Quarkonium Production within Jets

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Topical Workshop on QCD Structure of Nucleons in the Modern Era
UCLA, Los Angeles, CA
5/5/2017

NRQCD and Quarkonium Production

Fragmenting Jet Functions (FJFs)

Heavy Quarkonium FJFs

Recent LHCb Data on Quarkonia in Jets

TMD-dependent FJFs and Heavy Quarkonium

Non-Relativistic QCD (NRQCD) Factorization Formalism

Bodwin, Braaten, Lepage, PRD 51 (1995) 1125

$$\sigma(gg \to J/\psi + X) = \sum_n \sigma(gg \to c\bar{c}(n) + X)\langle \mathcal{O}^{J/\psi}(n)\rangle$$

$$n - {}^{2S+1}L_J^{(1,8)}$$

double expansion in α_s, v

NRQCD long-distance matrix element (LDME)

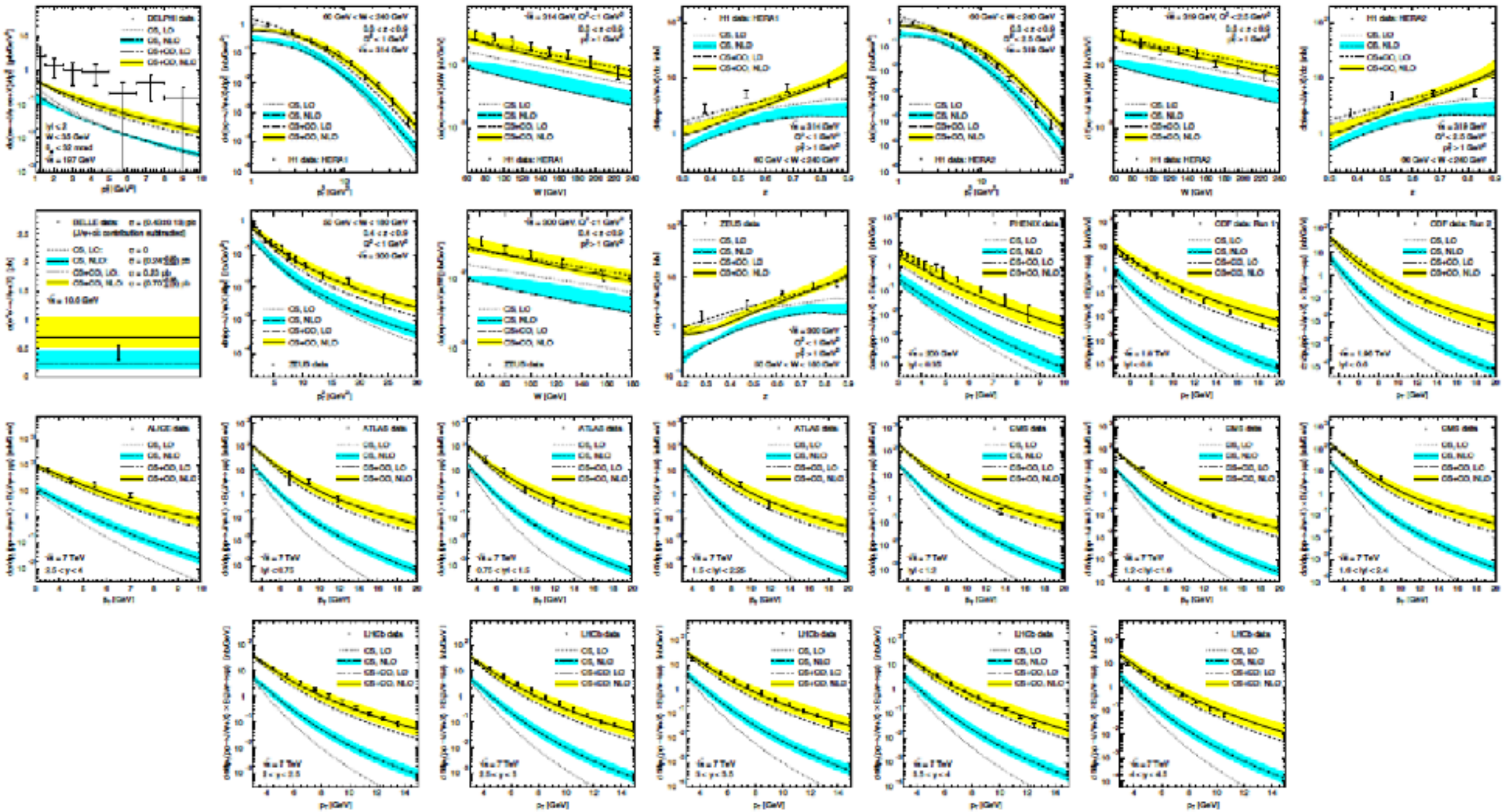
$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle \sim v^3$ CSM - lowest order in v

$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]})\rangle \sim v^7$

color-octet mechanisms

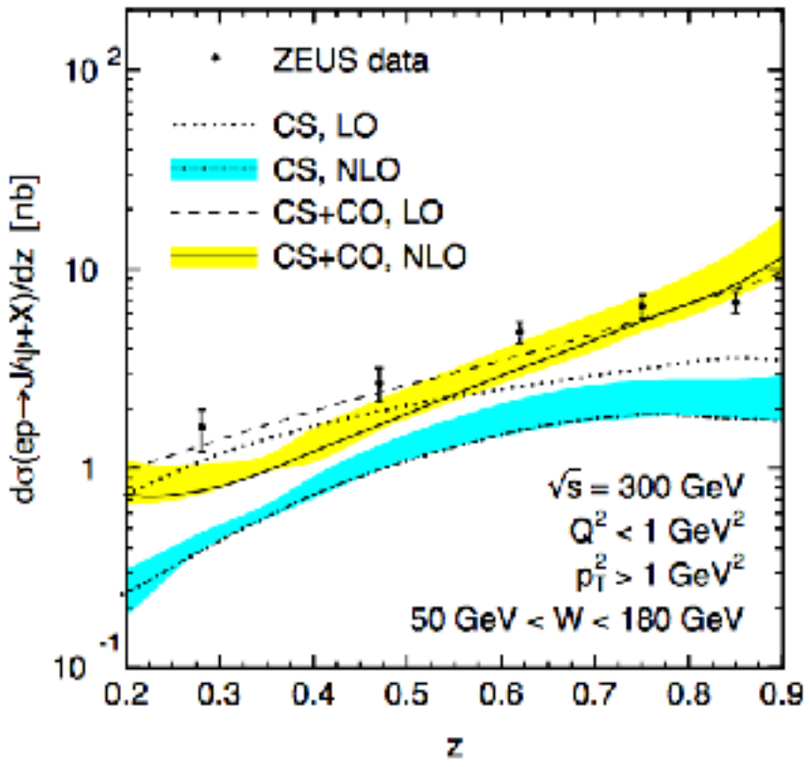
Global Fits with NLO CSM + COM

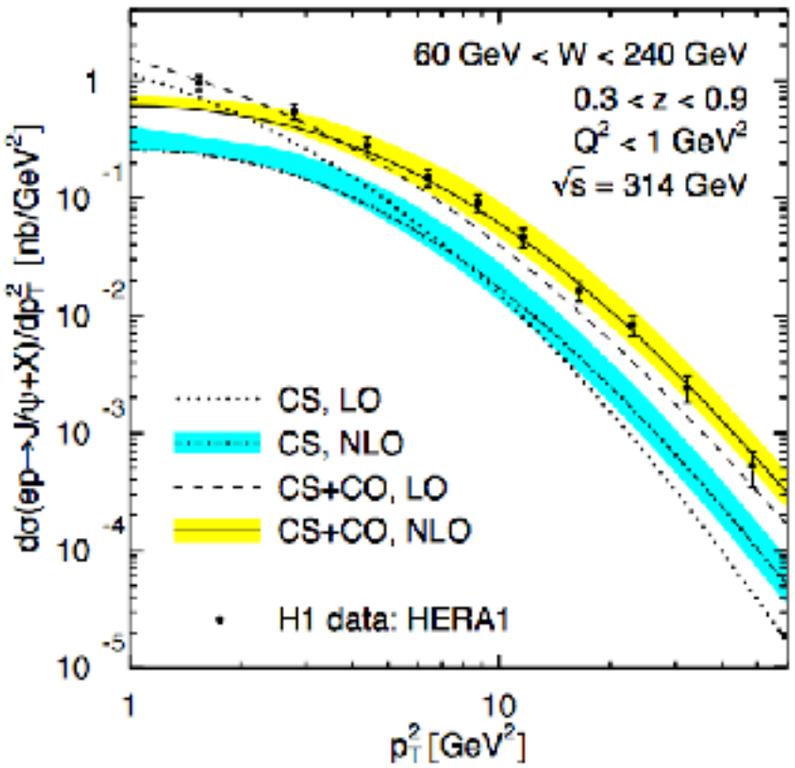
Butenschoen and Kniehl, PRD 84 (2011) 051501



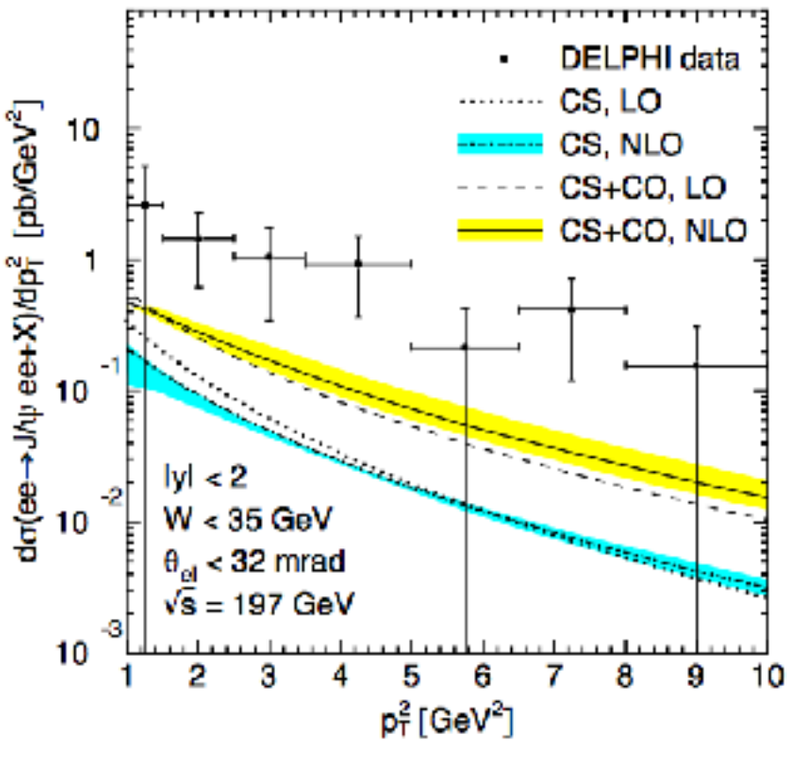
$e^+e^-, \gamma\gamma, \gamma p, p\bar{p}, pp \to J/\psi + X$ fit to 194 data points, 26 data sets

NLO: CSM + COM Required to Fit Data





$$ep \to J/\psi + X$$



$$\gamma^*\gamma^* \to J/\psi + X$$

Status of NRQCD approach to J/ψ Production

NLO: COM + CSM required for most processes

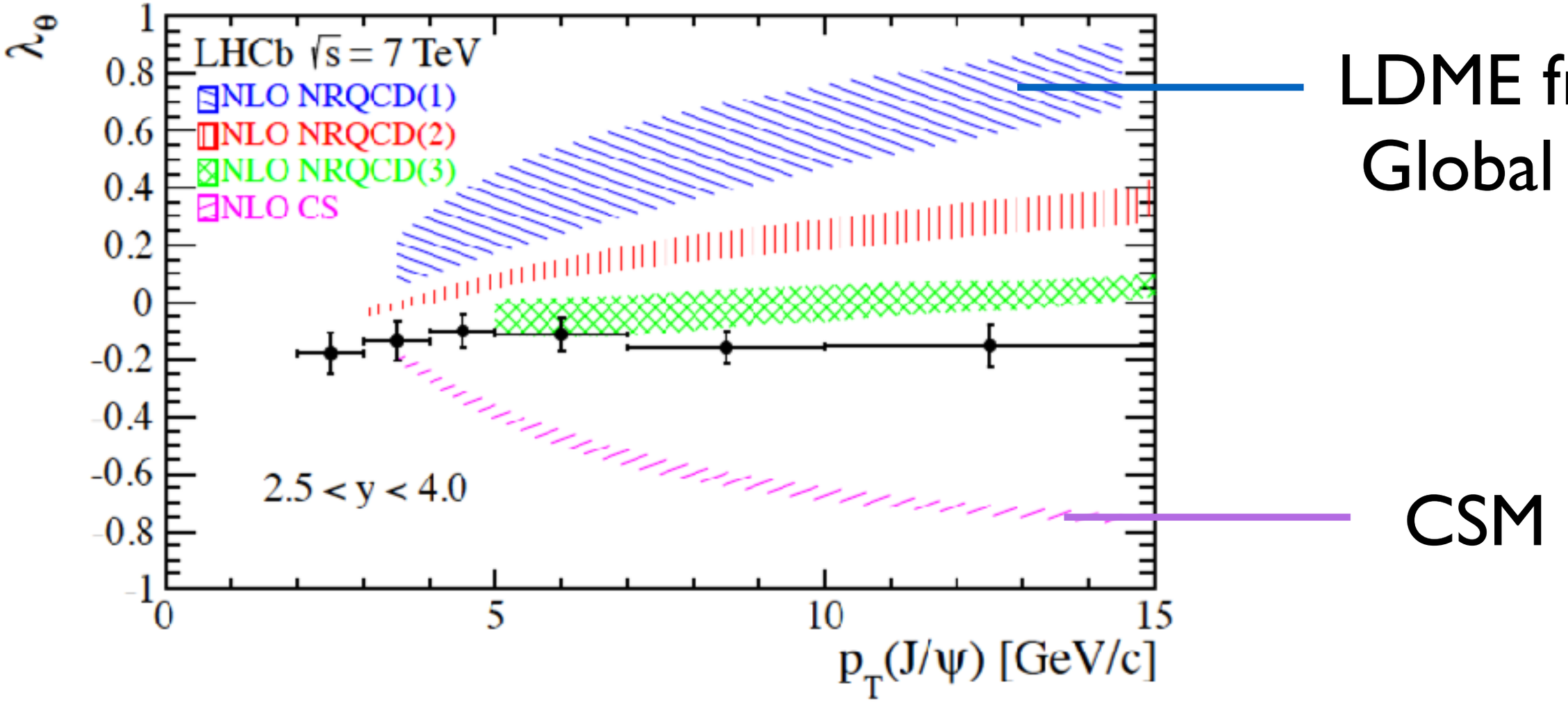
extracted LDME satisfy NRQCD v-scaling

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle = 1.32 \text{ GeV}^3$$

$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$	$(4.97 \pm 0.44) \times 10^{-2}$ GeV^3
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$	$(2.24 \pm 0.59) \times 10^{-3}$ GeV^3
$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$	$(-1.61 \pm 0.20) \times 10^{-2}$ GeV^5

$$\chi^2_{\text{d.o.f.}} = 857/194 = 4.42$$

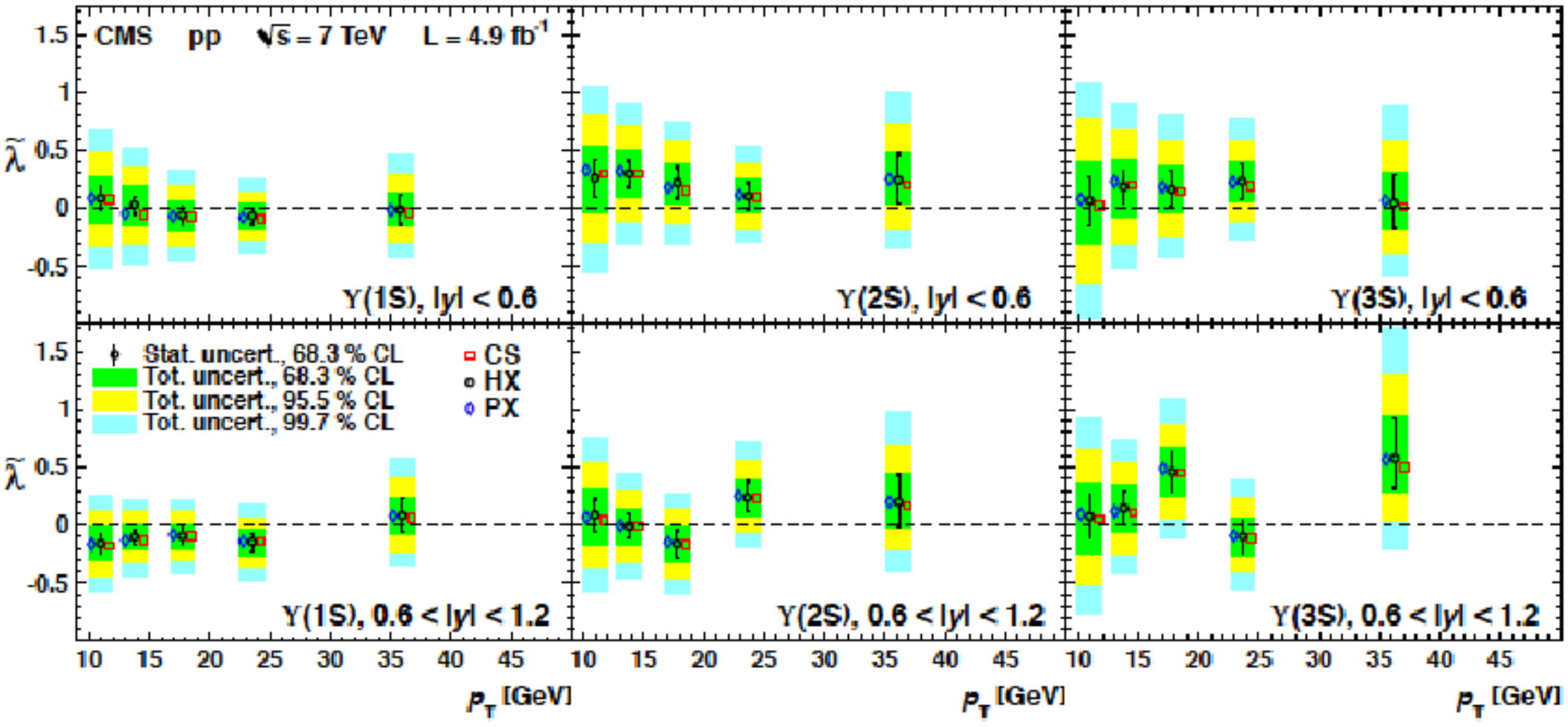
Polarization of J/ψ at LHCb



LDME from Global fits

CSM

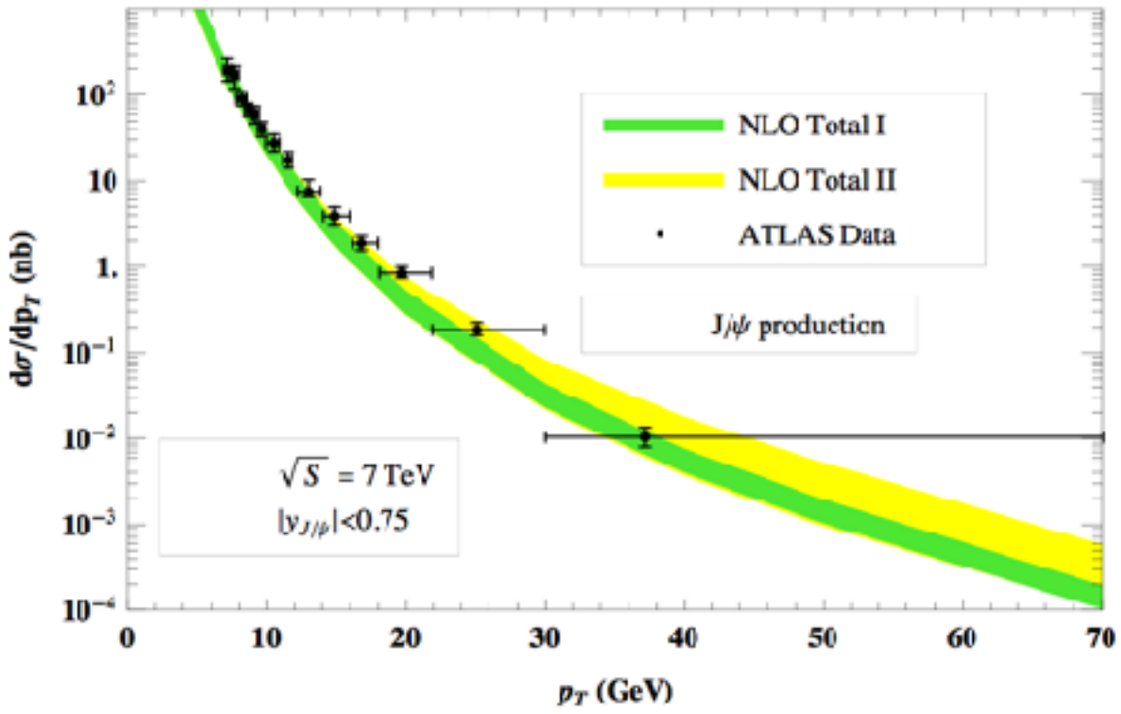
Polarization of Υ(nS) at CMS

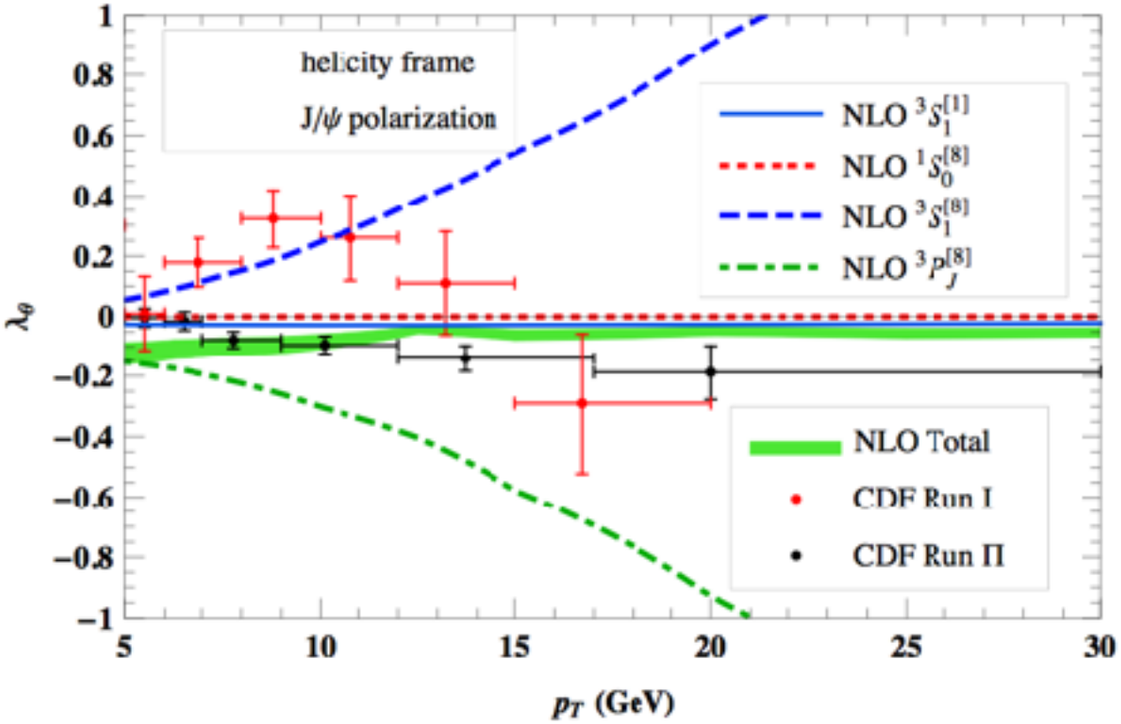


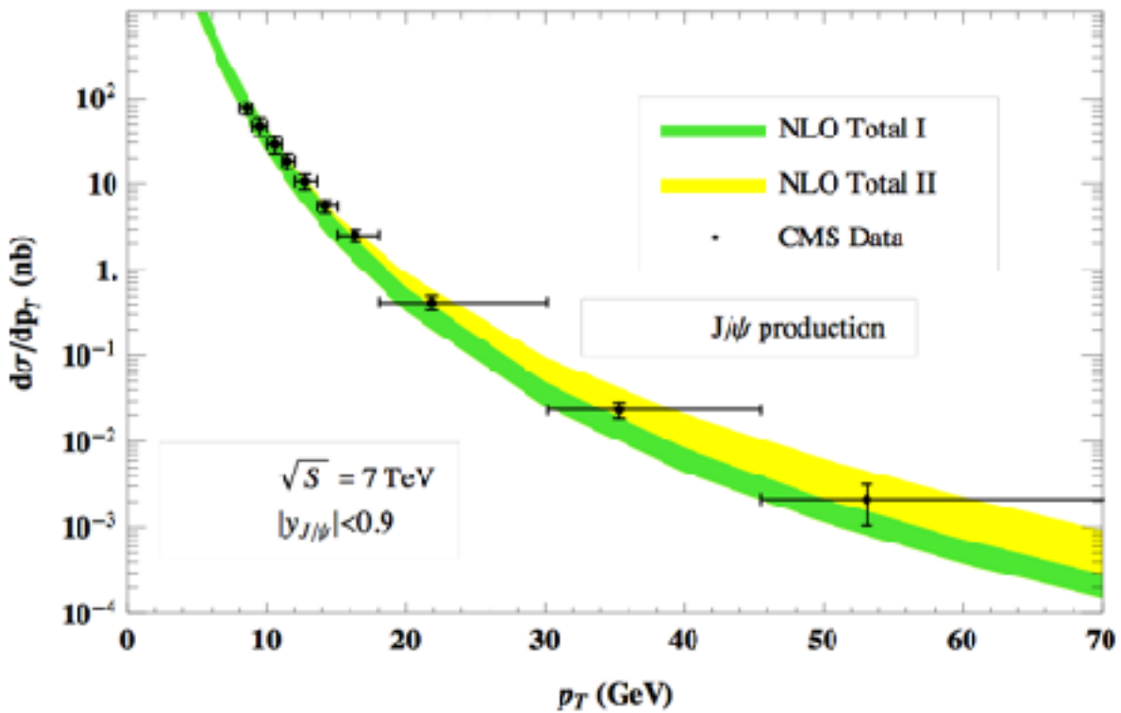
Recent Attempts to Resolve J/ψ Polarization Puzzle

simultaneous NLO fit to CMS, ATLAS high p_t production, polarization

Chao, et. al. PRL 108, 242004 (2012)







$\langle \mathcal{O}(^3S_1^{[1]}) \rangle$ GeV^3	$\langle \mathcal{O}(^1S_0^{[8]}) \rangle$ 10^{-2}GeV^3	$\langle \mathcal{O}(^3S_1^{[8]}) \rangle$ 10^{-2}GeV^3	$\langle \mathcal{O}(^3P_0^{[8]}) \rangle / m_c^2$ 10^{-2}GeV^3
1.16	8.9 ± 0.98	0.30 ± 0.12	0.56 ± 0.21
1.16	0	1.4	2.4
1.16	11	0	0

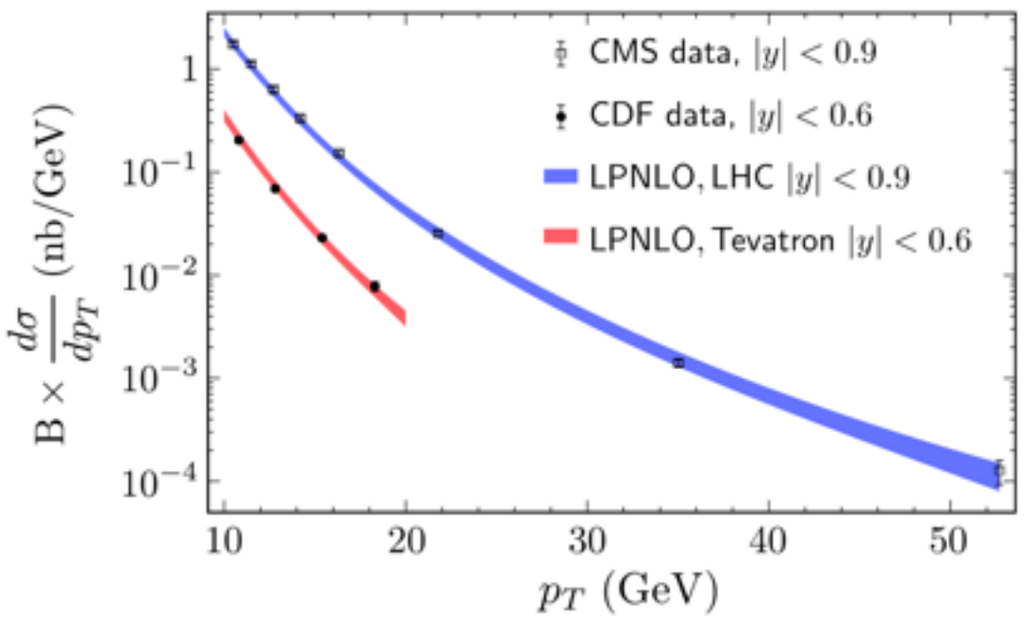
Recent Attempts to Resolve J/ψ Polarization Puzzle

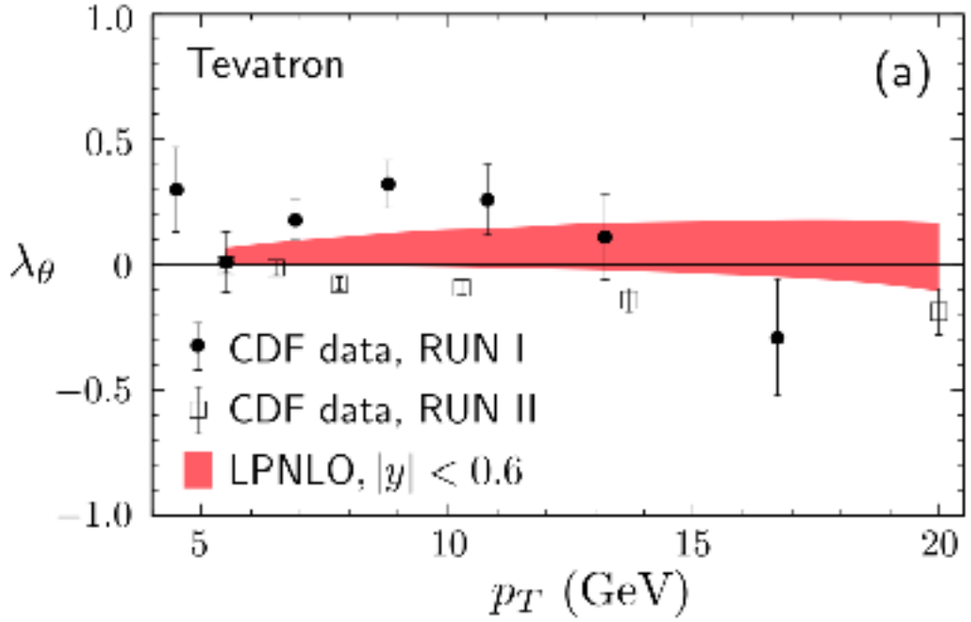
i) large p_t production at CDF

Bodwin, et. al., PRL 113, 022001(2014)

ii) resum logs of p_t/m_c using AP evolution

iii) fit COME to p_t spectrum, predict basically no polarization





Extracted COME **inconsistent** with global fits

$$\langle \mathcal{O}^{J/\psi}({}^1S_0^{(8)}) \rangle = 0.099 \pm 0.022\,\mathrm{GeV}^3$$

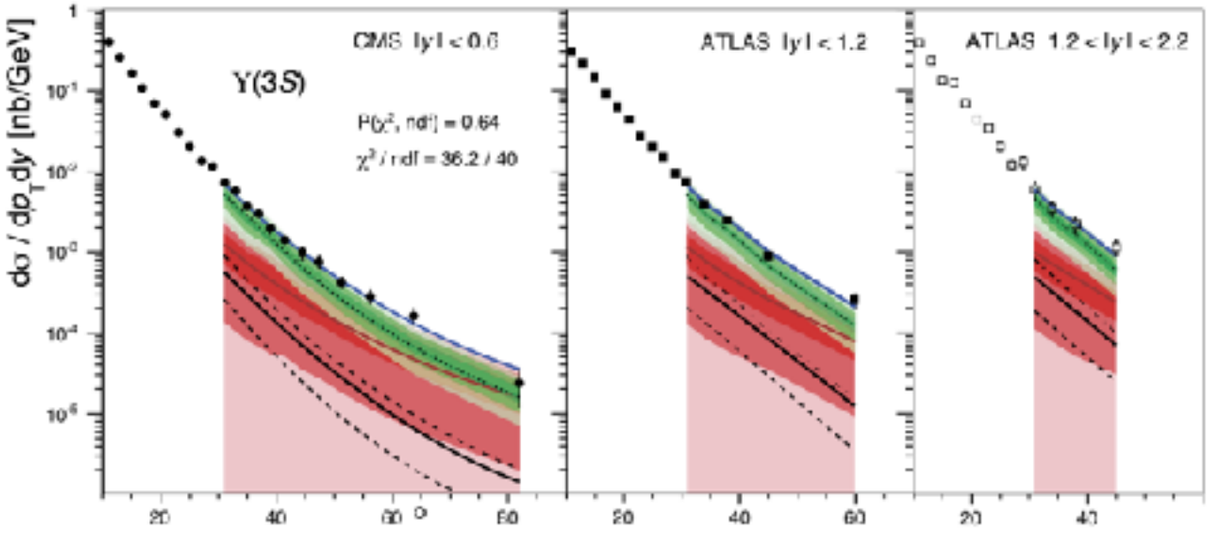
$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{(8)}) \rangle = 0.011 \pm 0.010\,\mathrm{GeV}^3$$

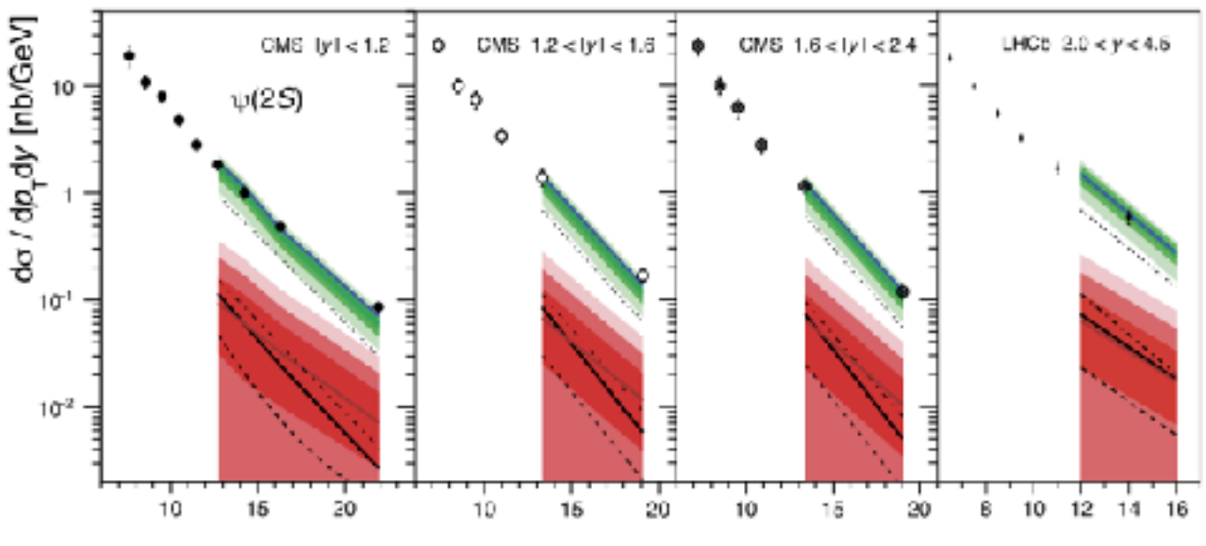
$$\langle \mathcal{O}^{J/\psi}({}^3P_0^{(8)}) \rangle = 0.011 \pm 0.010\,\mathrm{GeV}^5$$

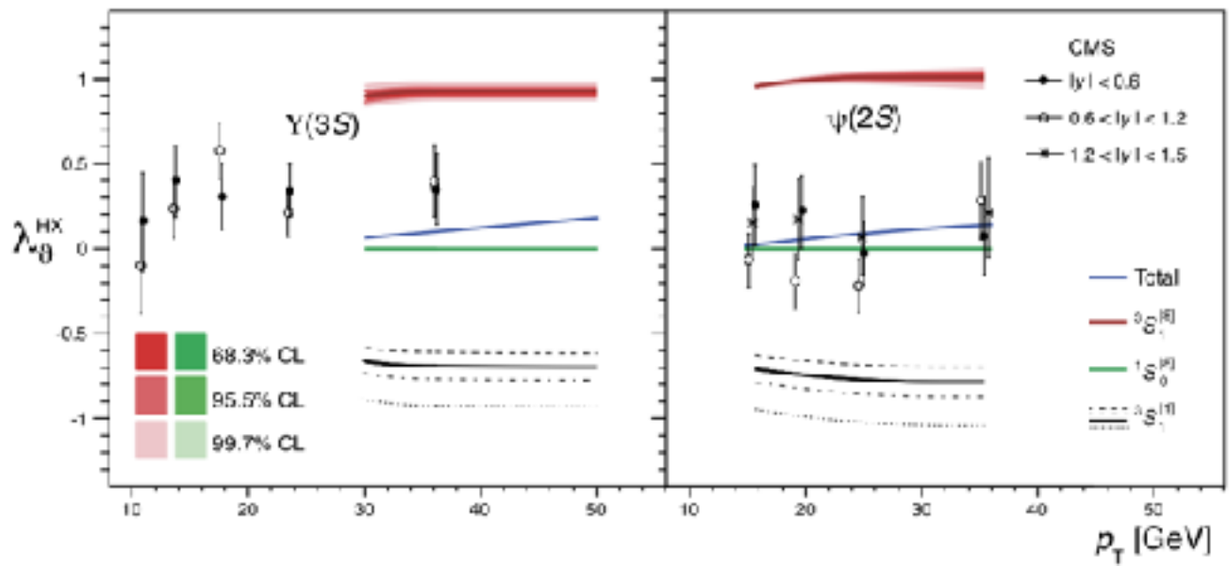
Recent Attempts to Resolve J/ψ Polarization Puzzle

Faccioli, et. al. PLB736 (2014) 98

Lourenco, et. al., NPA, in press





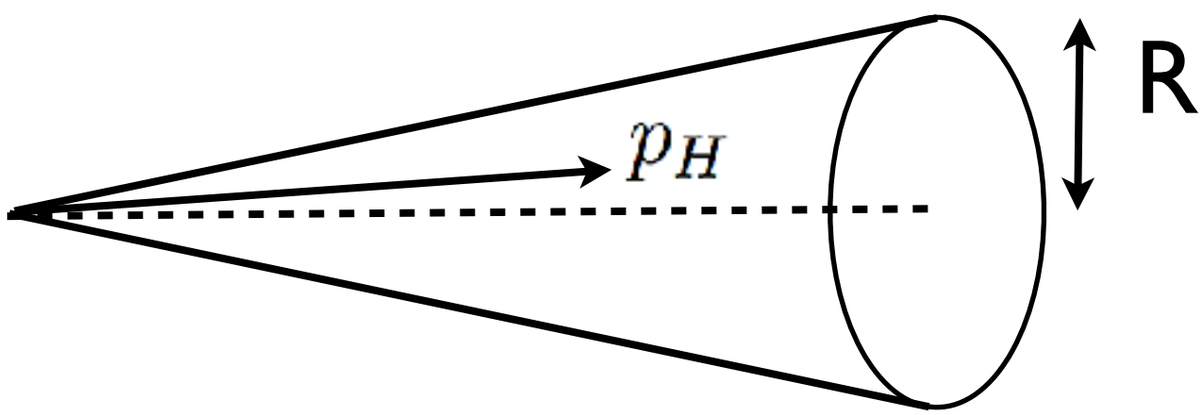


argue for $^1S_0^{(8)}$ dominance in both ψ(2S) & Υ(3S) production

Fragmenting Jet Functions

M. Procura, I. Stewart, PRD 81 (2010) 074009
A. Jain, M. Procura, W. Waalewijn, JHEP 1105 (2011) 035
A. Procura, W. Waalewijn, PRD 85 (2012) 114041

jets with identified hadrons



Jet Energy: E

$$p_H^+ = z p_{\rm jet}^+$$

cross sections determined by **fragmenting jet function (FJF):**

$$\mathcal{G}_g^h(E, R, \mu, z)$$

inclusive hadron production: fragmentation functions

$$\frac{1}{\sigma_0}\frac{d\sigma^h}{dz}(e^+e^- \to h\,X) = \sum_i \int_z^1 \frac{dx}{x}\, C_i(E_{\rm cm}, x, \mu)\; D_i^h(z/x, \mu)$$

jet cross sections: jet functions

$$d\sigma(E,R) = \int d\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell$$

$$\mathcal{G}_g^h(E,R,\mu,z) \longrightarrow D_i^h(z/x,\mu)\ ,\ J_\ell$$

relationship to jet function:

$$\sum_h \int_0^1 \mathrm{d}z\, z\, D_j^h(z,\mu) = 1$$

$$\longrightarrow \quad J_i(E,R,z,\mu) = \frac{1}{2}\sum_h \int \frac{dz}{(2\pi)^3}\, z\, \mathcal{G}_i^h(E,R,z,\mu)$$

cross section for jet w/ identified hadron from jet cross section

$$\frac{d\sigma}{dE} = \int d\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell J_i(E,R,\mu)$$

$$\longrightarrow \quad \frac{d\sigma}{dEdz} = \int d\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell\, \mathcal{G}_i^h(E,R,z,\mu)$$

relationship to fragmentation functions

$$\mathcal{G}_i^h(E,R,z,\mu) = \sum_i \int_z^1 \frac{\mathrm{d}z'}{z'} \mathcal{J}_{ij}(E,R,z',\mu) D_j^h\left(\frac{z}{z'},\mu\right)\left[1+\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{4E^2\tan^2(R/2)}\right)\right]$$

matching coefficients calculable in perturbation theory

$$\frac{\mathcal{J}_{gg}(E,R,z,\mu)}{2(2\pi)^3} = \delta(1-z) + \frac{\alpha_s(\mu)C_A}{\pi}\left[\left(L^2-\frac{\pi^2}{24}\right)\delta(1-z) + \hat{P}_{gg}(z)L + \hat{\mathcal{J}}_{gg}(z)\right]$$

$$\hat{\mathcal{J}}_{gg}(z) = \begin{cases} \hat{P}_{gg}(z)\ln z & z \le 1/2 \\ \frac{2(1-z+z^2)^2}{z}\left(\frac{\ln(1-z)}{1-z}\right)_+ & z \ge 1/2. \end{cases}$$

$$L = \ln[2E\tan(R/2)/\mu],$$

scale for $\mathcal{J}_{ij}(E,R,z,\mu)$

sum rule for matching coefficients

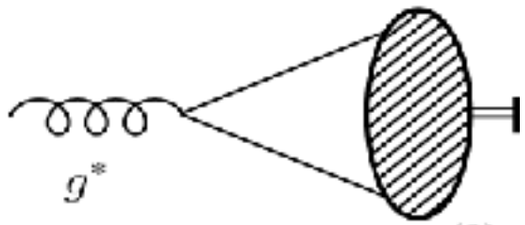
$$\sum_j \int_0^1 \mathrm{d}z\, z\, \mathcal{J}_{ij}(R,z,\mu) = 2(2\pi)^3\, J_i(R,\mu)$$

NRQCD fragmentation functions

Braaten, Yuan, PRD 48 (1993) 4230
Braaten, Chen, PRD 54 (1996) 3216
Braaten, Fleming, PRL 74 (1995) 3327

Perturbatively calculable **at the scale $2m_c$**

$$D_g^{\psi(8)}(z, 2m_c) = \frac{\pi\alpha_s(2m_c)}{3M_\psi^3}\langle O^\psi(^3S_1^{(8)})\rangle\delta(1-z)$$



$$D_g^{\psi(1)}(z, 2m_c) = \frac{5\alpha_s^3(2m_c)}{648\pi^2}\frac{\langle O^\psi(^3S_1^{(1)})\rangle}{M_\psi^3}\int_0^z dr\int_{(r+z^2)/2z}^{(1+r)/2} dy\frac{1}{(1-y)^2(y-r)^2(y^2-r)^2}$$
$$\sum_{i=0}^{2} z^i\left(f_i(r,y) + g_i(r,y)\frac{1+r-2y}{2(y-r)\sqrt{y^2-r}}\ln\frac{y-r+\sqrt{y^2-r}}{y-r-\sqrt{y^2-r}}\right),$$

Altarelli-Parisi evolution: $2m_c$ to 2E
tan(R/2)

FJF in terms of fragmentation function

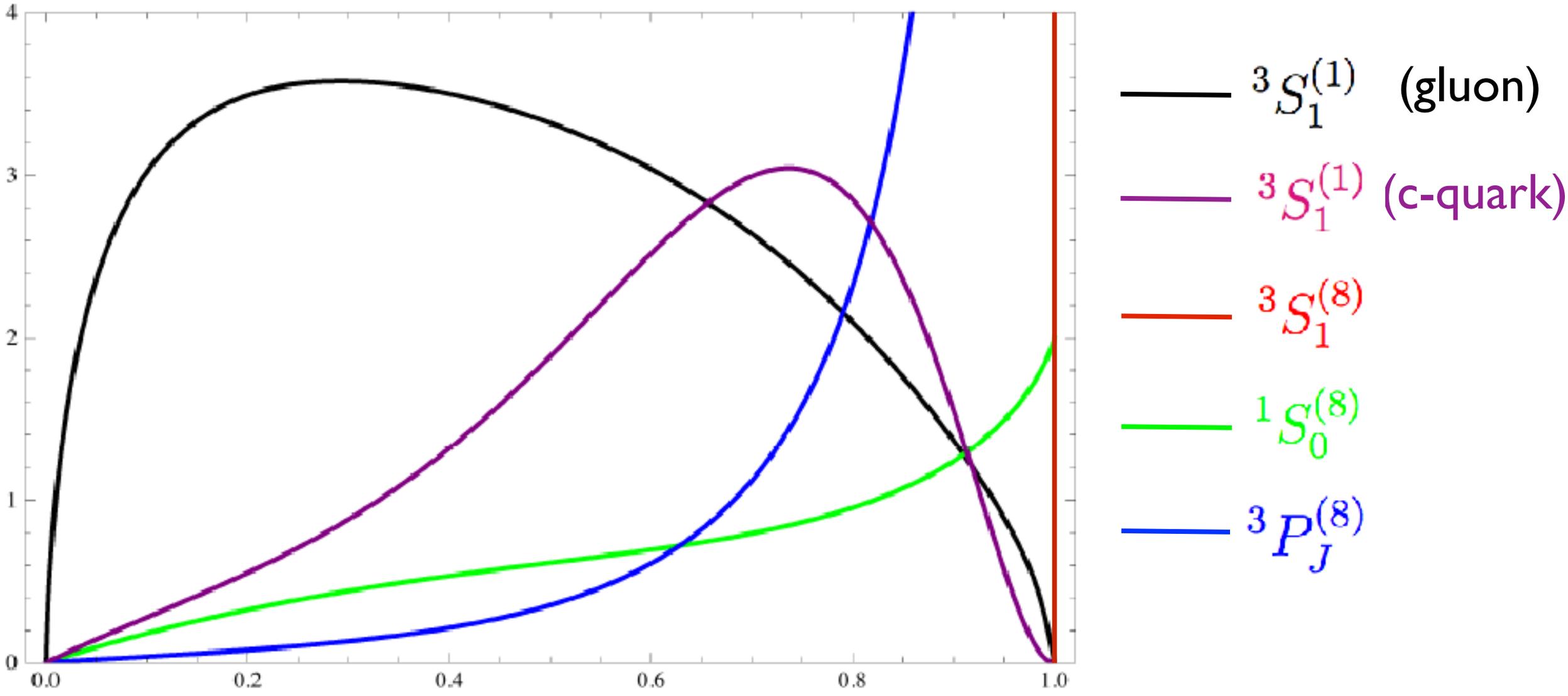
$$
\begin{aligned}
\mathcal{G}_g^{\psi}(E,R,z,\mu) \;=\; & D_{g\to\psi}(z,\mu)\left(1+\frac{C_A\alpha_s}{\pi}\left(L_{1-z}^2-\frac{\pi^2}{24}\right)\right) \\
& +\frac{C_A\alpha_s}{\pi}\left[\int_z^1\frac{dy}{y}\tilde{P}_{gg}(y)L_{1-y}D_{g\to\psi}\left(\frac{z}{y},\mu\right)\right. \\
& +2\int_z^1 dy\frac{D_{g\to\psi}(z/y,\mu)-D_{g\to\psi}(z,\mu)}{1-y}L_{1-y} \\
& \left.+\theta\left(\frac{1}{2}-z\right)\int_z^{1/2}\frac{dy}{y}\hat{P}_{gg}(y)\ln\left(\frac{y}{1-y}\right)D_{g\to\psi}\left(\frac{z}{y},\mu\right)\right]
\end{aligned}
$$

$$
L_{1-z}=\ln\left(\frac{2E\tan(R/2)(1-z)}{\mu}\right)
$$

For large E, FJF ~ NRQCD frag. function (at scale 2E tan(R/2))

$$
\mathcal{G}_g^h(E,R,\mu=2E\tan(R/2),z)\to D_g^{\psi}(z,2E\tan(R/2))+O(\alpha_s)
$$

NRQCD FF's (at scale $2m_c$)

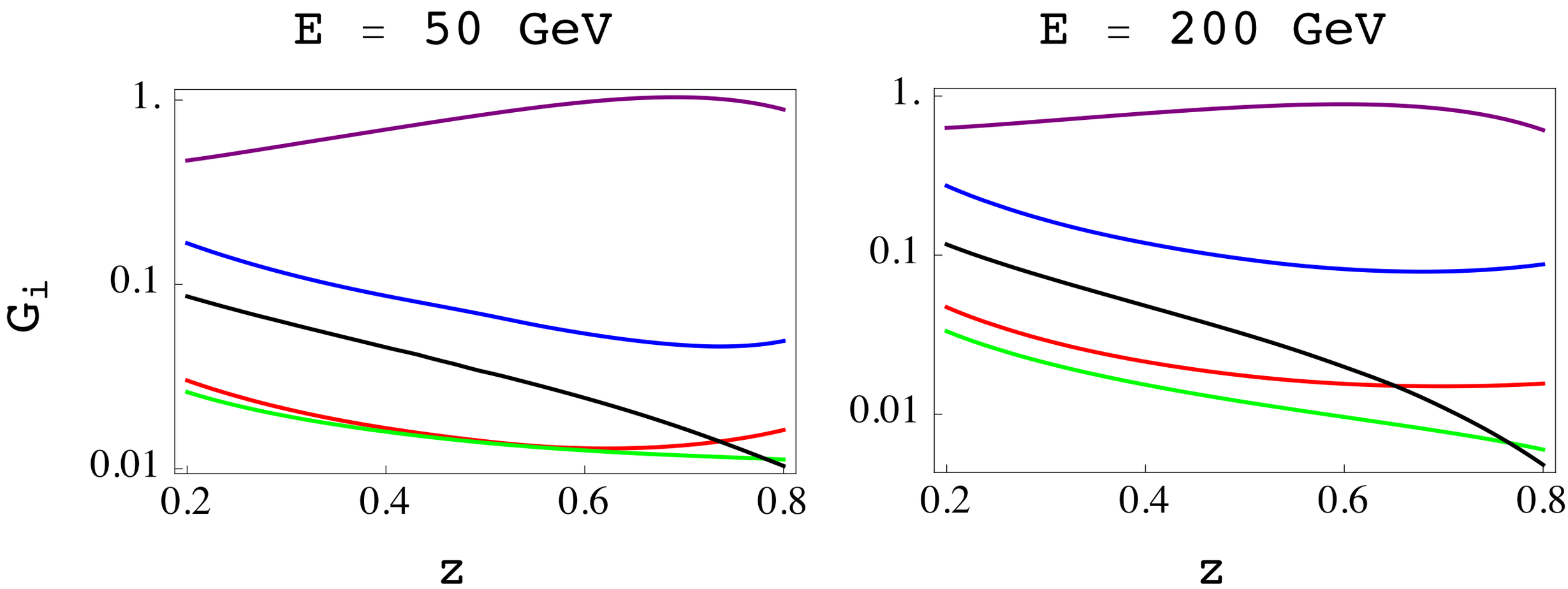


(normalization arbitrary)

Evolution to 2E tan(R/2) will soften discrepancies

FJF's at Fixed Energy vs. z

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003



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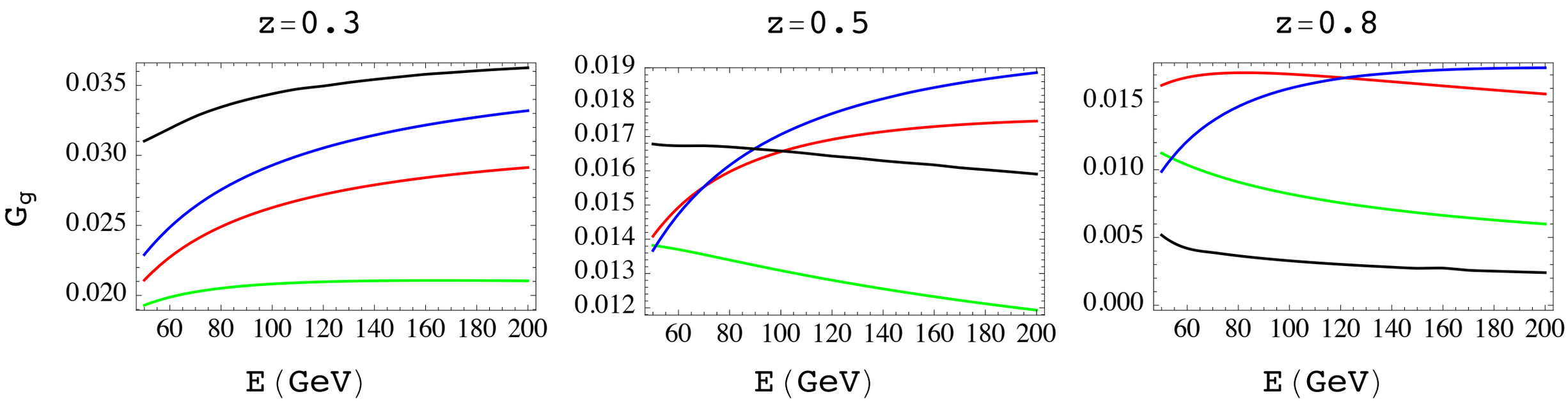
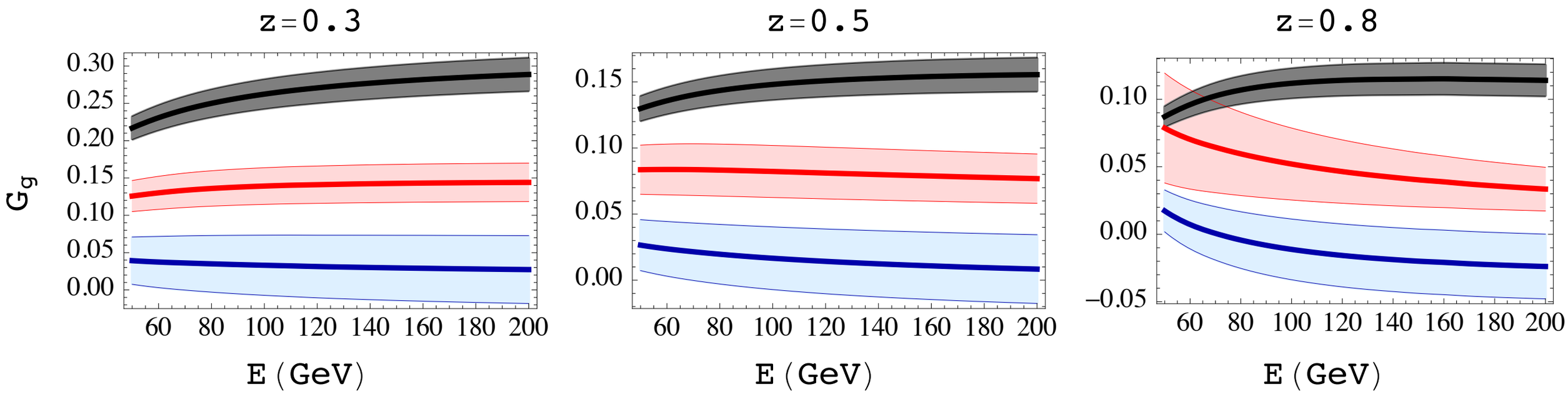


$^1S_0^{(8)}$ dominance predicts negative slope for z vs. E if z > 0.5

Gluon FJF for different extractions of LDME

fix z, vary energy



Butenschoen and Kniehl, PRD 84 (2011) 051501, arXiv:1105.0822

Bodwin, et. al. arXiv:1403.3612

Chao, et. al. PRL 108, 242004 (2012)

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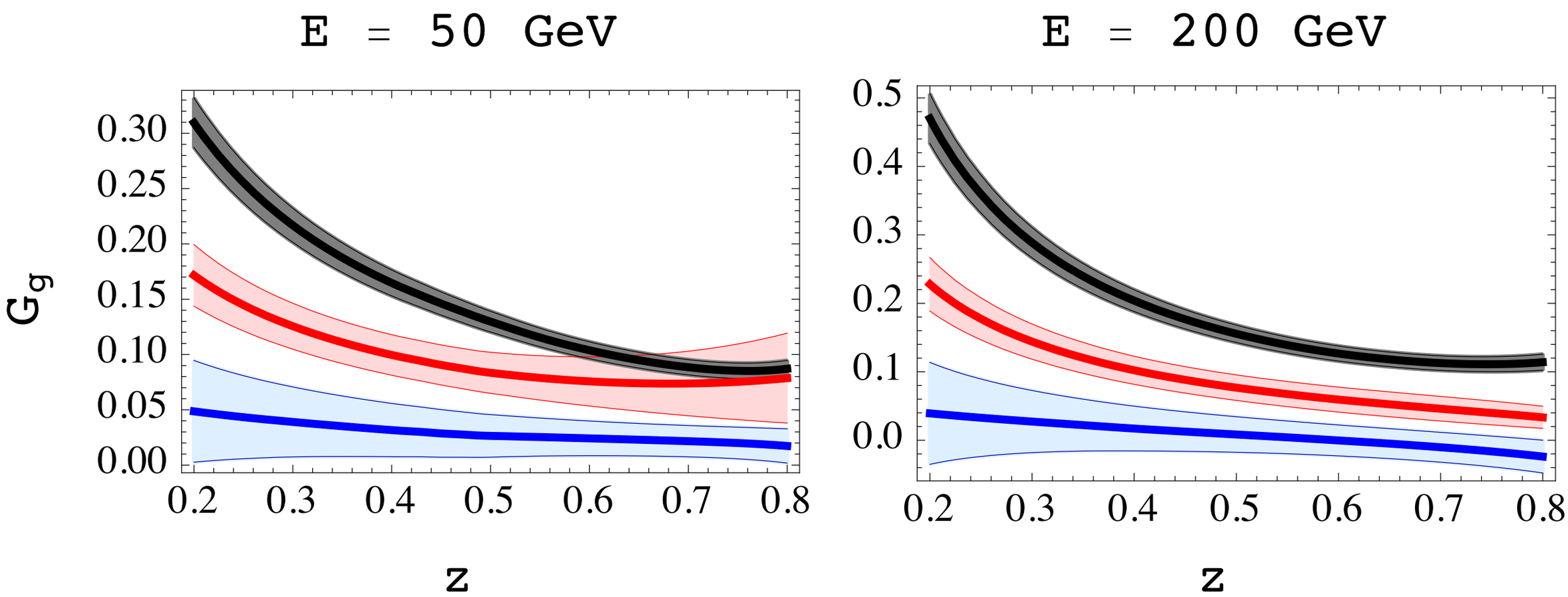
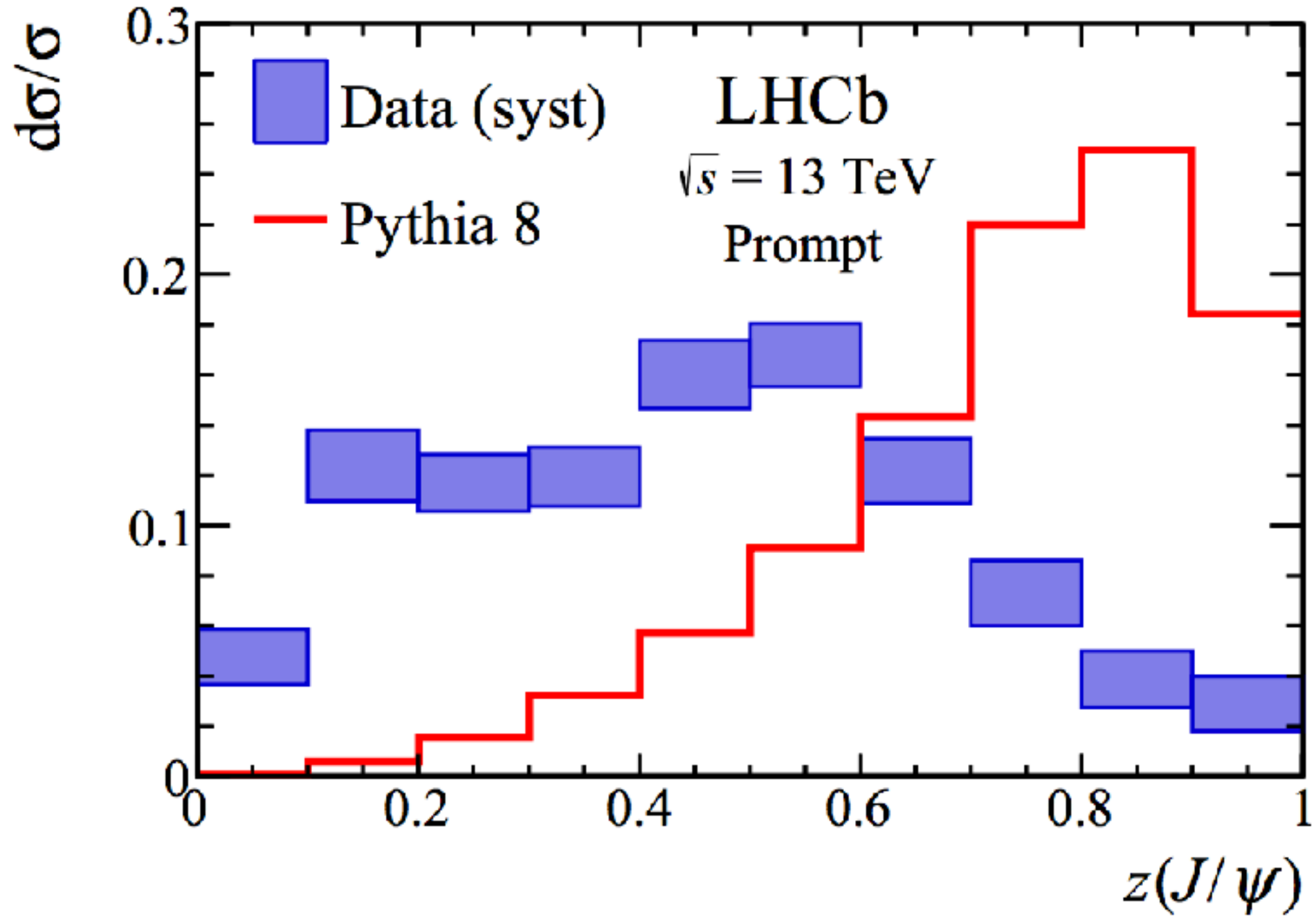


Recent Observations of Quarkonia within Jets

LHCb collaboration, arXiv:1701.05116



cuts: $2.5 < \eta_{\rm jet} < 4.0 \quad p_{T,jet} > 20\,{\rm GeV} \quad p(\mu) > 5\,{\rm GeV}$

This result was anticipated in:

Jets w/ Heavy Mesons: NLL' vs. Monte Carlo

(w/ R. Bain, L. Dai, A. Hornig, A.Leibovich, Y. Makris)

JHEP 1606 (2016) 121 (arXiv:1601.05815)

$$e^+e^- \to b\bar{b}$$

$\hookrightarrow$ B jet

$$e^+e^- \to q\bar{q}g$$

$\hookrightarrow$ J/ψ jet

$e^+e^- \longrightarrow$ Jets in SCET

S.D. Ellis, et.al., JHEP1011(2010) 101

$$d\sigma = H \times J^q \otimes J^{\bar{q}} \otimes J^g \otimes S$$

unmeasured jets:

E, R

measured jets:

angularity: $\tau_a = \frac{1}{\omega} \sum_i (p_i^+)^{1-a/2} (p_i^-)^{a/2}$

$$\omega = \sum_i p_i^- \qquad s = \omega^2 \tau_0$$

$e^+e^- \to$ Jets Formula (NLL')

$$\frac{1}{\sigma^{(0)}}\frac{d\sigma^{(i)}}{dz\,d\tau_a} = \sum_j \int_z^1 \frac{dx}{x} D_j(x;\mu_J) H_2(\mu_H) \left(\frac{\mu_H}{\omega}\right)^{\omega_H(\mu,\mu_H)} S^{\text{unmeas}}(\mu_\Lambda) J_\omega(\mu_R) \left(\frac{\mu_R}{\omega \tan\frac{R}{2}}\right)^{\omega_R(\mu,\mu_R)}$$
$$\times \left\{ \left[\delta_{ij}\delta(1-z/x)(1+f_S(\tau_a,\mu_S)) + f_J^{ij}(\tau_z, z/x;\mu_J)\right] \left(\frac{\mu_S \tan^{1-a}\frac{R}{2}}{\omega_1}\right)^{\omega_S(\mu,\mu_S)} \right.$$
$$\left. \times \left(\frac{\mu_J}{\omega}\right)^{(2-a)\omega_J(\mu,\mu_J)} \frac{1}{\Gamma[-\omega_J(\mu,\mu_J)-\omega_S(\mu,\mu_S)]} \frac{1}{\tau_a^{1+\omega_J(\mu,\mu_J)+\omega_S(\mu,\mu_S)}} \right\}_+$$
$$\times \exp\left[\mathcal{K}(\mu;\mu_H,\mu_R,\mu_J,\mu_S,\mu_\Lambda) + \gamma_E \Omega(\mu;\mu_J,\mu_S)\right] .$$

$e^+e^- \to$ Jets Formula (NLL')

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$$\times \left\{ \left[\delta_{ij}\delta(1-z/x)(1 + f_S(\tau_a,\mu_S)) + f_J^{ij}(\tau_z, z/x;\mu_J)\right] \left(\frac{\mu_S \tan^{1-a}\frac{R}{2}}{\omega_1}\right)^{\omega_S(\mu,\mu_S)} \right.$$

$$\left. \times \left(\frac{\mu_J}{\omega}\right)^{(2-a)\omega_J(\mu,\mu_J)} \frac{1}{\Gamma[-\omega_J(\mu,\mu_J) - \omega_S(\mu,\mu_S)]} \frac{1}{\tau_a^{1+\omega_J(\mu,\mu_J)+\omega_S(\mu,\mu_S)}} \right\}_+$$

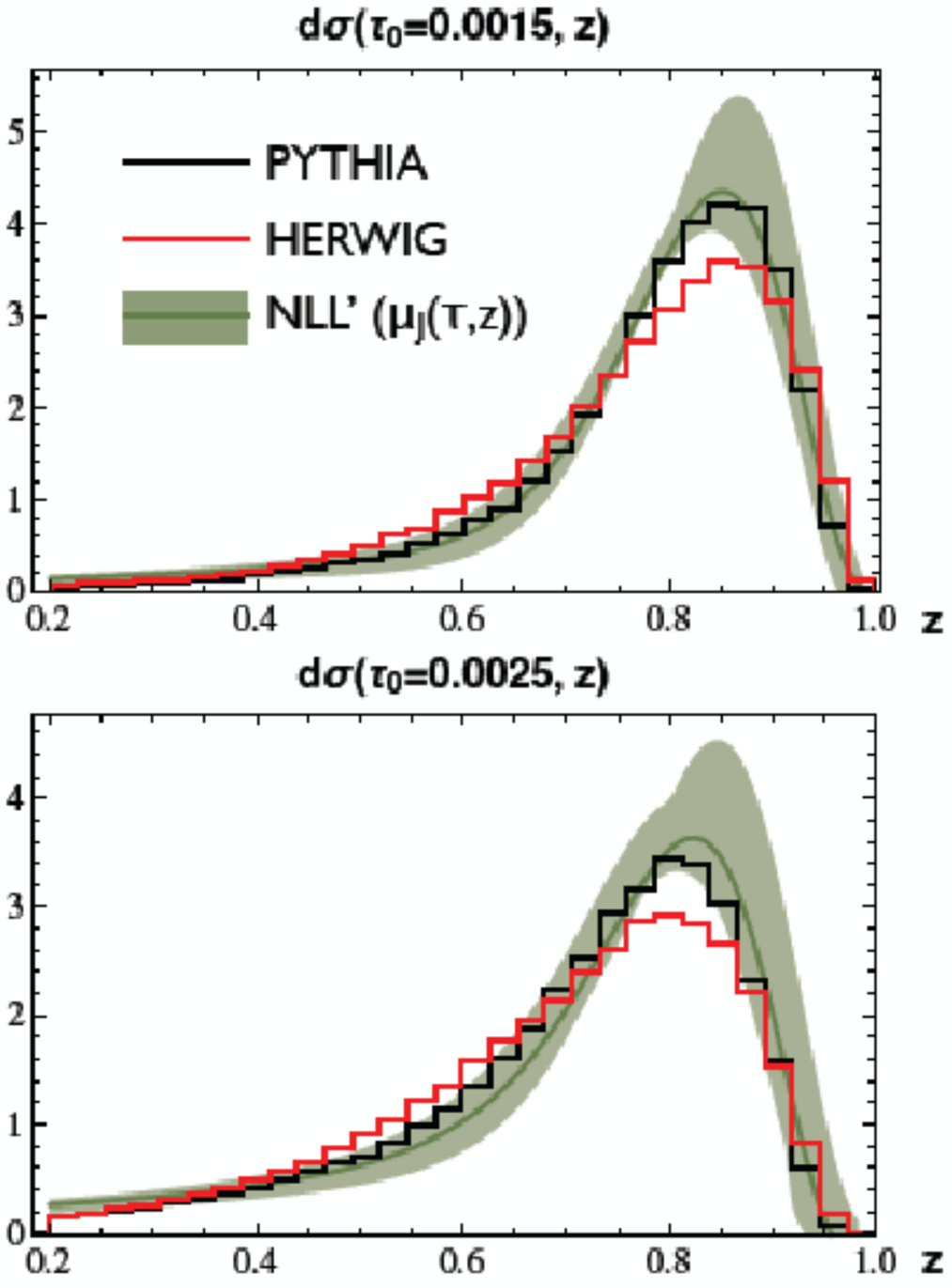
$$\times \exp\left[\mathcal{K}(\mu;\mu_H,\mu_R,\mu_J,\mu_S,\mu_\Lambda) + \gamma_E \Omega(\mu;\mu_J,\mu_S)\right].$$

NLL' includes these terms

NLL RGE evolution

NLL' vs. Monte Carlo

fixed τ_0, variable z



Monte Carlo Details

SISCone, $R = 0.6$, $N_{jets} = 2$

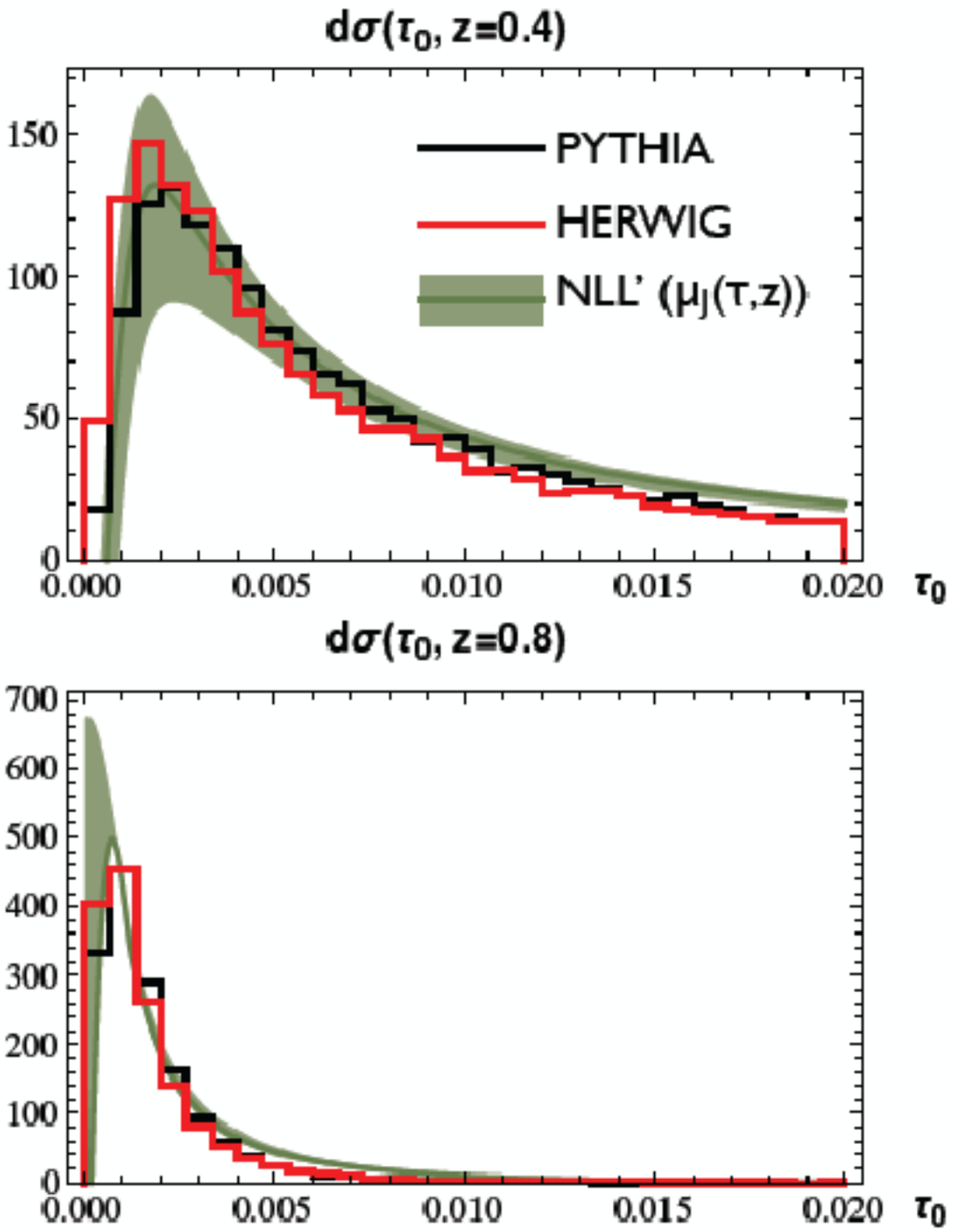
$$E_{jet} > \frac{E_{cm} - \Lambda}{N_{jets}}, \; E_{cm} = 250\text{GeV}$$

$\Lambda = 30\text{GeV}$

(HQ FF from LEP data) Kniehl, et. al. Phys.Rev. D77 (2008) 014011

NLL' vs. Monte Carlo

fixed z, variable τ_0



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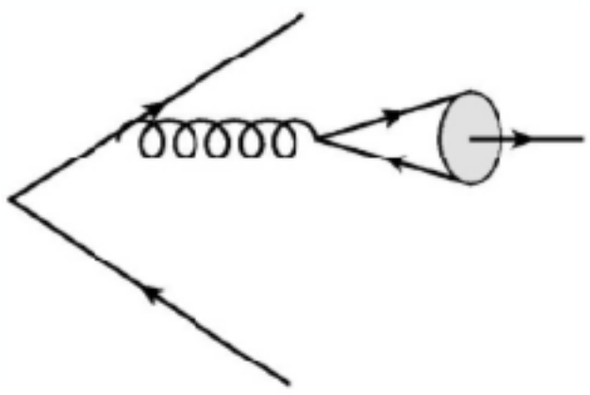
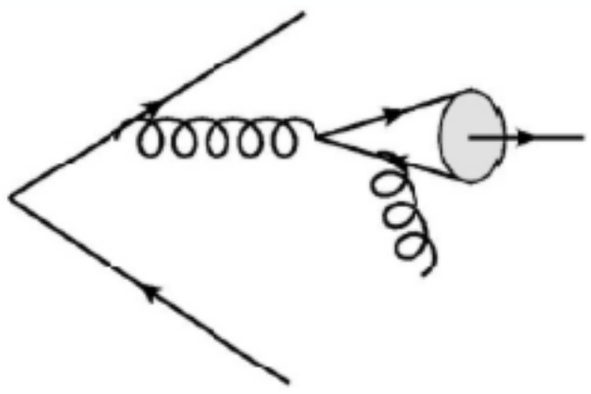
Madgraph + PYTHIA

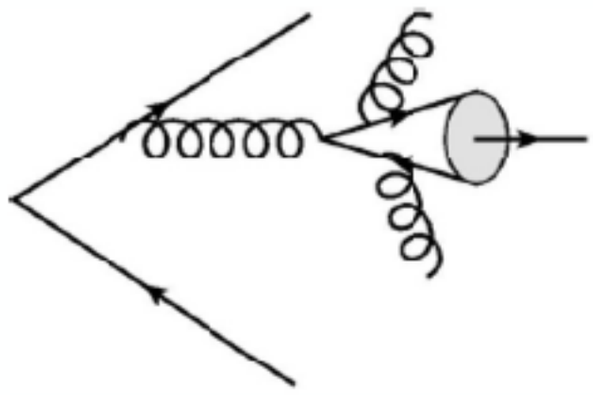
$e^+e^- \to b\bar{b}\,c\bar{c}\left[{}^3S_1^{(8)}\right]$



$e^+e^- \to b\bar{b}\,g\,c\bar{c}\left[{}^1S_0^{(8)}\right]$



$e^+e^- \to b\bar{b}\,g\,g\,c\bar{c}\left[{}^3S_1^{(1)}\right]$

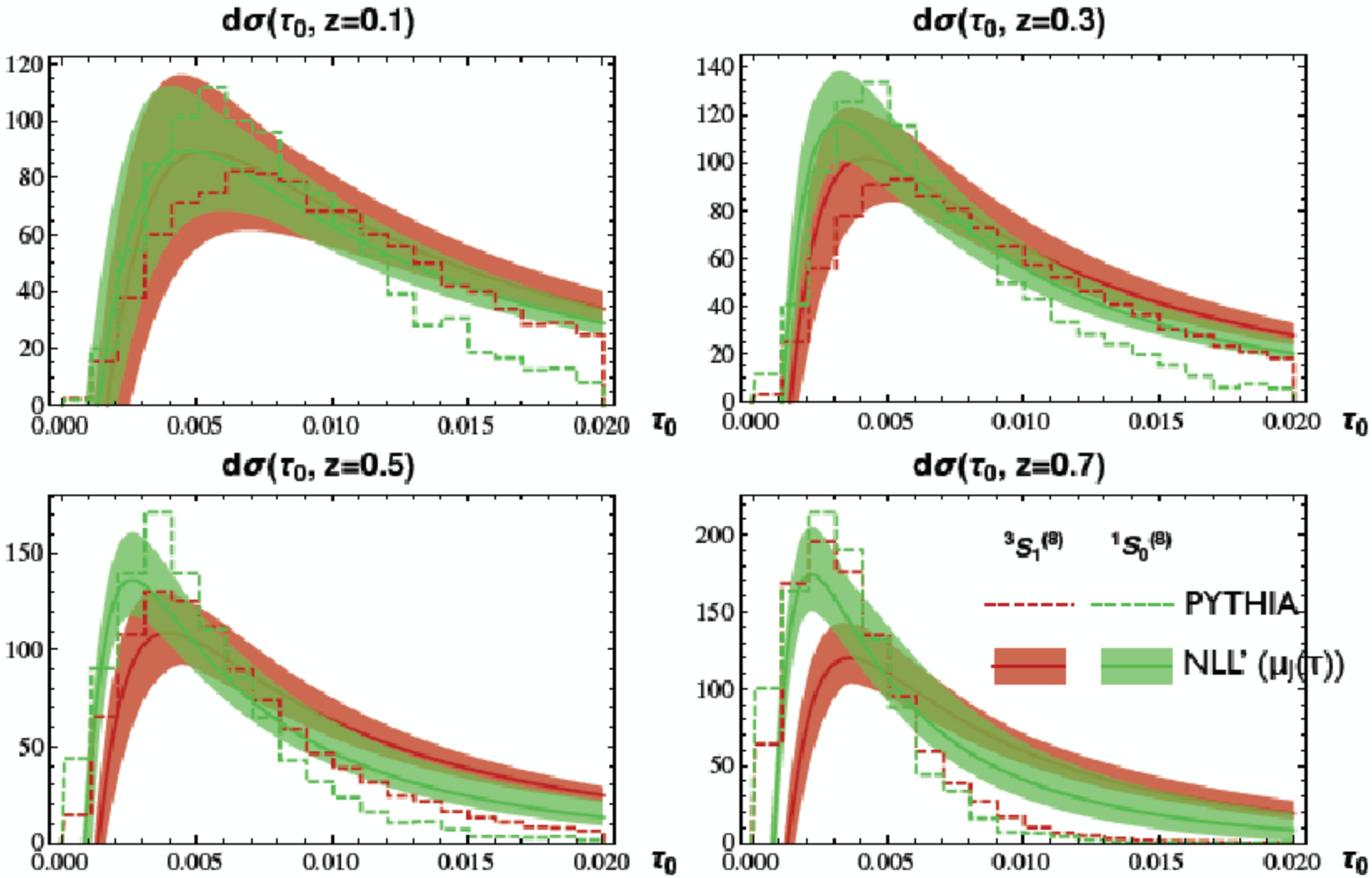


Force Madgraph to create J/ψ from gluon initiated jet

PYTHIA: parton shower, hadronization

NLL' vs. Monte Carlo

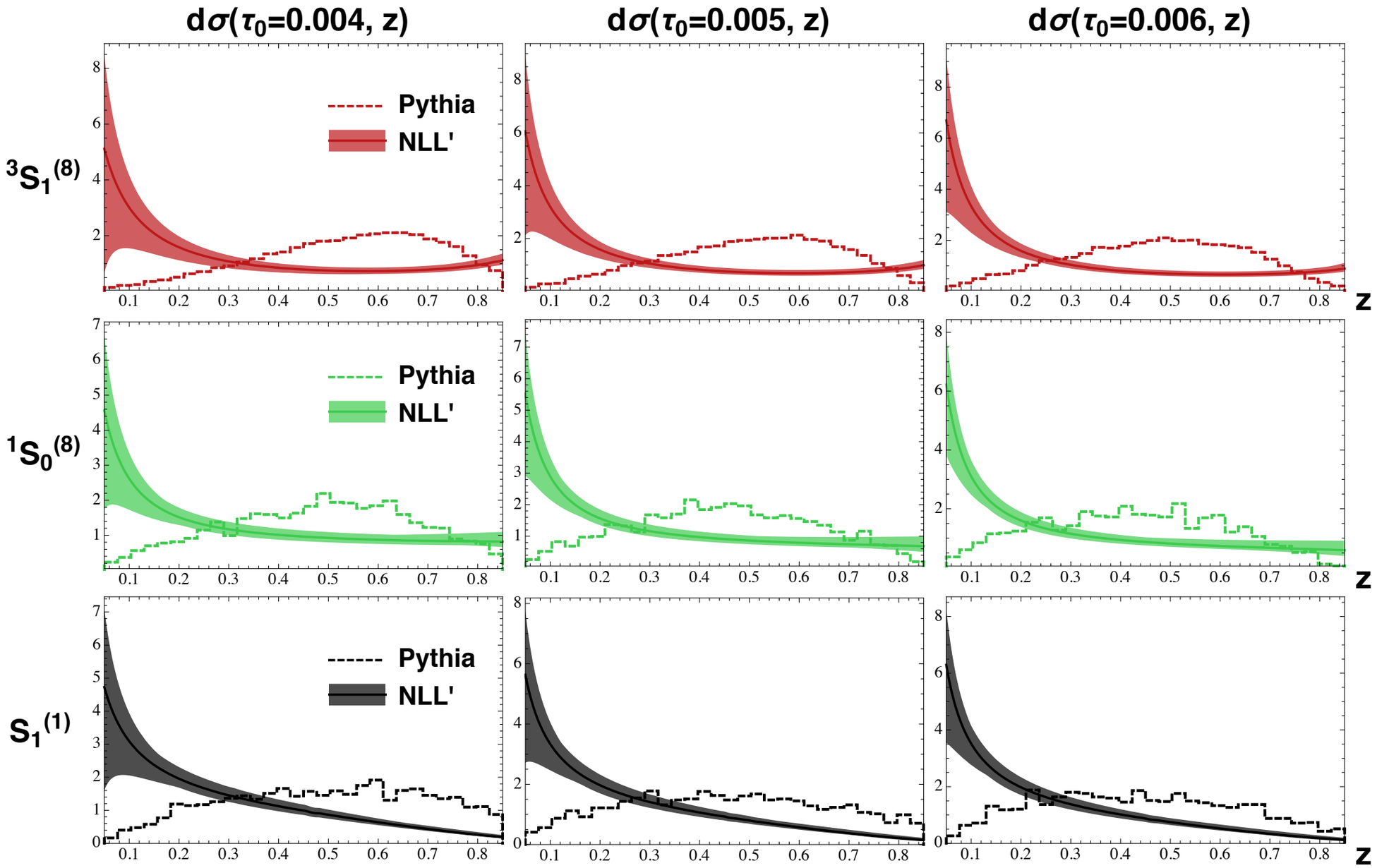
fixed z, variable τ_0



good agreement, some discrimination for large z

NLL' FJF vs. Pythia

R. Bain, L. Dai, A. Hornig, A. K. Leibovich, Y. Makris, T. Mehen JHEP 1606 (2016) 121

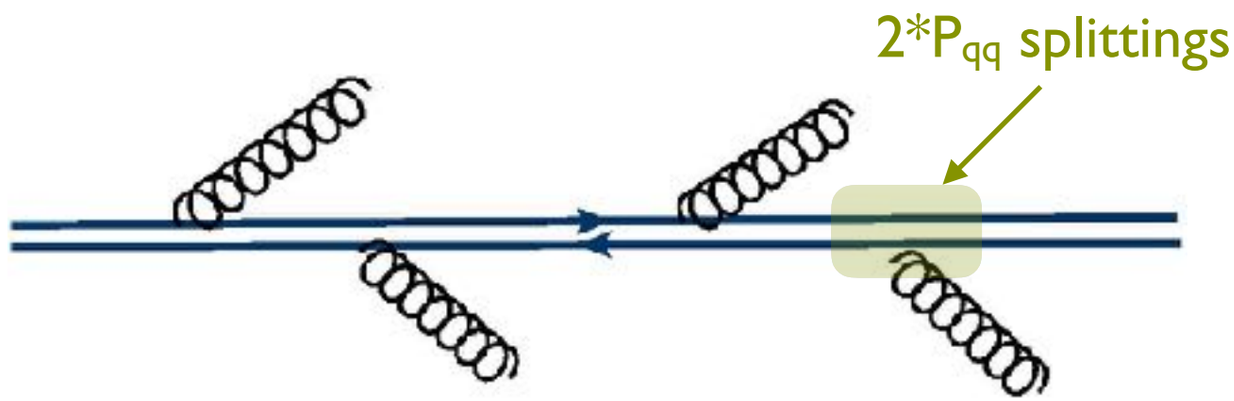


$e^+e^- \to \bar{q}qg \qquad E_{CM} = 250\,\mathrm{GeV} \qquad \tau_0 = s/\omega^2$

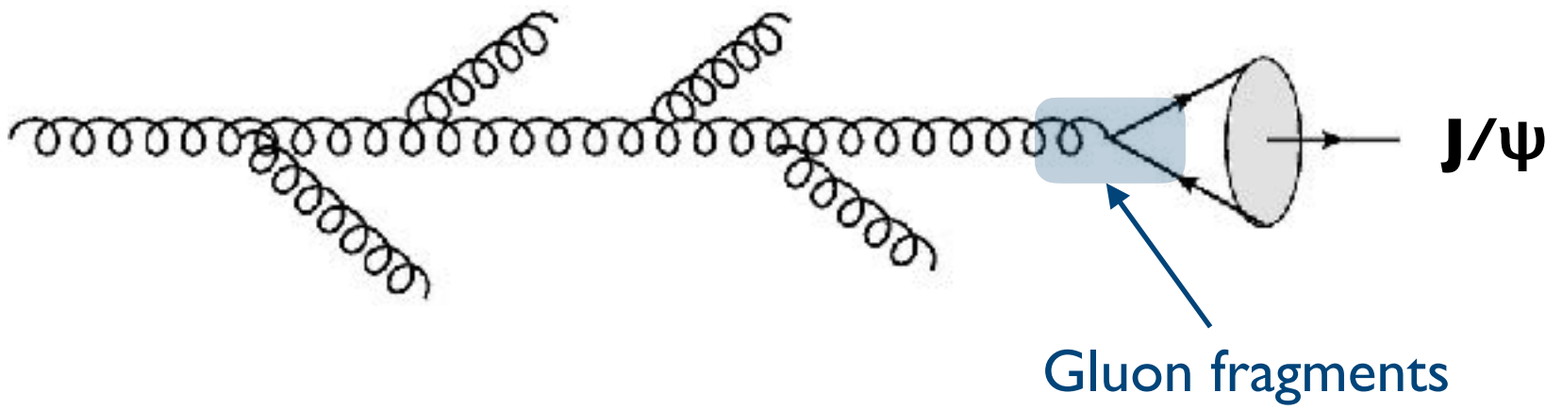
↳ jet w/ J/ψ

Explaining difference between NLL' vs Pythia

PYTHIA's model for showering color-octet $c\bar{c}$ pairs:



Physical picture of analytical calculation

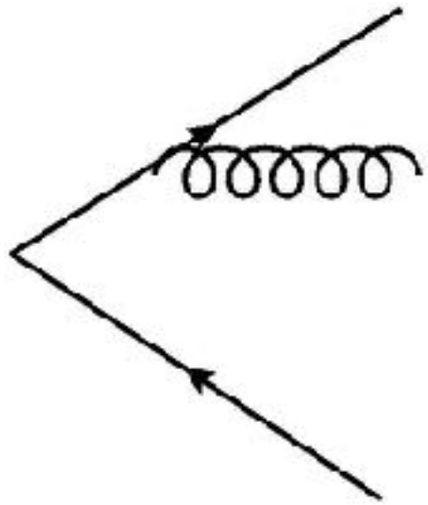


Pythia z distributions much harder than NLL' calculations

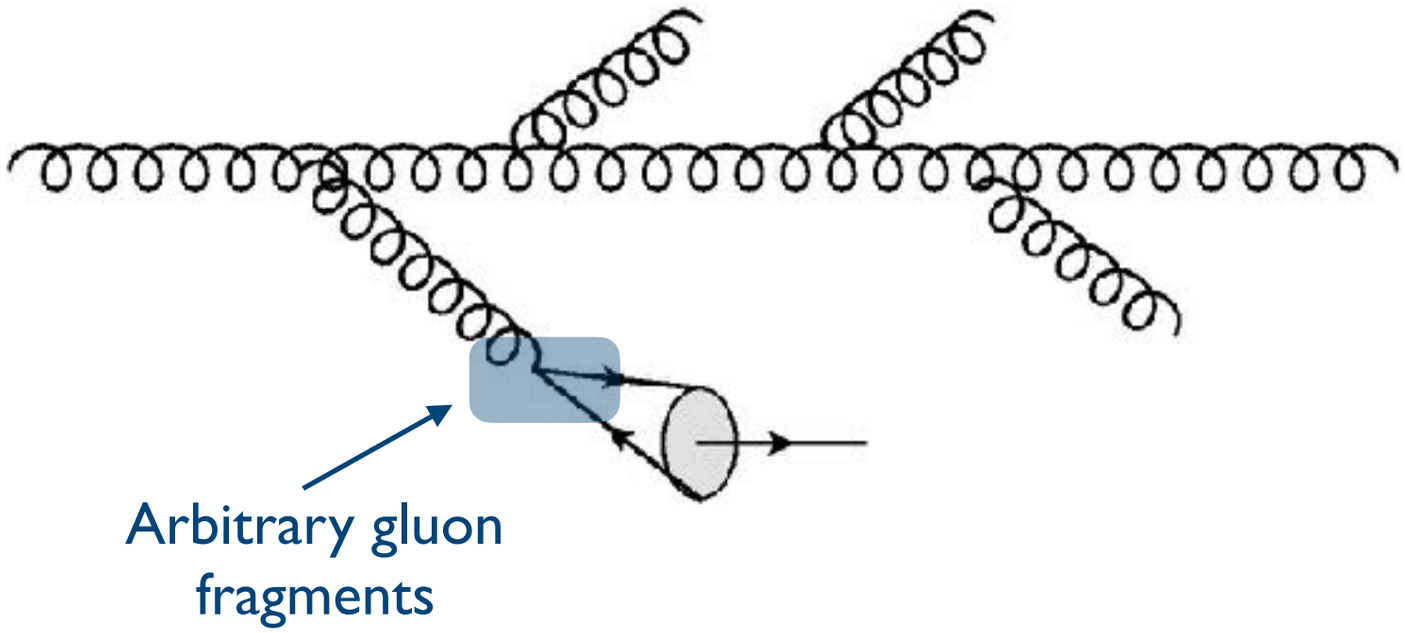
Gluon Fragmentation Improved PYTHIA (GFIP)

Madgraph 5

$$e^+e^- \to b\,\bar{b}\,g$$



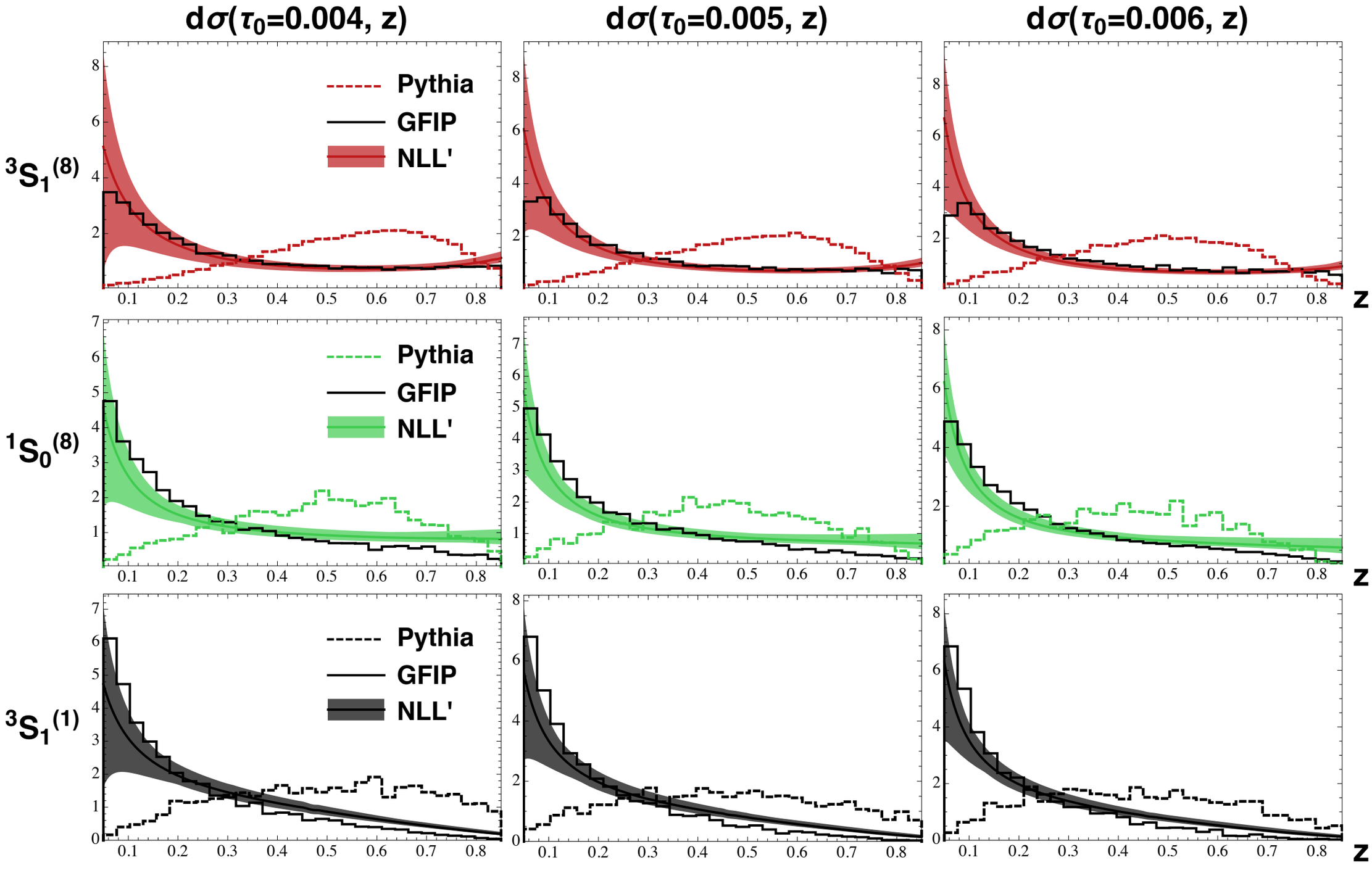
PYTHIA + Convolution



shower gluon with PYTHIA down to scale ~$2m_c$, no hadronization

convolve final state gluon distribution w/ NRQCD FFs

NLL', PYTHIA, and GFIP



GFIP and Recent LHCb Observations

R. Bain, L. Dai, A. K. Leibovich, Y. Makris, T. Mehen, submitted to PRL

generate events with hard c-quark , gluons

LHCb: pp collisions $\sqrt{s} = 13\,\mathrm{TeV}$ cuts:

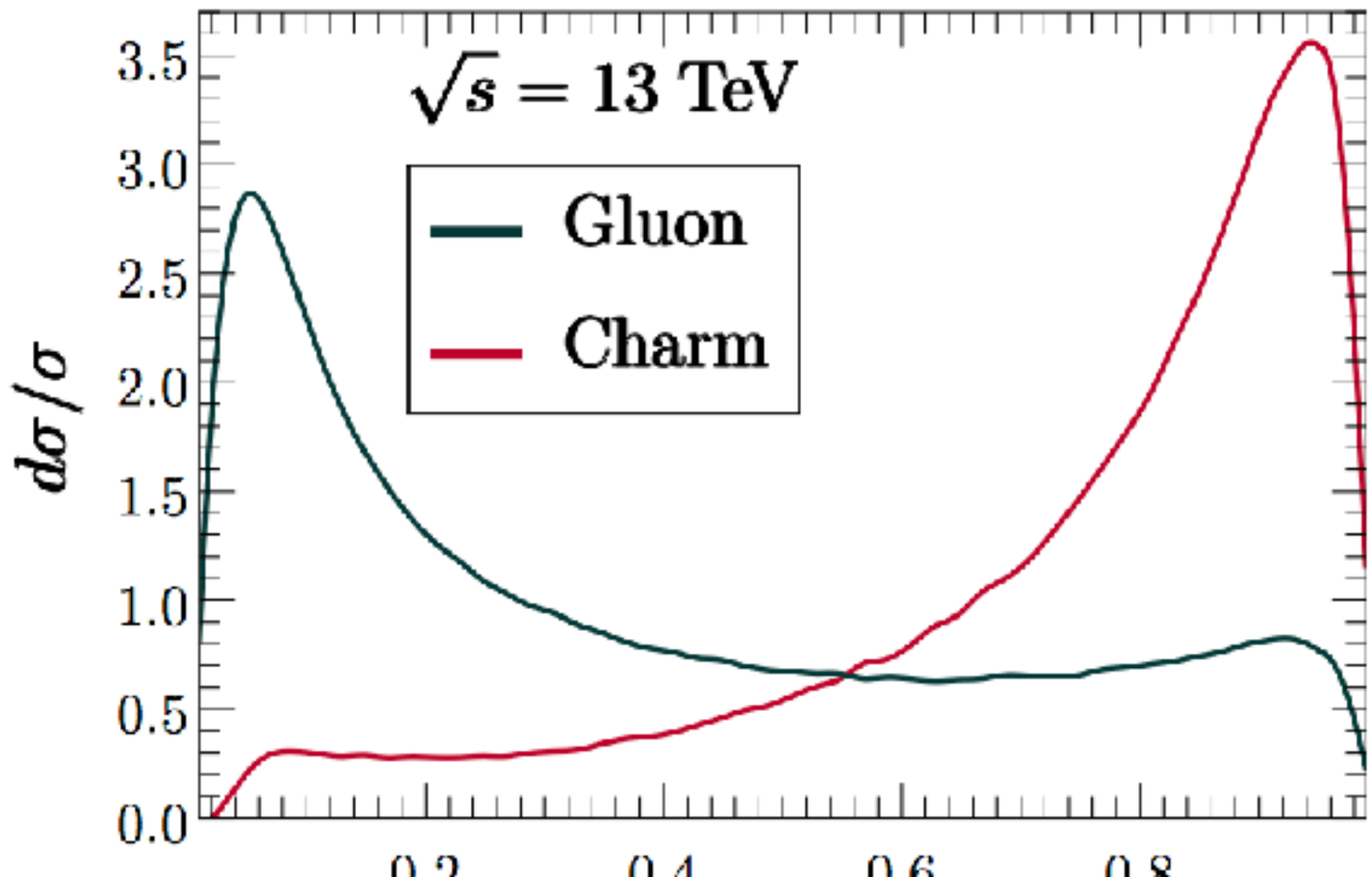
$2<\eta<4.5$

$R=0.5$

$p_{T,JET} < 20$ GeV

$p_\mu < 5$ GeV

evolve shower to scale ~$2m_c$



convolve w/ NRQCD FF for c quarks, gluons ~$2m_c$

LHCb data is normalized so $\sum_i \Delta z \left(\frac{d\sigma}{\sigma}\right)_i = \Delta z$

compare $0.1 < z < 0.9$

Use following three sets of LDMEs

	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle$ $\times$ GeV^3	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$ $\times 10^{-2}$ GeV^3	$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ $\times 10^{-2}$GeV^3	$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle/m_c^2$ $\times 10^{-2}$GeV^3
B & K [5, 6]	1.32 ± 0.20	0.224 ± 0.59	4.97 ± 0.44	-0.72 ± 0.88
Chao, et al. [12]	1.16 ± 0.20	0.30 ± 0.12	8.9 ± 0.98	0.56 ± 0.21
Bodwin et al. [13]	1.32 ± 0.20	1.1 ± 1.0	9.9 ± 2.2	0.49 ± 0.44

FJF and Recent LHCb Observations

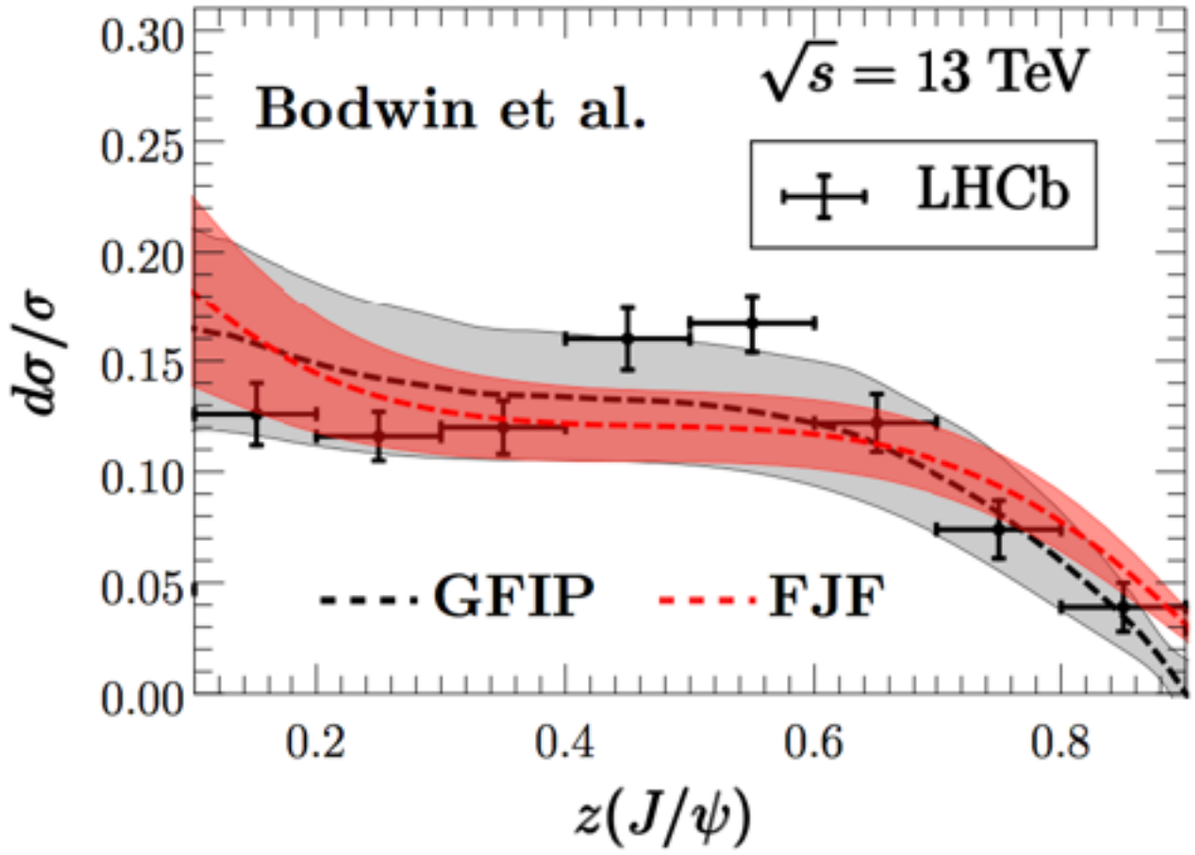
combine FJFs with hard events generated by Madgraph

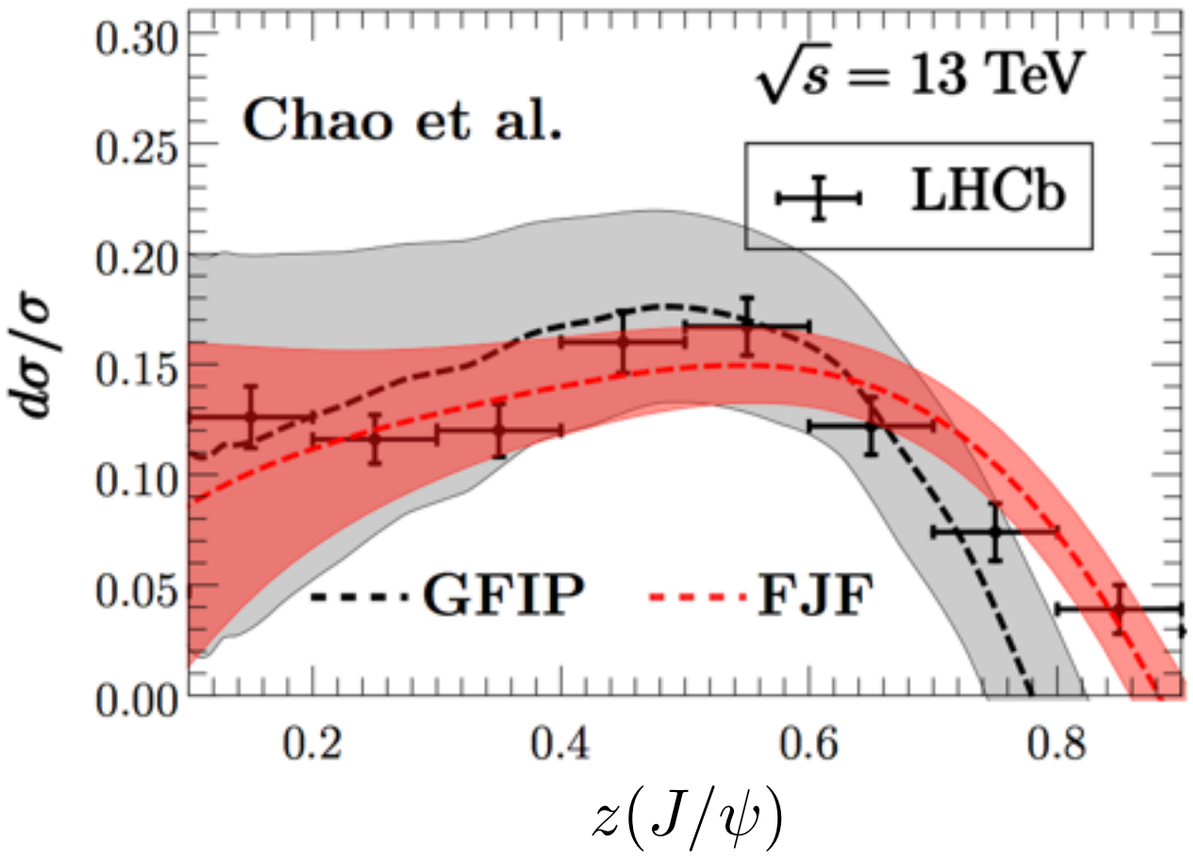
NRQCD FFs evolved from $2m_c$ to jet energy scale using DGLAP

factorization theorem with tree level hard function, trivial soft function, no NLL' resummation

FJF is only terms in factorization dependent on $z(J/\psi)$

Results

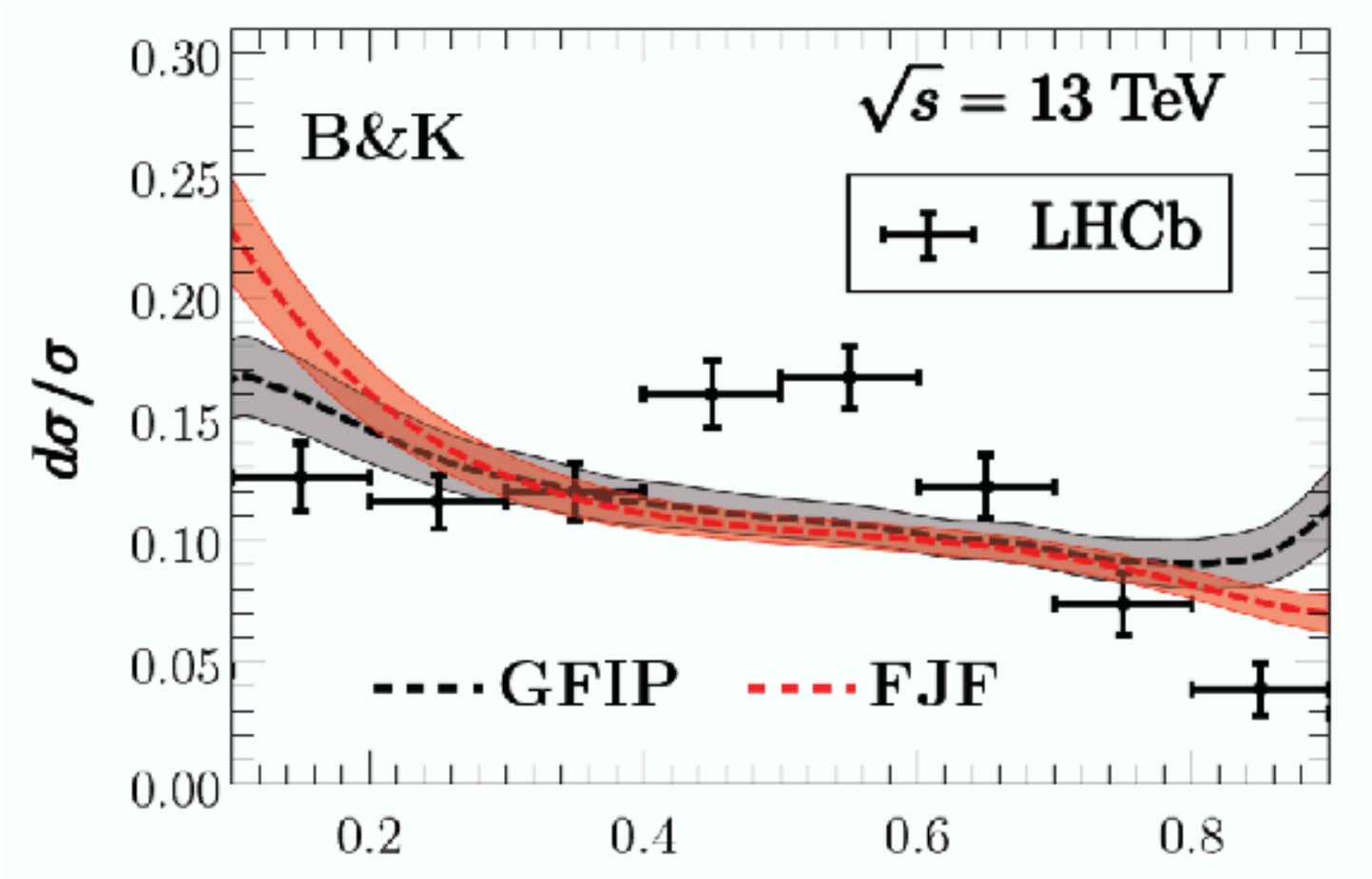




FJFs, GFIP consistent

LDME from fits high p_T agree with LHCb

Results



LDME from global fits:
poorer agreement with LHCb, better than PYTHIA

Fitting the LHCb Observations

LHCb data goes to zero as $z(J/\psi) \to 0$

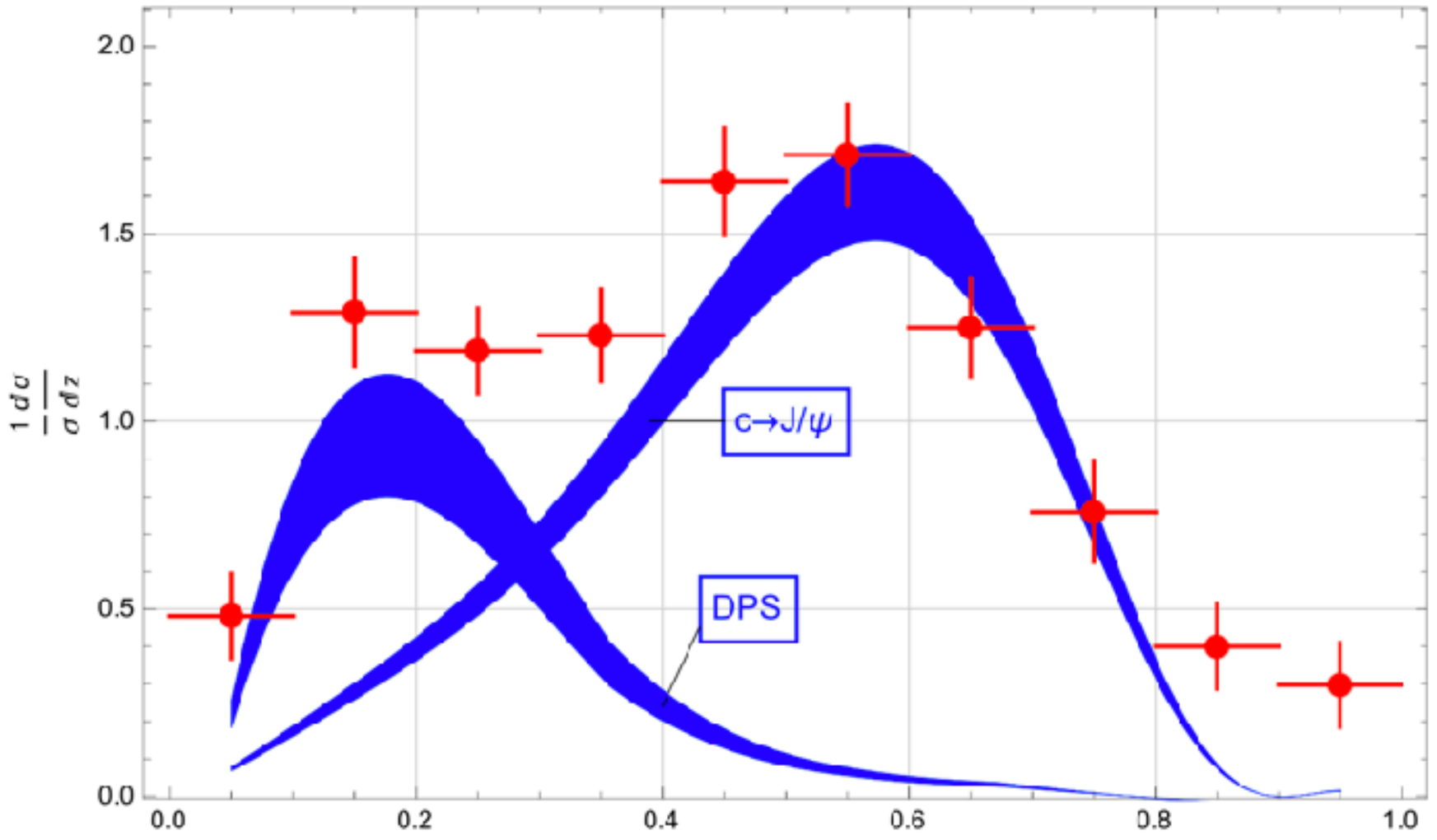
This is a property of ${}^3S_1^{(1)}, {}^1S_0^{(8)}$ FFs

but not ${}^3S_1^{(8)}, {}^3P_J^{(8)}$ FFs

Similar cancellation/suppression of ${}^3S_1^{(8)}, {}^3P_J^{(8)}$ FFs required to get vanishing high p_T polarization

Another Analysis of LHCb Observations

I. Belyaev, A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky. arXiv:1703.09081



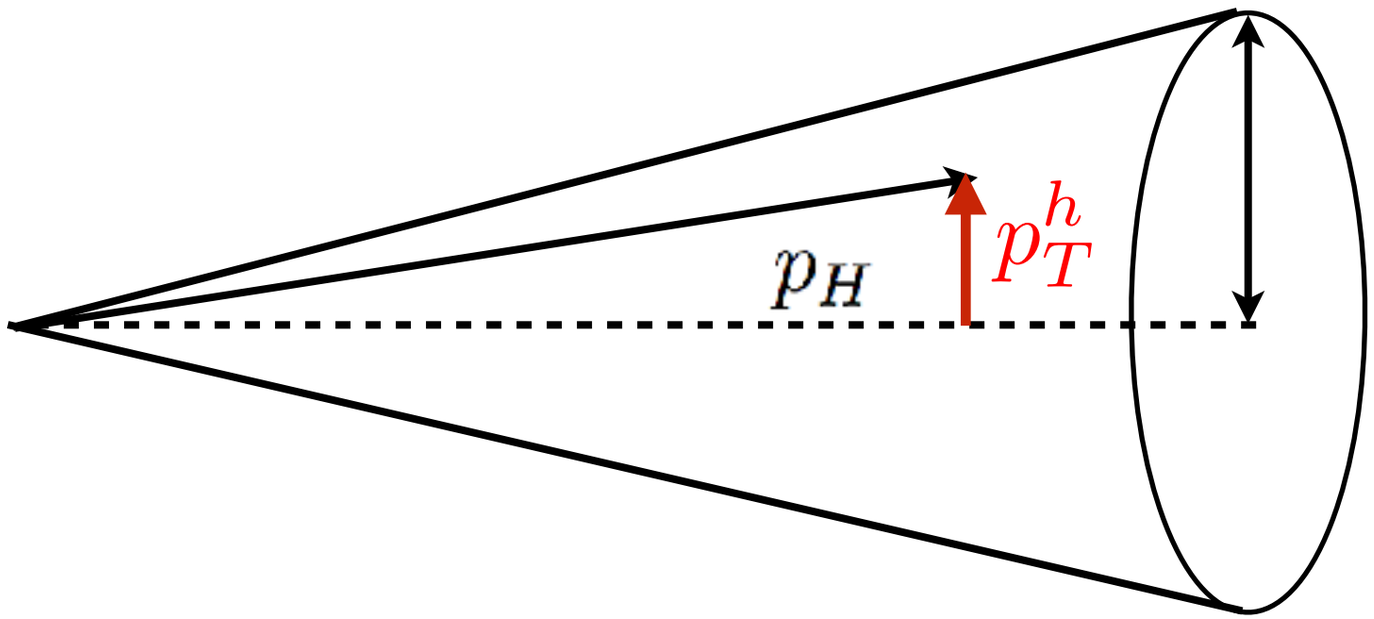
Color-singlet c-quark fragmentation only

double patron scattering at low z

Transverse Momentum Dependent FJFs

R. Bain, Y. Makris, TM, JHEP 1611 (2016) 144

jets with identified hadron: hadron z, p_T are both measured



R

Jet Energy: E

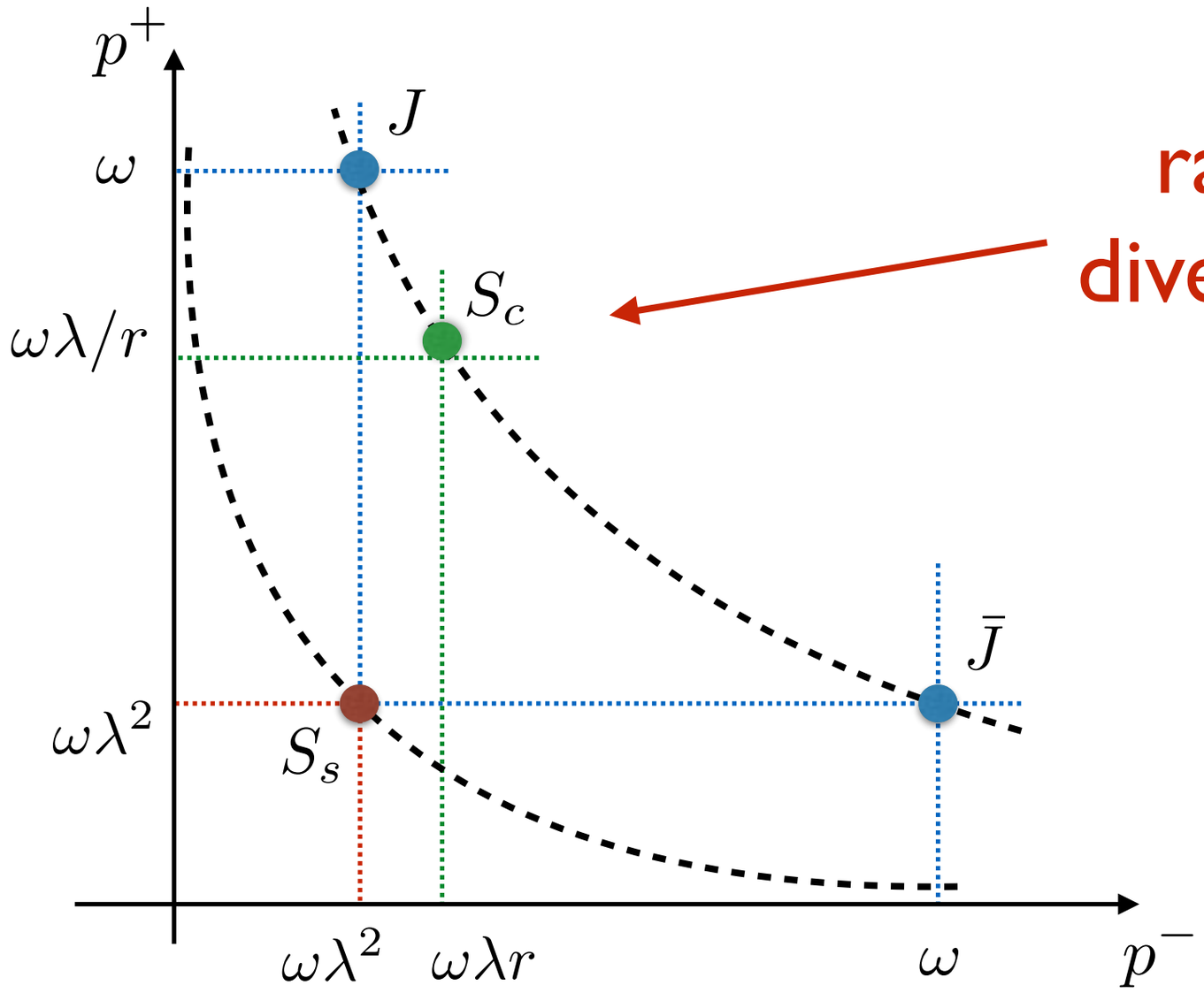
$$p_H^+ = z p_{\rm jet}^+$$

transverse momentum measured w/ rspt. to jet axis

jet axis ~ parton initiating jet if out of jet radiation is ultrasoft

$$\omega \gg p_T^h \gg \Lambda \gg \Lambda_{\rm QCD}$$

Scales in TMDFJF



rapidity divergences

$$p_c \sim \omega(\lambda^2, 1, \lambda) \qquad p_{cs} \sim p_h^\perp(r, 1/r, 1) \qquad p_{us} \sim \Lambda(1, 1, 1)$$

$$\lambda = p_h^\perp/\omega$$

Factorization Theorem

$$D_{q/h}(\mathbf{p}_\perp, z, \mu) = H_+(\mu) \times \left[\mathcal{D}_{q/h} \otimes_\perp S_C\right](\mathbf{p}_\perp, z, \mu)$$

$$H_+(\mu) = (2\pi)^2 N_c \, C_+^\dagger(\mu) C_+(\mu)$$

$$\mathcal{D}_{q/h}(\mathbf{p}_\perp^{\mathcal{D}}, z) \equiv \frac{1}{z}\sum_{X_n} \frac{1}{2N_c}\delta(p^-_{Xh;r})\delta^{(2)}(p^\perp_{Xh;r}) \, \mathrm{Tr}\left[\frac{\bar{n}\!\!\!/}{2}\langle 0|\delta_{\omega,\overline{\mathcal{P}}}\chi_n(0)\delta^{(2)}(\mathcal{P}_\perp^{X_n} + \mathbf{p}_\perp^{\mathcal{D}})|X_n h\rangle \times \langle X_n h|\bar{\chi}_n(0)|0\rangle\right]$$

$$\mathcal{D}_{i/h}(\mathbf{p}_\perp, z, \mu, \nu) = \int_z^1 \frac{dx}{x} \, \mathcal{J}_{i/j}(\mathbf{p}_\perp, x, \mu, \nu) D_{j/h}\left(\frac{z}{x}, \mu\right) \quad + \quad \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{|\mathbf{p}_\perp|^2}\right)$$

$$S_C(\mathbf{p}_\perp^S) \equiv \frac{1}{N_c}\sum_{X_{cs}} \mathrm{Tr}\left[\langle 0|V_n^\dagger(0)U_n(0)\delta^{(2)}(\mathcal{P}_\perp + \mathbf{p}_\perp^S)|X_{cs}\rangle\langle X_{cs}|U_n^\dagger(0)V_n(0)|0\rangle\right]$$

Anomalous Dimensions for RGE, RRGE

RGE

$$\gamma_\mu^{S_C}(\nu) = \frac{\alpha_s C_i}{\pi} \ln\left(\frac{\mu^2}{r^2\nu^2}\right)$$

$$\gamma_\mu^{\mathcal{D}}(\nu) = \frac{\alpha_s C_i}{\pi}\left(\ln\left(\frac{\nu^2}{\omega^2}\right) + \bar{\gamma}_i\right)$$

$$\gamma_\mu^{\mathcal{D}}(\nu) + \gamma_\mu^{S_C}(\nu) = \gamma_\mu^J = \frac{\alpha_s C_i}{\pi}\left(\ln\left(\frac{\mu^2}{r^2\omega^2}\right) + \bar{\gamma}_i\right)$$

Rapidity Renormalization Group

$$\gamma_\nu^{S_C}(p_\perp, \mu) = +(8\pi)\alpha_s C_i \, \mathcal{L}_0(\mathbf{p}_\perp, \mu^2)$$

$$\gamma_\nu^{\mathcal{D}}(p_\perp, \mu) = -(8\pi)\alpha_s C_i \, \mathcal{L}_0(\mathbf{p}_\perp, \mu^2)$$

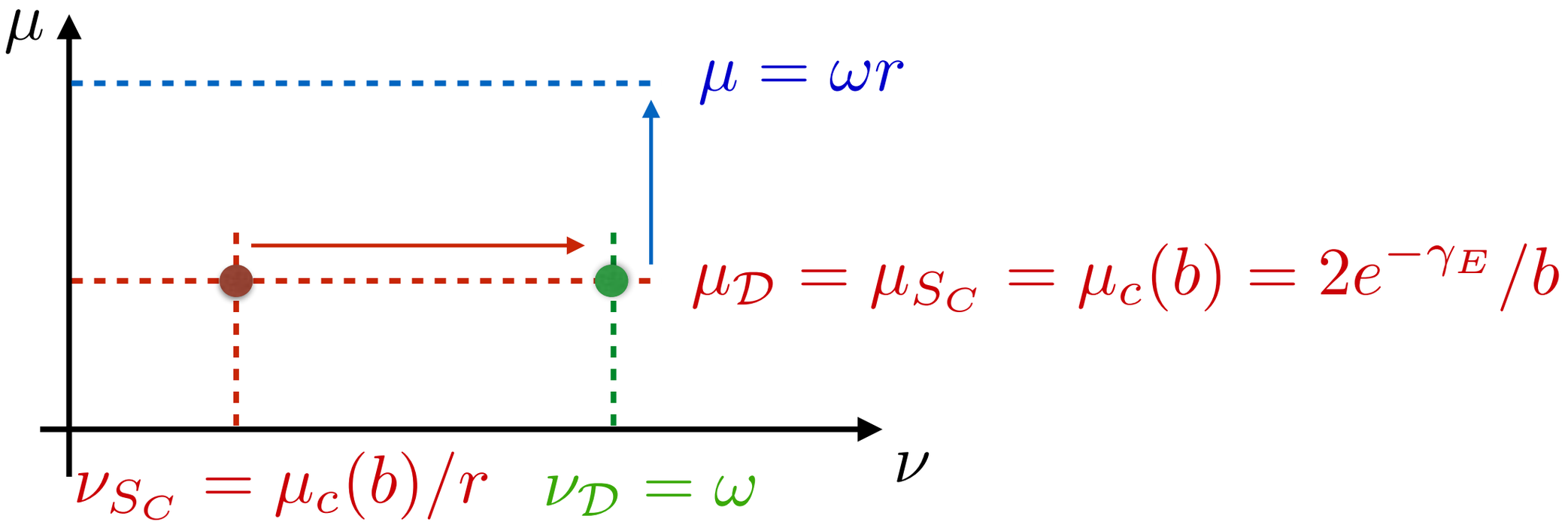
$$\gamma_\nu^{\mathcal{D}}(\mathbf{p}_\perp, \mu) + \gamma_\nu^{S}(\mathbf{p}_\perp, \mu) = 0$$

J-y. Chiu, A. Jain, D. Neill, I.Z. Rothstein, PRL108 (2012) 151601

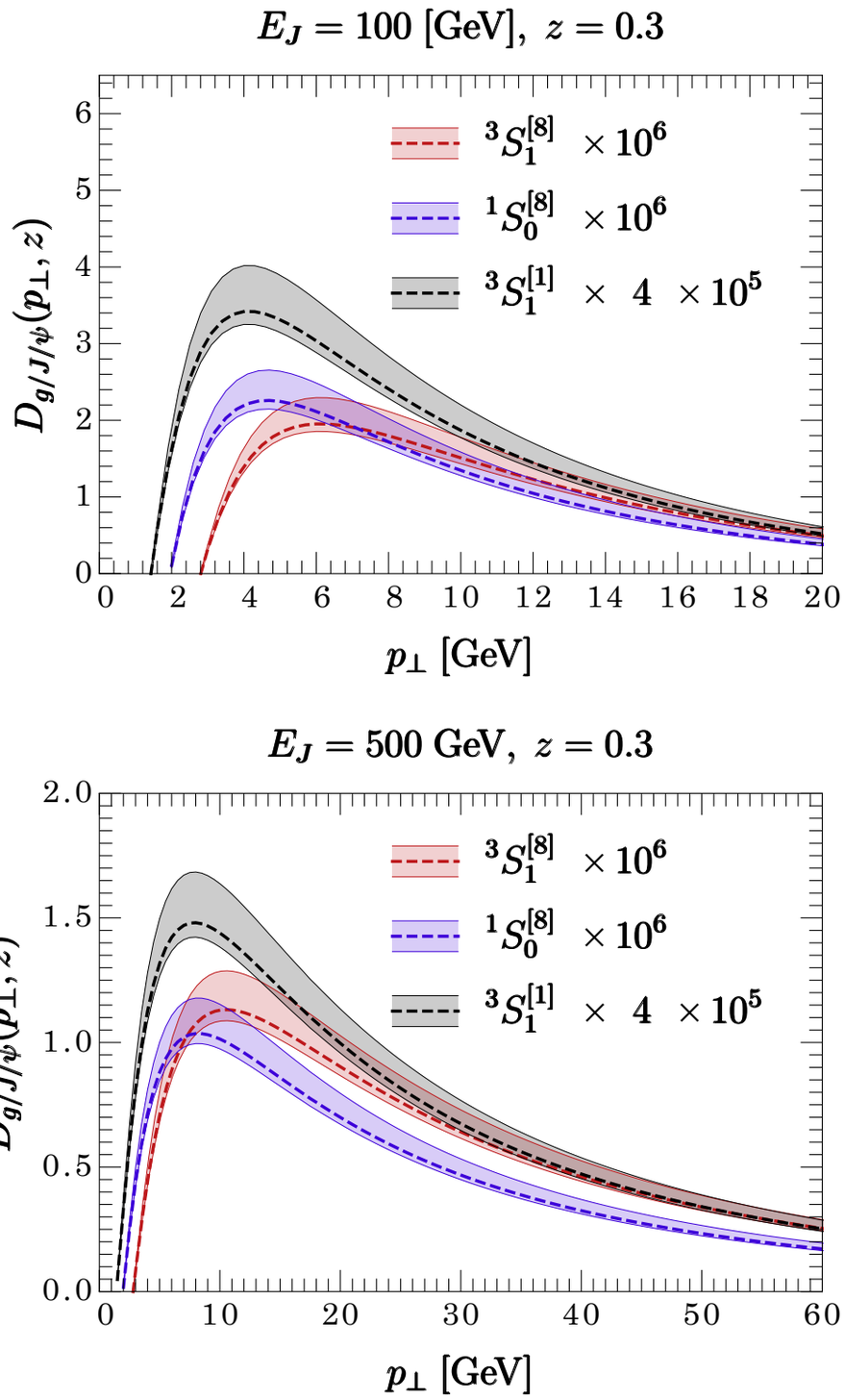
J-y. Chiu, A. Jain, D. Neill, I.Z. Rothstein, JHEP1205 (2012) 084

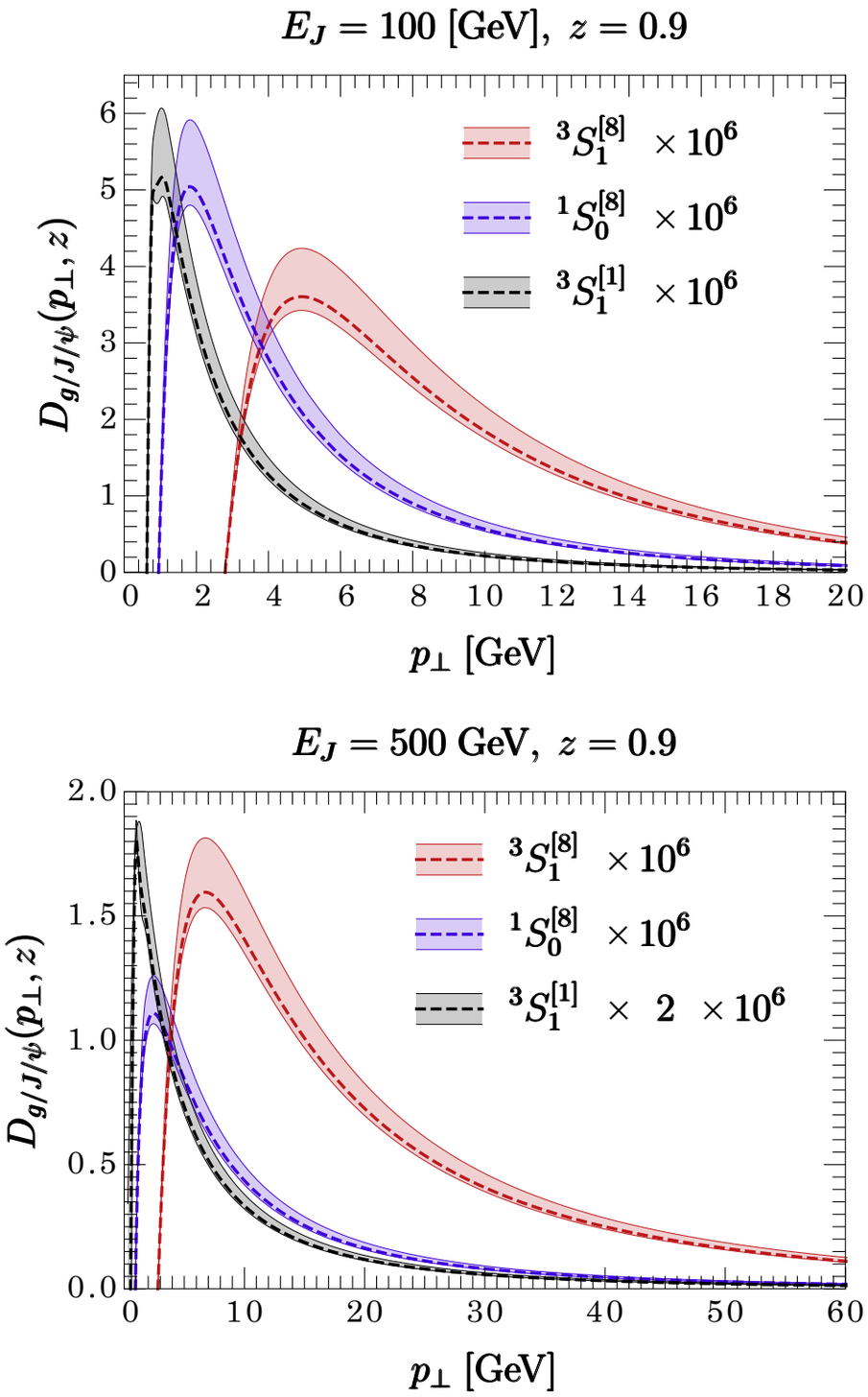
Solution to Evolution Eqs. in Fourier Space

$$D_{i/h}(p_\perp, z, \mu) = (2\pi)^2\, p_\perp \int_0^\infty db\; bJ_0(bp_\perp)\mathcal{U}_{S_C}(\mu, \mu_{S_C}, m_{S_C})\mathcal{U}_{\mathcal{D}}(\mu, \mu_{\mathcal{D}}, 1)$$
$$\times\, \mathcal{V}_{S_C}(b, \mu_{S_C}, \nu_{\mathcal{D}}, \nu_{S_C})\mathcal{FT}\left[\mathcal{D}_{i/h}(\mathbf{p}_\perp, z, \mu_{\mathcal{D}}, \nu_{\mathcal{D}}) \otimes_\perp S_C^i(\mathbf{p}_\perp, \mu_{S_C}, \nu_{S_C})\right]$$

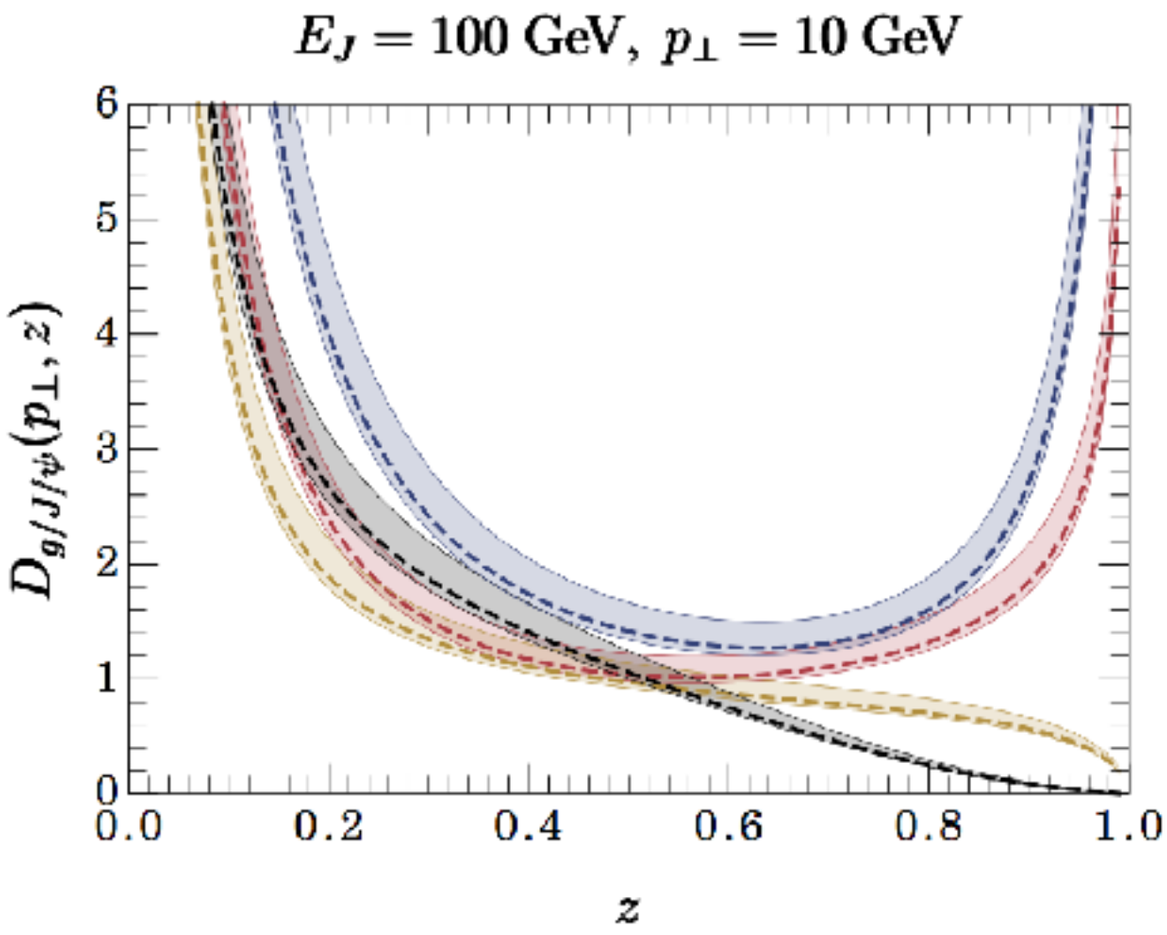


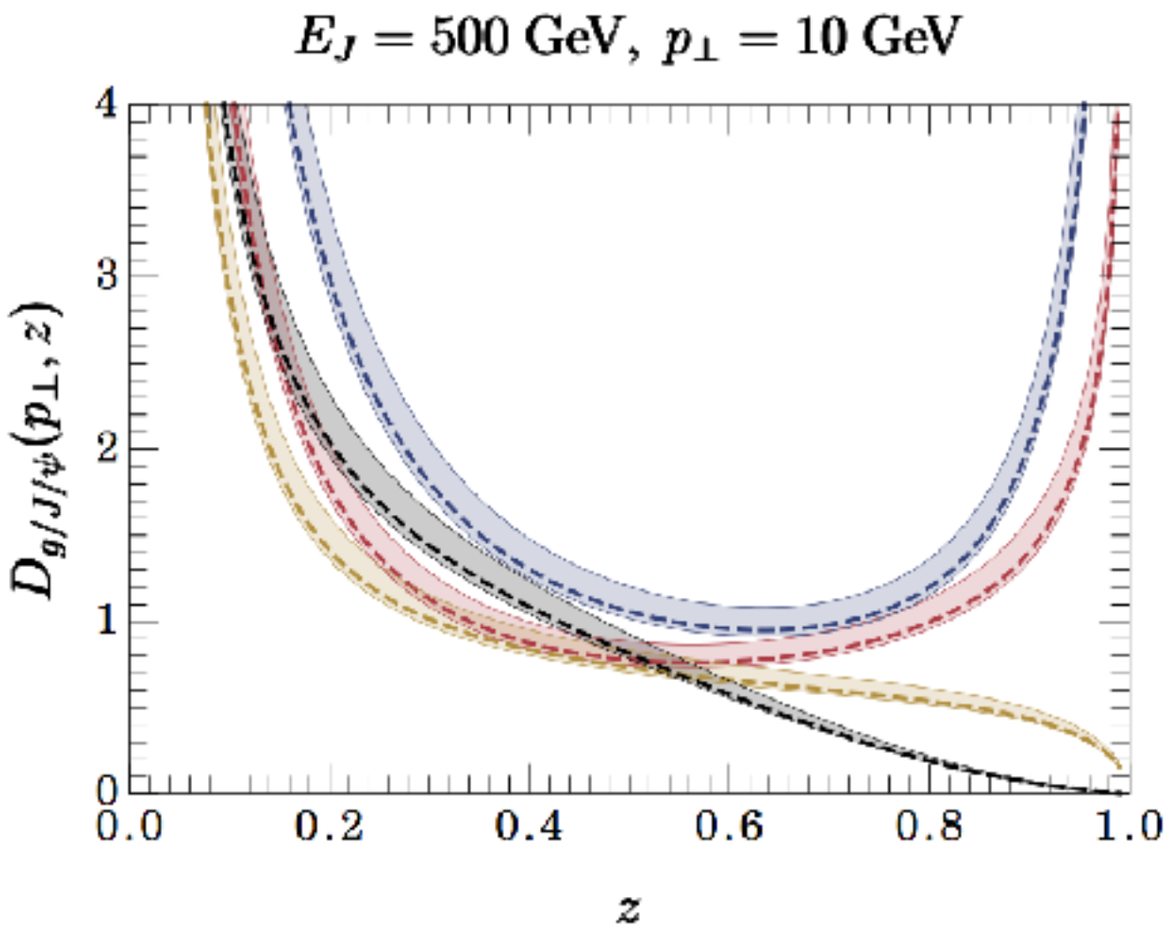
Application to Quarkonium Production





Application to Quarkonium Production

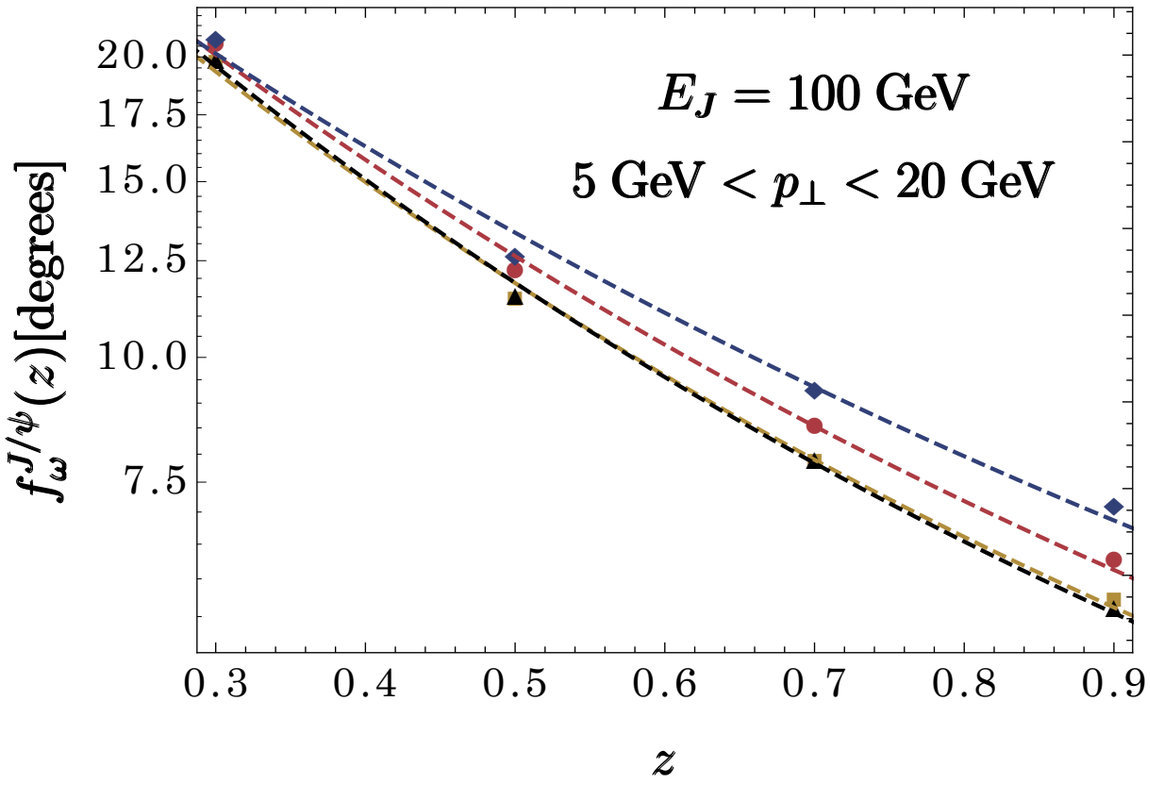


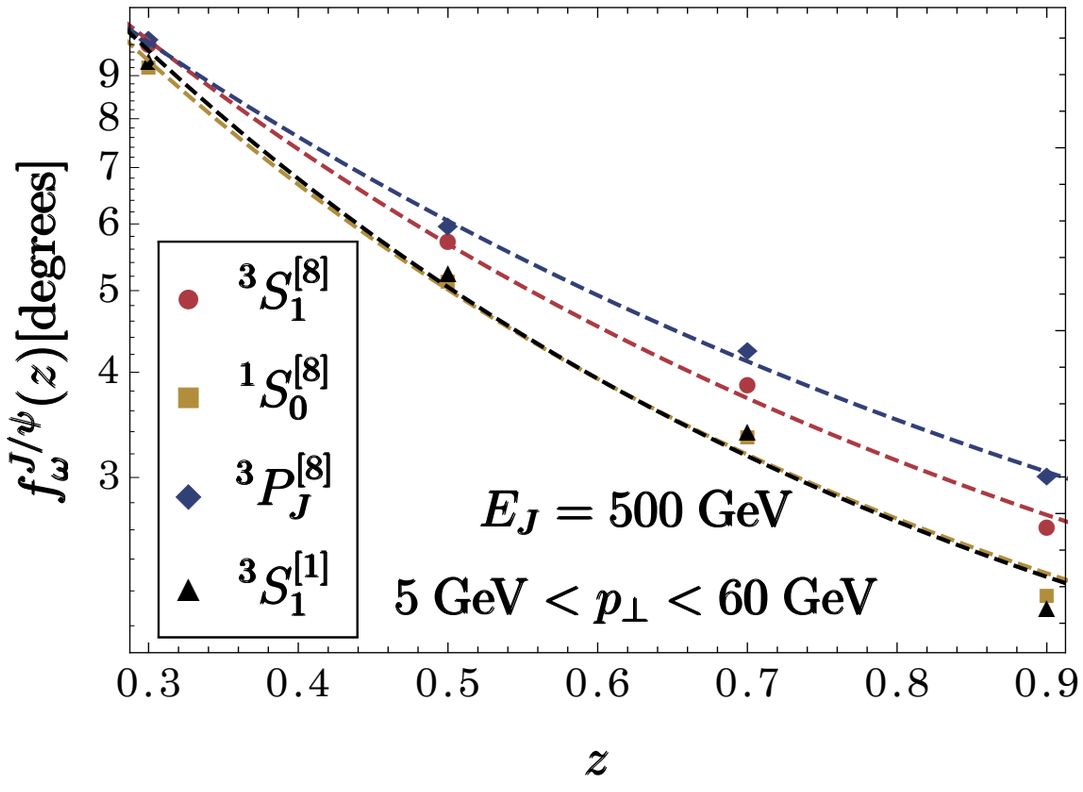


$^3S_1^{[8]} \times 10^6$ $^1S_0^{[8]} \times 10^6$ $^3P_J^{[8]} \times 3\,10^5$ $^3S_1^{[1]} \times 4\,10^5$

Application to Quarkonium Production

$$\langle\theta\rangle(z) \sim \frac{2\int dp_\perp\, p_\perp D_{g/h}(p_\perp, z, \mu)}{z\omega \int dp_\perp\, D_{g/h}(p_\perp, z, \mu)} \equiv f_\omega^h(z)$$





$E_J = 100$ GeV		
$^{2S+1}L_J^{[1,8]}$	C_0	C_1
$^3S_1^{[1]}$	3.92	0.92
$^3S_1^{[8]}$	3.86	0.84
$^1S_0^{[8]}$	3.88	0.90
$^3P_J^{[8]}$	3.75	0.74

$E_J = 500$ GeV		
$^{2S+1}L_J^{[1,8]}$	C_0	C_1
$^3S_1^{[1]}$	3.75	1.68
$^3S_1^{[8]}$	3.48	1.39
$^1S_0^{[8]}$	3.66	1.64
$^3P_J^{[8]}$	3.28	1.20

$$\ln(f(x)) = g(x; C_0, C_1) \text{ s.t. } g(x=0) = C_0$$

$$g_2(x) = C_0 \exp(-C_1 x)$$

Conclusions

measuring $Q\bar{Q}$ within jets, and using jet observables should provide insights into $Q\bar{Q}$ production

If $^1S_0^{(8)}$ mechanism dominates high p_T production FJF should have negative slope for z(E), for z>0.5

p_T-dependent quarkonium fragmenting jet functions (TMDFJFs)

p_T, theta of quarkonium in jet sensitive to NRQCD production mechanism

Analysis of LHCb Data

NRQCD + FJF, GFIP better than default PYTHIA

Confirms physical picture of NLL' calculations

z(J/ψ) prefers LDMEs from large p_T extractions

Similar cancellation/suppression of ${}^3S_1^{(8)}$, ${}^3P_J^{(8)}$ FFs

required to get vanishing high p_T polarization

is needed to get LHCb z(J/ψ) distribution

Backup

fragmentation function (QCD)

$$D_q^h(z) = z \int \frac{\mathrm{d}x^+}{4\pi}\, e^{ik^- x^+/2}\, \frac{1}{4N_c}\, \mathrm{Tr} \sum_X \langle 0|\bar{\not{n}}\, \Psi(x^+,0,0_\perp)|Xh\rangle\langle Xh|\bar{\Psi}(0)|0\rangle\big|_{p_h^\perp=0}$$

fragmentation function (SCET)

$$D_q^h\Big(\frac{p_h^-}{\omega},\mu\Big) = \pi\omega \int dp_h^+\, \frac{1}{4N_c}\, \mathrm{Tr} \sum_X \bar{\not{n}}\, \langle 0|[\delta_{\omega,\bar{\mathcal{P}}}\, \delta_{0,\mathcal{P}_\perp}\, \chi_n(0)]|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle$$

Jet function (SCET)

$$J_u(k^+\omega) = -\frac{1}{\pi\,\omega}\, \mathrm{Im} \int d^4x\, e^{ik\cdot x}\, i\, \langle 0|\, \mathrm{T}\, \bar{\chi}_{n,\omega,0_\perp}(0)\, \frac{\bar{\not{n}}}{4N_c}\, \chi_n(x)|0\rangle$$

fragmentation jet function (SCET)

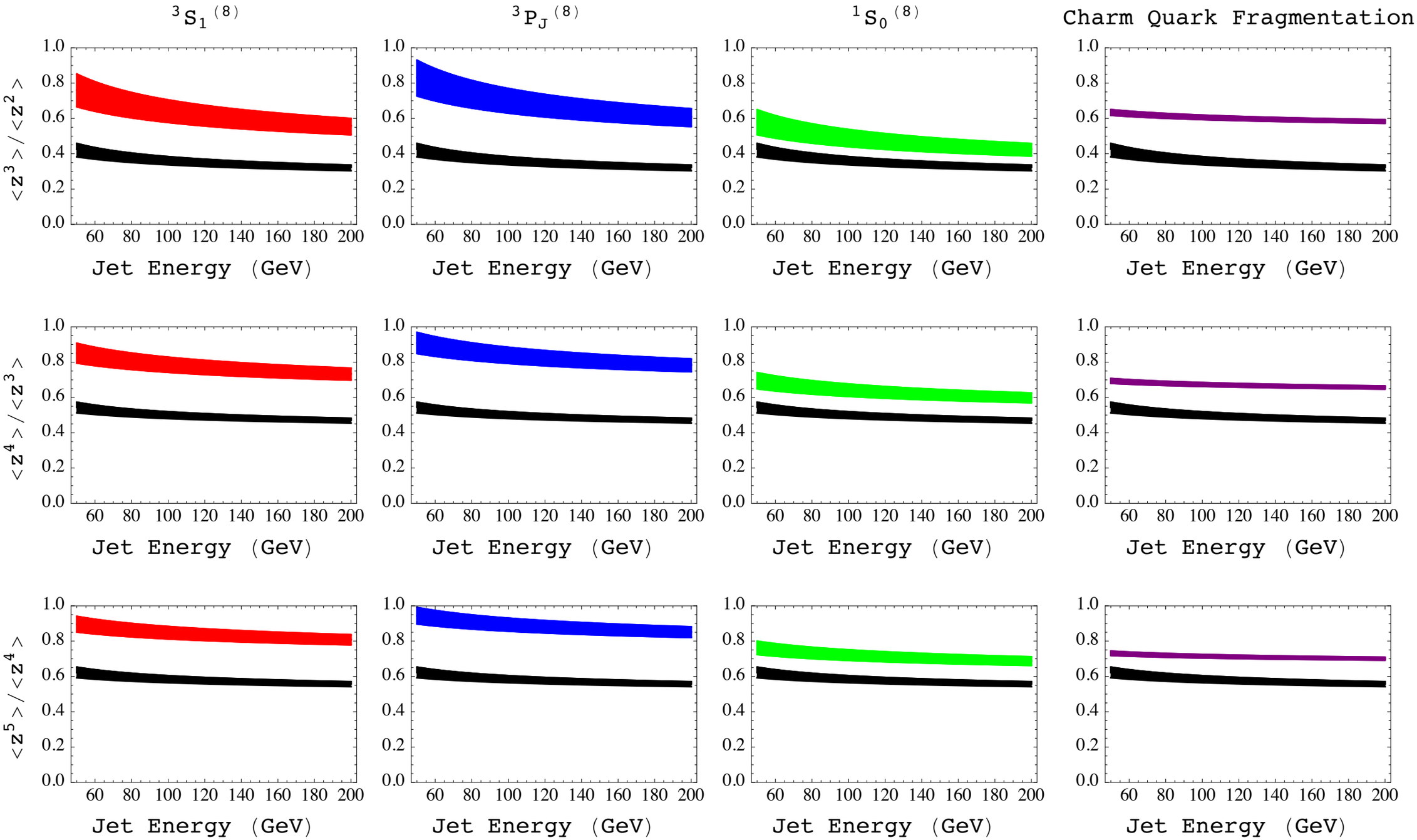
$$\mathcal{G}^h_{q,\mathrm{bare}}(s,z) = \int \mathrm{d}^4y\, \mathrm{e}^{\mathrm{i}k^+y^-/2} \int \mathrm{d}p_h^+ \sum_X \frac{1}{4N_c}\, \mathrm{tr}\left[\frac{\bar{\not{n}}}{2}\langle 0|[\delta_{\omega,\overline{\mathcal{P}}}\, \delta_{0,\mathcal{P}_\perp}\, \chi_n(y)]|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle\right]$$

$$\delta(p^+/z - P_H^+) \to \delta(p^+/z - P_H^+)\delta(p^- - s/p^+)$$

FF FJF

Ratios of Moments

$$E\tan(R/2) < \mu < 4E\tan(R/2)$$

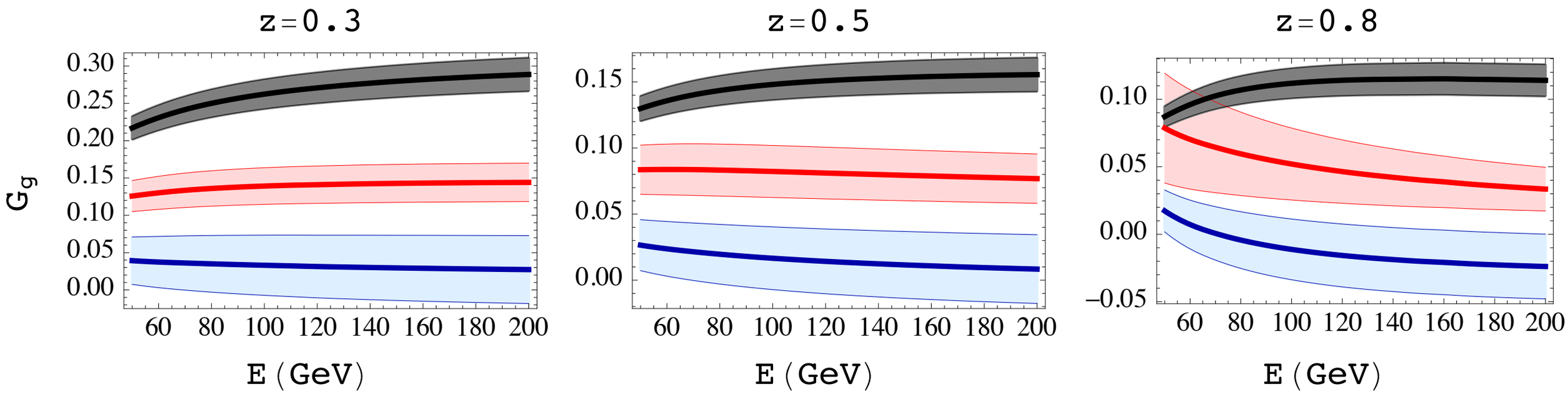


Ratios of Moments

$$\left.\frac{\langle z^{n+1}\rangle}{\langle z^{n}\rangle}\right|_{^3P_J^{(8)}} \approx \left.\frac{\langle z^{n+1}\rangle}{\langle z^{n}\rangle}\right|_{^3S_1^{(8)}} > \left.\frac{\langle z^{n+1}\rangle}{\langle z^{n}\rangle}\right|_{^1S_0^{(8)}} \approx \left.\frac{\langle z^{n+1}\rangle}{\langle z^{n}\rangle}\right|_{\mathrm{c-quark}} > \left.\frac{\langle z^{n+1}\rangle}{\langle z^{n}\rangle}\right|_{^3S_1^{(1)}}$$

Gluon FJF for different extractions of LDME

fix z, vary energy



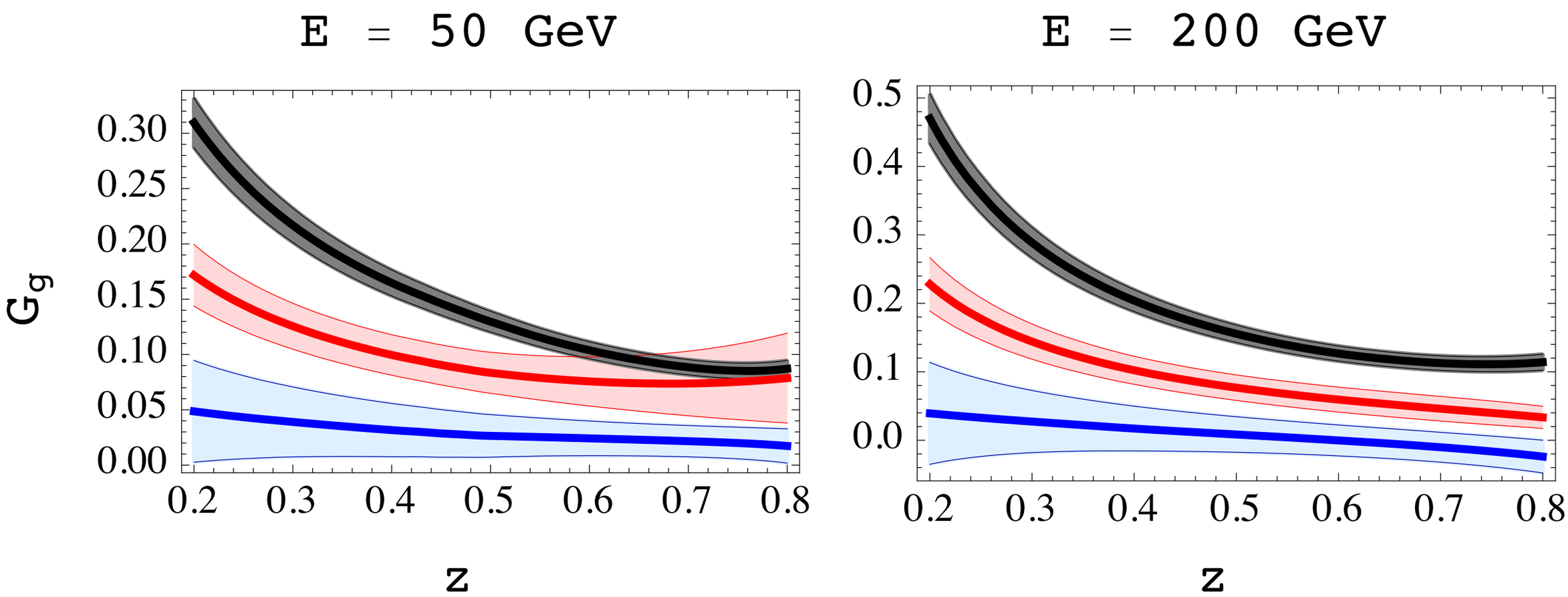
Butenschoen and Kniehl, PRD 84 (2011) 051501, arXiv:1105.0822

Bodwin, et. al. arXiv:1403.3612

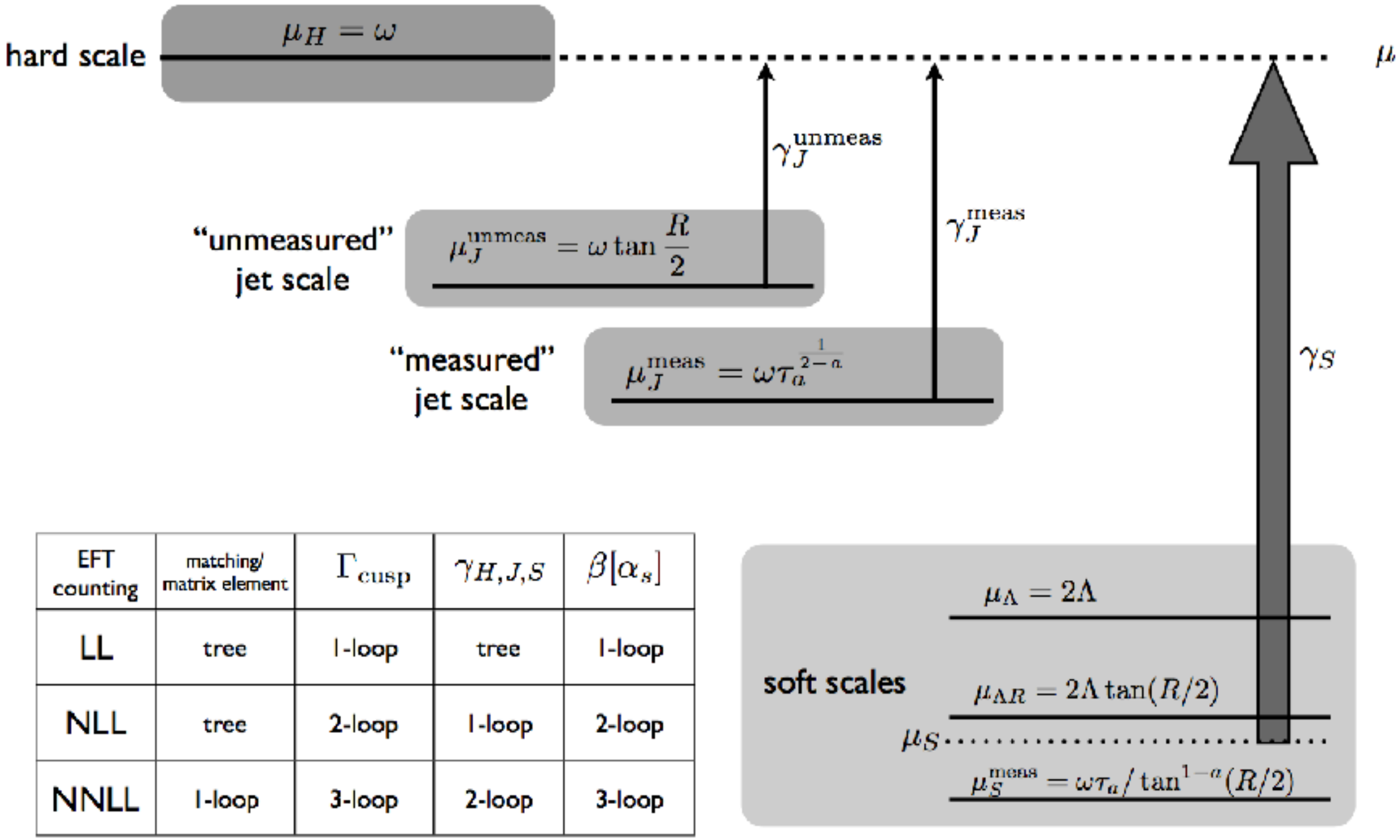
Chao, et. al. PRL 108, 242004 (2012)

Gluon FJF for different extractions of LDME

fix energy, vary z



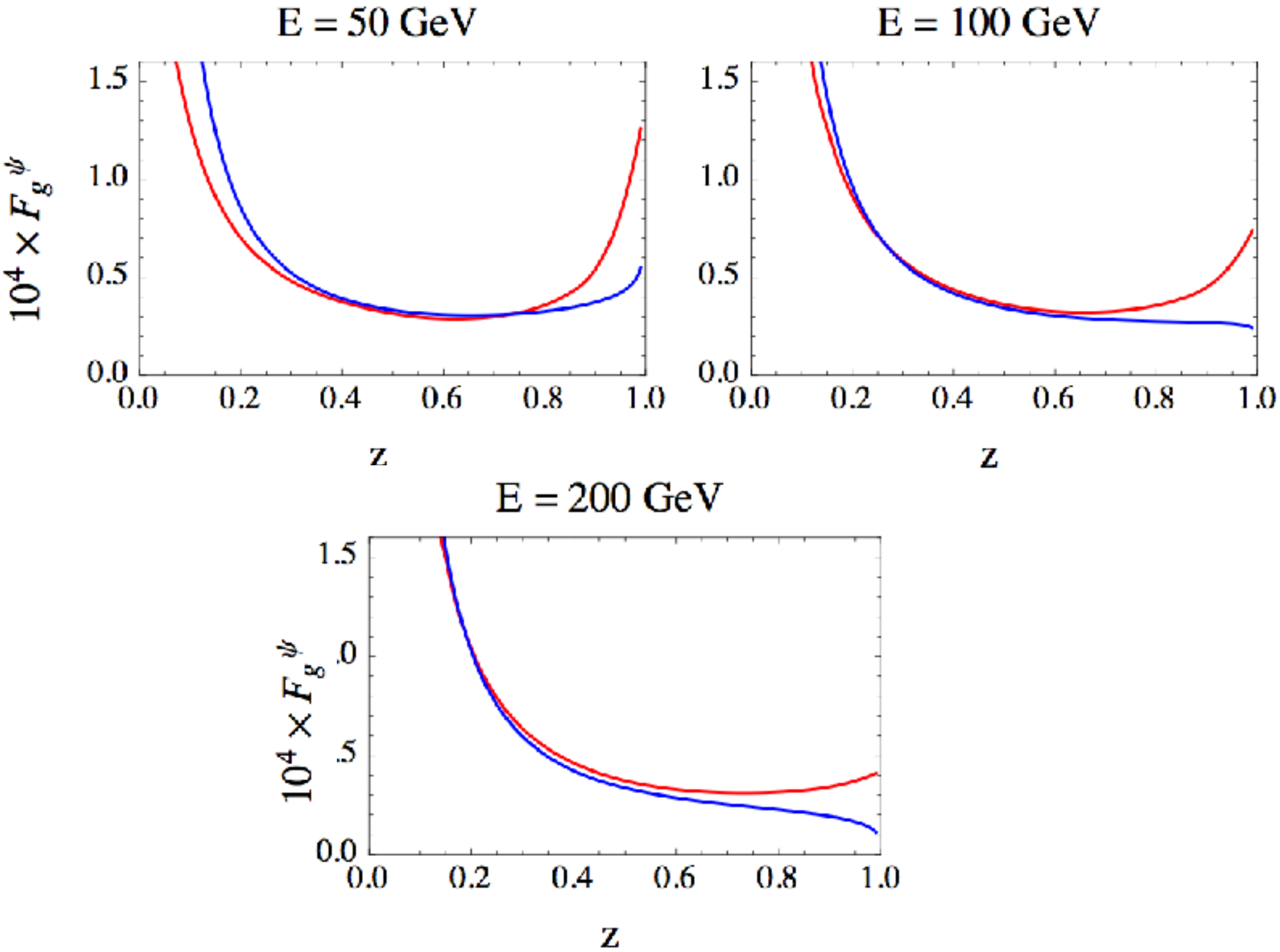
Scales in Jet Cross section



EFT counting	matching/ matrix element	$\Gamma_{\rm cusp}$	$\gamma_{H,J,S}$	$\beta[\alpha_s]$
LL	tree	1-loop	tree	1-loop
NLL	tree	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop

Color-Octet $^3S_1$ fragmentation function, FJF

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003

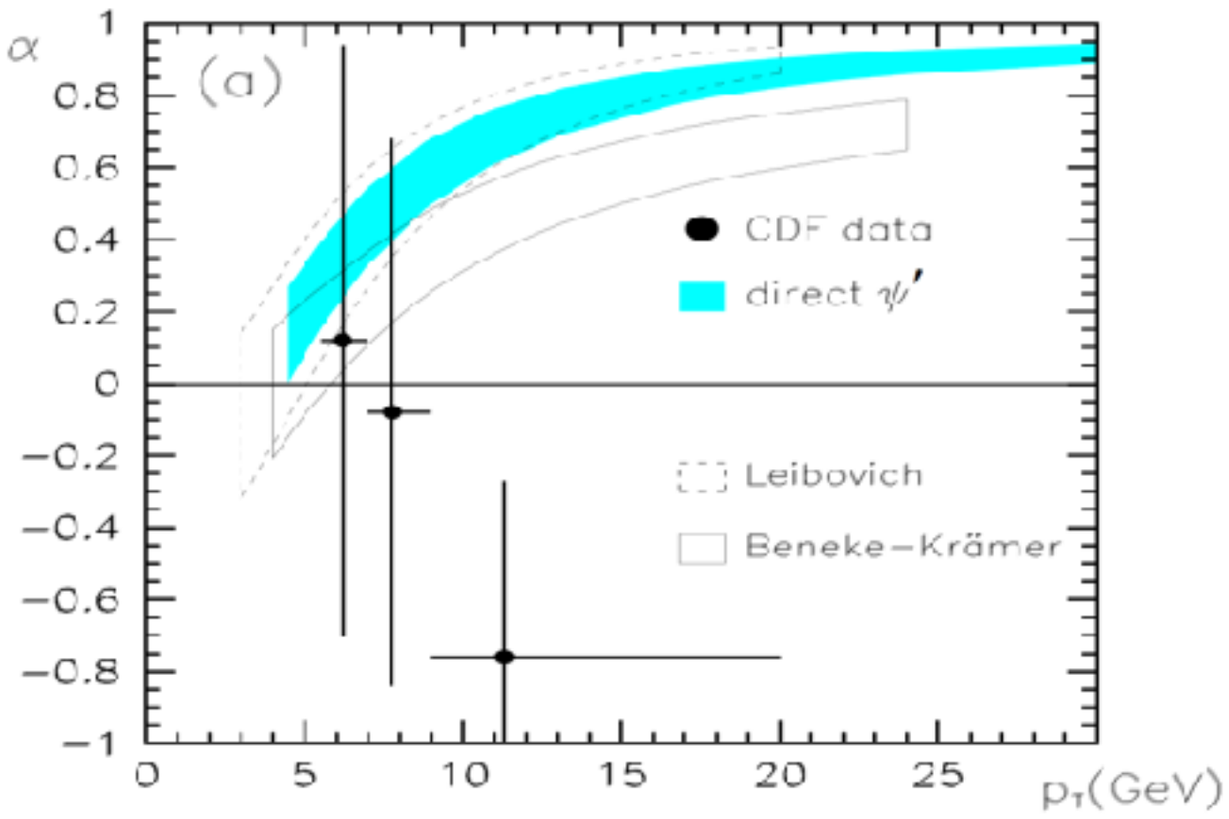


fragmentation function

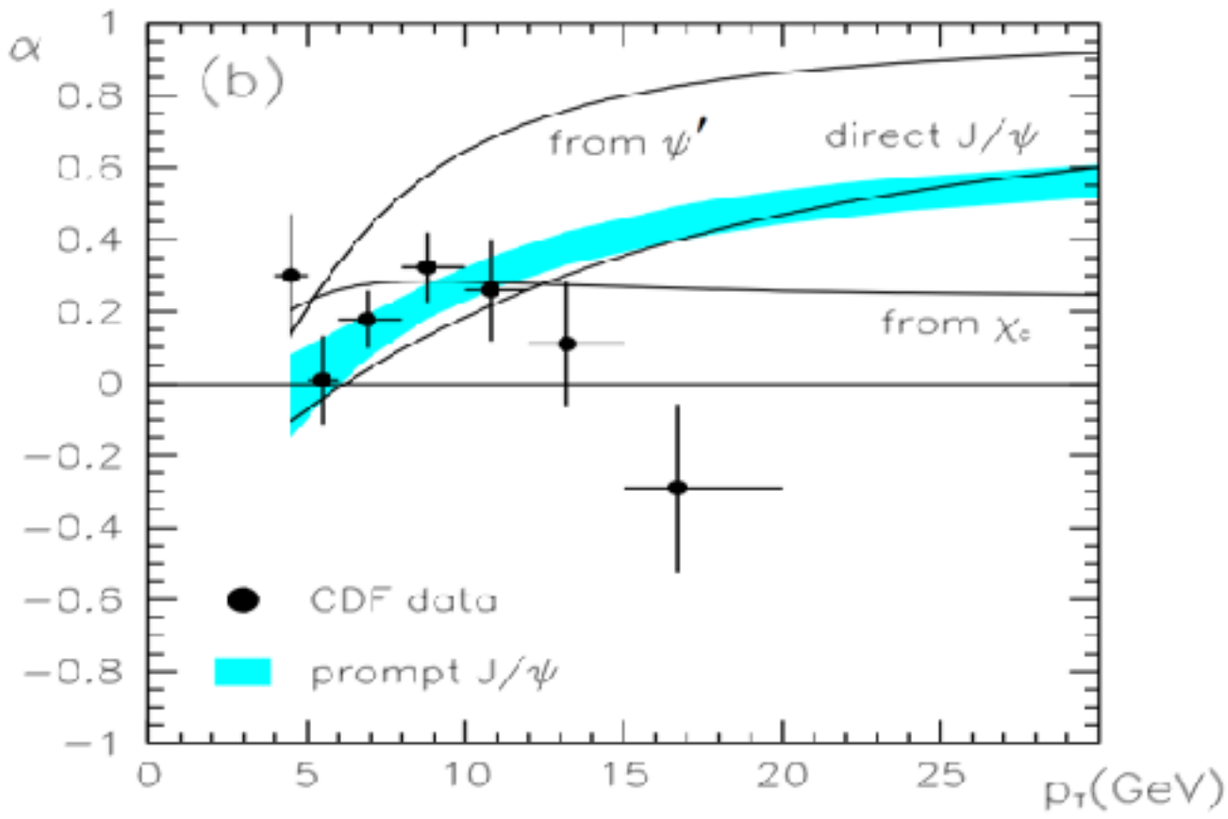
fragmenting jet function

Polarization Puzzle

$^3S_1^{[8]}$ fragmentation at large p_T predicts transversely polarized J/ψ, ψ'



ψ′



J/ψ

Braaten, Kniehl, Lee, 1999

$$D_{q/h}(\mathbf{p}_\perp, z, \mu) = \frac{1}{z}\sum_X \frac{1}{2N_c}\delta(p^-_{Xh;r})\delta^{(2)}(\mathbf{p}_\perp + \mathbf{p}^X_\perp)\,\mathrm{Tr}\left[\frac{\not{\bar{n}}}{2}\langle 0|\delta_{\omega,\overline{\mathcal{P}}}\chi^{(0)}_n(0)|Xh\rangle\right.$$

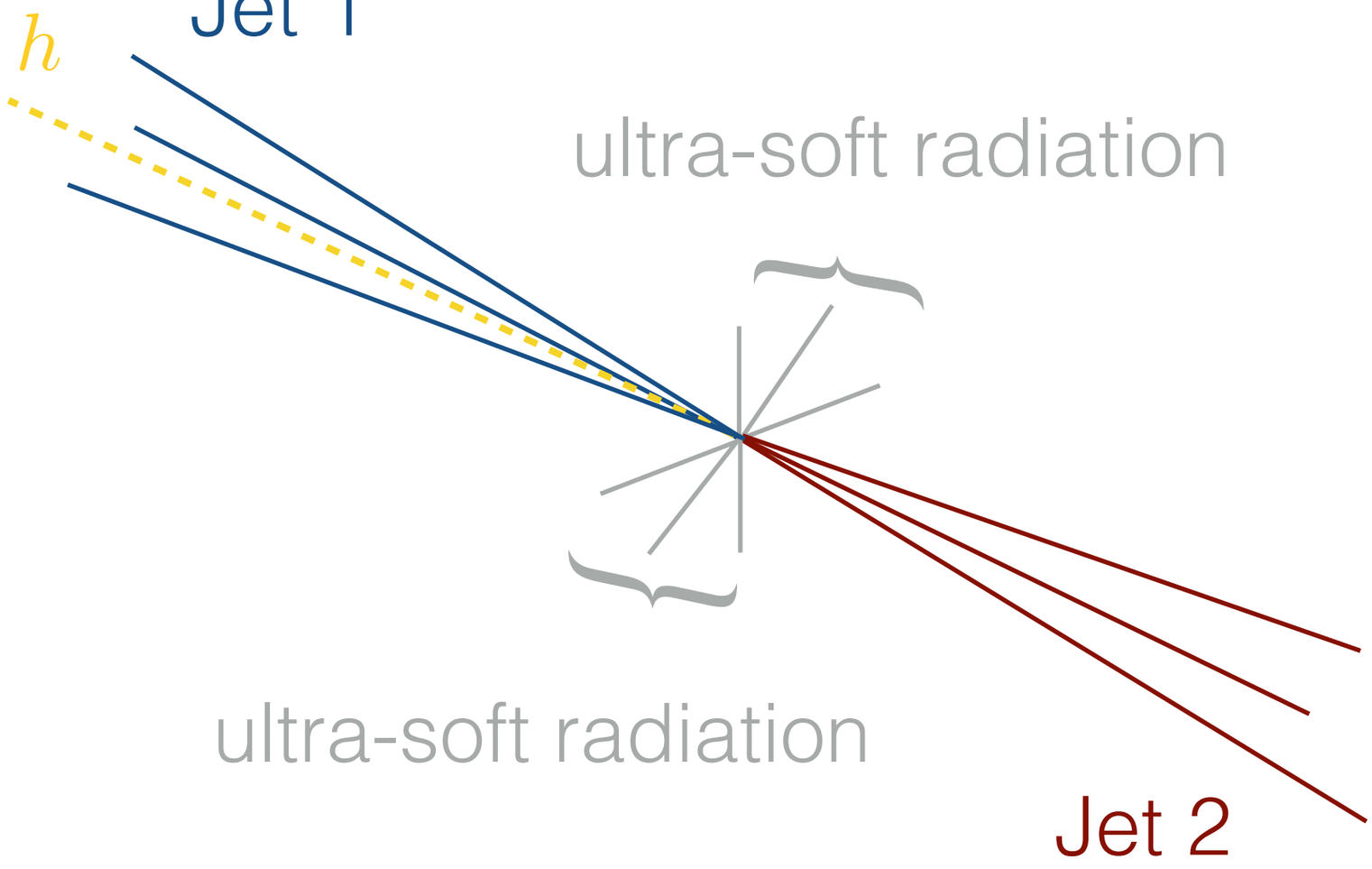
$$\left.\langle Xh|\bar{\chi}^{(0)}_n(0)|0\rangle\right]$$

$$\int d^2\mathbf{p}^h_\perp\; D_{q/h}(\mathbf{p}^h_\perp, z, \mu) = D_{q/h}(z, \mu)$$

Transverse Momentum Dependent FJFs

R. Bain, Y. Makris, TM, JHEP 1611 (2016) 144

$$\omega \gg p_h^\perp \gg \Lambda$$



$$D_{i/h}\left(z, p_h^\perp, \mu\right)$$

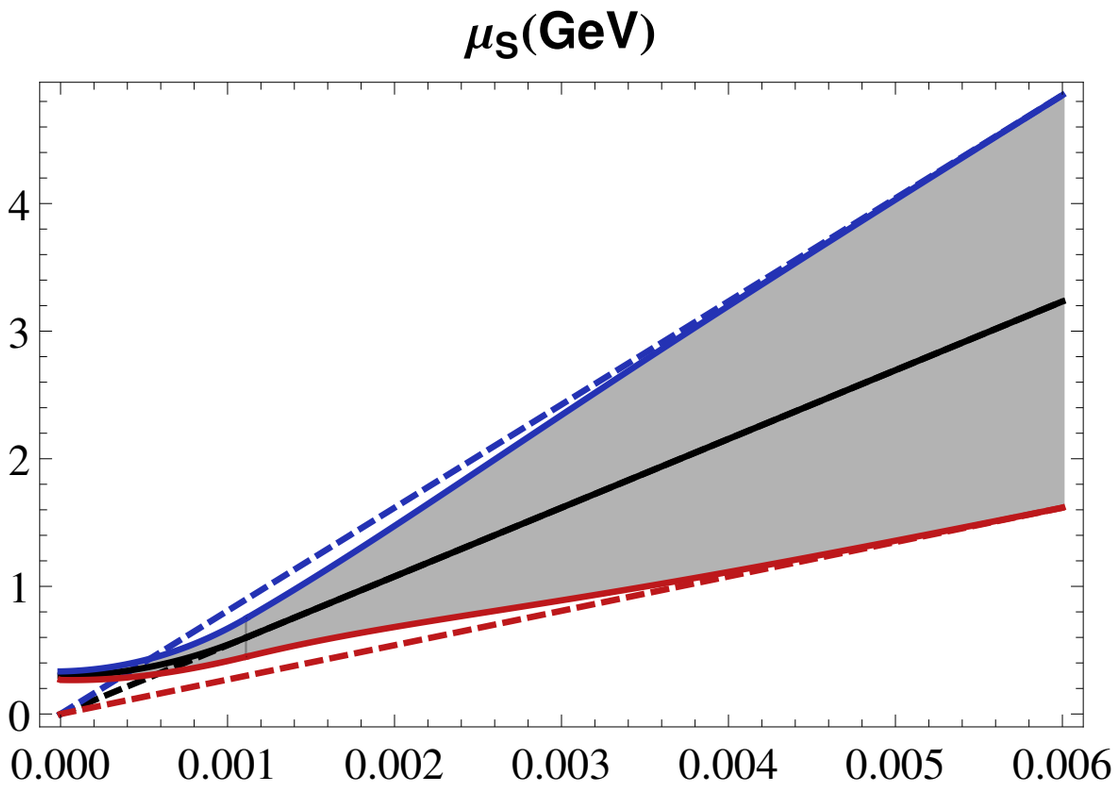
$$p_c \sim \omega(\lambda^2, 1, \lambda)$$

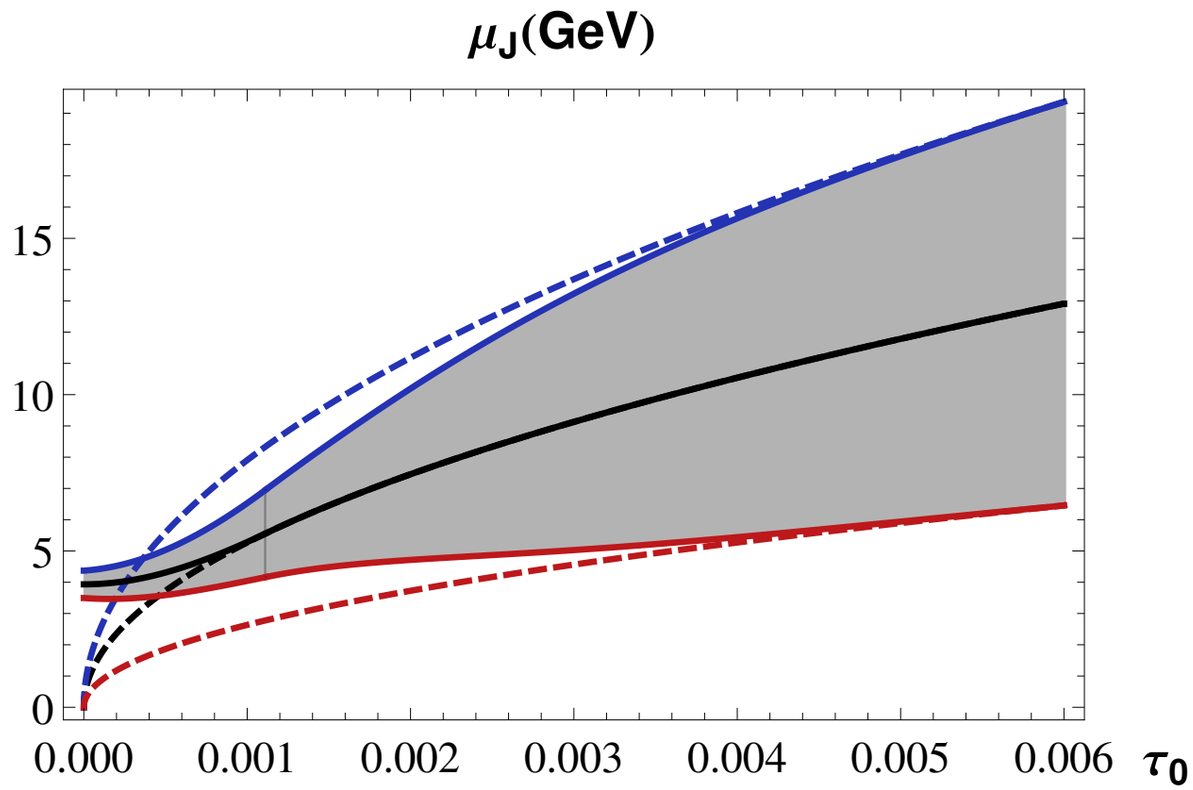
$$p_{cs} \sim p_h^\perp(r, 1/r, 1)$$

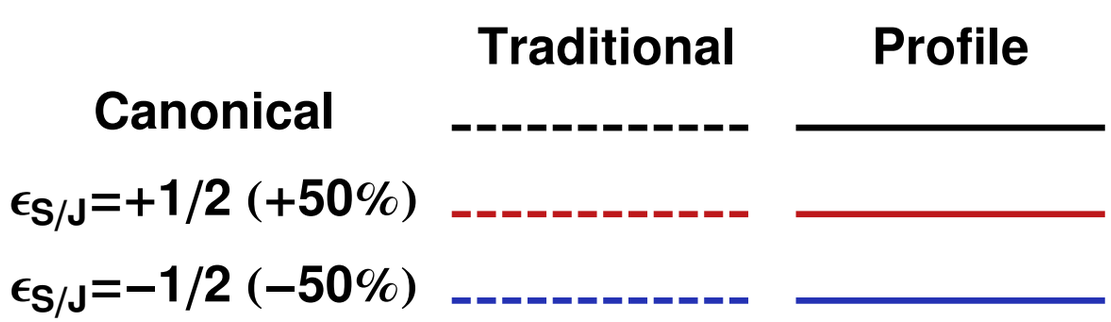
$$p_{us} \sim \Lambda(1, 1, 1)$$

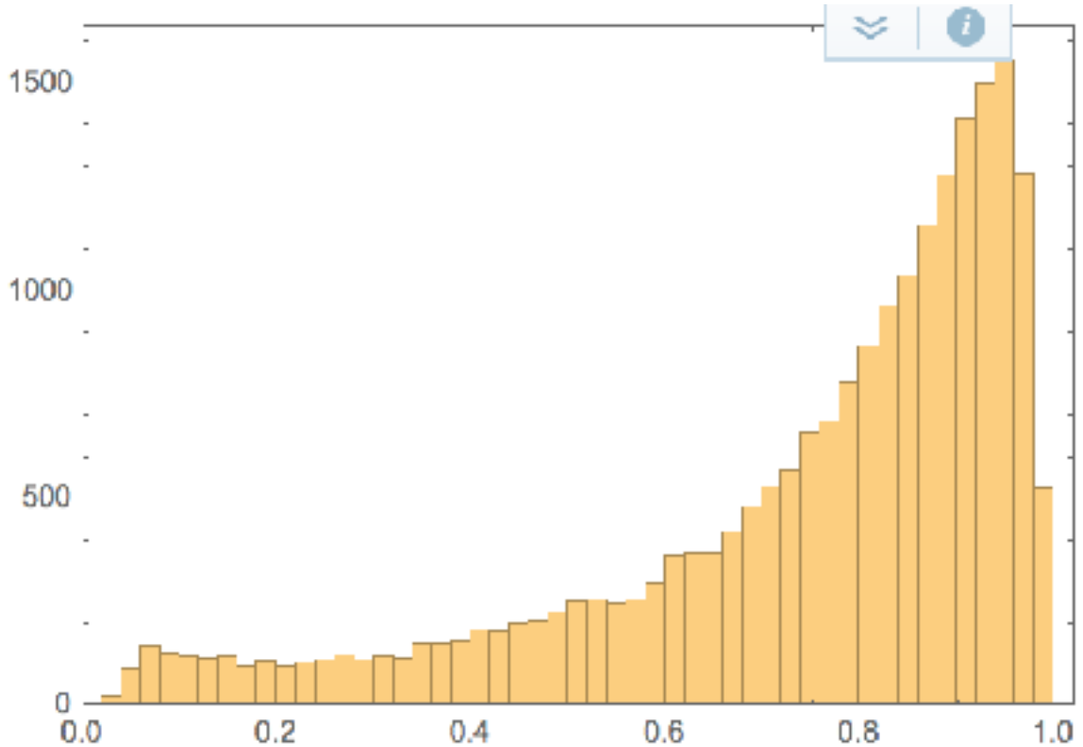
$$\lambda = p_h^\perp/\omega$$

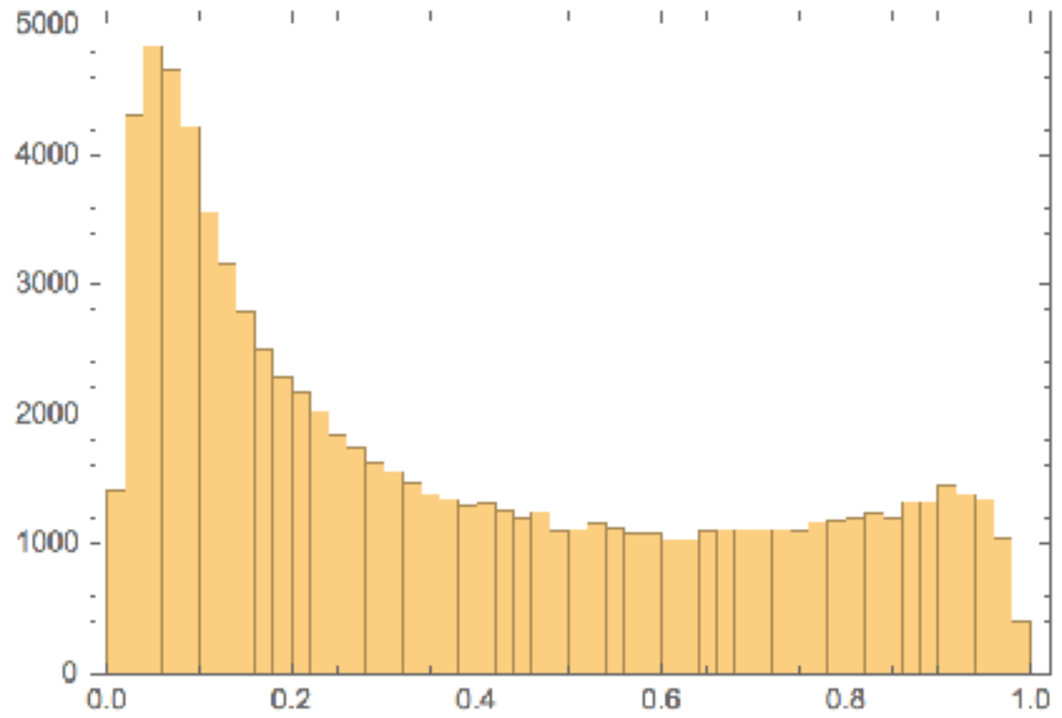
Profile Functions







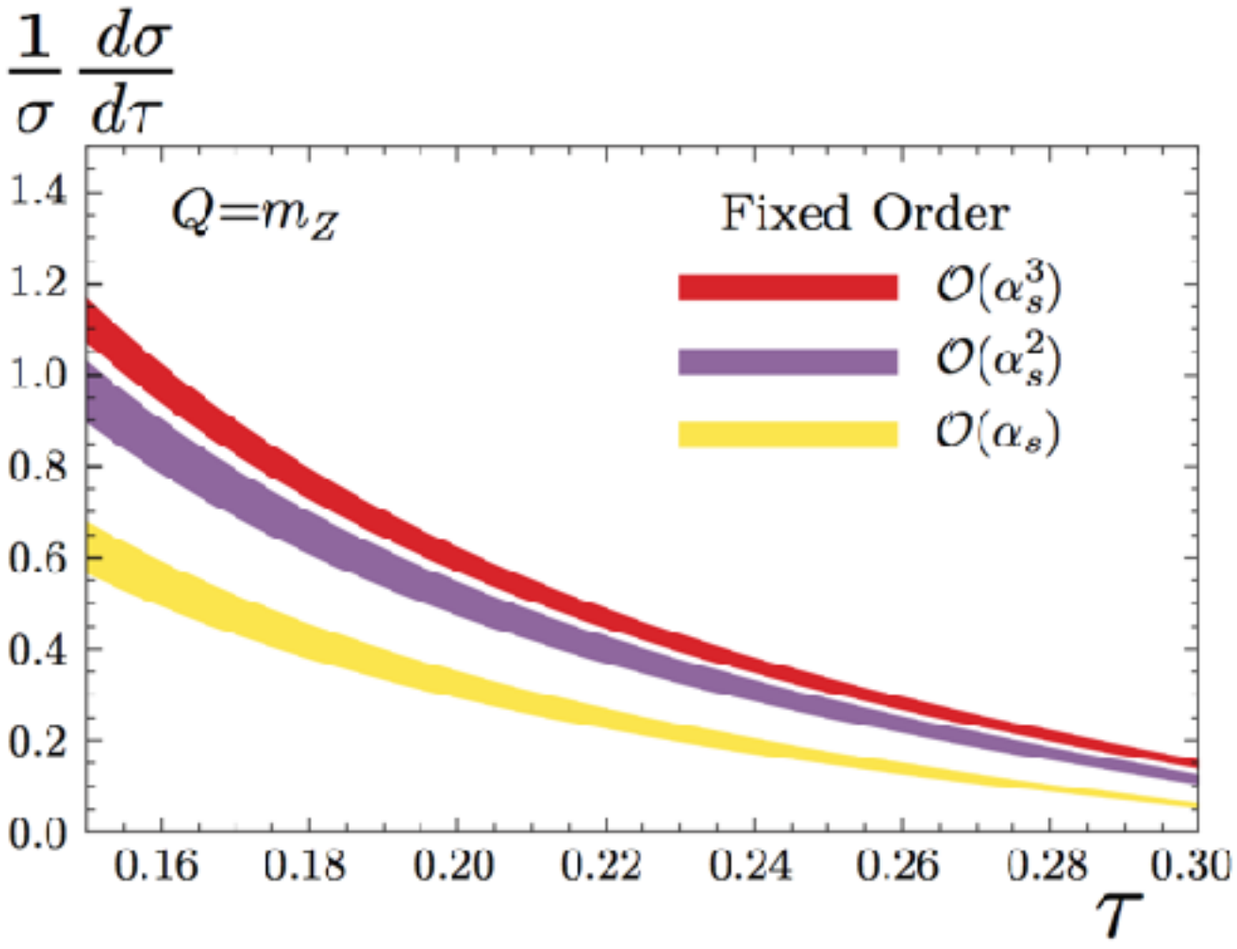


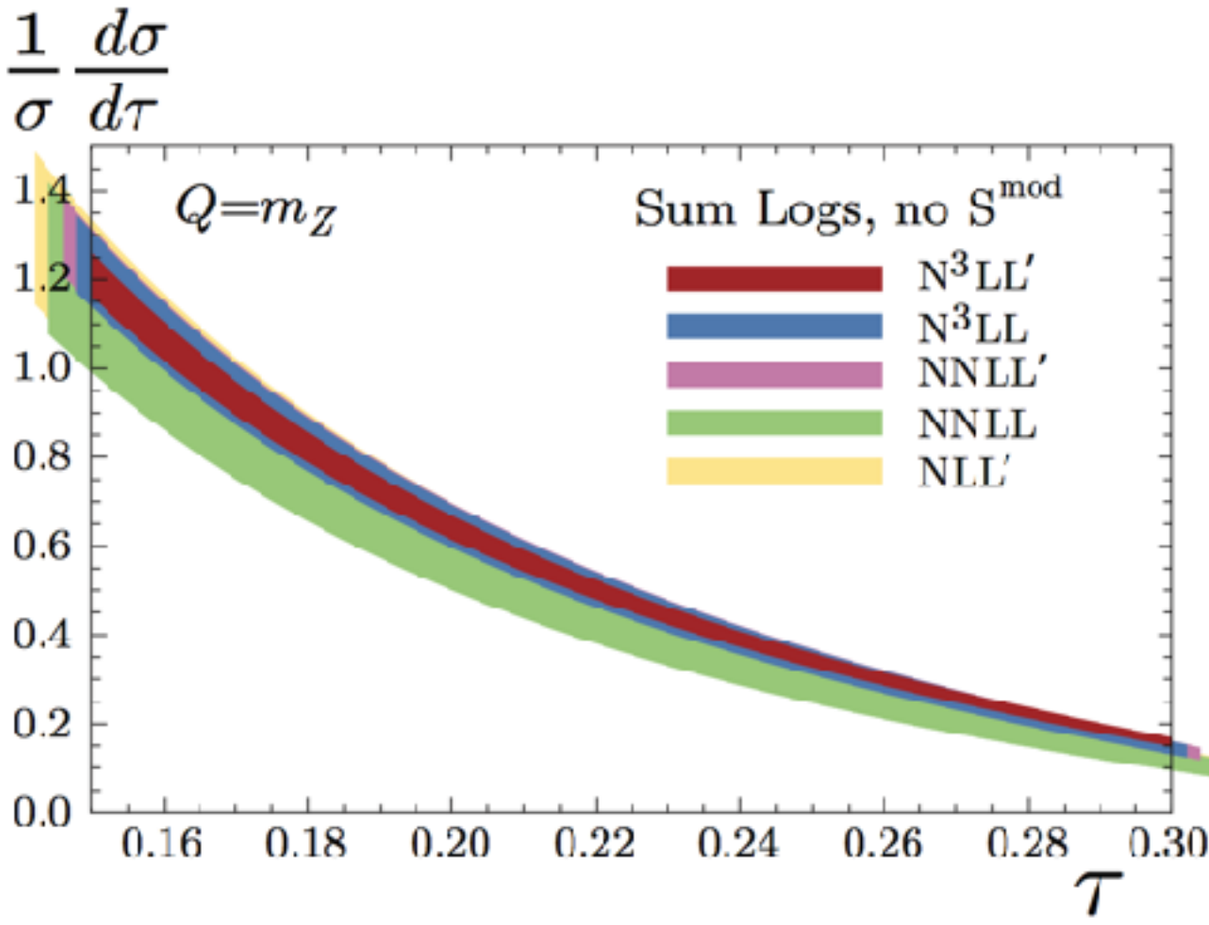


c distribution

g distribution

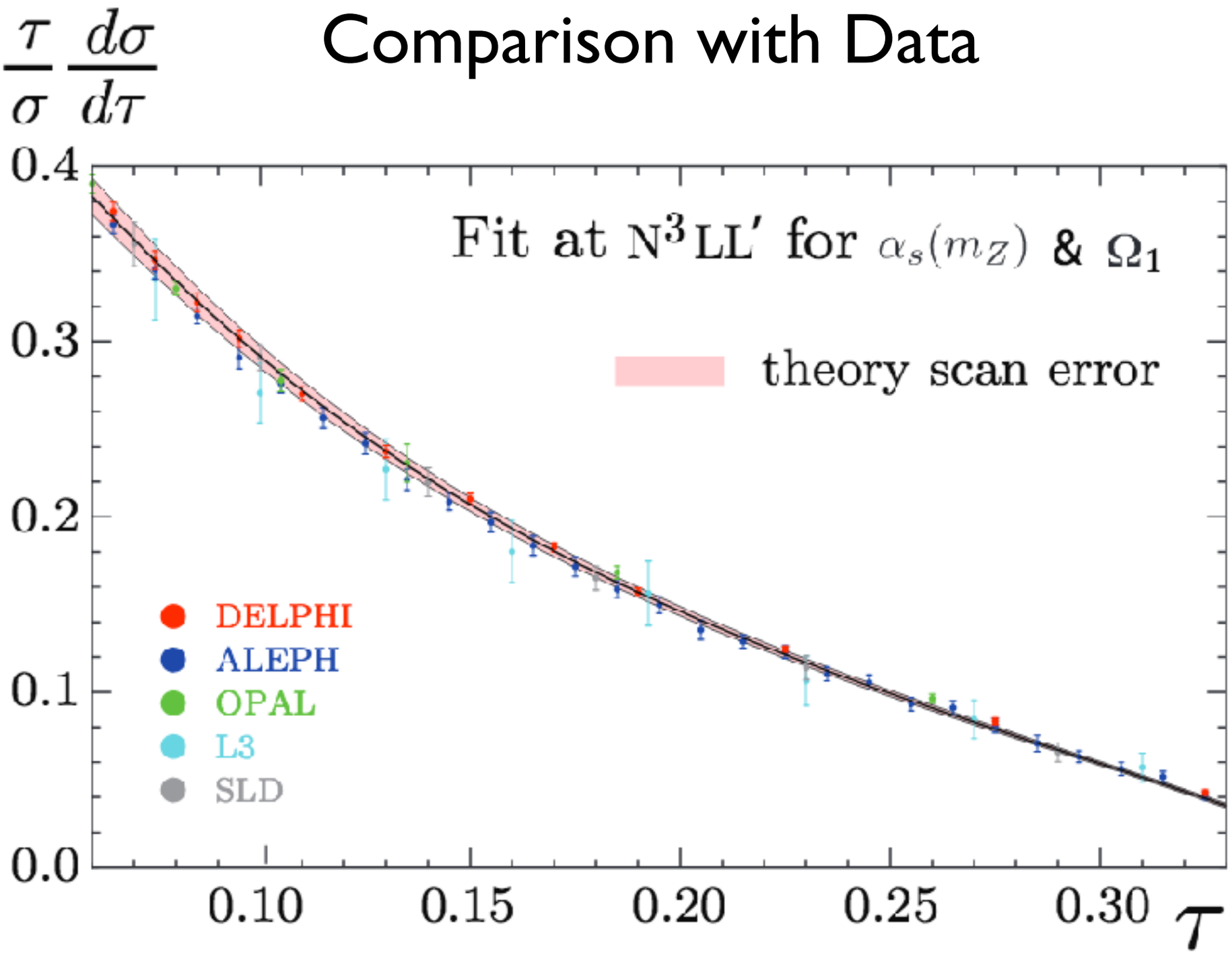
Importance of Resummation of Logarithms in Thrust





R. Abbate, et.al., Phys.Rev. D83 (2011) 074021

Comparison with Data



$$\alpha_s(M_Z) = 0.1135 \pm 0.0022$$

R. Abbate, et.al., Phys.Rev. D83 (2011) 074021