double pole}}{=} \left\{ \frac{1}{2}\left[\frac{\alpha_s}{2\pi}\frac{i\pi}{\epsilon}\left(\mathbf{T}_2\cdot\mathbf{T}_3 + \sum_{j=4}^{m} \mathbf{T}_2\cdot\mathbf{T}_j\right)\right]^2 \quad \frac{\alpha_s^2}{4\epsilon^2}\sum_{j=4}^{m} i f_{abc}\,\mathbf{T}_2^a\,\mathbf{T}_3^b\,\mathbf{T}_j^c\right\}\mathbf{Sp}^0\,\overline{\mathcal{M}}^0$$

- Fully reproduce strict-factorization violating terms in DIS or Drell-Yan like processes.
- Structure of exponentiated IR singularity achieved by the non-analytic rapidity regulator
- soft-glauber mixing and collinear-glauber mixing diagrams are separately rapidity divergent, more challenging to compute

Double splitting amplitude

Spectator-spectator Glauber interaction



Take $p_1^\mu = Q_1 n^\mu \quad p_2^\mu = Q_2 \bar{n}^\mu$

result depends three mass terms in 2d Euclidean space: $\vec{p}_{3,\perp}^{\,2} \quad \vec{p}_{4,\perp}^{\,2} \quad (\vec{p}_{3,\perp} + \vec{p}_{4,\perp})^2$

IR divergences cancel in the squared amplitude at α_s^4

double pole:

$$\left(\mathcal{M}_G^1\Big|_{\epsilon^{-1}}\right)\left(\mathcal{M}_G^1\Big|_{\epsilon^{-1}}\right)^* + \left(\mathcal{M}_G^2\Big|_{\epsilon^{-2}}\right)\left(\mathcal{M}_G^0\Big|_{\epsilon^{0}}\right)^* + \left(\mathcal{M}_G^0\Big|_{\epsilon^{0}}\right)\left(\mathcal{M}_G^2\Big|_{\epsilon^{-2}}\right)^* = 0$$

single pole:

$$\left(\mathcal{M}_G^1\Big|_{\epsilon^{-1}}\right)\left(\mathcal{M}_G^1\Big|_{\epsilon^{0}}\right)^* + \left(\mathcal{M}_G^2\Big|_{\epsilon^{-1}}\right)\left(\mathcal{M}_G^0\Big|_{\epsilon^{0}}\right)^* = 0$$

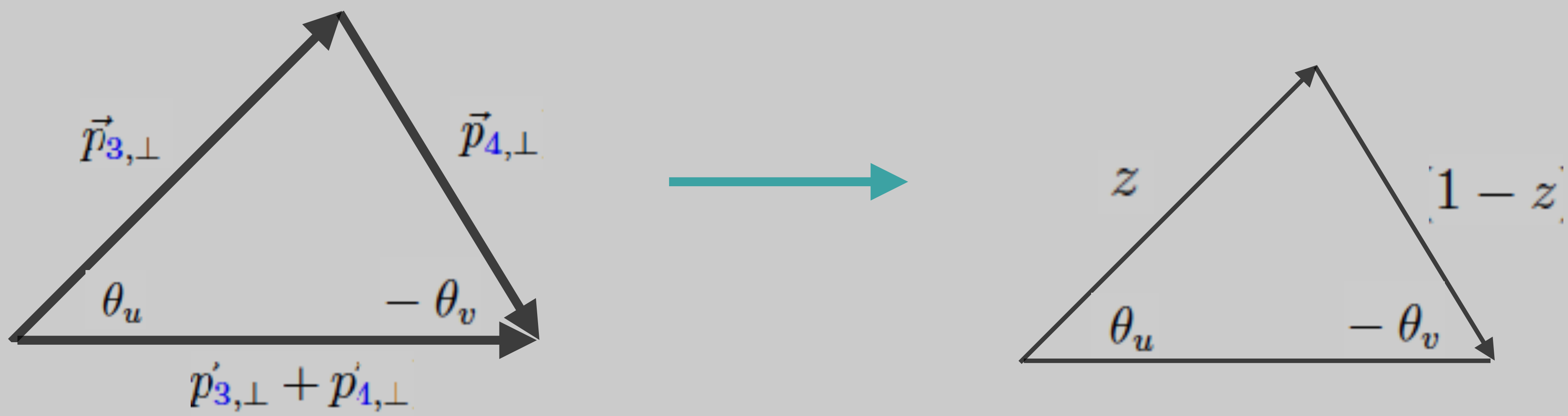
(work in progress)

Scale invariance of the matrix element

depend only on dimensionless parameter (u, v)
conveniently parametrized by (z , $\bar{z}$) (Chevez,Duhr)

$$z\bar{z} \equiv u, \quad (1-z)(1-\bar{z}) \equiv v$$

z



$$q_T^2 \equiv (\vec{p}_{3,\perp} + \vec{p}_{4,\perp})^2, \quad u \equiv \frac{\vec{p}_{3,\perp}^{\,2}}{q_T^2}, \quad v \equiv \frac{\vec{p}_{4,\perp}^{\,2}}{q_T^2}$$

$$z = \sqrt{u}e^{i\theta_u} = 1 - \sqrt{v}e^{i\theta_v}$$

phase-space integral $\displaystyle \int d^2p_{3,\perp}d^2p_{4,\perp} = \frac{1}{8}\int d^2q_T\, q_T^2 \int dz d\bar{z}$

need to handle the singularity at $z = 0, 1$

(work in progress)

add and subtract a subtraction term $|\mathcal{M}|^2_{\rm sub}(\epsilon)$

$$\sum_{h_3,h_4}\left\{|\mathcal{M}^1_G|^2+\mathcal{M}^2_G\cdot(\mathcal{M}^0_G)^*+\mathcal{M}^0_G\cdot(\mathcal{M}^2_G)^*-|\mathcal{M}|^2_{\rm sub}\right\}$$
$$=\left(\frac{\alpha_s^2}{4}\right)(\mathbf{T}_3\cdot\mathbf{T}_4)^2\sum_{h_3,h_4}\left\{|\mathcal{M}^0_G(\{h_i\})|^2\left[\delta_{h_3,h_4}\ln u\ln v+\delta_{h_3,(-h_4)}\frac{1}{2}(u+v-1)\ln^2\left(\frac{u}{v}\right)\right]\right\}+\mathcal{O}(\epsilon)$$

carrying out the phase-space integral for fixed qT:

$$\frac{1}{8}q_T^2\int dz d\bar{z}\sum_{h_3,h_4}\left\{|\mathcal{M}^1_G|^2+\mathcal{M}^2_G\cdot(\mathcal{M}^0_G)^*+\mathcal{M}^0_G\cdot(\mathcal{M}^2_G)^*-|\mathcal{M}|^2_{\rm sub}\right\}$$
$$=\left(\frac{\alpha_s^2}{4}\right)(\mathbf{T}_3\cdot\mathbf{T}_4)^2\sum_{h_3,h_4}\left|\mathcal{M}^0_G(\{h_i\})\right|^2\pi\zeta_3+\mathcal{O}(\epsilon)\qquad=-\int|\mathcal{M}|^2_{\rm sub}$$

- Glauber cancels in the cross section (in accordance with CSS)
  - Cancellation requires integrating over the entire complex z-plane

(work in progress)

Outlook

- handle and cancel the IR divergences in various subtraction schemes
- the resummation of large logarithms requires a quantitative study of factorization violation
- regulator independent definition of Glauber
- determine whether Glauber has physical implications on the observable

Thank you!