



# PDFs for LHC Physics

J. Huston

Michigan State University

IPPP Durham University



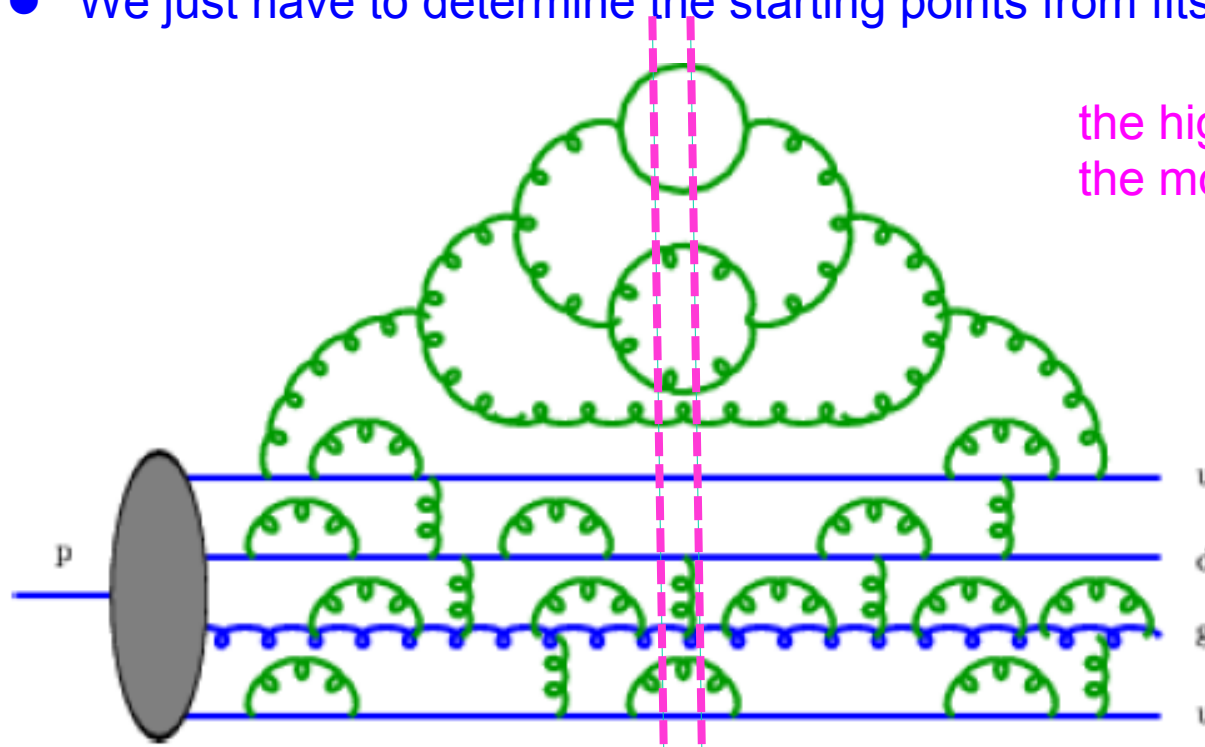
Make PDFs  
Great Again



# Hadrons



- The proton is a dynamical object; the structure observed depends on the time-scale ( $Q^2$ ) of the observation
- But we know how to calculate this variation (DGLAP) (at LO, NLO, NNLO)
- We just have to determine the starting points from fits to data



the higher the value of  $Q^2$ ,  
the more detail we examine

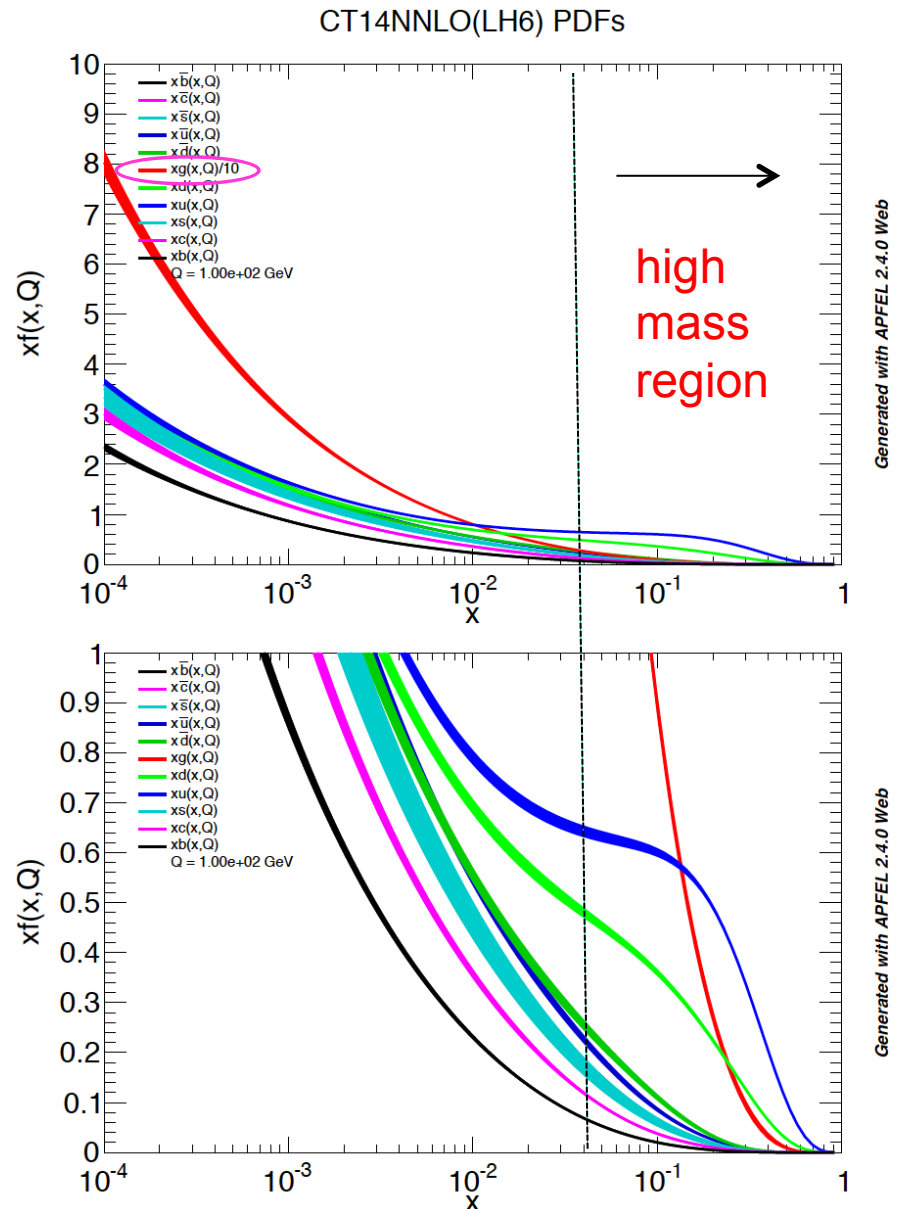
$f_i(x, Q^2)$  = number density of partons  $i$   
at momentum fraction  $x$  and probing scale  $Q^2$



# Parton distribution functions and global fits

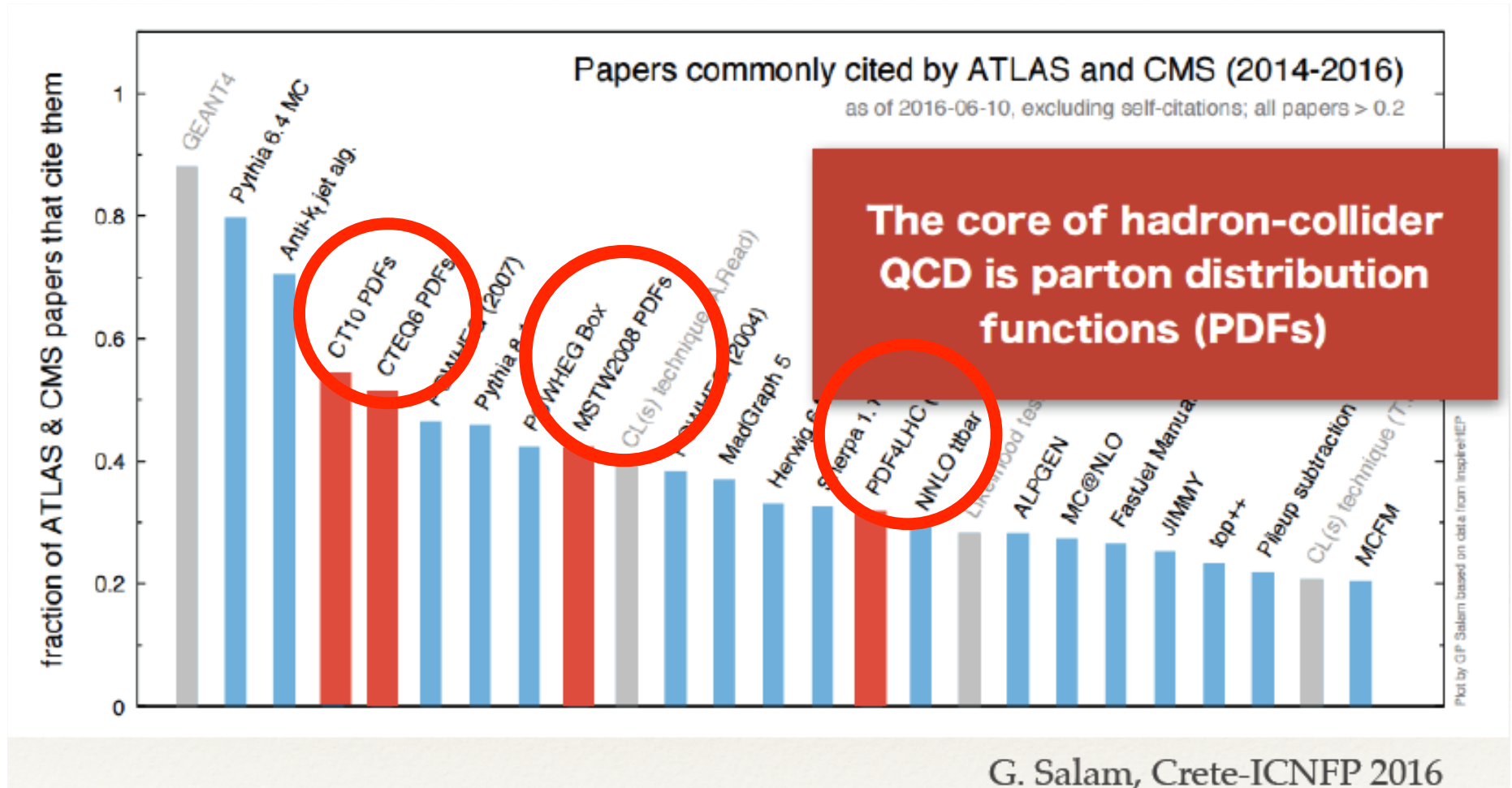


- Calculation of production cross sections at the LHC relies upon knowledge of PDF's in the relevant kinematic region
- PDFs are determined by global analyses of data from DIS, DY and jet production... now adding additional LHC processes such as  $t\bar{t}$  production,  $Z p_T$ ,  $W/Z/\text{photon} + c$ , etc
- PDF fitting groups come out with new PDF sets as new data/technology warrants, at LO, NLO and NNLO
  - ◆ *ABM12*
  - ◆ *CT14*
  - ◆ *HERAPDF2.0*
  - ◆ *MMHT2014*
  - ◆ *NNPDF3.0/3.1*





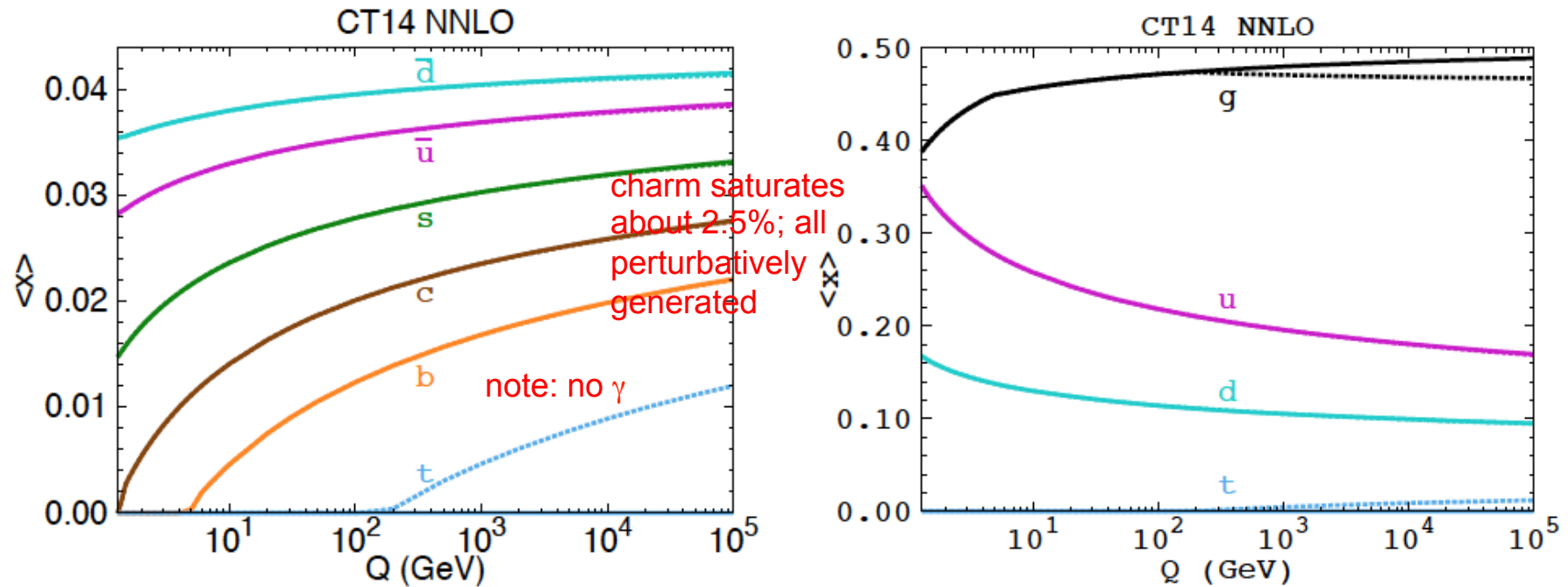
# PDFs are important



...at least to my citation index



# Momentum carried by partons



**Fig. 6.10** The momentum fractions carried by the CT14 NNLO quark and gluon distributions, as a function of  $Q$ . The gluon distribution in the right figure is shown without (with) the presence of a top quark PDF.

Don't usually define top quarks as initial state partons, but could. May be important for 100 TeV collider.

# LHC

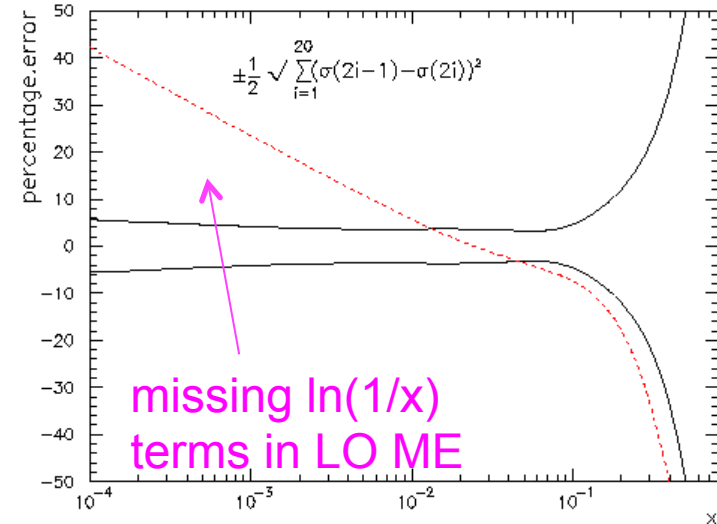
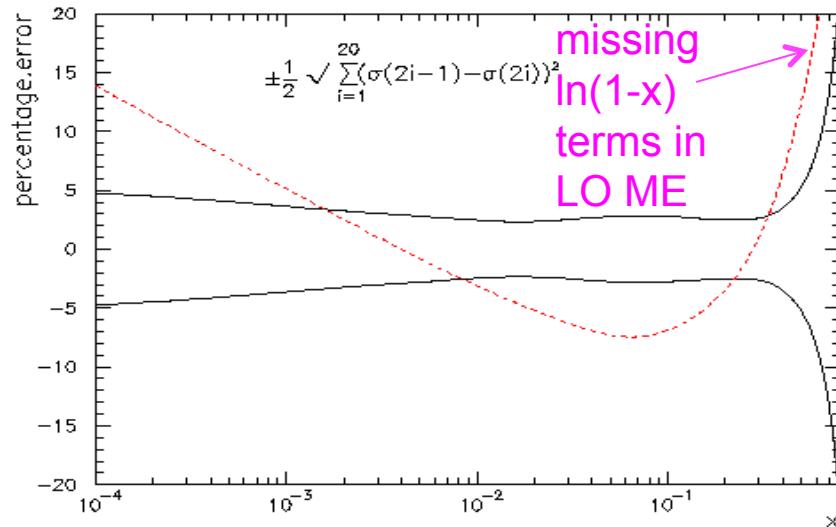
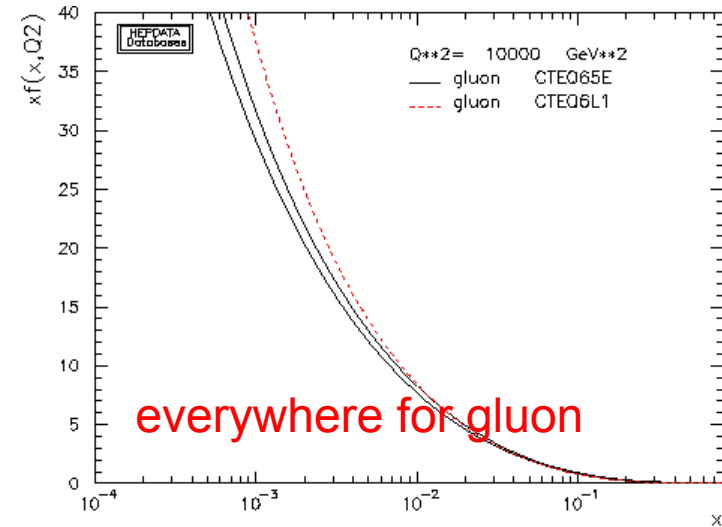
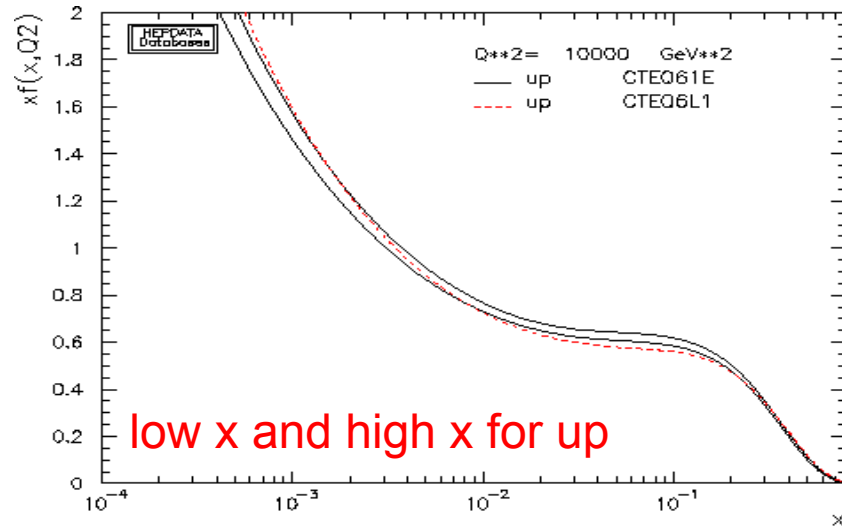
~

- We can determine PDFs at LO (not very well), NLO and NNLO  
not reliable at LHC
- These PDFs are evaluated in the relevant expressions for the hard scattering cross sections we are interested

$$\sigma = \sum_{a,b} \int_0^1 dx_1 f_{a/A}(x_1, \mu_F^2) \int_0^1 dx_2 f_{b/B}(x_2, \mu_F^2) \left\{ \int d\hat{\sigma}_{ab}^{LO}(\alpha_s) \Theta_{\text{obs}}^{(m)} + \alpha_s(\mu_R^2) \left[ \int (d\hat{\sigma}_{ab}^V(\alpha_s, \mu_R^2) + d\hat{\sigma}_{ab}^C(\alpha_s, \mu_F^2)) \Theta_{\text{obs}}^{(m)} + \int d\hat{\sigma}_{ab}^R(\alpha_s) \Theta_{\text{obs}}^{(m+1)} \right] \right\} + \dots$$



# How non-reliable are LO PDFs?



Differences between NLO and NNLO PDFs typically much smaller.





# Lessons



- Don't believe in predictions using LO PDFs unless you have checked at NLO or NNLO
- (Don't believe)<sup>n</sup> \* LO PDF error sets

\*where n is a large number



# LHC

- We can determine PDFs at LO (not very well), NLO and NNLO
- These PDFs are evaluated in the relevant expressions for the hard scattering cross sections we are interested

$$\sigma = \sum_{a,b} \int_0^1 dx_1 f_{a/A}(x_1, \mu_F^2) \int_0^1 dx_2 f_{b/B}(x_2, \mu_F^2) \left\{ \int d\hat{\sigma}_{ab}^{LO}(\alpha_s) \Theta_{\text{obs}}^{(m)} + \alpha_s(\mu_R^2) \left[ \int (d\hat{\sigma}_{ab}^V(\alpha_s, \mu_R^2) + d\hat{\sigma}_{ab}^C(\alpha_s, \mu_F^2)) \Theta_{\text{obs}}^{(m)} + \int d\hat{\sigma}_{ab}^R(\alpha_s) \Theta_{\text{obs}}^{(m+1)} \right] \right\} + \dots$$

- In addition to the PDFs themselves, it is often useful to

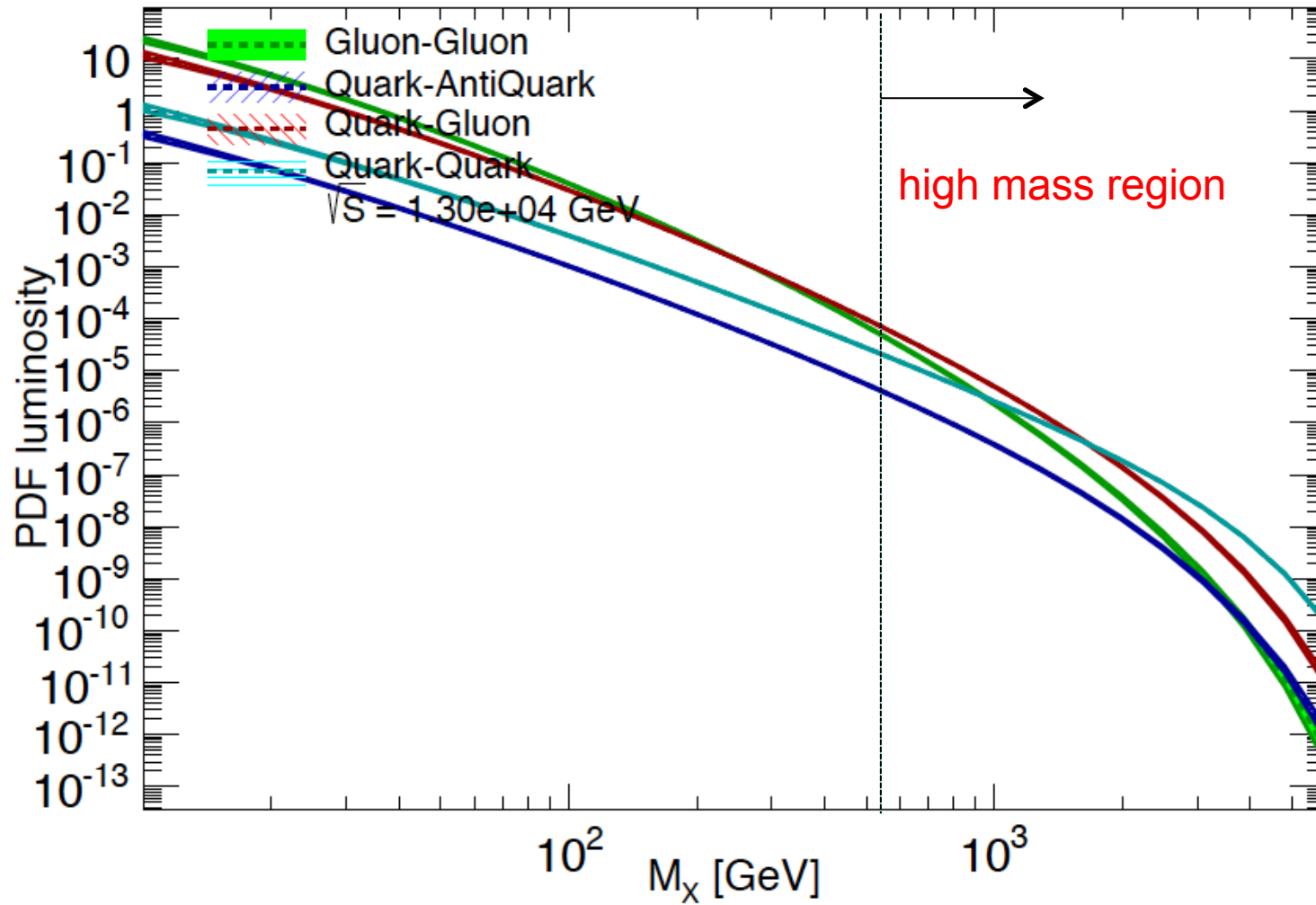
$$\frac{dL_{ij}}{d\hat{s} dy} = \frac{1}{s} \frac{1}{1 + \delta_{ij}} [f_i(x_1, \mu) f_j(x_2, \mu) + (1 \leftrightarrow 2)] . \text{ ...or integrated over } y$$



# PDF luminosities



CT14 NNLO luminosities



Generated with APFEL 2.4.0 Web



# PDFs: pre-history



gluon-gluon and gluon-quark luminosities in reasonable, but again not perfect, agreement for CT10, MSTW08 and NNPDF2.3 for full range of invariant masses

HERAPDF1.5 uncertainties larger in general

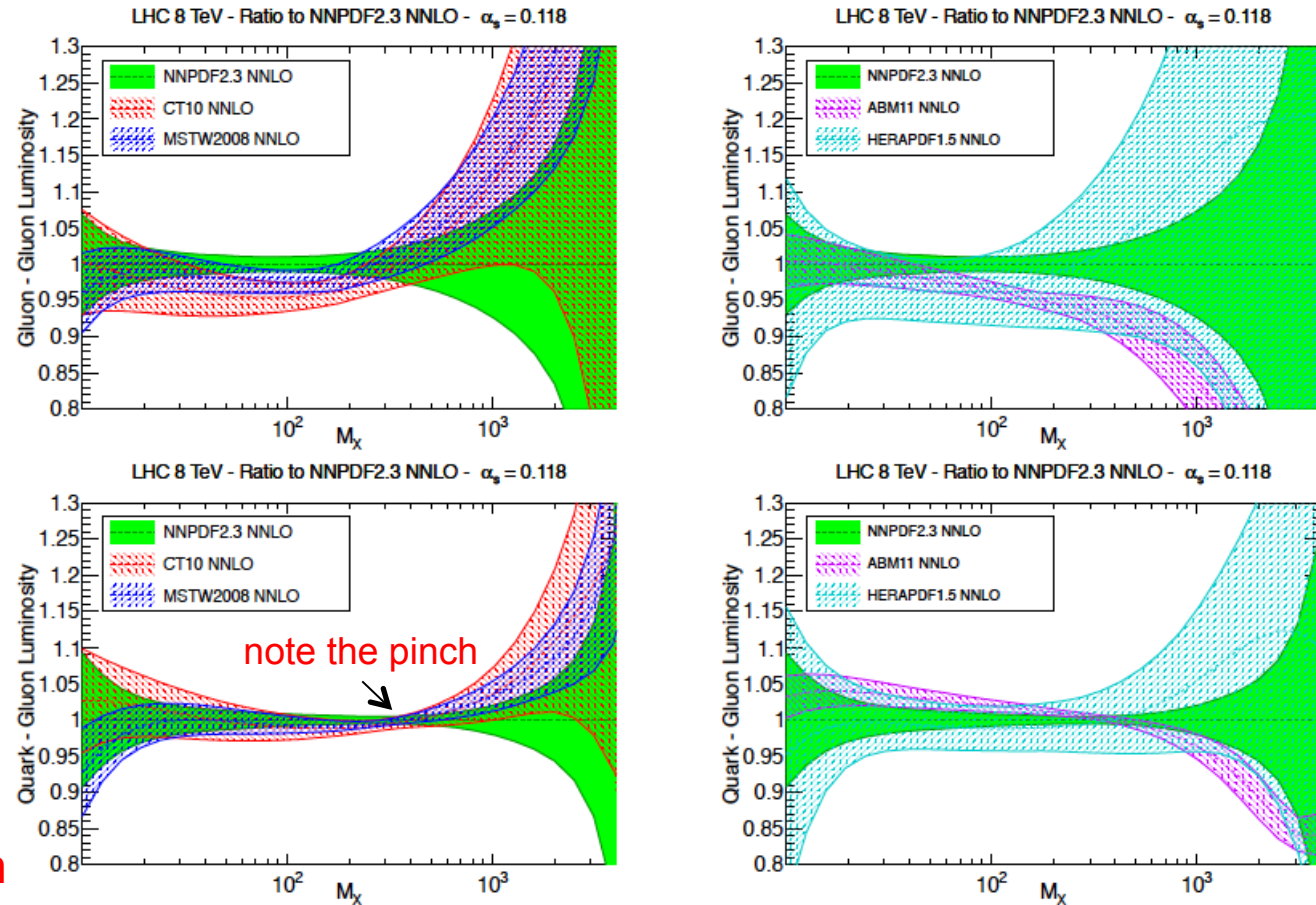


Figure 6: The gluon-gluon (upper plots) and quark-gluon (lower plots) luminosities, Eq. (2), for the production of a final state of invariant mass  $M_X$  (in GeV) at LHC 8 TeV. The left plots show the comparison between NNPDF2.3, CT10 and MSTW08, while in the right plots we compare NNPDF2.3, HERAPDF1.5 and MSTW08. All luminosities are computed at a common value of  $\alpha_s = 0.118$ .

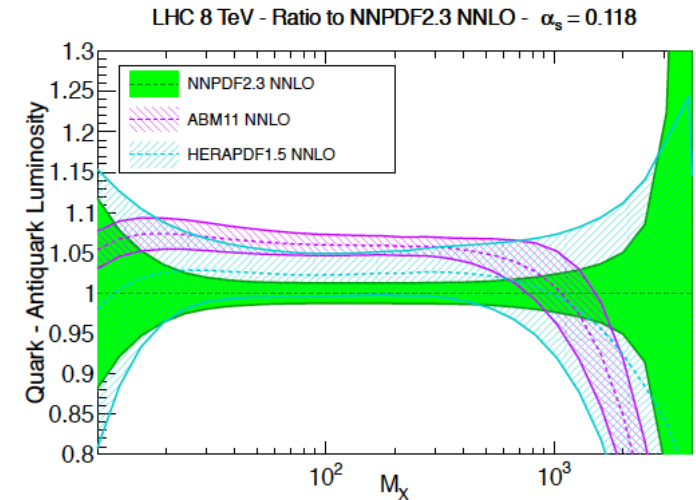
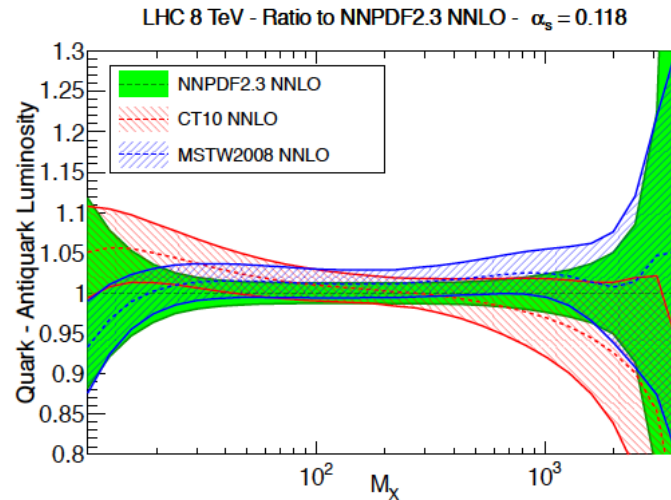


# PDF luminosities

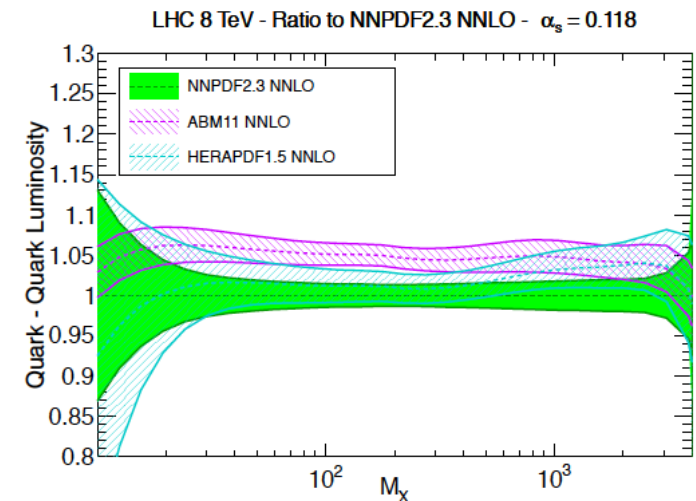
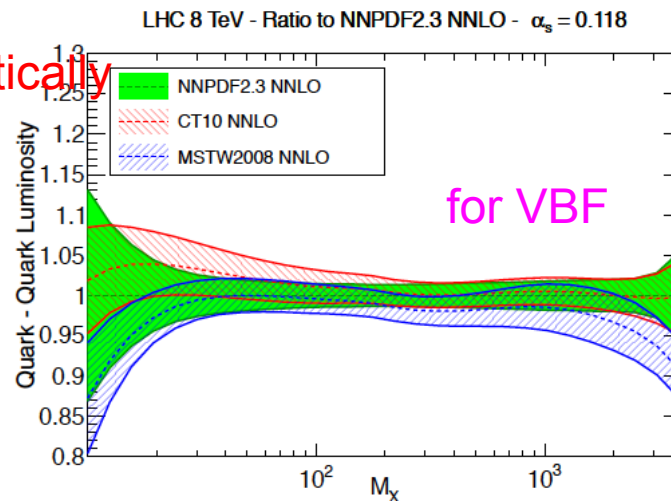


## quark-quark and quark-antiquark

quark-antiquark luminosities for CT10, MSTW08 and NNPDF2.3 overlap almost 100% in W/Z range



ABM11 systematically larger at small mass, then falls off more rapidly at high mass

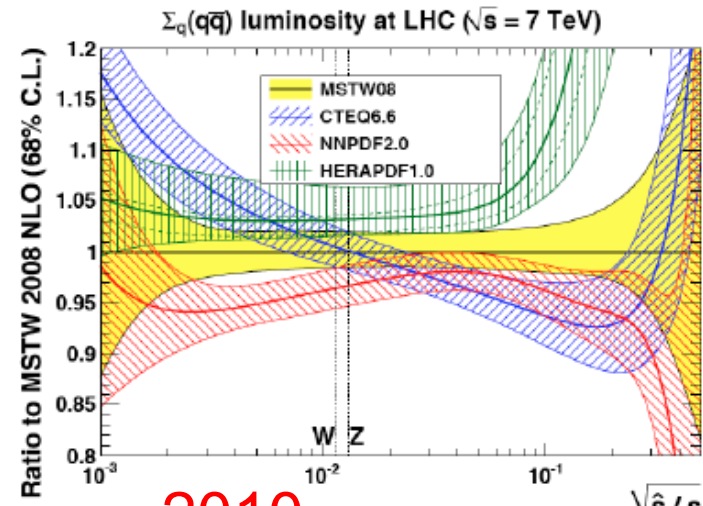




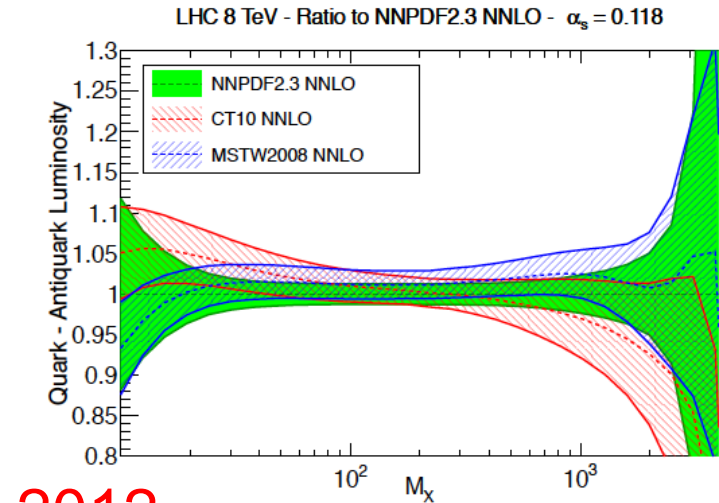
# Uncertainties had improved



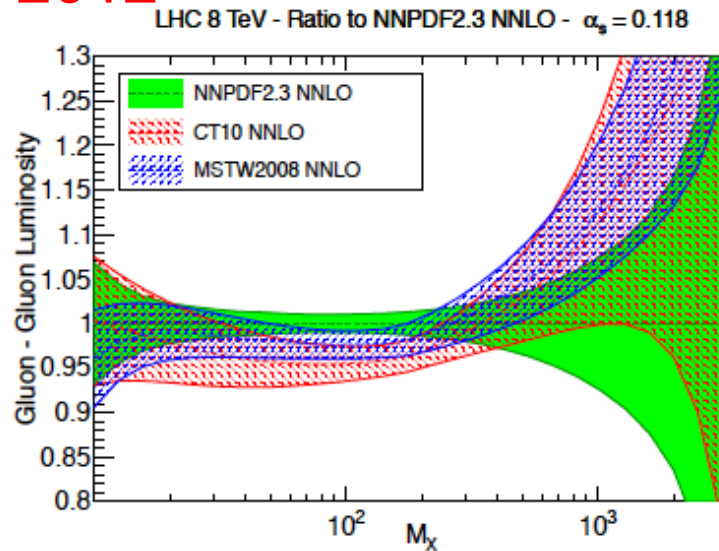
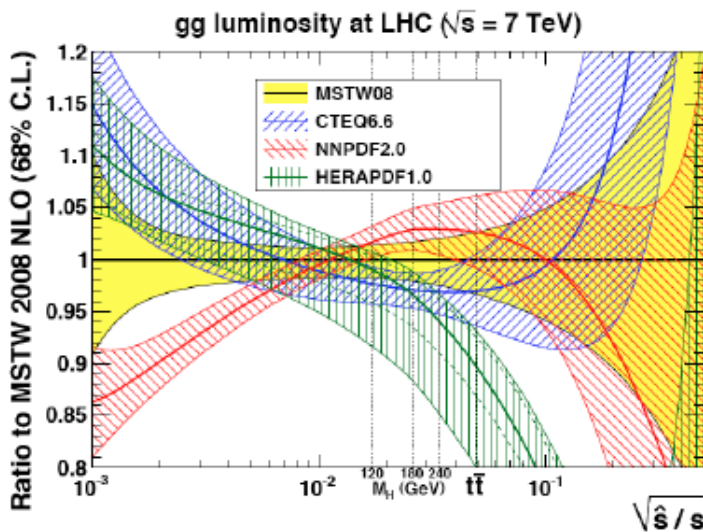
- ...with additional data and in going from NLO to NNLO



2010



2012

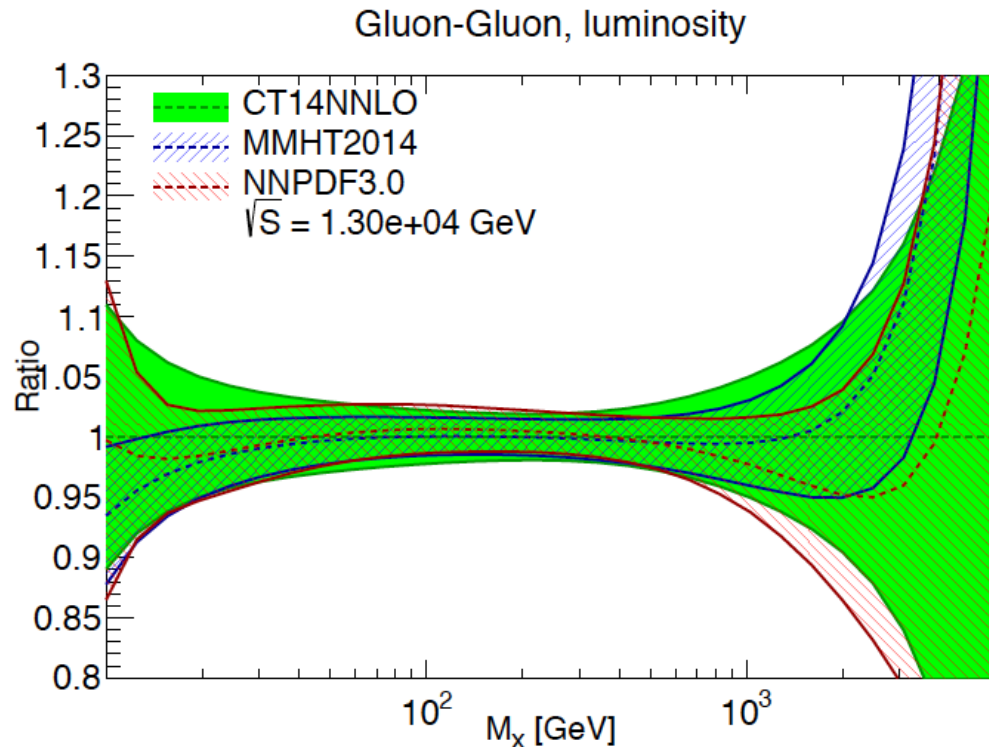




# PDFs: the next generation



- NNPDF3.0 (arXiv:1410.8849)
- MMHT14 (arXiv:1412.3989)
- CT14 (arXiv:1506.07443)
- HERAPDF2.0 (arXiv:1506.06042)
- The gg PDF luminosities for the first three PDFs are in good agreement with each other in the *precision physics* mass range, less so at very high mass



NNPDF down by 2-2.5%, CT14 up by ~1%,  
MMHT14 down by ~0.5%

partially data, partially corrections in  
fitting code, partially changes  
in fitting procedures

lead to new PDF4LHC recommendations

## PDF4LHC recommendations for LHC Run II

Jon Butterworth<sup>1</sup>, Stefano Carrazza<sup>2</sup>, Amanda Cooper-Sarkar<sup>3</sup>, Albert De Roeck<sup>4,5</sup>, Joël Feltesse<sup>6</sup>, Stefano Forte<sup>2</sup>, Jun Gao<sup>7</sup>, Sasha Glazov<sup>8</sup>, Joey Huston<sup>9</sup>, Zahari Kassabov<sup>2,10</sup>, Ronan McNulty<sup>11</sup>, Andreas Morsch<sup>4</sup>, Pavel Nadolsky<sup>12</sup>, Voica Radescu<sup>13</sup>, Juan Rojo<sup>14</sup> and Robert Thorne<sup>1</sup>.

<sup>1</sup>*Department of Physics and Astronomy, University College London,  
Gower Street, London WC1E 6BT, UK.*

<sup>2</sup>*TIF Lab, Dipartimento di Fisica, Università di Milano and INFN, Sezione di Milano,  
Via Celoria 16, I-20133 Milano, Italy*

<sup>3</sup>*Particle Physics, Department of Physics, University of Oxford,  
1 Keble Road, Oxford OX1 3NP, UK.*

<sup>4</sup>*PH Department, CERN, CH-1211 Geneva 23, Switzerland*

<sup>5</sup>*Antwerp University, B2610 Wilrijk, Belgium*

<sup>6</sup>*CEA, DSM/IRFU, CE-Saclay, Gif-sur-Yvette, France*

<sup>7</sup>*High Energy Physics Division, Argonne National Laboratory,  
Argonne, Illinois 60439, U.S.A.*

<sup>8</sup>*Deutsches Elektronen-Synchrotron (DESY),  
Notkestrasse 85, D-22607 Hamburg, Germany.*

<sup>9</sup>*Department of Physics and Astronomy, Michigan State University,  
East Lansing, MI 48824 U.S.A.*

<sup>10</sup>*Dipartimento di Fisica, Università di Torino and INFN, Sezione di Torino,  
Via Pietro Giuria 1, I-10125 Torino, Italy*

<sup>11</sup>*School of Physics, University College Dublin Science Centre North,  
UCD Belfield, Dublin 4, Ireland*

<sup>12</sup>*Department of Physics, Southern Methodist University, Dallas, TX 75275-0181, U.S.A.*

<sup>13</sup>*Physikalisches Institut, Universität Heidelberg, Heidelberg, Germany.*

<sup>14</sup>*Rudolf Peierls Centre for Theoretical Physics, 1 Keble Road,  
University of Oxford, OX1 3NP Oxford, UK*





# A comparison of ggF at NNLO

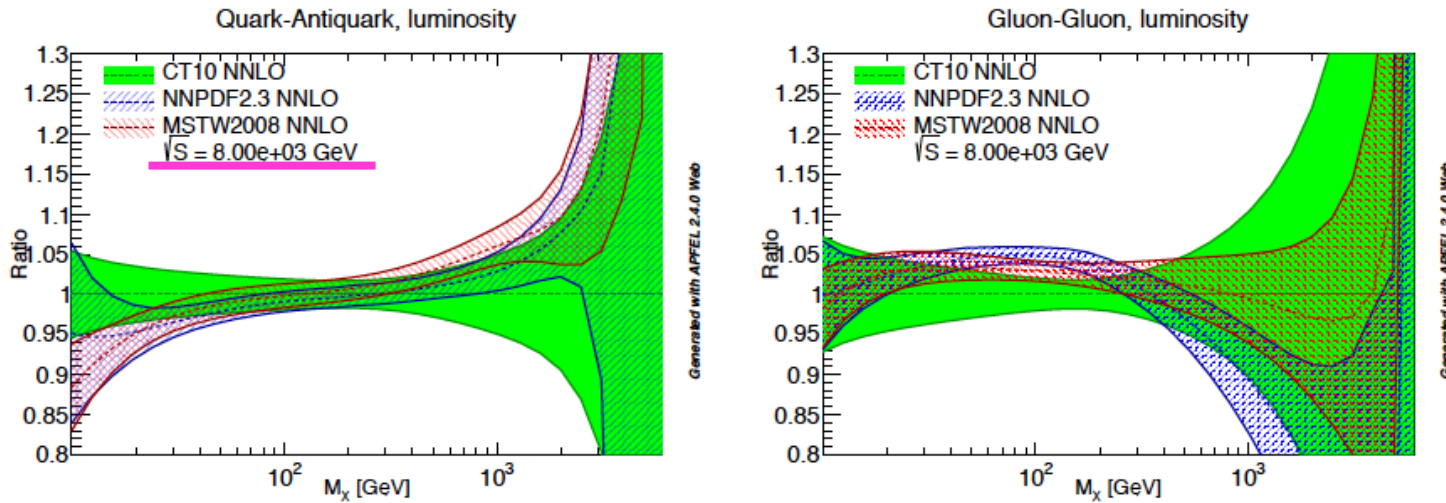


	CT14	MMHT2014	NNPDF3.0
<b>scale = <math>m_H</math></b>			
8 TeV	18.66 pb -2.2% +2.0%	18.65 pb -1.9% +1.4%	18.77 pb -1.8% +1.8%
13 TeV	42.68 pb -2.4% +2.0%	42.70 pb -1.8% +1.3%	42.97 pb -1.9% +1.9%

The PDF uncertainty using this new generation of PDFs (2-3%) is similar in size to the NNLO scale uncertainty and to the  $\alpha_s(m_Z)$  uncertainty.



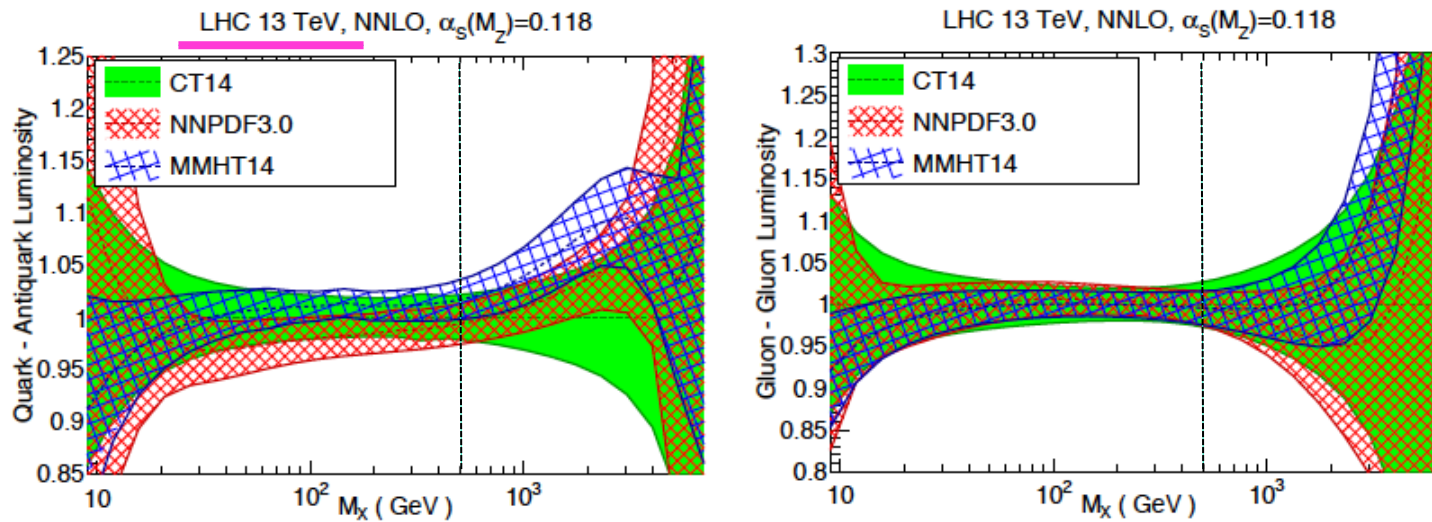
# Progress with recent PDFs



Note in particular the changes in the gg luminosity, especially important in the Higgs mass region

Figure 1: Comparison of the  $q\bar{q}$  (left) and  $gg$  (right) PDF luminosities at the LHC 8 TeV for CT10, MSTW2008 and NNPDF2.3. Results are shown normalized to the central value of CT10.

Note also differences remaining in high mass region





# Progress with recent PDFs

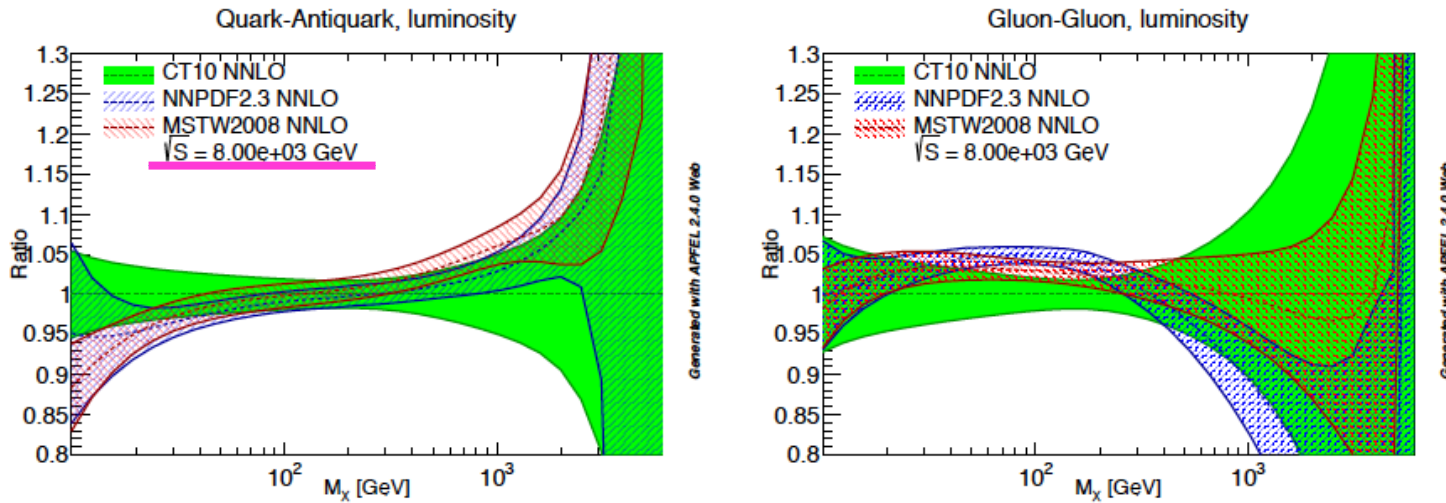
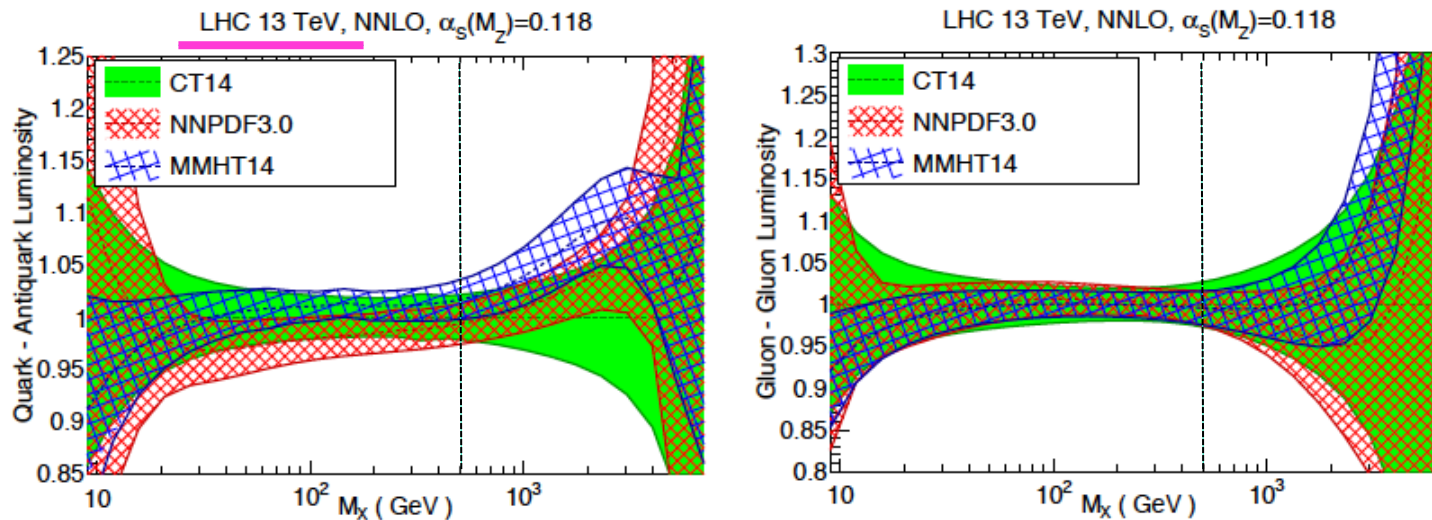


Figure 1: Comparison of the  $q\bar{q}$  (left) and  $gg$  (right) PDF luminosities at the LHC 8 TeV for CT10, MSTW2008 and NNPDF2.3. Results are shown normalized to the central value of CT10.

The  $gg$  precision has improved, but the  $qQ$  has not.

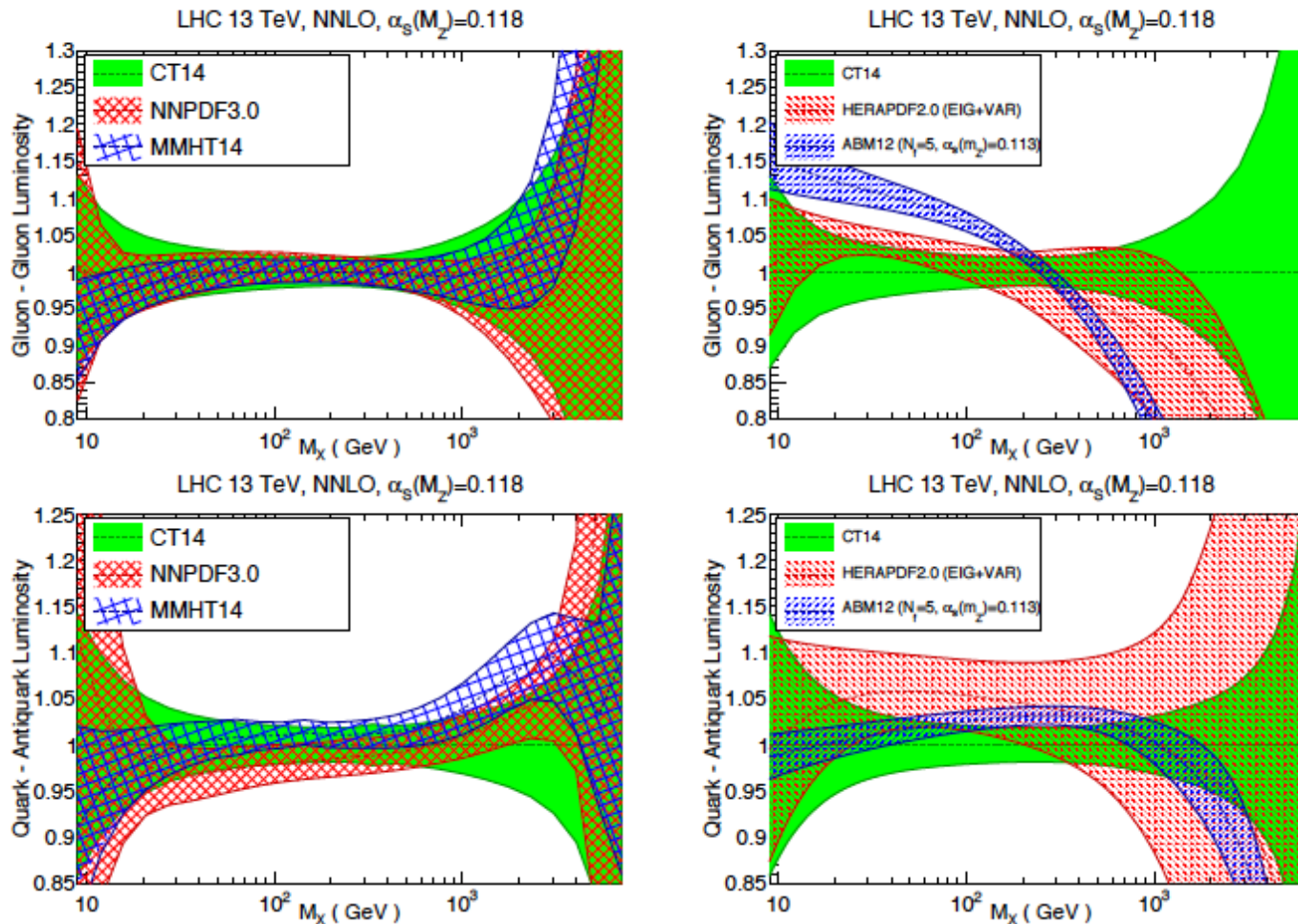
We hope (and think) we are making progress, but next generation of PDFs could lead to somewhat different behavior, either data or formalism.

The variation from generation to generation is related to the accuracy of the PDF sets





# Other new sets out as well



behavior for  
HERAPDF2.0  
and ABM12  
somewhat  
different

HERAPDF2.0  
uncertainties  
tend to be  
larger

ABM12  
uncertainties  
tend to be smaller

Figure 5: Comparison of the gluon-gluon (upper plots) and quark-antiquark (lower plots) PDF luminosities from the CT14, MMHT14 and NNPDF3.0 NNLO sets (left plots) and from the NNPDF3.0, ABM12 and HERAPDF2.0 NNLO sets (right plots), for a center-of-mass energy of 13 TeV, as a function of the invariant mass of the final state  $M_X$ .



# Three main uses of PDFs at LHC



1. Assessment of the total uncertainty on a cross section based on the available knowledge of PDFs, *e.g.*, when computing the cross section for a process that has not been measured yet (such as supersymmetric particle production cross-sections), or for estimating acceptance corrections on a given observable. This is also the case of the measurements that aim to verify overall, but not detailed, consistency with Standard Model expectations, such as when comparing theory with Higgs measurements.
2. Assessment of the accuracy of the PDF sets themselves or of related Standard Model parameters, typically done by comparing theoretical predictions using individual PDF sets to the most precise data available.
3. Input to the Monte Carlo event generators used to generate large MC samples for LHC data analysis.

**For 2), use individual PDF sets.**

For 1), a more general uncertainty requires more than the use of 1 PDF set.

For 3), may want to use an average of PDF sets. This point seems to be confusing to some, *i.e.* you can use PDF4LHC15 PDFs for MC generation.



# What PDFs to use?



1. The PDF sets to be combined should be based on a global dataset, including a large number of datasets of diverse types (deep-inelastic scattering, vector boson and jet production, ...) from fixed-target and colliders experiments (HERA, LHC, Tevatron).
2. Theoretical hard cross sections for DIS and hadron collider processes should be evaluated up to two QCD loops in  $\alpha_s$ , in a general-mass variable-flavor number scheme with up to  $n_f^{\max} = 5$  active quark flavors.<sup>1</sup> Evolution of  $\alpha_s$  and PDFs should be performed up to three loops, using public codes such as HOPPET [105] or QCDNUM [106], or a code benchmarked to these.
3. The central value of  $\alpha_s(m_Z^2)$  should be fixed at an agreed common value, consistent with the PDG world-average [107]. This value is currently chosen to be  $\alpha_s(m_Z^2) = 0.118$  at both NLO and NNLO.<sup>2</sup> For the computation of  $\alpha_s$  uncertainties, two additional PDF members corresponding to agreed upper and lower values of  $\alpha_s(m_Z^2)$  should also be provided. This uncertainty on  $\alpha_s(m_Z^2)$  is currently assumed to be  $\delta\alpha_s = 0.0015$ , again the same at NLO and NNLO.
4. All known experimental and procedural sources of uncertainty should be properly accounted for. Specifically, it is now recognized that the PDF uncertainty receives several contributions of comparable importance: the measurement uncertainty propagated from the experimental data, uncertainties associated with incompatibility of the fitted experiments, procedural uncertainties such as those related to the functional form of PDFs, the handling of systematic errors, etc. Sets entering the combination must account for these through suitable methods, such as separate estimates for additional model and parametrization components of the PDF uncertainty [9], tolerance [6, 10], or closure tests [11].



# Monte Carlo representation



- So based on the criteria on the previous slide, we use CT14, MMHT2014 and NNPDF3.0, with the option of adding additional sets in future upgrades if they satisfy the listed criteria
- In the previous recommendation, we used an envelope of 3 PDF sets; envelope determined by outliers
- Given the level of agreement of the 3 PDFs that will be used, try for a more relevant statistical approach
- Generate Monte Carlo replicas, equal numbers from error PDF sets of CT14, MMHT2014 and NNPDF3.0 using Thorne-Watt procedure
  - ◆ replicas generated from Hessian eigenvectors for CT14 and MMHT14 assuming a Gaussian probability distribution
  - ◆ this will lead to a more statistical representation of the uncertainty than the envelope procedure used previously



# Aside



...a different opinion, basically stating that all PDFs should be used for a general estimate of the total uncertainty

arXiv:1603.08906v2 [hep-ph] 8 Aug 2016

## A Critical Appraisal and Evaluation of Modern PDFs

A. Accardi<sup>a,b</sup>, S. Alekhin<sup>c,d</sup>, J. Blümlein<sup>e</sup>, M.V. Garzelli<sup>c</sup>, K. Lipka<sup>f</sup>,  
W. Melnitchouk<sup>b</sup>, S. Moch<sup>c</sup>, J.F. Owens<sup>g</sup>, R. Plačakytė<sup>f</sup>, E. Reya<sup>h</sup>, N. Sato<sup>b</sup>, A. Vogt<sup>i</sup>  
and O. Zenaiev<sup>f</sup>

<sup>a</sup> *Hampton University, Hampton, VA 23668, USA*

<sup>b</sup> *Jefferson Lab, Newport News, VA 23606, USA*

<sup>c</sup> *II. Institut für Theoretische Physik, Universität Hamburg  
Luruper Chaussee 149, D-22761 Hamburg, Germany*

<sup>d</sup> *Institute for High Energy Physics  
142281 Protvino, Moscow region, Russia*

<sup>e</sup> *Deutsches Elektronensynchrotron DESY  
Platanenallee 6, D-15738 Zeuthen, Germany*

<sup>f</sup> *Deutsches Elektronensynchrotron DESY  
Notkestraße 85, D-22607 Hamburg, Germany*

<sup>g</sup> *Florida State University, Tallahassee, FL 32306, USA*

<sup>h</sup> *Institut für Physik, Technische Universität Dortmund  
D-44221 Dortmund, Germany*

<sup>i</sup> *Department of Mathematical Sciences, University of Liverpool  
Liverpool L69 3BX, United Kingdom*

### Abstract:

We review the present status of the determination of parton distribution functions (PDFs) in the light of the precision requirements for the LHC in Run 2 and other future hadron colliders. We provide brief reviews of all currently available PDF sets and use them to compute cross sections for a number of benchmark processes, including Higgs boson production in gluon-gluon fusion at the LHC. We show that the differences in the predictions obtained with the various PDFs are due to particular theory assumptions made in the fits of those PDFs. We discuss PDF uncertainties in the kinematic region covered by the LHC and on averaging procedures for PDFs, such as advocated by the PDF4LHC15 sets, and provide recommendations for the usage of PDF sets for theory predictions at the LHC.





# The result



## Gluon-Fusion Higgs production, LHC 13 TeV

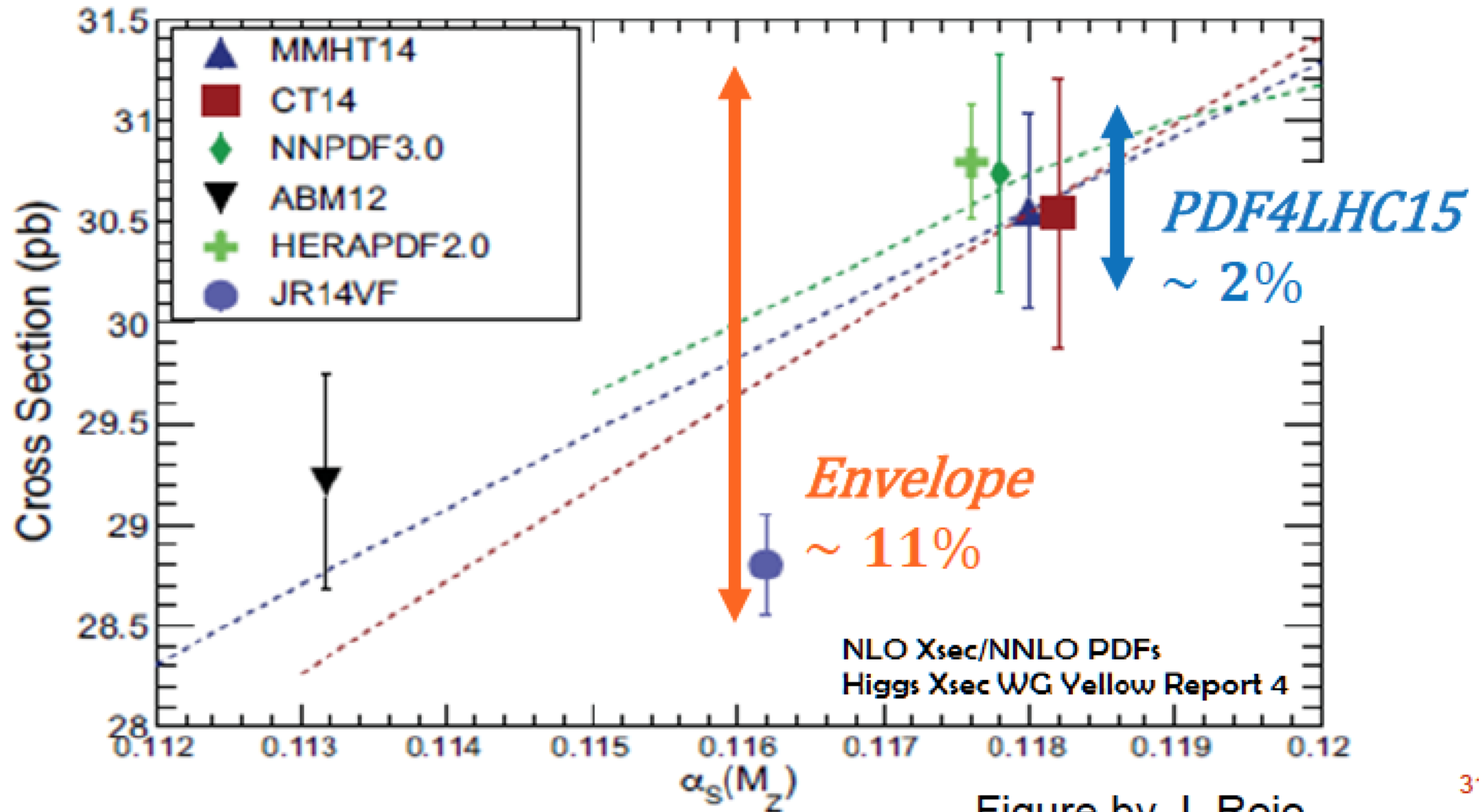
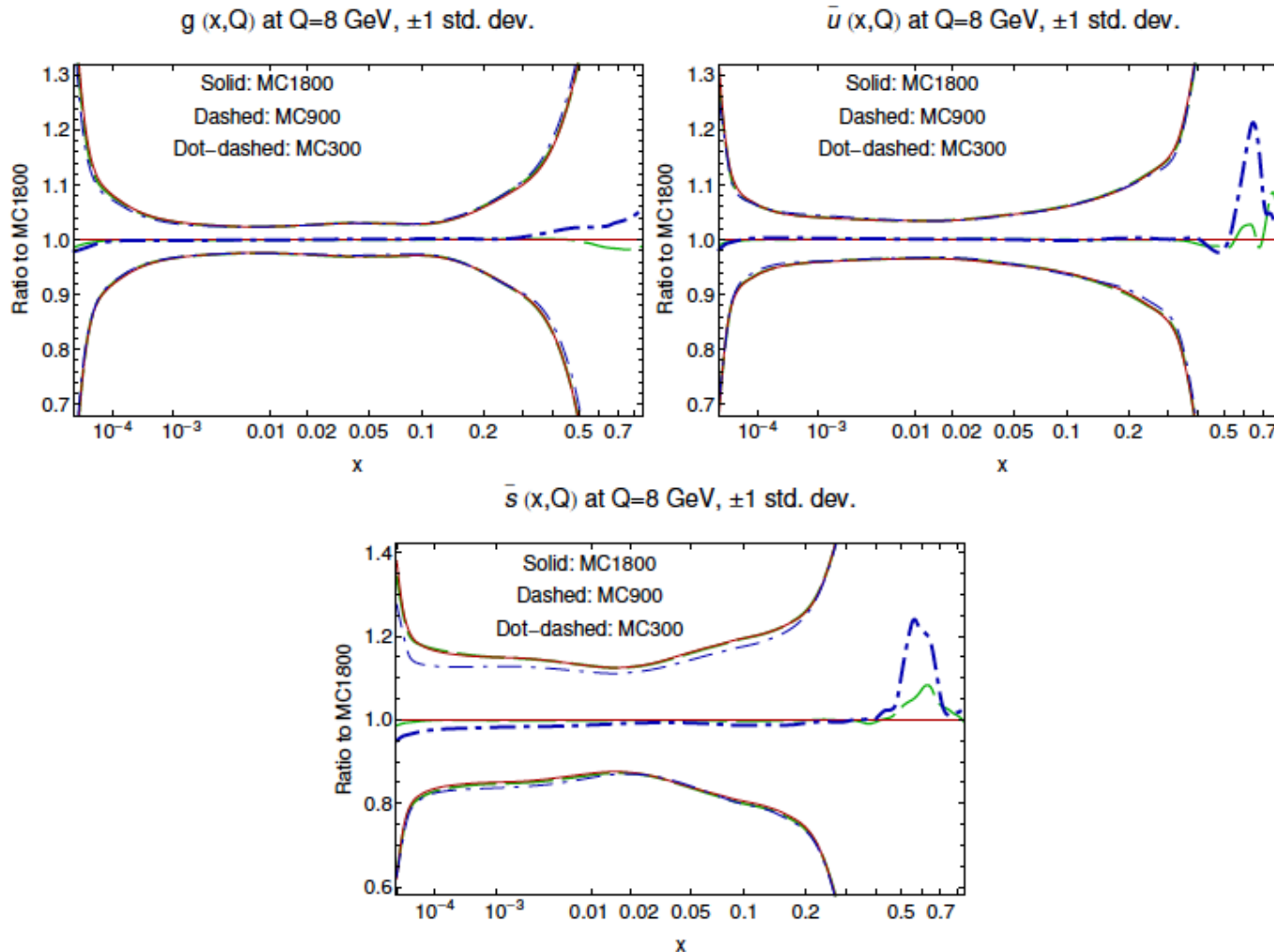


Figure by J. Rojo



# Monte Carlo replicas



900 replicas  
seems enough

->MC900  
or  
PDF4LHC\_prior

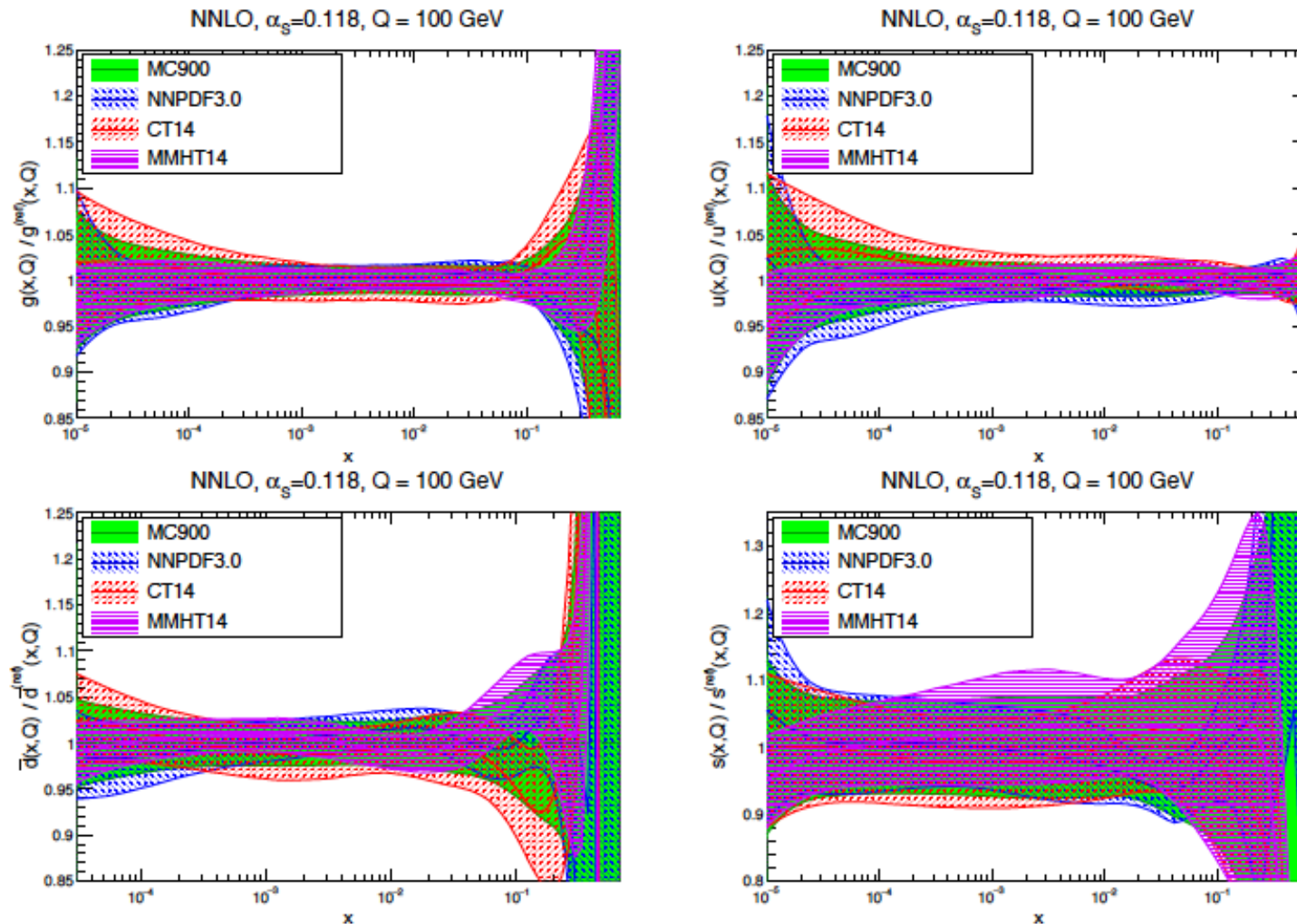
note that here we  
are trying for  
*precision*

the *accuracy* is  
another question.  
that is outside  
the realm of  
choosing a given  
number of  
replicas

Figure 7: Comparison of central values and uncertainties for the MC combination of CT14, MMHT14 and NNPDF3.0 for different values of  $N_{\text{rep}}$ , 300, 600 and 900, denoted by MC300, MC900 and MC1800 respectively.



# MC900



Note that MC900 is not the envelope of the 3 PDF error bands

The PDF error bands themselves are similar for the precision physics region, but not for low mass/high mass

Figure 8: Comparison of the MC900 PDFs with the sets that enter the combination: CT14, MMHT14 and NNP3.0 at NNLO. We show the gluon and the up, anti-down and strange quarks at  $Q = 100$  GeV. Results are normalized to the central value of MC900.



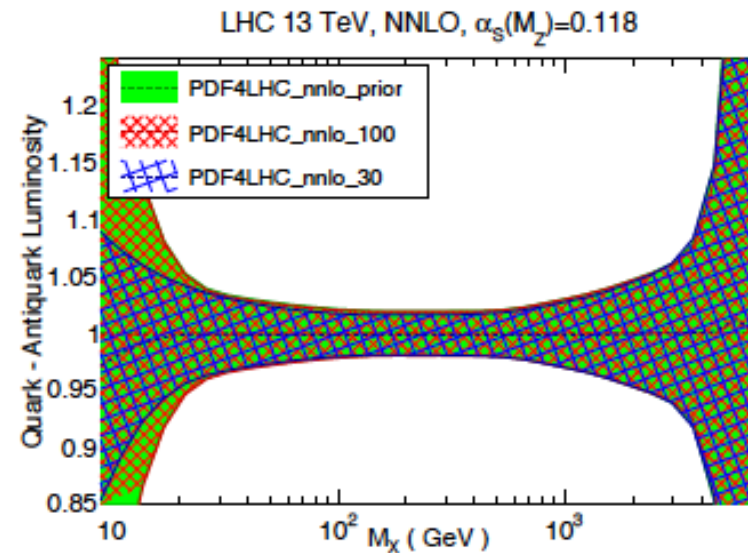
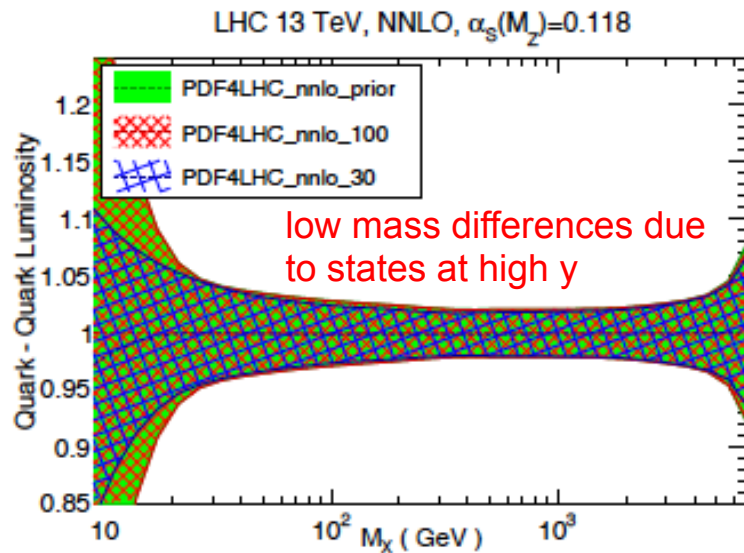
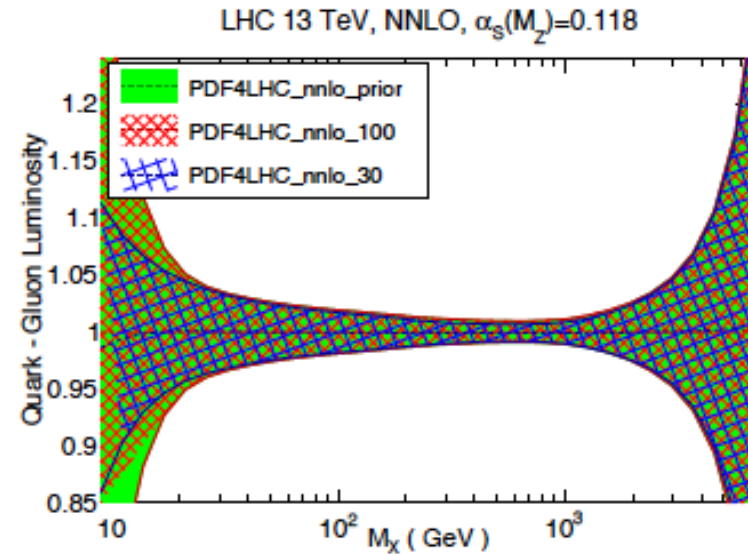
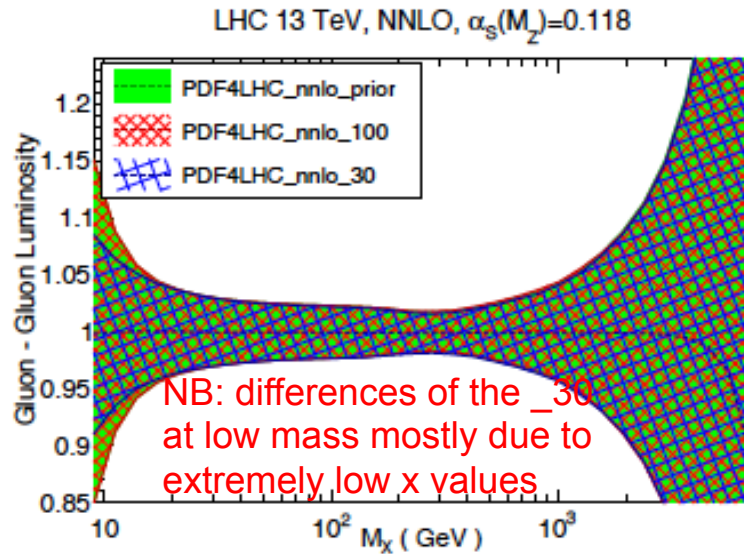
# Reduced sets



- 900 error PDFs are too much for general use
- We would like to reduce this number while still maintaining as much information on the uncertainties and on correlations between PDF uncertainties as possible
- We have settled on 3 techniques/outputs
  - ◆ Compressed Monte Carlo PDFs (PDF4LHC15\_nnlo(nlo)\_mc)
    - ▲ 100 PDF error sets; preserve non-Gaussian errors
  - ◆ META Hessian PDFs (PDF4LHC15\_nnlo(nlo)\_30)
    - ▲ 30 PDF error sets using METAPDF technique; Gaussian (symmetric) errors
  - ◆ MCH Hessian PDFs (PDF4lhc15\_nnlo(nlo)\_100)
    - ▲ 100 PDF error sets using MCH technique; Gaussian (symmetric errors)
- The META technique is able to more efficiently reproduce the uncertainties when using a limited number (30) of error PDFs
- The MCH technique best reproduces the uncertainties of the 900 MC set prior->**precision, not accuracy**



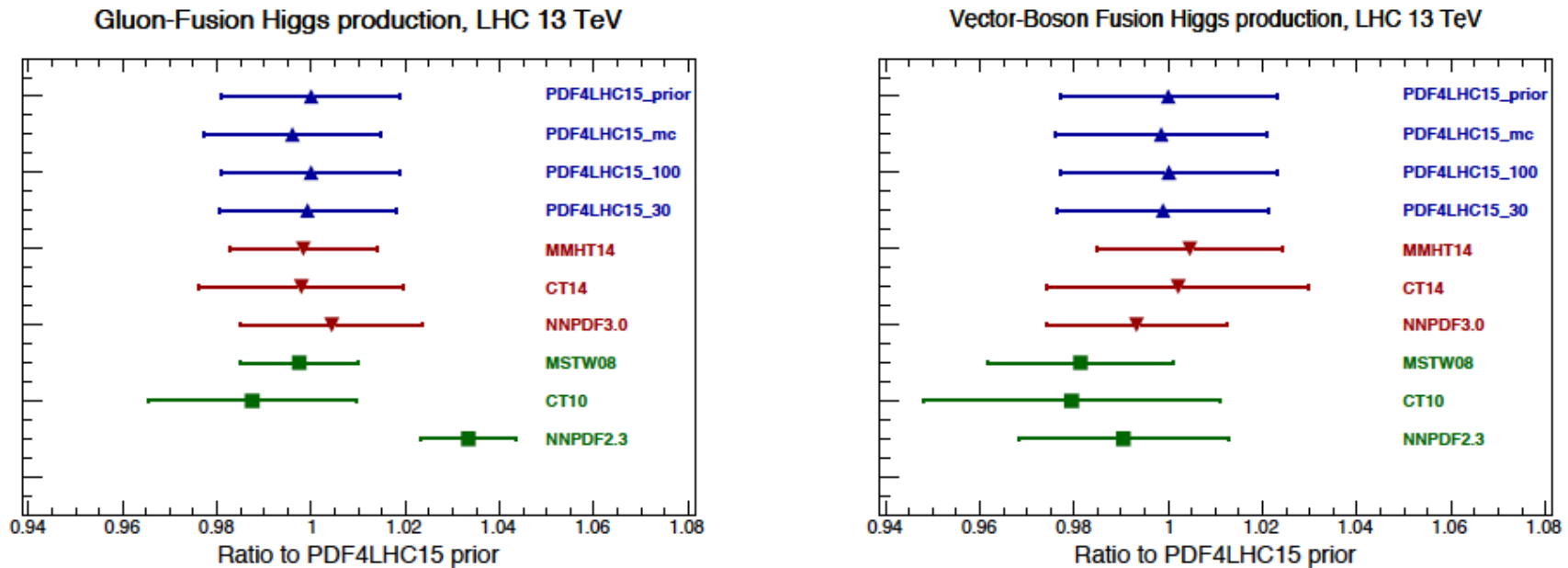
# Some comparisons: Hessian sets



...again, high mass uncertainty is smaller than envelope of 3 input PDFs



# Some comparisons for Higgs production



**Fig. 6.32** A comparison of the predictions for Higgs boson production through  $gg$  fusion (left) and vector boson fusion (right) is shown for a center-of-mass energy of 13 TeV,

As no one has been able to find any discernible difference between the `_30` and `_100` PDF sets in ATLAS, the `_30` tend to be used for convenience.



# (Relatively) New



## ● Photon PDFs

- ◆ the photon is a constituent of the proton just as quarks and gluons are
- ◆ it also evolves just as quarks and gluons do, but with Abelian splitting kernels
- ◆ it's much smaller than the other PDFs and there are fewer experimental handles to try to estimate its size
- ◆ but as it has implications for high mass physics, such as VV (or for a hypothetical particle at 750 GeV which could have been produced by a  $\gamma\gamma$  initial state), or EW corrections for just about any LHC final state, it's something we have to understand better

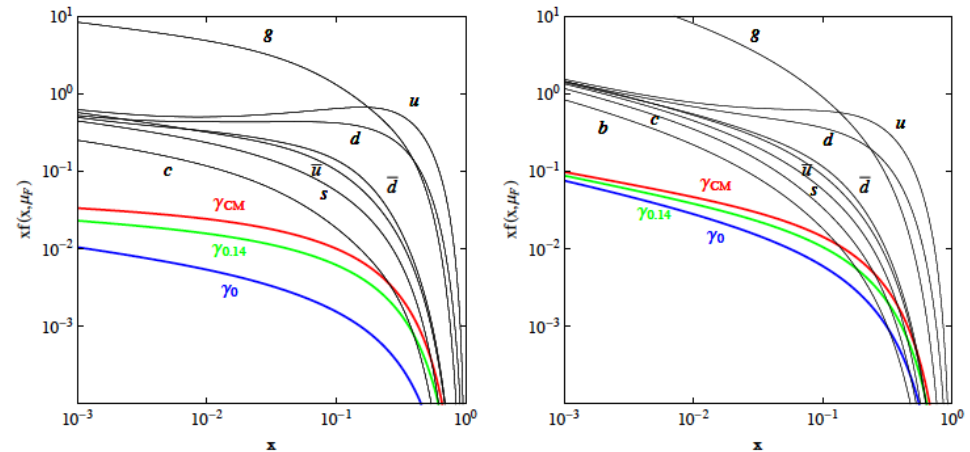
The evolution of the PDFs,  $f(x, \mu_F)$ , including QED contributions at leading order (LO) and QCD contributions at higher orders, is described by the equations:

$$\frac{df_{q_i}}{dt} = \frac{\alpha_s}{2\pi} \left( \sum_j (P_{q_i q_j} \circ f_{q_j} + P_{q_i g} \circ f_g) + P_{qg} \circ f_g \right) + \frac{\alpha}{2\pi} e_i^2 \left( \tilde{P}_{qq}^{(0)} \circ f_{q_i} + \tilde{P}_{q\gamma}^{(0)} \circ f_\gamma \right)$$

$$\frac{df_{\bar{q}_i}}{dt} = \frac{\alpha_s}{2\pi} \left( \sum_j (P_{\bar{q}_i \bar{q}_j} \circ f_{\bar{q}_j} + P_{\bar{q}_i g} \circ f_g) + P_{g\bar{q}} \circ f_g \right) + \frac{\alpha}{2\pi} e_i^2 \left( \tilde{P}_{q\bar{q}}^{(0)} \circ f_{\bar{q}_i} + \tilde{P}_{q\gamma}^{(0)} \circ f_\gamma \right)$$

$$\frac{df_g}{dt} = \frac{\alpha_s}{2\pi} \left( P_{gg} \circ f_g + \sum_i P_{gq} \circ (f_{q_i} + f_{\bar{q}_i}) \right) \quad (2)$$

$$\frac{df_\gamma}{dt} = \frac{\alpha}{2\pi} \left( \tilde{P}_{\gamma\gamma}^{(0)} \circ f_\gamma + \sum_i e_i^2 \tilde{P}_{\gamma q}^{(0)} \circ (f_{q_i} + f_{\bar{q}_i}) \right),$$



arXiv:1509.02905

FIG. 1: Plots of  $xf(x, \mu_F)$  for  $\mu_F = 3.2$  GeV (left) and  $\mu_F = 85$  GeV (right). Three representative photon PDFs are plotted: the “Current Mass” photon PDF ( $\gamma_{CM}$ , red), and photon PDFs with initial photon momenta fractions of  $p_0^\gamma = 0$  and 0.14% ( $\gamma_0$ , blue, and  $\gamma_{0.14}$ , green, respectively). The effect of the different initial photon PDFs on the quark and gluon PDFs is imperceptible in these plots.



# Photon PDFs



- MRST were the first
  - ◆ parametrize inelastic\* contribution to the photon at initial scale  $Q_0$  as

$$f_{\gamma/p}(x, Q_0) = \frac{\alpha}{2\pi} \left( A_u e_u^2 \tilde{P}_{\gamma q} \circ u^0(x) + A_d e_d^2 \tilde{P}_{\gamma q} \circ d^0(x) \right)$$

- ◆  $P_{\gamma q_0} f_0(x)$  is the convolution of the quark to photon splitting function with the primordial quark distribution
  - ◆ define  $A_i = \ln(Q^2/Q_i^2)$ , and setting  $Q_i$  to current quark masses; alternatively use constituent quark masses
- CT14qed followed a similar approach, but fitting to DIS data with isolated photons from ZEUS that allowed a constraint on the total photon momentum

- NNPDF2.3 used a more general photon parametrization, allowing photon to be fit to data (W,Z, Drell-Yan); this implicitly includes an elastic component as well

\*There is also an elastic component for the photon in which the proton remains intact. See, for example, arxiv:1607.04635.

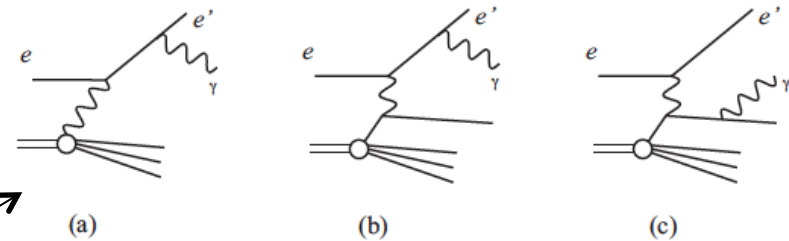
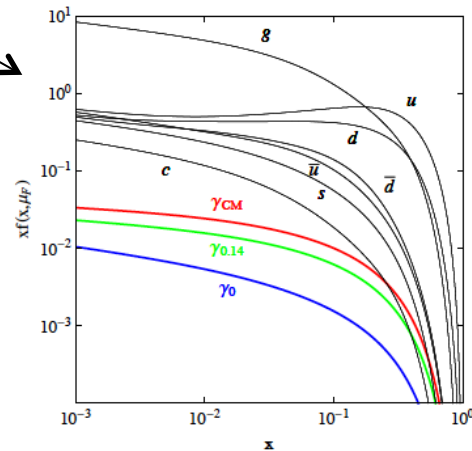


FIG. 3: Amplitudes for the process  $ep \rightarrow e\gamma + X$ . For each diagram shown there is an additional diagram where the photon is emitted off the initial-state lepton or quark.

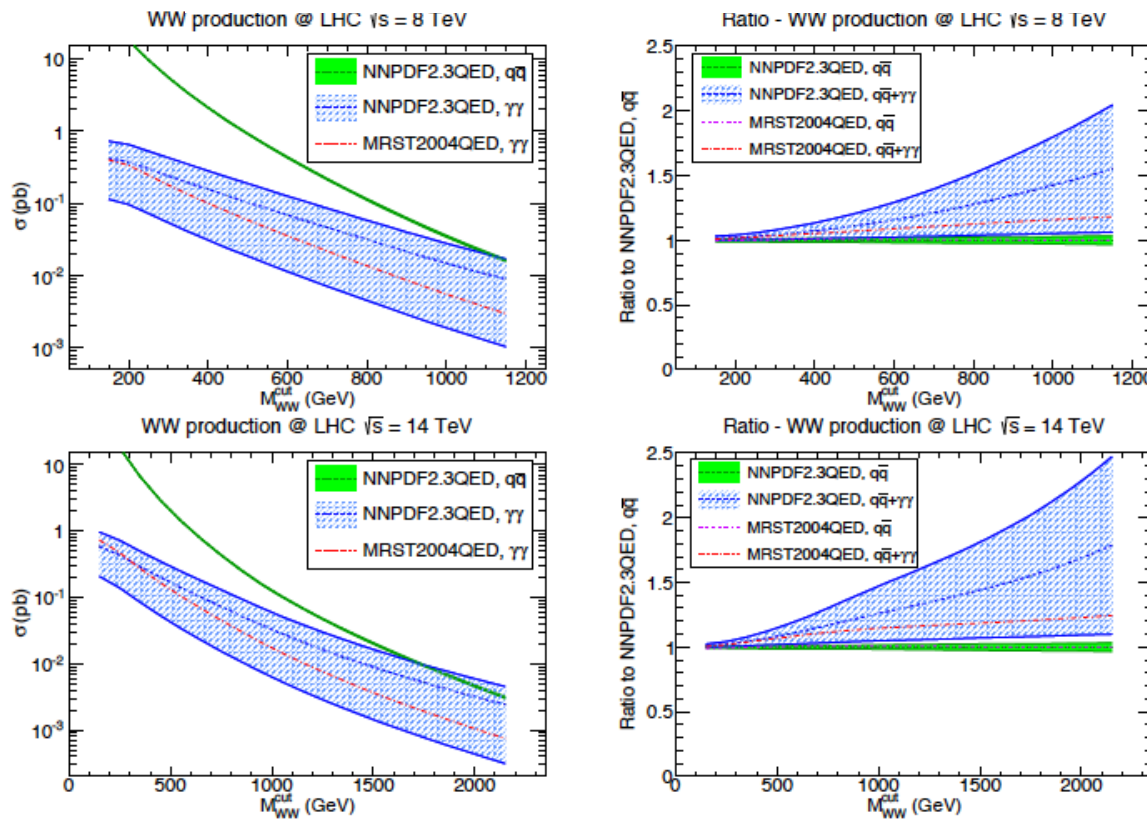
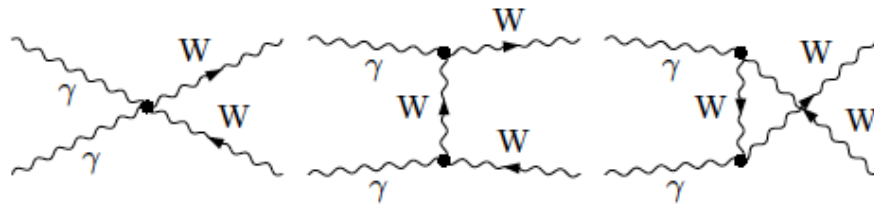


fit constrains the photon PDF;  $\gamma_{CM}$  doesn't fit the data; data fit well for current quark prescription with  $\gamma$  momentum fraction (at  $Q_0$ )=0.1%; 90%CL from 0 to 0.14%





# NNPDF2.3qed



appreciable  
fraction of WW cross  
section at large  
mass due to  $\gamma\gamma$   
initial state

arxiv:1308.0598

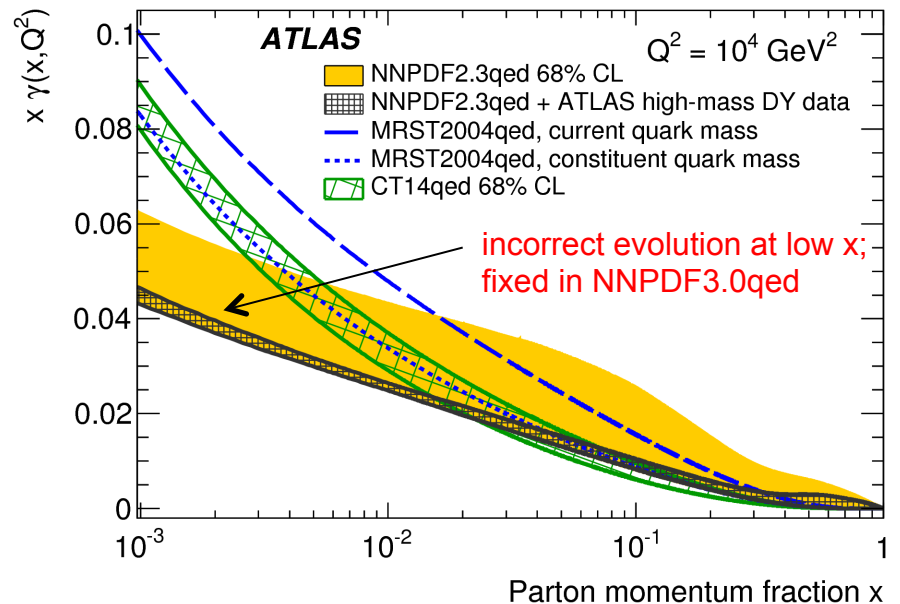
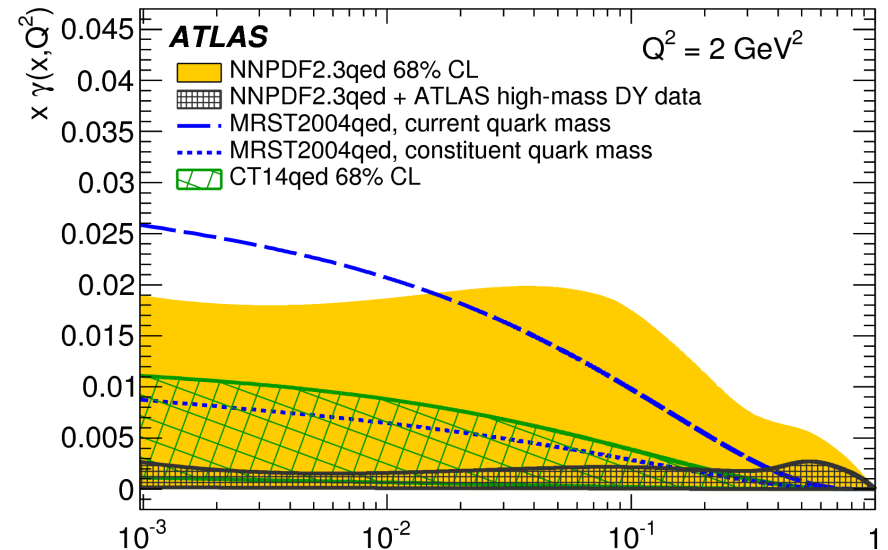
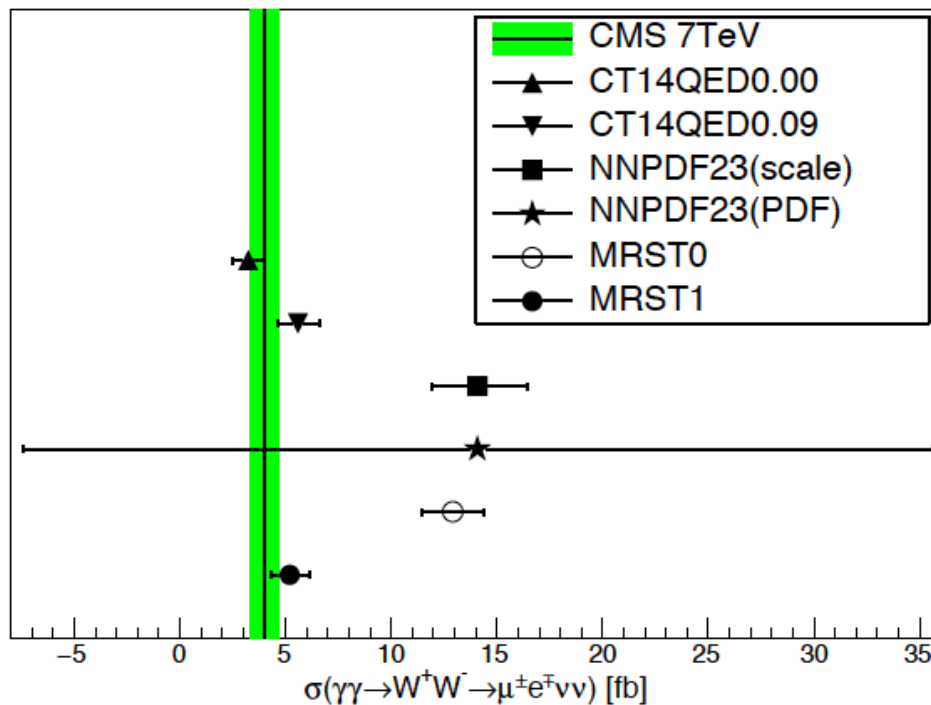
Figure 25: Photon-induced and quark-induced Born-level contributions to the production of a  $W$  pair with mass  $M_{WW} > M_{WW}^{\text{cut}}$  plotted as a function of  $M_{WW}^{\text{cut}}$  at the LHC 8 TeV (top) and LHC 14 TeV (bottom), computed with the code of Ref. [64] and NNPDF2.3QED NLO and MRST2004QED PDFs.



...but



- ATLAS fit to higher statistics Drell-Yan data prefers photon distribution at lower end of NNPDF2.3qed uncertainty band, << central value
- Also, arXiv:1603.04874



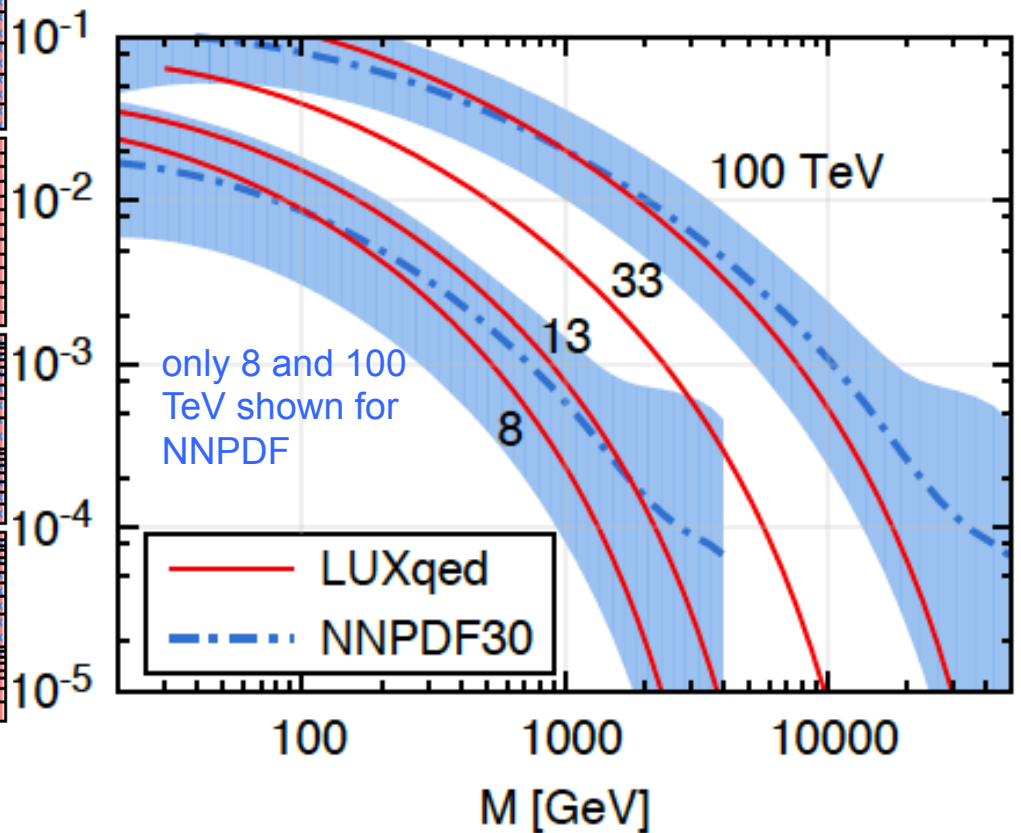
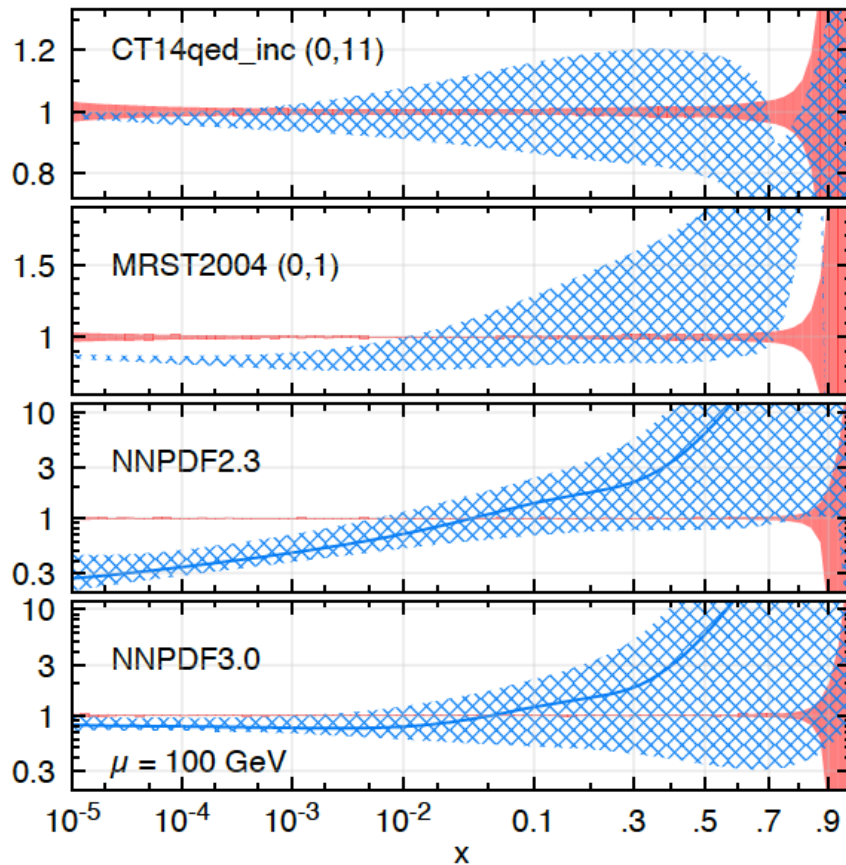


# How bright is the photon?: arXiv:1607.04266



Can define the  $\overline{\text{MS}}$  photon PDF in terms of proton structure functions, resulting in a constraint of the photon PDF at the level of 1-2% over a broad range of  $x$ .

ratio with respect to LUXqed



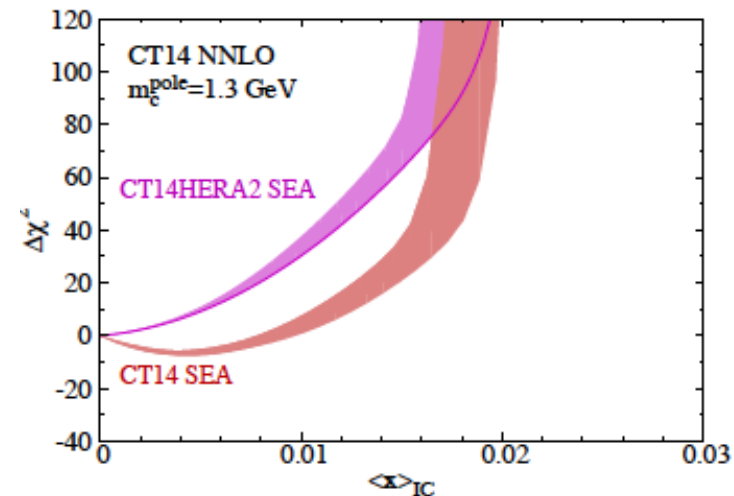
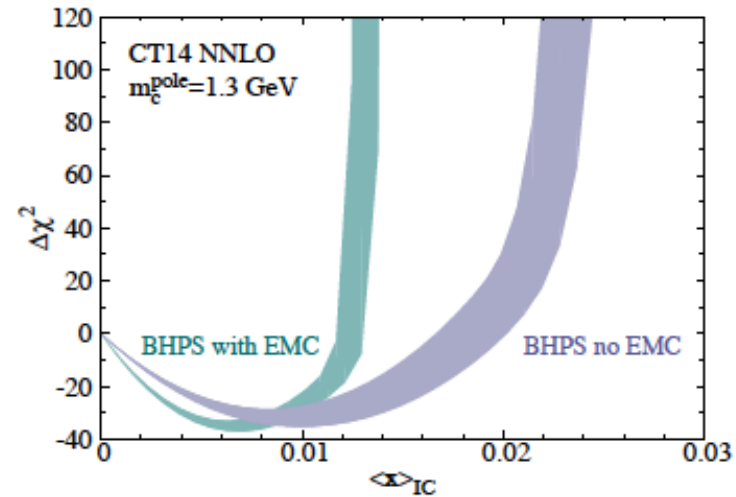


# Impacts of fitted charm at LHC



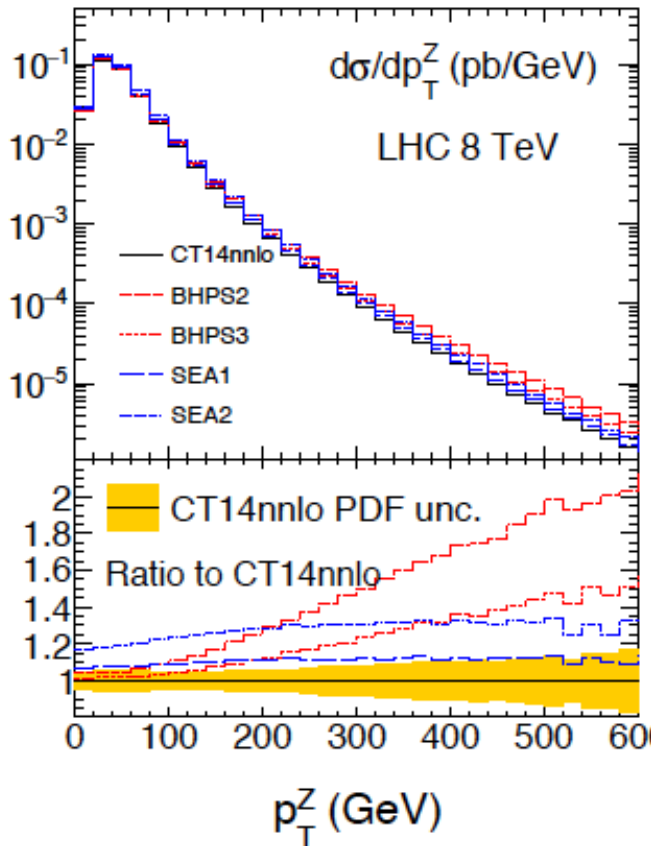
- Can you fit charm with the same freedom as up, down, gluon, etc?
- Suffers from lack of data to constrain it (similar to the photon case)
  - ◆ EMC data potentially sensitive but lack the kind of correlated error information present in modern-day experiments
- No factorization theorem for fitted charm, i.e. fitted charm determined from one process may not be valid for another
- Impacts for any charm-related cross section but potentially also for cross sections like Higgs ggF
- CTEQ PDF fits have traditionally found a modest improvement in  $\chi^2$  by inclusion of **intrinsic** charm

arxiv:1707.00657



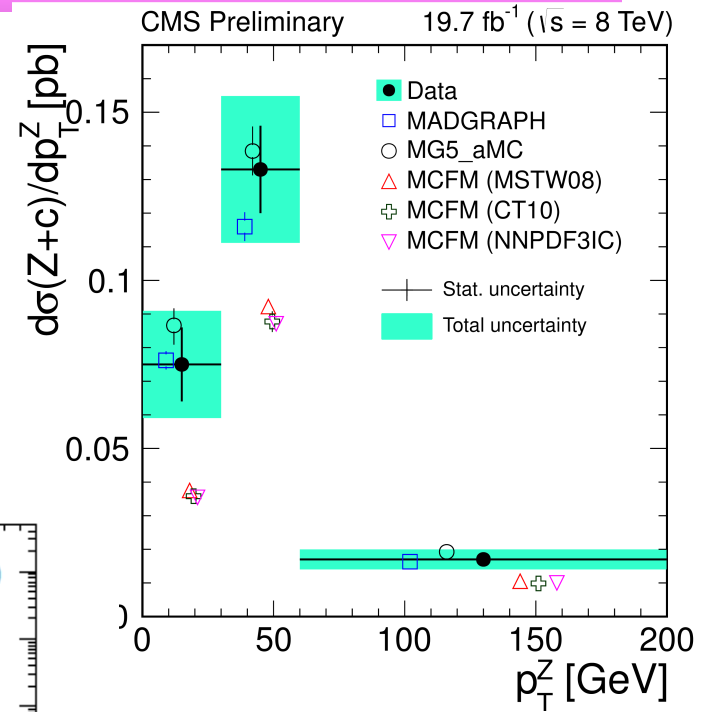
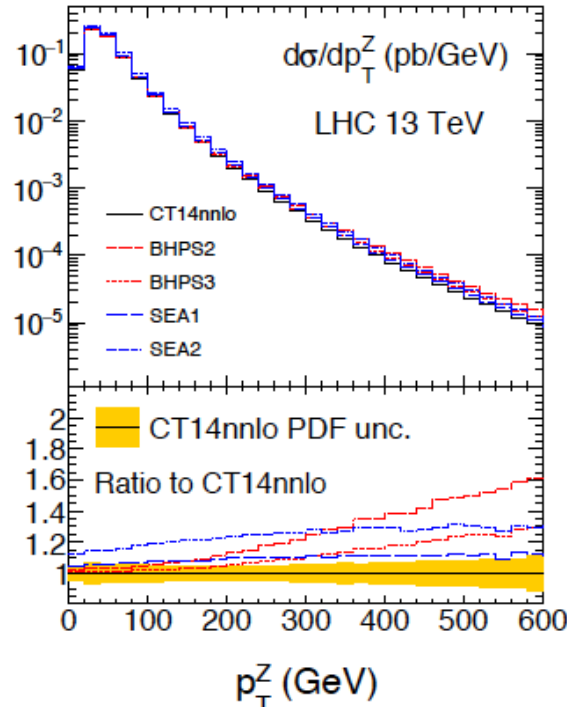


# Further investigations



NLO

no discrimination  
in LHC data so far,  
nor in Tevatron;  $\gamma$   
+charm at 8 TeV  
comes closest



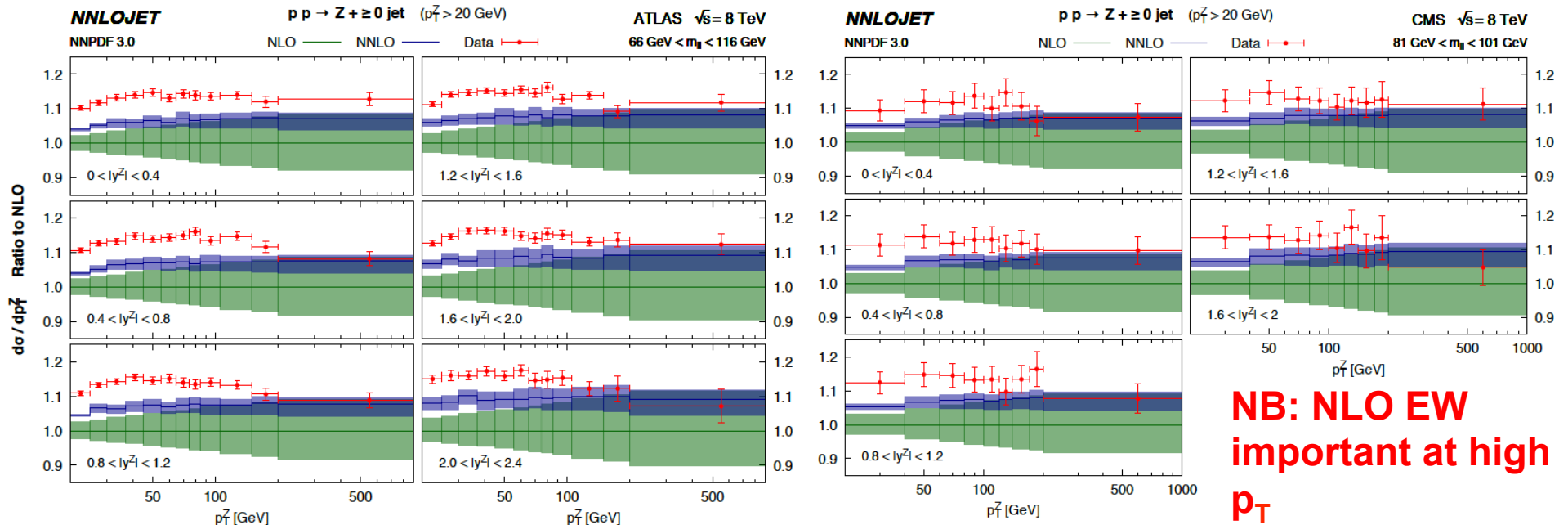
Sherpa: parton showers  
dilute impact of intrinsic  
charm



# Z $p_T$ (arXiv:1605.04295)



- ATLAS, CMS Z  $p_T$  data seem to be above NNLO prediction
  - ◆ better agreement if normalize to the Z cross section
- These distributions are very precise at both the experimental and theoretical levels
- The data is being included in the next round of global PDF fits, and will be sensitive to the gluon distribution in the range for ggF Higgs





# Dijets



- One of key processes for perturbative QCD
  - ◆ covers largest kinematic range with jets produced in the multi-TeV range
  - ◆ EW effects very important in this range
- Only process currently included in global fits not known at NNLO
  - ◆ now it has been calculated
- Current experimental precision on the order of 5-10% for jets from 200 GeV/c to 1 TeV/c
- Would like better precision for theory
  - ◆ so need NNLO QCD and NLO EW
- We also need a better understanding of the impact of parton showers on the fixed order cross section

Process	State of the Art	Desired
$t\bar{t}$	$\sigma_{\text{tot}}(\text{stable tops}) @ \text{NNLO QCD}$ $d\sigma(\text{top decays}) @ \text{NLO QCD}$ $d\sigma(\text{stable tops}) @ \text{NLO EW}$	$d\sigma(\text{top decays}) @ \text{NNLO QCD} + \text{NLO EW}$
$t\bar{t} + j(j)$	$d\sigma(\text{NWA top decays}) @ \text{NLO QCD}$	$d\sigma(\text{NWA top decays}) @ \text{NNLO QCD} + \text{NLO EW}$
$t\bar{t} + Z$	$d\sigma(\text{stable tops}) @ \text{NLO QCD}$	$d\sigma(\text{top decays}) @ \text{NLO QCD} + \text{NLO EW}$
single-top	$d\sigma(\text{NWA top decays}) @ \text{NLO QCD}$	$d\sigma(\text{NWA top decays}) @ \text{NNLO QCD} + \text{NLO EW}$
dijet	$d\sigma @ \text{NNLO QCD (g only)}$ $d\sigma @ \text{NLO EW (weak)}$	$d\sigma @ \text{NNLO QCD} + \text{NLO EW}$
$3j$	$d\sigma @ \text{NLO QCD}$	$d\sigma @ \text{NNLO QCD} + \text{NLO EW}$
$\gamma + j$	$d\sigma @ \text{NLO QCD}$ $d\sigma @ \text{NLO EW}$	$d\sigma @ \text{NNLO QCD} + \text{NLO EW}$

IPPP/16/110, MPP

NNLO QCD predictions for single jet inclusive production at the LHC

J. Currie<sup>a</sup>, E.W.N. Glover<sup>a</sup>, J. Pires<sup>b</sup>

<sup>a</sup> Institute for Particle Physics Phenomenology, University of Durham, Durham DH1 3LE, England

<sup>b</sup> Max-Planck-Institut für Physik, Föhringer Ring 6 D-80805 Munich, Germany

We report the first calculation of fully differential jet production in all partonic channels at next-to-next-to leading order (NNLO) in perturbative QCD and compare to the available ATLAS 7 TeV data. We discuss the size and shape of the perturbative corrections along with their associated scale variation across a wide range in jet transverse momentum,  $p_T$ , and rapidity,  $y$ . We find significant effects, especially at low  $p_T$ , and discuss the possible implications for Parton Distribution Function fits.

→ topic for Les Houches

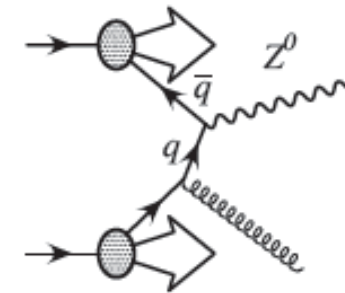


# NNLO

- How to distribute the calculation?
- For calculations like  $W/Z + n$  jets, Higgs +  $n$  jets, all at NLO, use ROOT ntuples
- Processes at NNLO, such as H+jet, and  $W/Z$ +jet, use MCFM?
- Inclusive jet production not amenable to above techniques, so may use applgrid/fastNLO grids
- Such an approach useful/needed for global PDF fits
  - ◆ right now we are using NNLO/NLO K-factors

## NNLOJET (and APPLfast)

- Semi-automated calculation of cross sections at NNLO from the IPPP, Zurich, ETH and others
  - Gehrman-De Ridder *et al* [arXiv: 1607.01749](https://arxiv.org/abs/1607.01749)
  - See talk from **Alex Huss** tomorrow
- APPLfast-NNLO
  - Developers from **NNLOJET**, **APPLgrid** and **fastNLO**
  - A single, combined interface for NNLOJET with both APPLgrid and fastNLO



- Many processes implemented in NNLOJET
  - Developing a generic interface for **all available** processes
  - Concentrating on Z + jets at NNLO for the initial development and proof-of-concept

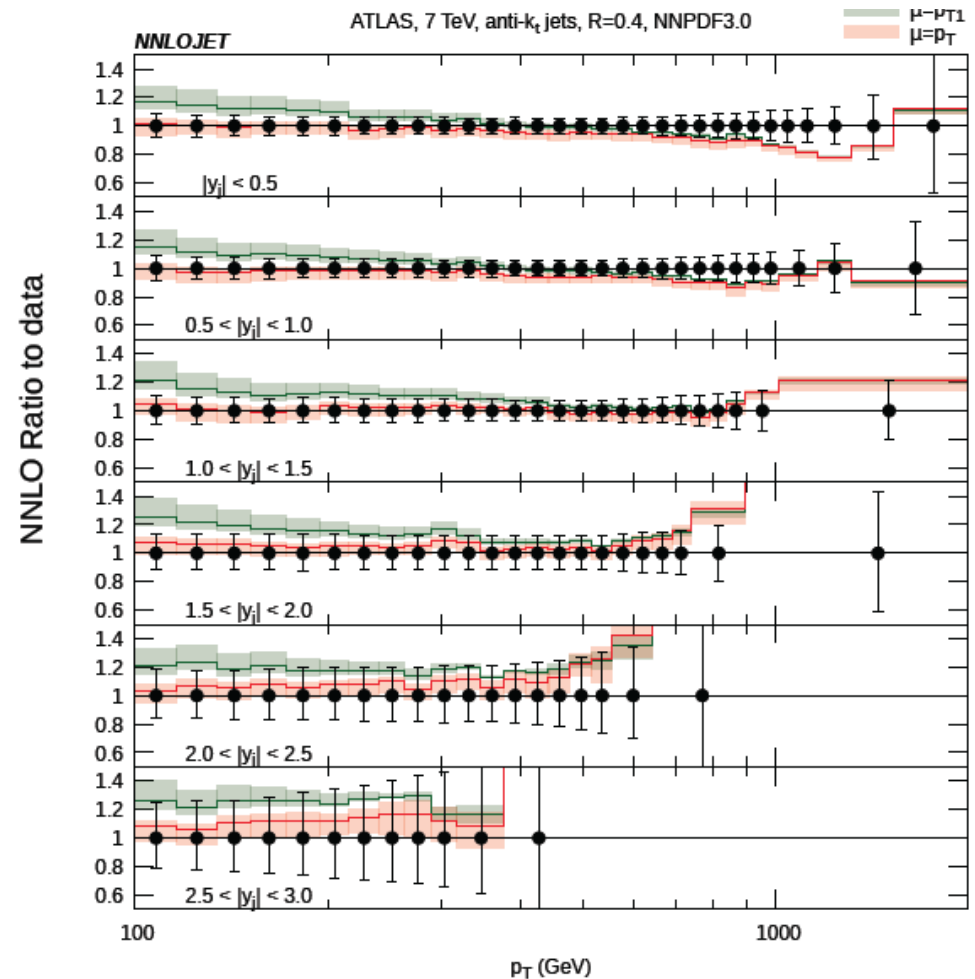




# NNLO



- Basically only process included in NNLO PDF fits not (until now) at NNLO
- Scale dependence greatly reduced...but sizeable differences between  $p_{T}^{\text{jet}}$  and  $p_{T}^{\text{leadjet}}$
- Which scale choice is preferable? Difference larger than the nominal factor of 2 variation
- Religious wars: my opinion is that since this is an inclusive calculation, you should use an inclusive scale
  - ◆ this is not a classical Monte Carlo
  - ◆ NB: ATLAS uses  $p_{T}^{\text{leadjet}}$

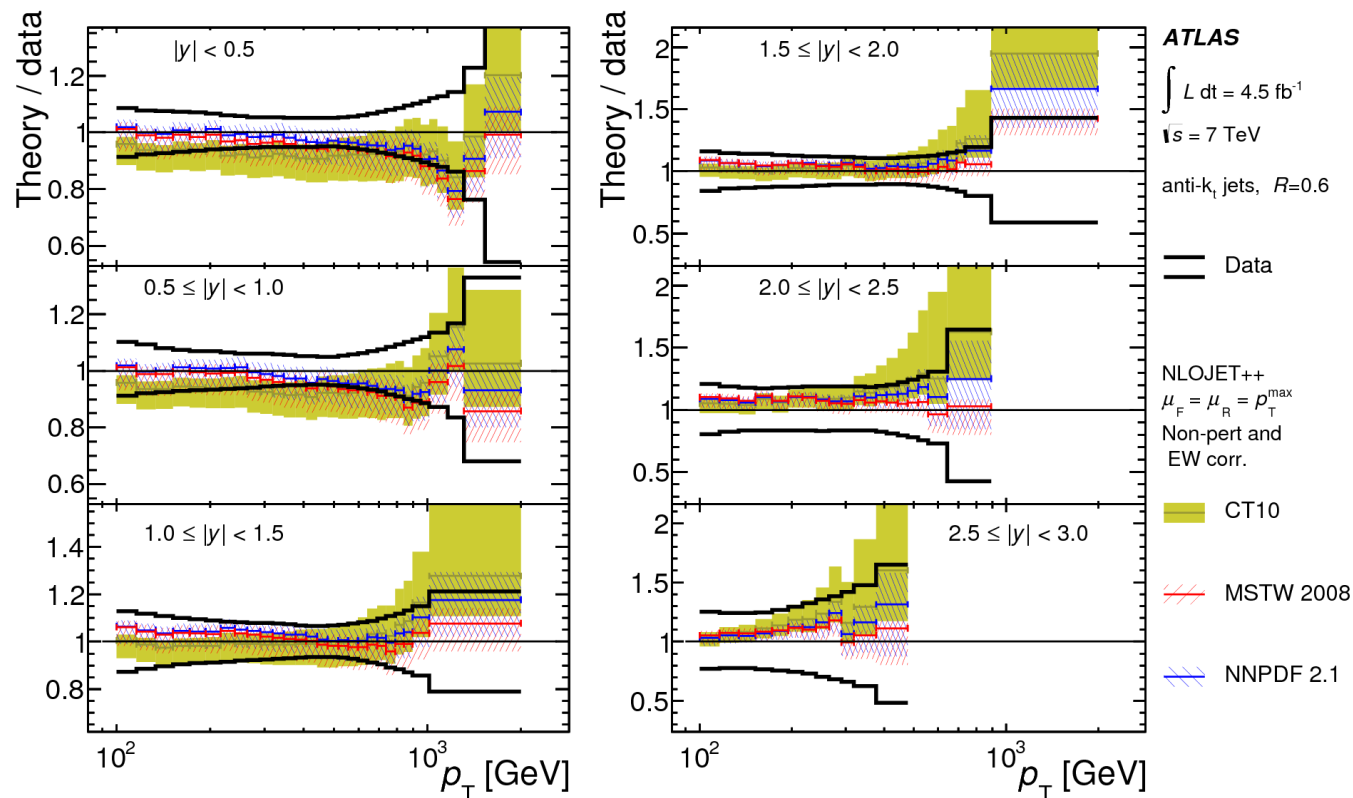




# ATLAS 7 (and 8) TeV jet data



- Impossible to get a good  $\chi^2$  when fitting all rapidity intervals simultaneously, although each rapidity interval by itself gives a good  $\chi^2$  -> correlations? 8 TeV data has the same problem
- If only one y interval is chosen, which one? Do the other rapidity intervals provide the same constraint? If not, then how can the data be used?
- In general, ATLAS jet data prefers a weaker gluon at high x

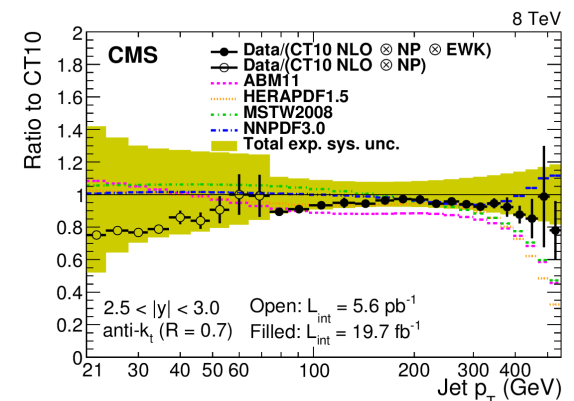
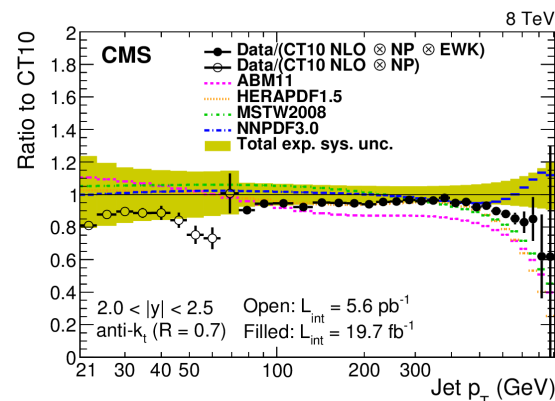
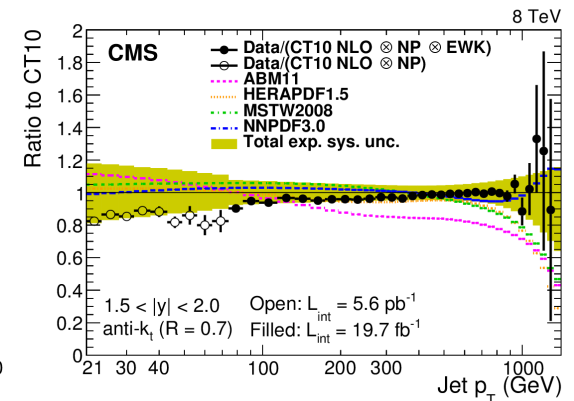
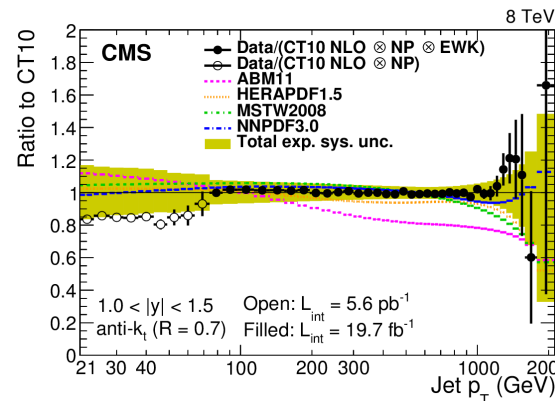
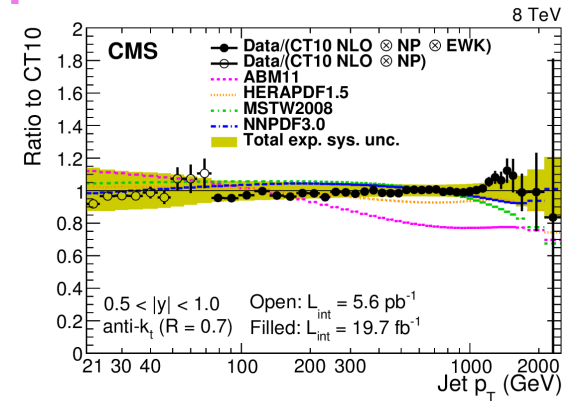
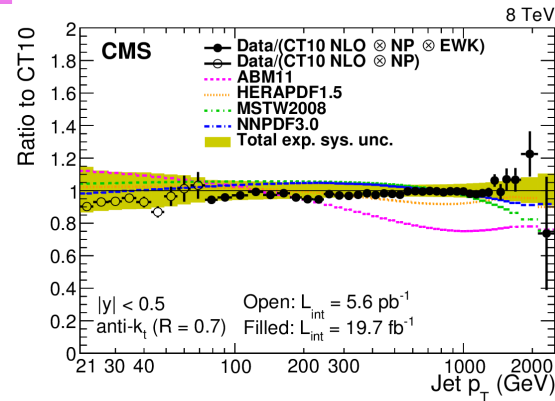




# CMS 8 TeV jet data



- CT10 has a harder gluon than CT14
- CMS data seems happy with that
- I'm happy with that
- ...but may point out a tension between the ATLAS and CMS jet data sets; if so, high  $x$  gluon uncertainty may not be reduced by these data sets
- ...being included in CT17

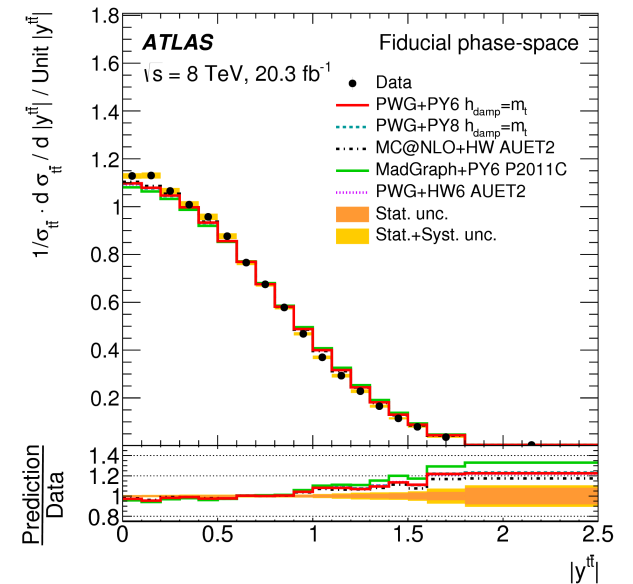
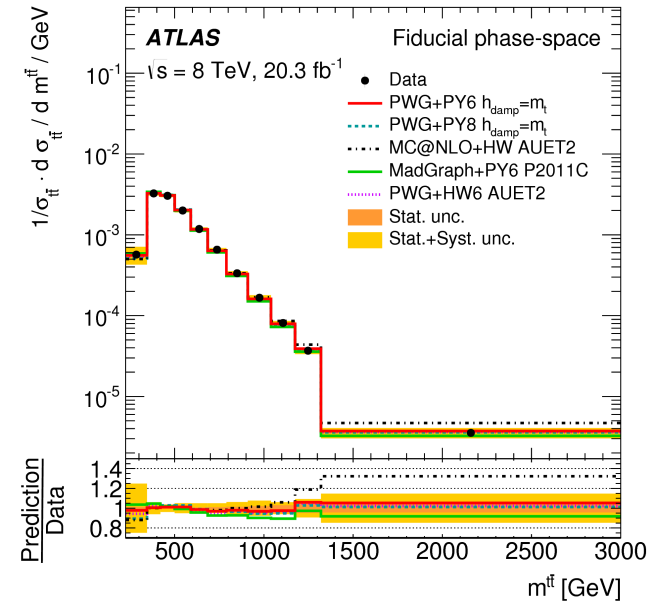
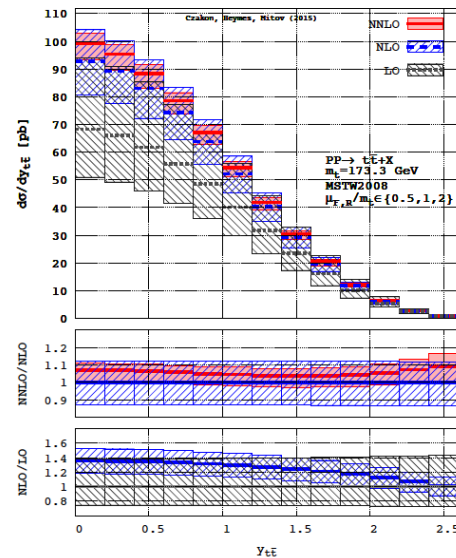
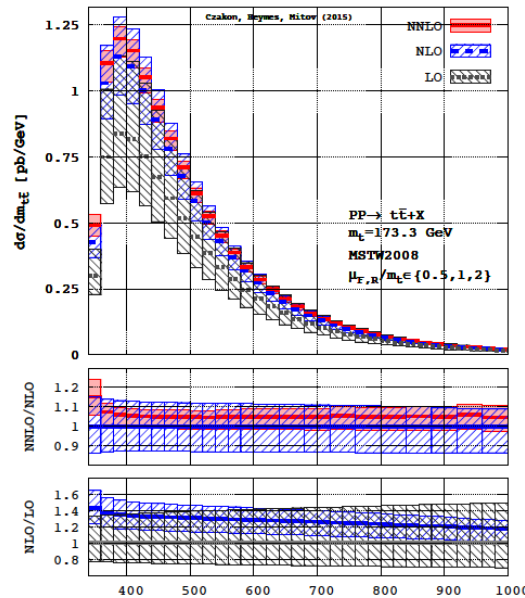




# tT differential data



- tT differential cross sections provide a great handle on the high x gluon distribution
  - may settle the struggle between ATLAS and CMS jet data
- Recent calculation by Czakon, Heyes and Mitov; arXiv:1511.00549
- How can the predictions differ by so much at high mass, rapidity
- If it's because of the parton shower/matching, how can that be?
- Topic for Les Houches





# Top distributions



- There are several distributions measured by ATLAS and CMS that have information on the high  $x$  gluon
  - ◆  $m_{t\bar{T}}, y_{t\bar{T}}$  directly
  - ◆  $y_{t,T}, p_T^{t\bar{T}}$  indirectly
- Only one distribution should be used, unless a correlation model can be developed
  - ◆ which one?
  - ◆ do they give the same answer? if not, do we understand why? how can you claim a decrease in uncertainty if you pick and choose the variables that give the answer (and constraints) you want?
- ATLAS and CMS have different trends; in this case, ATLAS favors harder gluon (than NNPDF3.0) at high  $x$ , CMS weaker gluon
- In general, the ATLAS and CMS top results are in tension internally, and with each other (the latter more so in the case of normalized distributions where the experimental errors are smaller)
- This is similar to the tension that exists between the ATLAS and CMS jet data, although there the tension is in the opposite direction
- If tension, then gluon PDF uncertainty may not decrease and may even increase

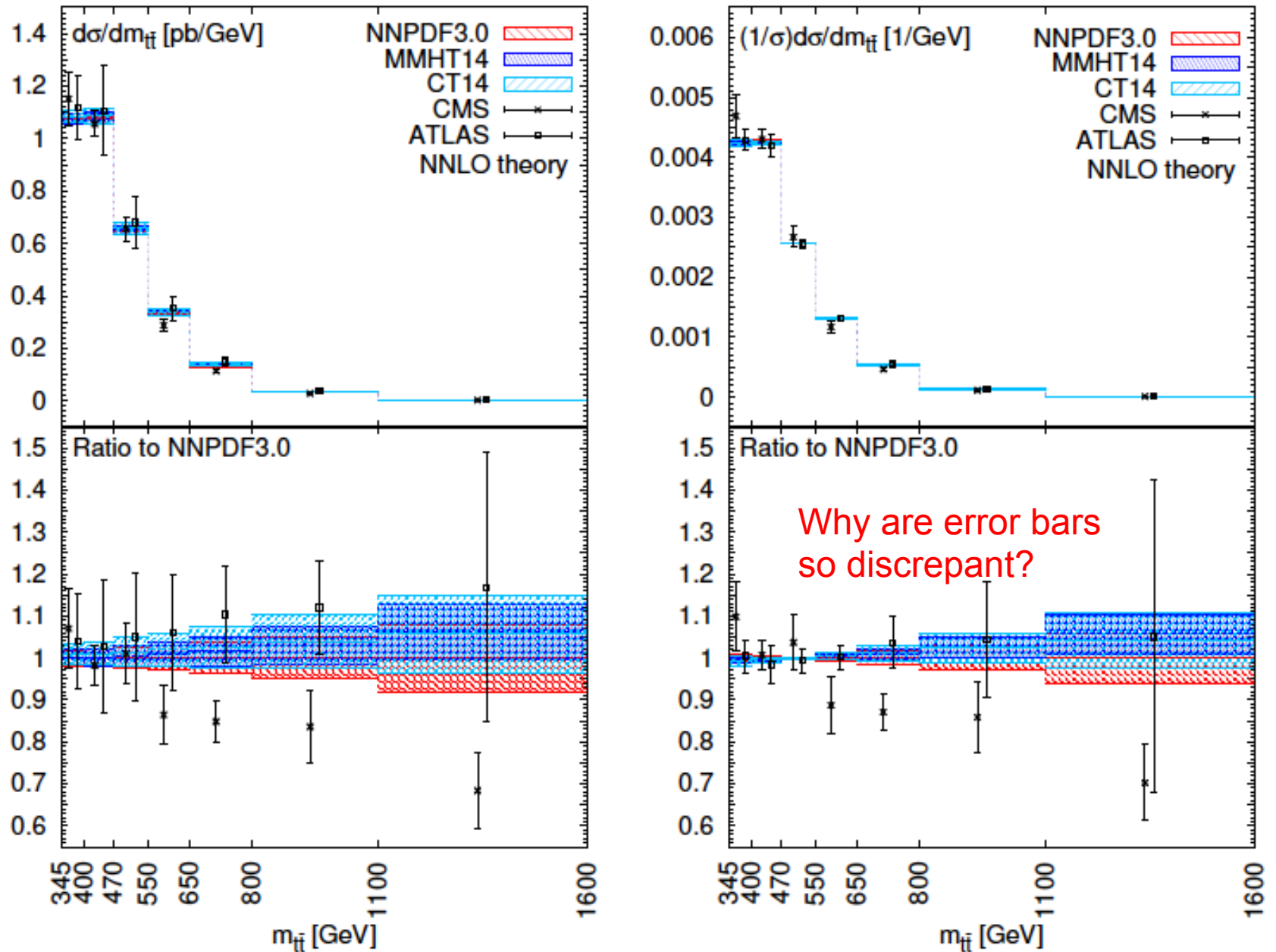


Figure 10: Same as Fig. 4 for the invariant mass distribution of the top-antitop pair,  $m_{\bar{t}t}$ .



dataset	Fit ID									
	1	2	3	4	5	6	7	8	9	10
ATLAS $d\sigma/dp_T^t$	2.37	2.30	<b>1.99</b>	2.36	2.24	2.23	2.09	2.18	2.34	2.24
ATLAS $d\sigma/dy_t$	0.93	0.80	0.74	<b>1.09</b>	0.76	0.76	0.86	0.69	0.76	0.66
ATLAS $d\sigma/dy_{t\bar{t}}$	2.44	2.03	1.96	2.59	<b>1.32</b>	2.32	2.11	1.74	1.26	1.80
ATLAS $d\sigma/dm_{t\bar{t}}$	4.27	4.47	4.68	4.14	4.92	<b>4.02</b>	4.34	4.79	4.98	4.99
ATLAS $(1/\sigma)d\sigma/dp_T^t$	2.93	3.97	3.29	4.36	5.22	4.35	<b>2.96</b>	4.26	4.92	5.68
ATLAS $(1/\sigma)d\sigma/dy_t$	5.00	3.17	2.47	6.36	1.55	2.93	3.94	<b>1.68</b>	<b>1.45</b>	1.10
ATLAS $(1/\sigma)d\sigma/dy_{t\bar{t}}$	9.69	5.59	5.89	8.95	2.68	5.73	6.73	3.57	<b>2.17</b>	3.73
ATLAS $(1/\sigma)d\sigma/dm_{t\bar{t}}$	2.30	2.80	3.31	2.67	3.96	4.21	3.09	3.68	3.77	<b>2.98</b>
ATLAS $\sigma_{t\bar{t}}$	0.12	<b>0.10</b>	0.21	0.10	0.10	0.12	<b>0.36</b>	<b>0.29</b>	<b>0.26</b>	0.10
CMS $d\sigma/dp_T^t$	3.50	3.46	<b>2.60</b>	3.50	3.03	3.00	2.85	3.11	3.24	2.92
CMS $d\sigma/dy_t$	3.48	3.71	4.05	<b>2.66</b>	4.18	3.49	3.38	4.23	4.43	4.99
CMS $d\sigma/dy_{t\bar{t}}$	1.36	1.13	1.00	1.32	<b>0.89</b>	0.86	1.00	1.01	1.04	1.24
CMS $d\sigma/dm_{t\bar{t}}$	7.07	6.27	5.79	6.33	5.09	<b>5.11</b>	6.00	5.57	5.21	4.31
CMS $(1/\sigma)d\sigma/dp_T^t$	4.31	4.00	3.39	4.28	3.65	3.59	<b>3.56</b>	<b>3.57</b>	3.73	3.48
CMS $(1/\sigma)d\sigma/dy_t$	3.66	4.10	4.45	3.10	4.98	4.06	3.65	<b>4.76</b>	<b>5.13</b>	6.09
CMS $(1/\sigma)d\sigma/dy_{t\bar{t}}$	1.59	1.20	1.06	1.73	0.94	1.01	1.20	0.99	<b>1.05</b>	1.32
CMS $(1/\sigma)d\sigma/dm_{t\bar{t}}$	12.0	10.8	9.81	11.1	8.72	8.72	10.3	9.15	9.27	<b>7.27</b>
CMS $\sigma_{t\bar{t}}$	0.10	<b>0.35</b>	0.26	0.19	0.32	0.21	<b>0.11</b>	<b>0.10</b>	<b>0.15</b>	<b>0.35</b>

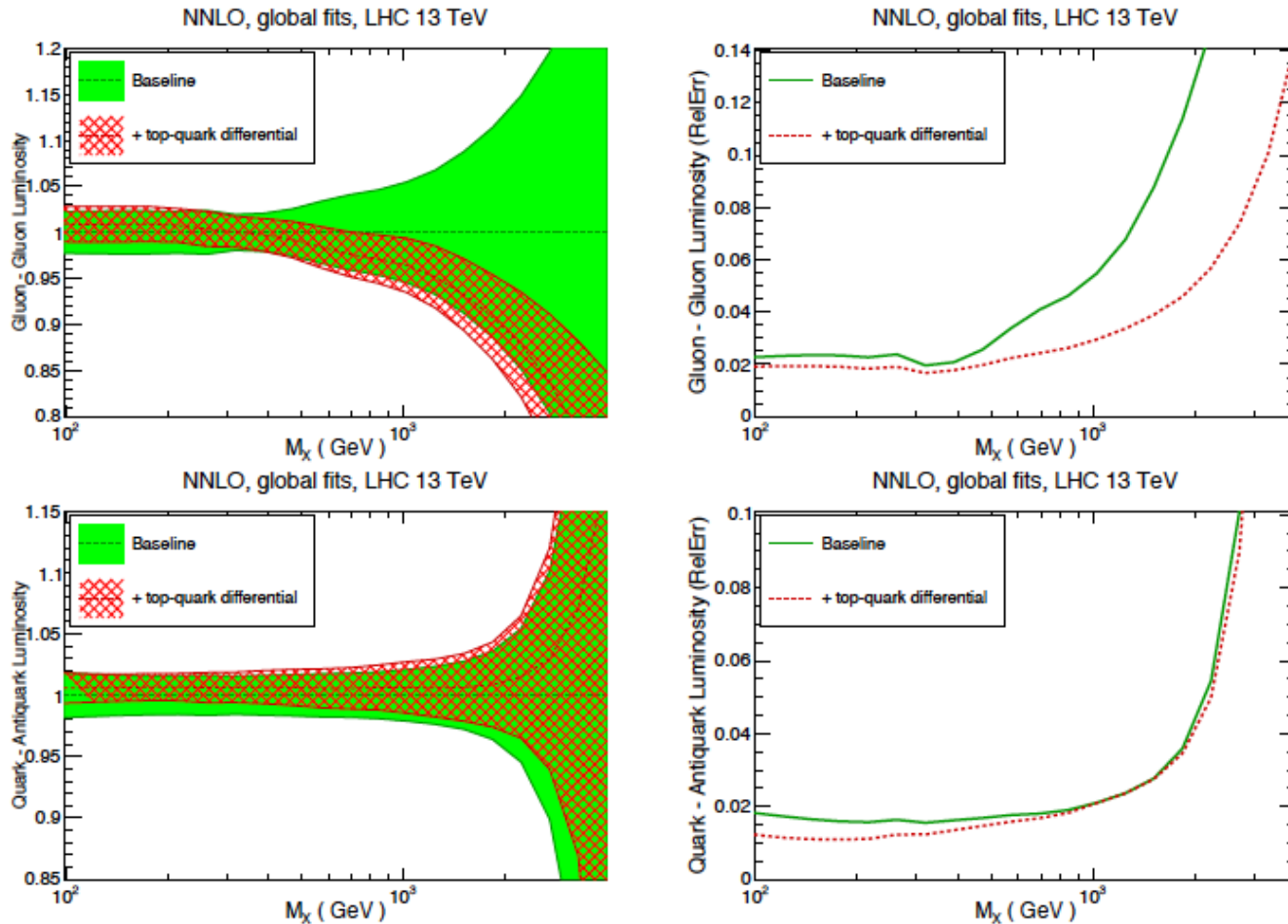
final fit includes only these cross sections

...and not these (for example)

Table 7: Same as Table 6 for the global fits.



# Resultant fit



I think more detailed studies investigating the degree of compatibility between the different observables within ATLAS and within CMS, and between the two experiments, is needed before any strong conclusion can be determined

Figure 15: The gluon-gluon (upper) and quark-antiquark (lower) NNLO luminosities (left) and their relative  $1\text{-}\sigma$  PDF uncertainties (right) at the LHC with  $\sqrt{s} = 13$  TeV. We compare the global baseline fit with the fit including the optimal combination of LHC top-quark pair differential data.





- NNPDF3.1 claims a significant reduction in PDF uncertainty, mostly due to inclusion of fitted charm and collider data
- Because of the tensions mentioned earlier, I don't know if CT17 (or the new MMHT) will come to the same conclusion

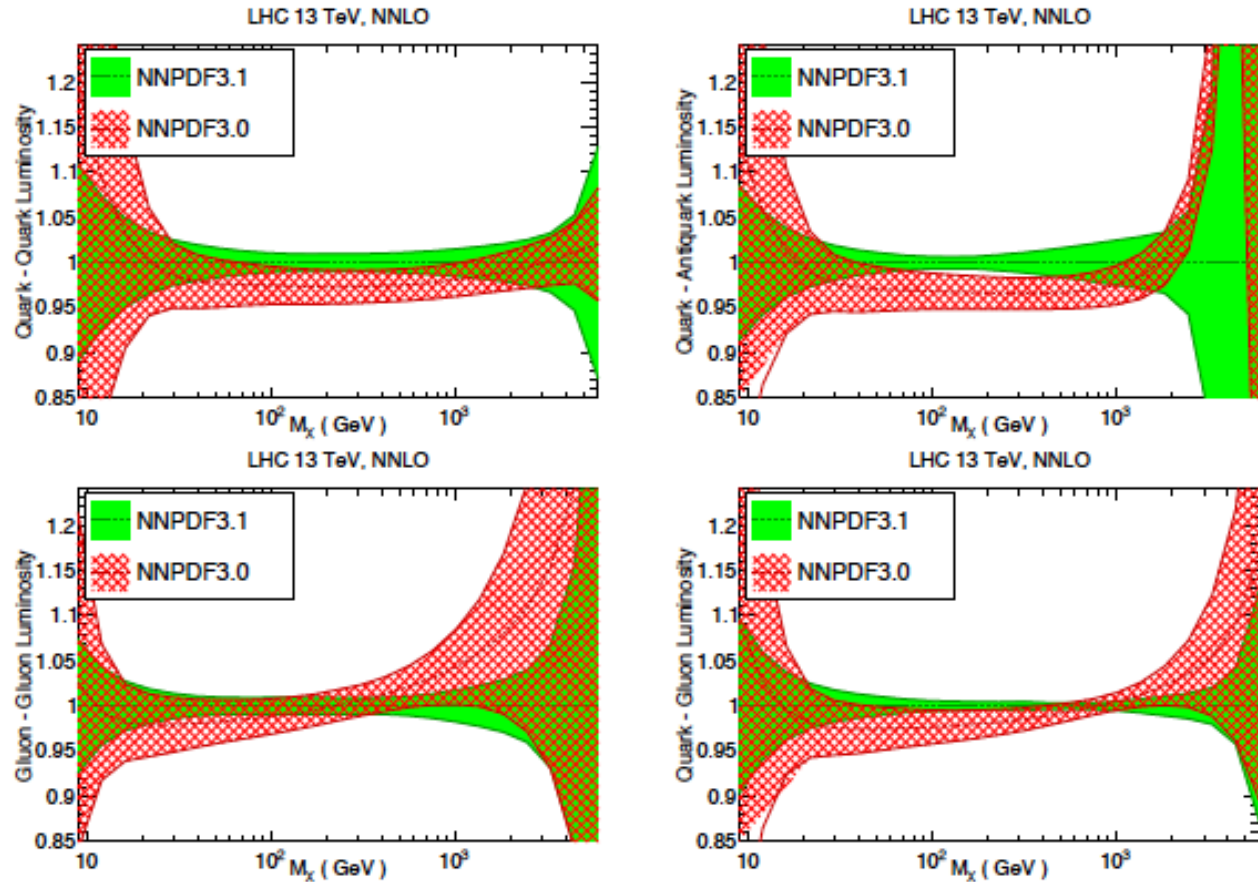


Figure 5.8: Comparison of parton luminosities with the NNPDF3.0 and NNPDF3.1 NNLO PDF sets for the LHC 13 TeV. From left to right and from top to bottom quark-antiquark, quark-quark, gluon-gluon and quark-gluon PDF luminosities are shown. Results are shown normalized to the central value of NNPDF3.1.



# CT17



## Included experiments:

- Combined HERA1+2 DIS
- LHCb 7 TeV Z, W muon rapidity dist.
- LHCb 8 TeV Z, W muon rapidity dist.
- ATLAS 7 TeV inclusive jet
- CMS 7 TeV inclusive jet (extended  $y$  range)
- ATLAS 7 TeV Z  $p_T$  dist.
- LHCb 13 TeV Z rapidity dist.
- CMS 8 TeV Z  $p_T$  and rapidity dist. (double diff.)
- CMS 8 TeV W, muon asymmetry dist.
- ATLAS 7 TeV W/Z, lepton(s) rapidity dist.
- CMS 7,8 TeV  $t\bar{t}$  differential dist.
- ATLAS 7,8 TeV  $t\bar{t}$  differential dist.

- Previous LHC data included in CT14 are superseded by updated Run 1 or Run 2 results.
- adding new data, especially on Z boson  $p_T$  and top quark differential distributions.



# 8 is enough



- PDF4LHC15\_30 is a general purpose LHC set
- Can re-diagonalize eigenvector set to look for directions most sensitive to a particular class of physics, for example Higgs physics
- In that case, 8 PDFs are sufficient
- Could also do the same for any cross section

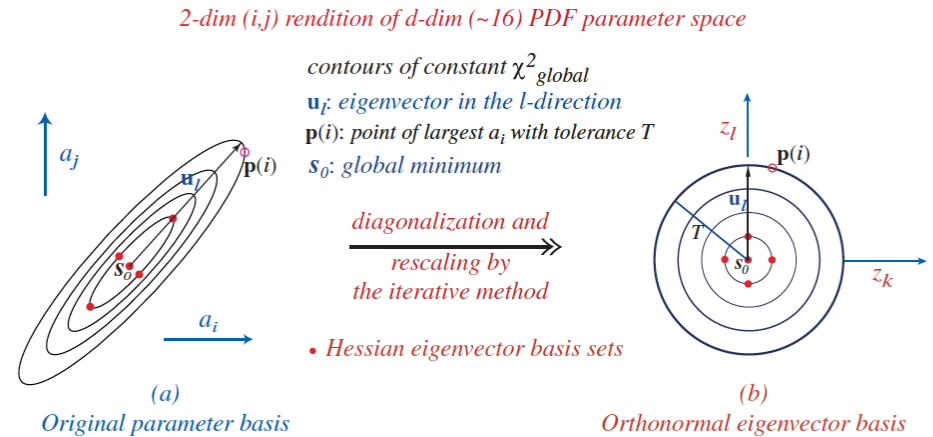


Fig. 6.13 A schematic representation of the transformation from the pdf parameter basis to the orthonormal eigenvector basis.

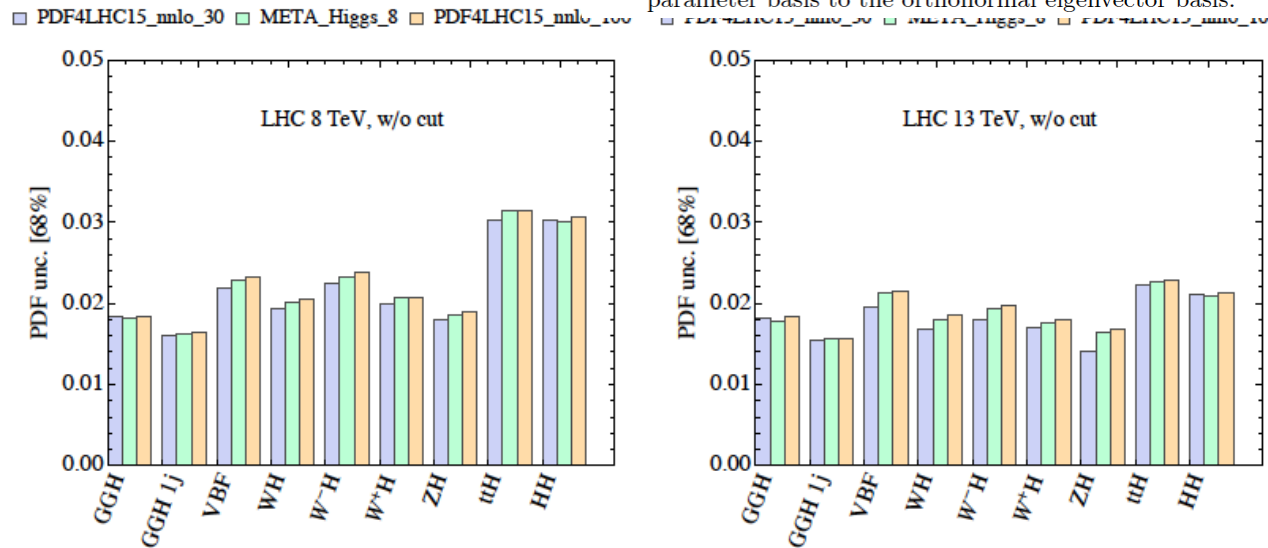


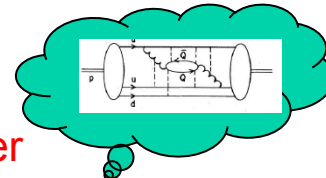
FIG. 1: Predictions on the total PDF uncertainties comparing NNLO PDFs of PDF4LHC15 30 set, reduced set (with 8 eigenvectors), and PDF4LHC15 100 set.



# Summary



- First, let me summarize what I didn't talk about
  - ◆ the combined HERA1+2 data set was released after this last generation of PDF sets
  - ◆ all PDF groups have included the data in a new round of (private) fits, and find that it doesn't change the results obtained with using HERA1 data alone
- ...and this
  - ◆ sometimes data is included in PDF fits not directly, but by re-weighting; I think this is typically not done correctly, and over-estimates the effect of the data->work in progress
- It appears that the photon PDF is fairly-well constrained now, and fairly small
- The idea of a large *intrinsic/fitted* charm component still needs more study, both theoretical and experimental
  - ◆ LHC data should be able to tell us
  - ◆ Stan may have to keep dreaming for a bit longer
- PDF fitting continues to grow in sophistication and in the amount of LHC data included in the fits
  - ◆ still hard to fight the precision of the DIS data
  - ◆ ATLAS, CMS, LHCb data have to agree in order to reduce the current size of PDF uncertainties
  - ◆ some PDFs, such as charm, strange, photon, and the high  $x$  gluon still have large uncertainties, but with further data/improvements, should improve





# Summary



- There's a wealth of new data from the LHC along with new calculations at NNLO that should allow more detailed knowledge of PDFs and of cross sections at the LHC
- One problem is the use of such calculations in global PDF fits where thousands of iterations are required
  - ◆ for CT, we are using a combination of applgrid and fastNLO for the NLO matrix elements, with NNLO/NLO K-factors, along with parallelization of the computations
  - ◆ It will be possible to directly use applgrid/fastNLO such as what NNLOJET is working on
- In any case, the impact of the LHC data on global PDF fits requires a great deal of study and interaction with the experimenters, especially in cases where the experiments disagree and where different observables (or even rapidity regions) within the the same experiment disagree
- Next PDF4LHC meeting March 7, 2017 at CERN
  - ◆ updates of PDF fits, data sets
  - ◆ discussion of incorporation of scale uncertainties in PDF fits
  - ◆ correlations among data sets
  - ◆ ...



# ...meanwhile, the book



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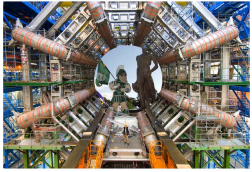


# Now on LHAPDF



LHAPDF6 grid	Pert order	ErrorType	$N_{\text{mem}}$	$\alpha_s(m_Z^2)$
PDF4LHC15_nnlo_mc	NNLO	replicas	100	0.118
PDF4LHC15_nnlo_100	NNLO	symmhessian	100	0.118
PDF4LHC15_nnlo_30	NNLO	symmhessian	30	0.118
PDF4LHC15_nnlo_mc_pdfas	NNLO	replicas+as	102	mem 0:100 $\rightarrow$ 0.118 mem 101 $\rightarrow$ 0.1165 mem 102 $\rightarrow$ 0.1195
PDF4LHC15_nnlo_100_pdfas	NNLO	symmhessian+as	102	mem 0:100 $\rightarrow$ 0.118 mem 101 $\rightarrow$ 0.1165 mem 102 $\rightarrow$ 0.1195
PDF4LHC15_nnlo_30_pdfas	NNLO	symmhessian+as	32	mem 0:30 $\rightarrow$ 0.118 mem 31 $\rightarrow$ 0.1165 mem 32 $\rightarrow$ 0.1195
PDF4LHC15_nnlo_asvar	NNLO	-	1	mem 0 $\rightarrow$ 0.1165 mem 1 $\rightarrow$ 0.1195

Table 5: Summary of the combined NNLO PDF4LHC15 sets with  $n_f^{\text{max}} = 5$  that are available from LHAPDF6. The corresponding NLO sets are also available. Members 0 and 1 of PDF4LHC15\_nnlo\_asvar coincide with members 101 and 102 (31 and 32) of PDF4LHC15\_nnlo\_mc\_pdfas and PDF4LHC15\_nnlo\_100\_pdfas (PDF4LHC15\_nnlo\_30\_pdfas). Recall that in LHAPDF6 there is always a zeroth member, so that the total number of PDF members in a given set is always  $N_{\text{mem}} + 1$ . See text for more details.



# Recommendations



## 1. Comparisons between data and theory for Standard Model measurements

**Recommendations:** Use *individual PDF sets*, and, in particular, as many of the modern PDF sets [5–11] as possible.

**Rationale:** Measurements such as jet production, vector-boson single and pair production, or top-quark pair production, have the power to constraining PDFs, and this is best utilized and illustrated by comparing with many individual sets.

As a rule of thumb, *any measurement that potentially can be included in PDF fits* falls in this category.

The same recommendation applies to the *extraction of precision SM parameters*, such as the strong coupling  $\alpha_s(m_Z^2)$  [75,124], the  $W$  mass  $M_W$  [125], and the top quark mass  $m_t$  [126] which are directly correlated to the PDFs used in the extraction.

## 2. Searches for Beyond the Standard Model phenomena

**Recommendations:** Use the PDF4LHC15\_mc sets.

**Rationale:** BSM searches, in particular for *new massive particles in the TeV scale*, often require the knowledge of PDFs in regions where available experimental constraints are limited, notably close to the hadronic threshold where  $x \rightarrow 1$  [127]. In these extreme kinematical regions the PDF uncertainties are large, the *Monte Carlo combination of PDF sets is likely to be non-Gaussian*. *c.f.* Figs. 10 and 11.





3. Calculation of PDF uncertainties in situations when computational speed is needed, or a more limited number of error PDFs may be desirable

**Recommendations:** Use the PDF4LHC15\_30 sets.

**Rationale:** In many situations, PDF uncertainties may affect the extraction of physics parameters. From the point of view of the statistical analysis, it might be useful in some cases to *limit the number of error PDFs* that need to be included in such analyses. In these cases, use of the PDF4LHC15\_30 sets may be most suitable.

In addition, the calculation of *acceptances, efficiencies or extrapolation factors* are affected by the corresponding PDF uncertainty. These quantities are only a moderate correction to the measured cross-section, and thus a mild loss of accuracy in the determination of PDF uncertainties in these corrections is acceptable, while computational speed can be an issue. In these cases, use of the PDF4LHC15\_30 sets is most suitable.

However, in the cases when PDF uncertainties turn out to be substantial, we recommend to cross-check the PDF estimate by comparing with the results of the PDF4LHC15\_100 sets.

4. Calculation of PDF uncertainties in precision observables

**Recommendation:** Use the PDF4LHC15\_100 sets.

**Rationale:** For several LHC phenomenological applications, the highest accuracy is sought for, with, in some cases, the need to *control PDF uncertainties to the percent level*, as currently allowed by the development of high-order computational techniques in the QCD and electroweak sectors of the Standard Model.

Whenever the highest accuracy is desired, the PDF4LHC15\_100 set is most suitable.



Pedagogical text about their use has been added



## 6.2 Formulae for the calculation of PDF and PDF+ $\alpha_s$ uncertainties

For completeness, we also collect in this report the explicit formulae for the calculation of PDF and combined PDF+ $\alpha_s$  uncertainties in LHC cross-sections when using the PDF4LHC15 combined sets. Let us assume that we wish to estimate the PDF+ $\alpha_s$  uncertainty of given cross-section  $\sigma$ , which could be a total inclusive cross-section or any bin of a differential distribution.

First of all, to compute the PDF uncertainty, one has to evaluate this cross-section  $N_{\text{mem}} + 1$  times, where  $N_{\text{mem}}$  is the number of error sets (either symmetric eigenvectors or MC replicas) of the specific combined set,

$$\sigma^{(k)}, \quad k = 0, \dots, N_{\text{mem}}, \quad (19)$$

so in particular  $N_{\text{mem}} = 30$  in PDF4LHC15\_30 and  $N_{\text{mem}} = 100$  in PDF4LHC15\_100 and PDF4LHC15\_mc.

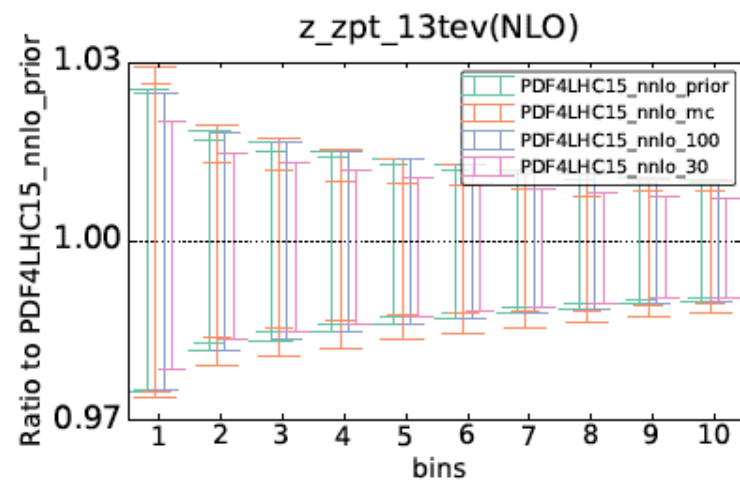
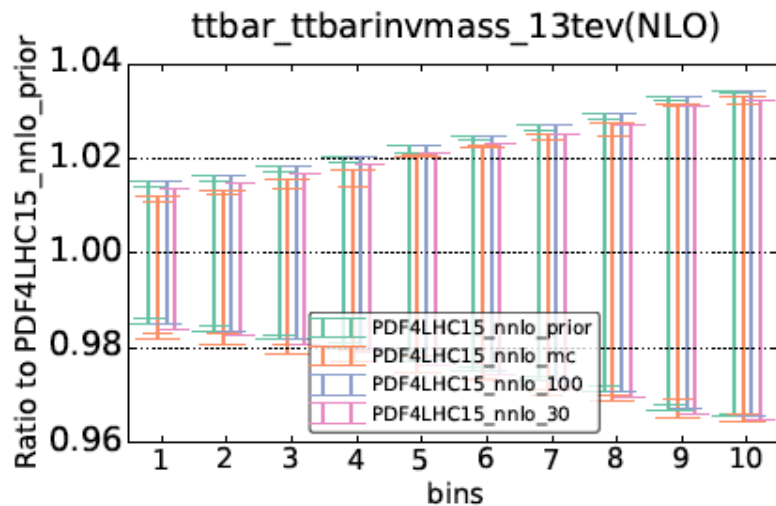
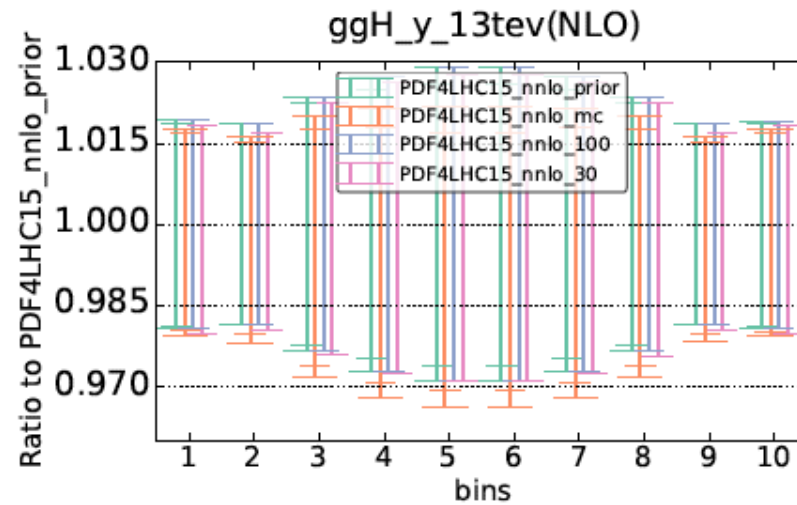
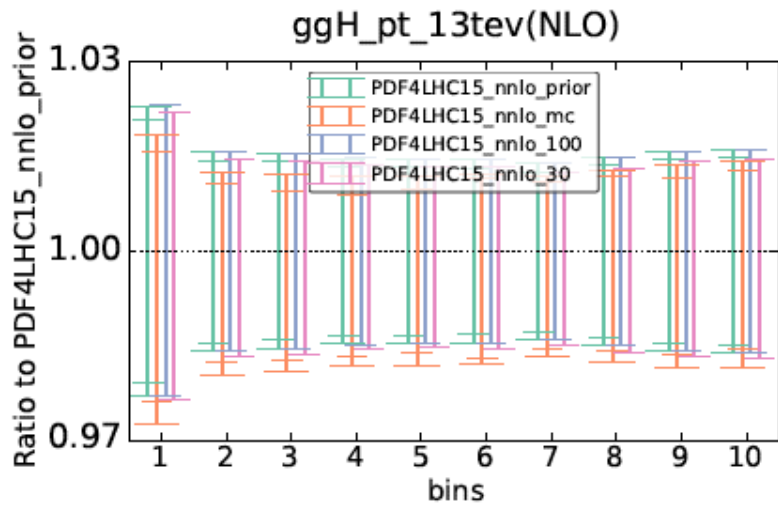
**PDF uncertainties for Hessian sets.** In the case of the Hessian sets, PDF4LHC15\_30 and PDF4LHC15\_100, the master formula to evaluate the PDF uncertainty is given by

$$\delta^{\text{pdf}} \sigma = \sqrt{\sum_{k=1}^{N_{\text{mem}}} (\sigma^{(k)} - \sigma^{(0)})^2}, \quad (20)$$

This uncertainty is to be understood as a 68% confidence level. From this expression it is also easy to determine the contribution of each eigenvector  $k$  to the total Hessian PDF uncertainty. ...continues with discussion of MC PDFs



# Application to cross sections





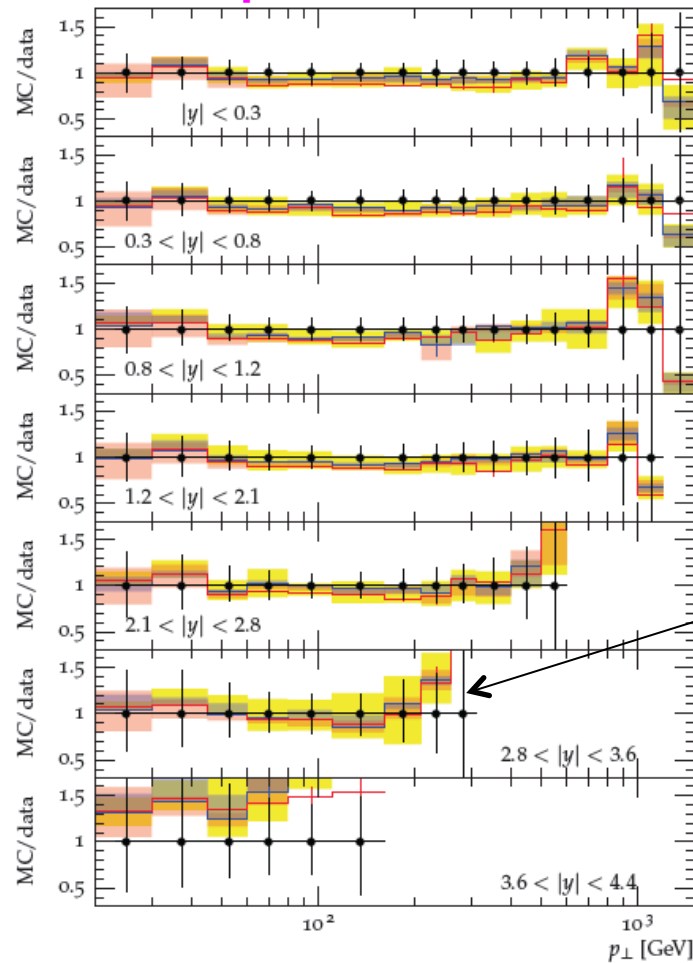
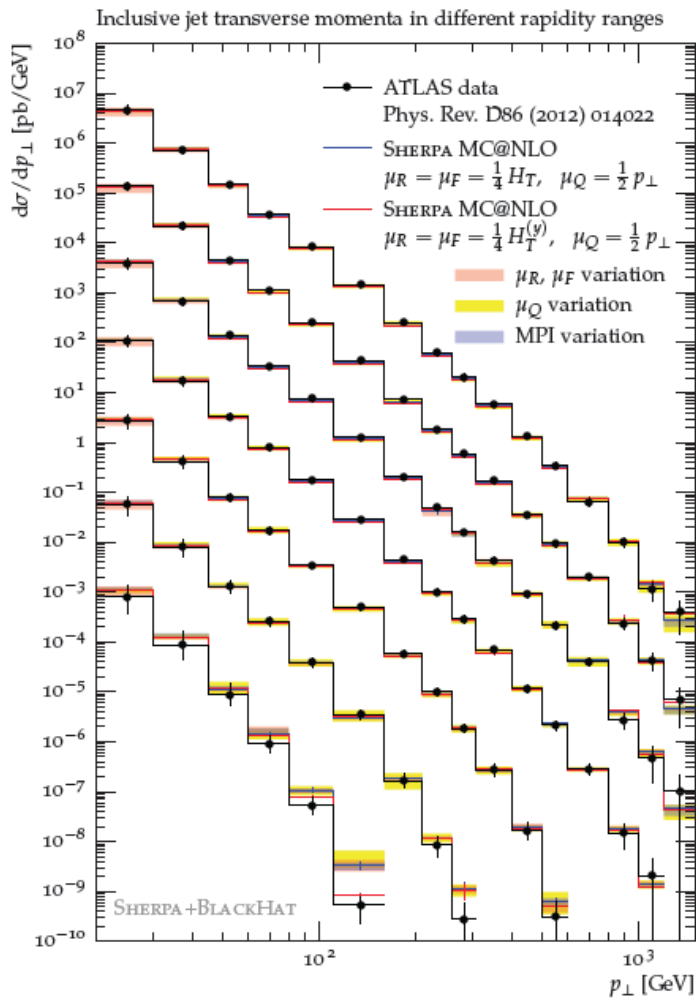
# Inclusive jet production: Les Houches project



- We also need a better understanding of the impact of parton showers on the fixed order cross section

Sherpa MC@NLO seems to do a good job in describing ATLAS data (but PDF dependent statement)

Compare to fixed order with same PDF



resummation scale uncertainties seem small except at extremes of phase space (as expected)