Computation and application of TMD observables

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What are TMD observables?

- Measuring the transverse momentum of a final state particle(s) which recoils against QCD radiation

- Two main objectives for studying these observables

  - Precision calculations for probing Higgs and BSM physics

    e.g Higgs , Drell-Yan transverse spectrum at large Q

- Non-perturbative : Three dimensional structure of hadrons

  - Need observables that are sensitive to non-pertubative physics
    e.g. Drell-Yan at low Q, Groomed TMDFJF

Outline

- Part I ： A fast and accurate technique for perturbative resummation of TMD observables

  In collaboration with: D. Kang, C. Lee

- Part II : Groomed TMDFJF observables

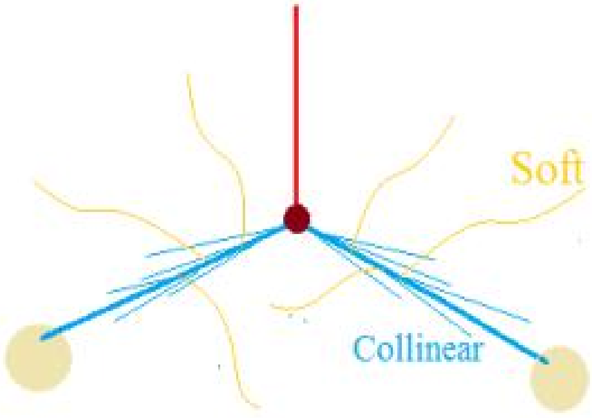
  In collaboration with: Y. Makris, D. Neill

Part I

Transverse spectrum of gauge bosons

P+P $\to$ H+X, P+P $\to$ $l^+ + l^-$ +X.

$p \sim Q\,(1, 1, \lambda)$

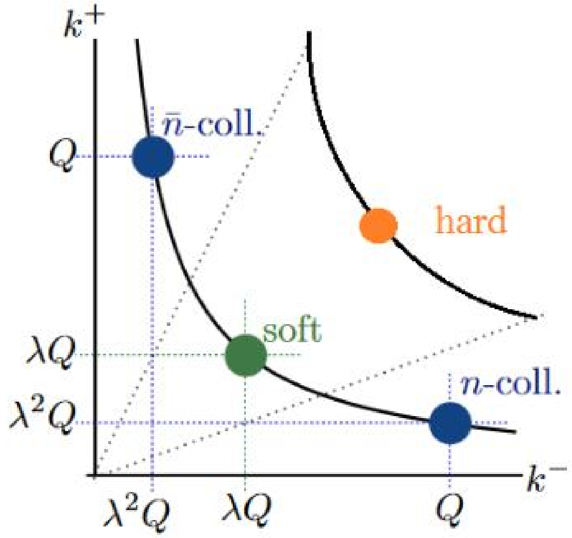


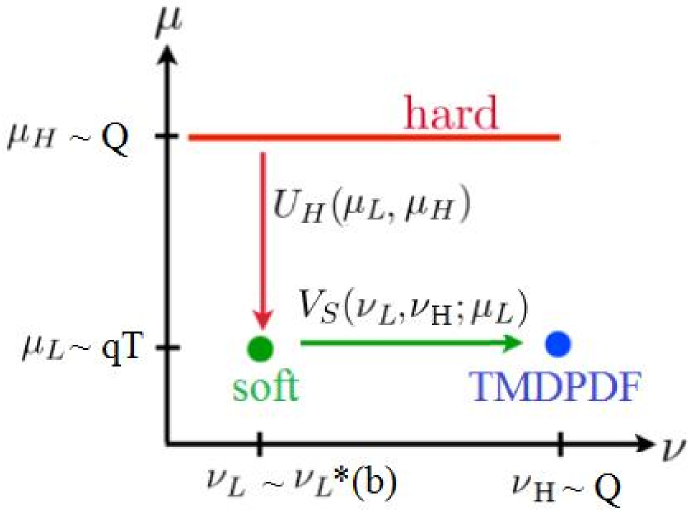
$$p_c \sim \; Q\,(1, \lambda^2, \lambda)\,,$$
$$p_{\bar{c}} \sim \; Q\,(\lambda^2, 1, \lambda)\,,$$
$$p_s \sim \; Q\,(\lambda, \lambda, \lambda)\,,$$
$$\lambda = qT / Q$$

EFT for resummation of large logs of qT/Q

$$\frac{d\sigma}{d^2q_T dy} = \sigma_0 C_t^2(M_t^2,\mu) H(Q^2;\mu) \int d^2\vec{q}_{Ts} d^2\vec{q}_{T1} d^2\vec{q}_{T2} \delta^2(\vec{q_T} - (\vec{q_T}_s + \vec{q}_{T1} + \vec{q}_{T2}))$$
$$\times\, S(\vec{q}_{Ts};\mu,\nu) f_1^{\perp}\Big(\vec{q}_{T1}, x_1, p^-;\mu,\nu\Big) f_2^{\perp}\Big(\vec{q}_{T2}, x_2, p^+;\mu,\nu\Big)\,,$$

Transverse spectrum of gauge bosons





$$\frac{d\sigma}{dq_T^2 dy} \propto H\left(\frac{\mu}{Q}\right) \int bdbJ_0(bq_T) S(b,\mu,\nu) f_1^{\perp}(x_1, b, \mu, \nu, Q) f_2^{\perp}(x_2, b, \mu, \nu, Q)$$

- CSS resumation scheme: Choose μ ~ 1/b, v ~1/b , Landau pole cut off , numerical implementation of cross section

- Implement a resummation scheme with v chosen in b space, μ in momentum space.
- Obtain an analytical expression for the cross section

Resummed spectrum

- **Resummed cross section**

$$\frac{d\sigma}{dq_T^2 dy} = \frac{\sigma_0}{2} C_t^2(M_t^2, \mu_T) H(Q^2, \mu_H) U(\mu_L, \mu_H, \mu_T) I_b(q_T, Q; \mu_L, \nu_L^*, \nu_H)$$

$$I_b(q_T, Q; \mu_L, \nu_L^*, \nu_H) \equiv \int_0^\infty db\, b J_0(b q_T) \widetilde{F}(b, x_1, x_2, Q; \mu_L, \nu_L^*, \nu_H) V_\Gamma(\nu_L^*, \nu_H; \mu_L)$$

Fixed order terms   resummed exponent

$$V_\Gamma = C e^{-A \ln^2(\mu_L b_0 \chi)}$$

Quadratic in log b

$$I_b^0 = \int_0^\infty db\, b J_0(b q_T)\, e^{-A \ln^2(\Omega b)}$$

$$\Omega \equiv \mu_L e^{\gamma_E} \chi / 2.$$

$$b_0 = \frac{b e^{\gamma_E}}{2}$$

- **Fixed order terms can be obtained from this master integral by taking derivatives**

Computing the master integral

- **Mellin Barnes representation of the Bessel function**

$$J_0(z) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dt \frac{\Gamma[-t]}{\Gamma[1+t]}\left(\frac{1}{2}z\right)^{2t}$$

- **Integral in b space can be done exactly**

$$I_b^0 = \int_0^\infty db\, b J_0(bq_T)\, e^{-A\ln^2(\Omega b)} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dt\, \frac{\Gamma[-t]}{\Gamma[1+t]}\int_0^\infty db\, b\left(\frac{bq_T}{2}\right)^{2t} e^{-A\ln^2(\Omega b)}$$

$$I_b^0 = \frac{2}{iq_T^2}\frac{e^{-AL^2}}{\sqrt{\pi A}}\int_{c-i\infty}^{c+i\infty} dt\, \frac{\Gamma[-t]}{\Gamma[1+t]} e^{\frac{1}{A}(t-t_0)^2}$$

$$t_0 = -1 + AL \qquad\qquad L = \ln(2\Omega/q_T)$$

Expansion in Hermite basis

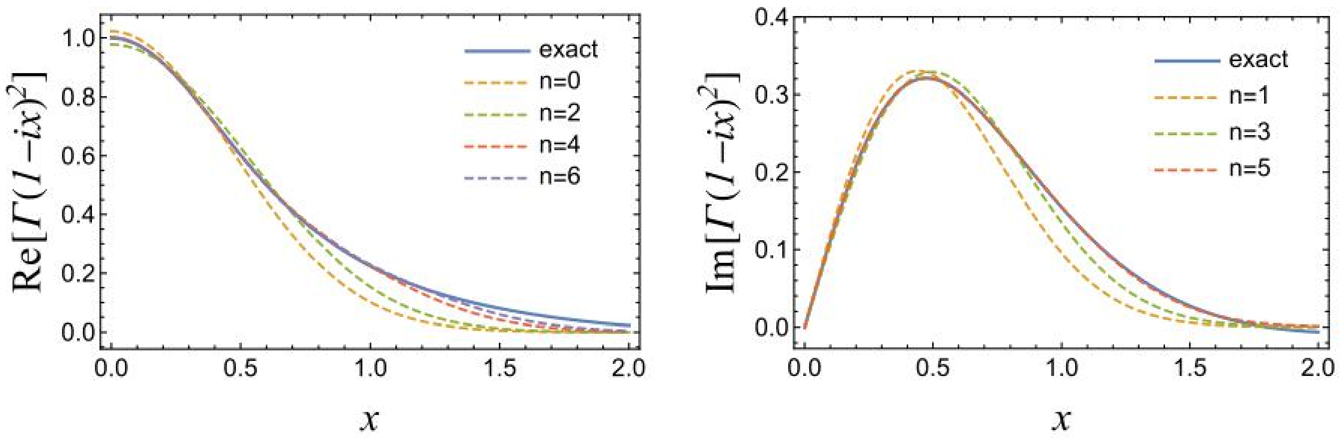
$$I_b^0 = \frac{2}{\pi q_T^2 \sqrt{\pi A}} \,\mathrm{Im}\Big\{ e^{-A(L-i\pi/2)^2} \int_{-\infty}^{\infty} dx\, \Gamma(-c-ix)^2 e^{-\frac{1}{A}\left[x+\frac{A\pi}{2}-i(c-t_0)\right]^2} \Big\}$$

Writing t= c + i x

$$A \sim \Gamma_{cusp}(\alpha_s(\mu))$$

We make a choice c= -1. $\Gamma(-c-ix)^2$ **contributes only in the region -2 > x >-2**

$$\Gamma(1-ix)^2 = e^{-a_0 x^2} \sum_{n=0}^{\infty} c_{2n} H_{2n}(\alpha x) + \frac{i\gamma_E}{\beta} e^{-b_0 x^2} \sum_{n=0}^{\infty} c_{2n+1} H_{2n+1}(\beta x)$$



Computing the master integral

$$I_b^0 = \frac{2}{\pi q_T^2} \sum_{n=0}^{\infty} \mathrm{Im}\left\{ c_{2n} \mathcal{H}_{2n}(\alpha, a_0) + \frac{i\gamma_E}{\beta} c_{2n+1} \mathcal{H}_{2n+1}(\beta, b_0) \right\}$$

- **An explicit all orders expression for the master integral**

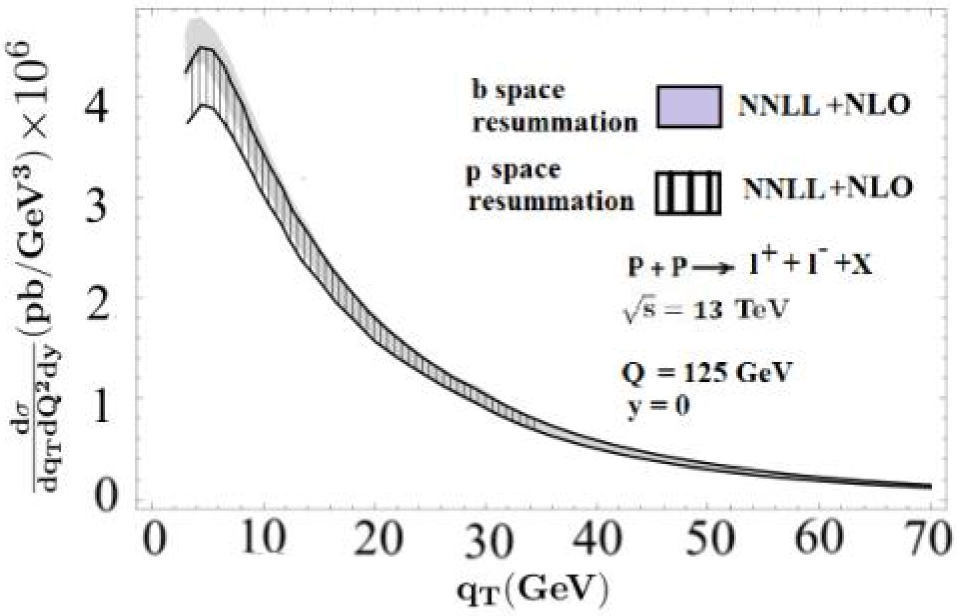
$$\mathcal{H}_n(\alpha, a_0) = \mathcal{H}_0(\alpha, a_0) \frac{(-1)^n n!}{(1+a_0 A)^n} \sum_{m=0}^{\lfloor n/2 \rfloor} \frac{1}{m!} \frac{1}{(n-2m)!} \left\{ [A(\alpha^2 - a_0) - 1](1 + a_0 A) \right\}^m (2\alpha z_0)^{n-2m}$$

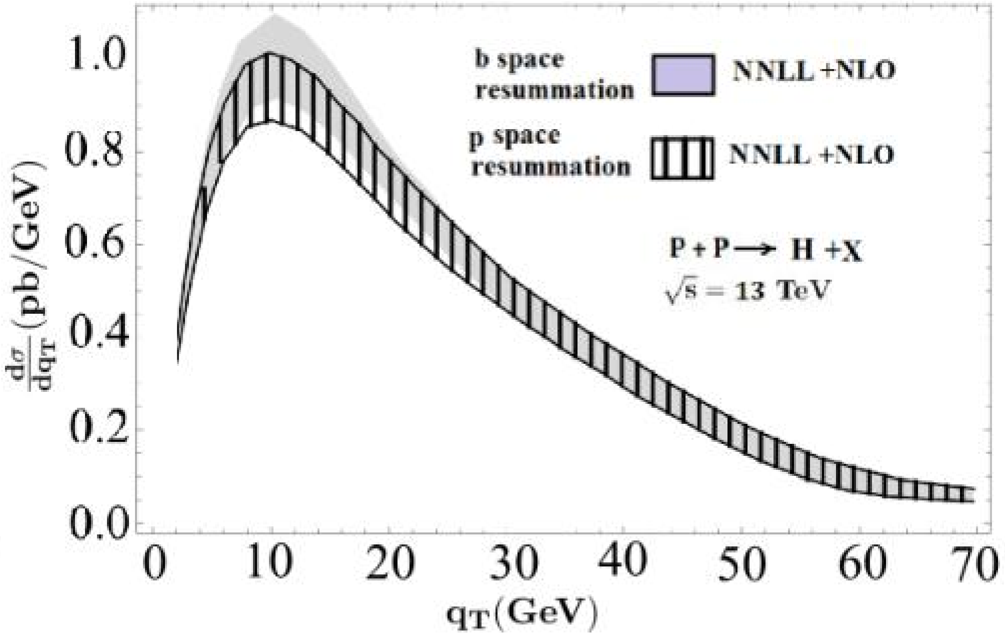
$$z_0 = A(\pi/2 + iL)$$

$$\frac{d\sigma}{dq_T^2 dy} = \frac{1}{2} \sigma_0 C_t^2(M_t^2, \mu_T) H(Q^2, \mu_H) U(\mu_H, \mu_T)\, C \times \sum_{n=0}^{\infty} \sum_{k=0}^{2n} \left(\frac{\alpha_s(\mu_{\text{run}})}{4\pi} \right)^n \tilde{F}_k^{(n)}\, I_b^k(q_T)$$

$$I_b^k = \left[\hat{\partial}_\chi \right]^k I_b^0 \qquad I_b^0 = \int_0^\infty db\, b J_0(bq_T)\, e^{-A \ln^2(\Omega b)} \qquad \Omega \equiv \mu_L e^{\gamma_E} \chi/2.$$

Comparison with other schemes



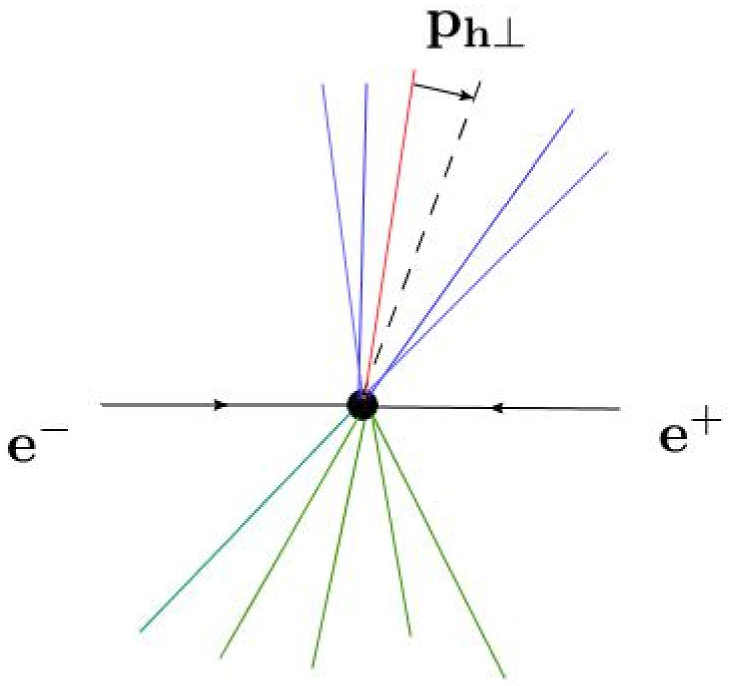


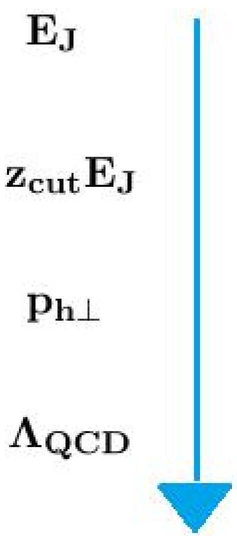
Summary and outlook

- The first (semi) analytic expression for resummed cross section of the transverse spectrum of gauge bosons
- Accuracy of the result can be improved systematically
- The technique can be extended easily to other TMD observables.
- Include non-perturbative models for extracting non-perturbative physics

Part II

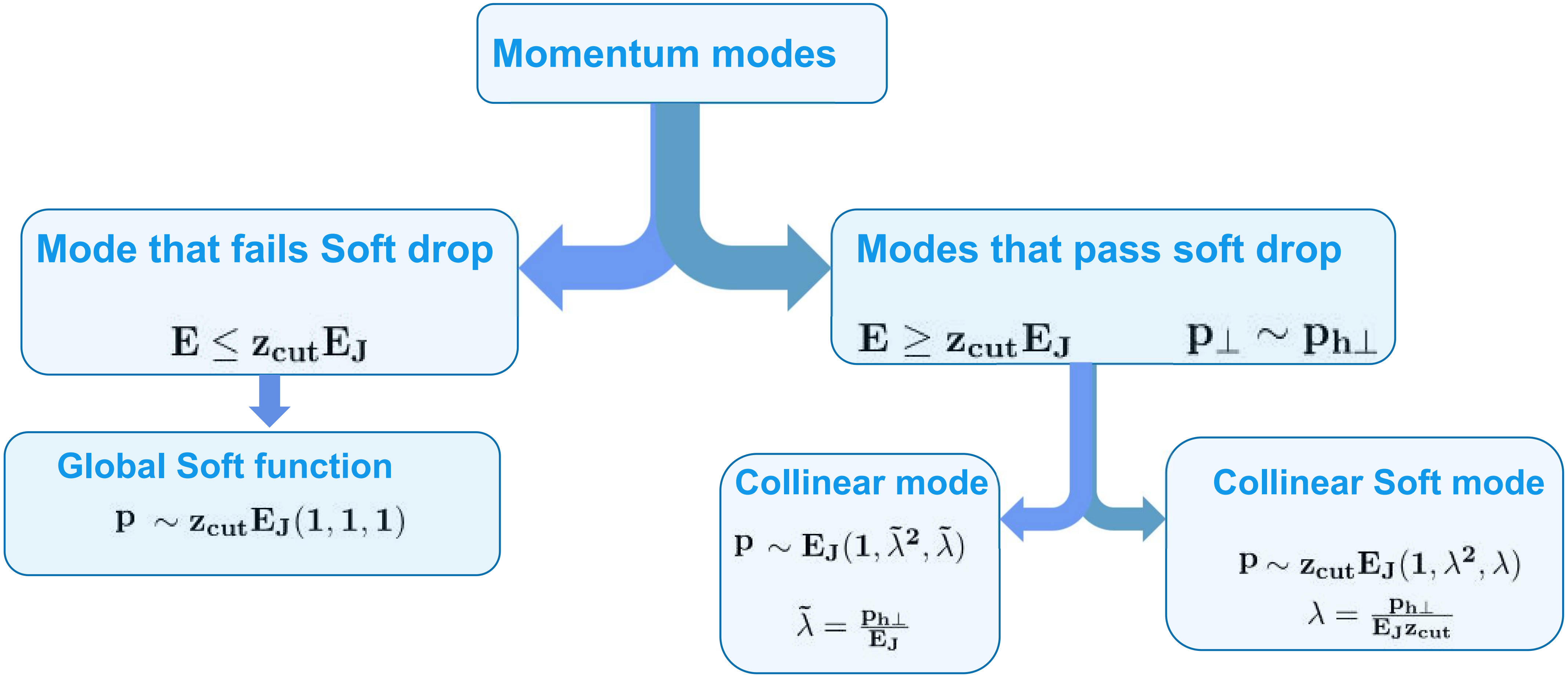
Groomed TMDFJF observable





- Measure the transverse momentum of an identified hadron in the jet w.r.t. the jet axis.
- Implement Soft drop grooming on the jet with an energy cut $z_{cut}\, E_J$
- Implement an EFT to resum logarithm of $p_\perp/E_J$

Factorization



Factorization

$$\frac{d\sigma}{dz_h d^2 p_{h\perp}} = \sigma_0(E_J) S(z_{cut} E_J, R) J_q(R, E_J, z_{cut}, p_{h\perp}, z_h)$$

$$J_q(R, E_J, z_{cut}, p_{h\perp}, z_h) = \sum_{X \in \mathrm{Jet}(R)} \frac{1}{2N_c} \delta(2E_J - p_X^- + p_h^-) \delta^{(2)}(\vec{p}_{h\perp} + \vec{p}_{X^{SD}\perp}) \mathrm{tr}\left[\frac{\bar{n}}{2} \langle 0|\chi_n(0)|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle\right]$$

- **Factorize the TMDFJF into collinear and collinear soft functions**

$$J_i\left(z_h, \vec{p}_{h\perp}, z_{cut}, R, E_J\right) = \int d^2 k_{c\perp} d^2 k_{s\perp} \delta^2(p_{h\perp} + k_{c\perp} + k_{s\perp}) \mathcal{D}^{\perp}_{i/h}\left(z_h, E_J, \vec{k}_{c\perp}\right) S_i\left(\vec{k}_{s\perp}, z_{cut}\right) + \ldots$$

Factorization

$$\mathcal{D}_{q/h}^{\perp}\left(z_h, E_J, \vec{k}_{c\perp}\right) = \sum_X \frac{1}{2N_c}\delta(2E_J - p_X^- - p_h^-)\delta^{(2)}(\vec{k}_{c\perp} - \vec{p}_{X\perp})\mathrm{tr}\left[\frac{\bar{n}}{2}\langle 0|\chi_n(0)|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle\right]$$

- **Soft drop condition can be expanded away for collinear modes**

$$S_i^{\perp}\left(\vec{k}_{s\perp}, z_{cut}\right) = \frac{1}{N_i}\mathrm{tr}\langle 0|T\{S_n^i S_{\bar{n}}^i\}(0)\delta^{(2)}\left(\vec{k}_{s\perp} - \vec{\mathbb{P}}_{\perp}^{SD}\right)\bar{T}\{S_n^i S_{\bar{n}}^i\}(0)|0\rangle$$

$$S_k^i(x) = P\mathrm{exp}\left(ig\int_0^{\infty} ds\, k\cdot A^a(x+sk)\mathbf{T}_i^a\right)$$

Factorization

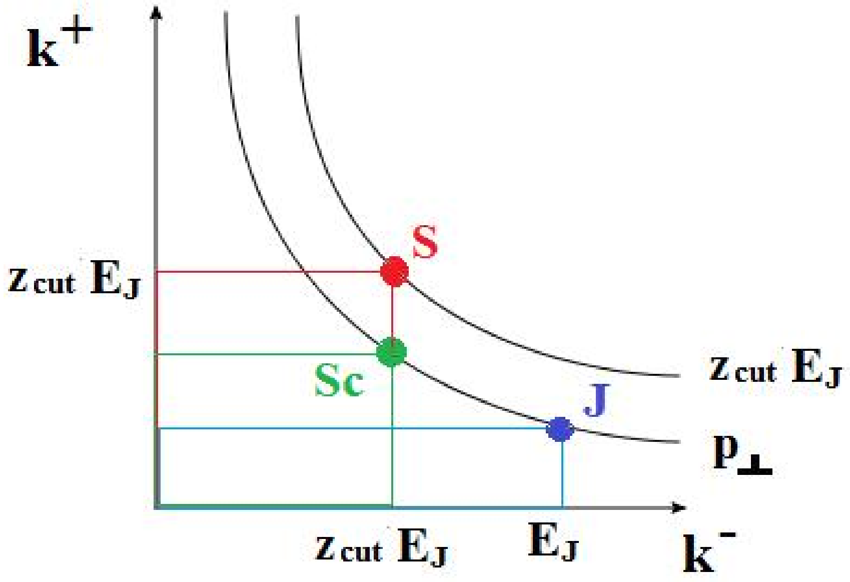
- **Matching onto ordinary Fragmentation functions.**

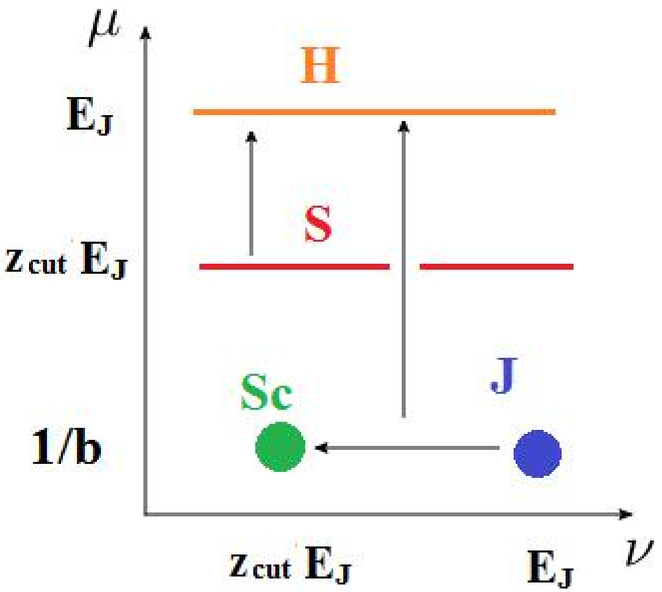
$$\mathcal{D}^{\perp}_{i/h}\left(z_h, E_J, \vec{k}_{c\perp}\right) = \int_{z_h}^{1} \frac{dx}{x} \mathcal{J}_{ij}\left(x, E_J, \vec{k}_{c\perp}\right) D_{j/h}\left(\frac{z_h}{x}\right)$$

$$D_{q/h}\left(z_h, E_q\right) = \sum_X \frac{z_h}{2N_c p_q^-} \delta(1 - z_X - z_h) \mathrm{tr}\left[\frac{\bar{n}}{2} \langle 0|\chi_n(0)|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle\right]$$

$$\frac{d\sigma}{dz_h d^2 p_{h\perp}} = \sigma_0(E_J) S(z_{cut} E_J, R)$$
$$\times \sum_i \int d^2 k_{s\perp} d^2 k_{c\perp} S_i^{\perp}\left(\vec{k}_{s\perp}, z_{cut}\right) \int_{z_h}^{1} \frac{dx}{x} \mathcal{J}_{ij}\left(x, E_J, \vec{k}_{c\perp}\right) D_{j/h}\left(\frac{z_h}{x}\right) \delta^2(p_{h\perp} + k_{c\perp} + k_{s\perp})$$

Anomalous dimensions





$$\mu \frac{d}{d\mu}\gamma_\nu = \Gamma_{\text{cusp}}(\alpha_s(\mu))$$

$$\gamma_\nu^J(\mu) = 2\int_{1/b_0}^{\mu} d\ln\mu' \Gamma_{\text{cusp}}(\alpha_s(\mu')) + \gamma^{RS}(1/b_0)$$

$$\gamma_\mu^D = \Gamma_{\text{cusp}}(\alpha_s(\mu)) \log\left(\frac{\nu^2}{\omega^2}\right) + \gamma_J^f(\alpha_s(\mu))$$

$$\gamma_\mu^{\text{Sc}} = -\Gamma_{\text{cusp}}(\alpha_s(\mu)) \log\left(\frac{\nu^2}{\omega^2 z_{cut}^2}\right) + \gamma_{\text{Sc}}^f(\alpha_s(\mu))$$

Resummation

$$U = e^{\left[\int_{\mu_L}^{\mu_H} d\log(\mu)\left(\gamma_J^f + \gamma_{CS}^f\right) + \ln(z_{cut}^2)\left(\int_{1/b_0}^{\mu_H} d\ln\mu' \Gamma_{\text{cusp}}(\alpha_s(\mu')) + \gamma^{RS}(1/b_0)\right)\right]}$$

All order rapidity anomalous dimension

- This leads to terms of the form $(\ln z_{cut})\alpha_s^n \ln^n(\mu_H b)$ in the exponent
- For an ungroomed jet we have instead $\alpha_s^n \ln^{n+1}(\mu_H b)$
- At large values of b, the ungroomed jet exponent damps out the non perturbative physics.

The groomed jet observable is more sensitive to non-perturbative physics

How to access non-perturbative physics

- **Step I : An accurate prediction of the p⊥ shape in the perturbative region.**

$$\frac{d\sigma}{dz_h d^2 p_{h\perp}} = \mathcal{N}(E_J, z_{cut}, R)\tilde{U}(p_{h\perp})D_{q/h}(z_h, \mu_L = p_{h\perp})$$

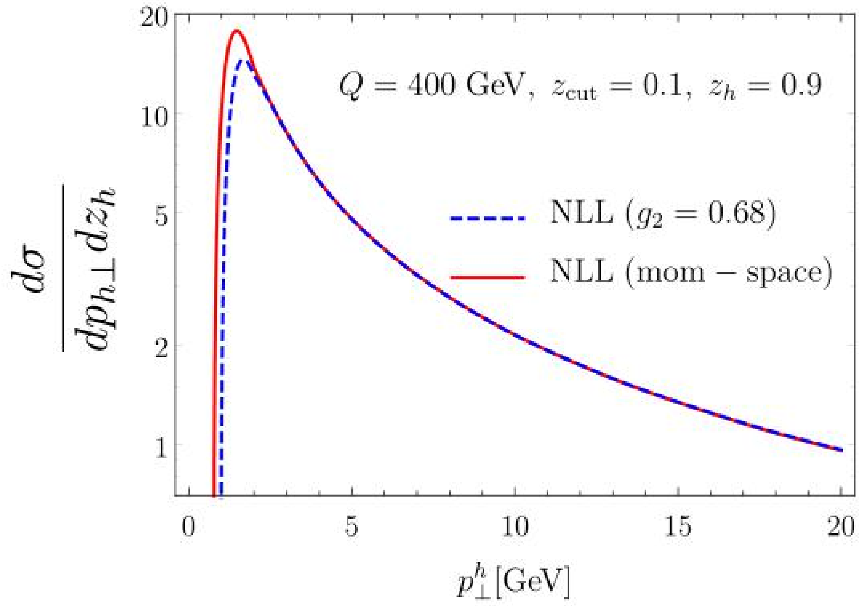
Cross section at NLL

$$\tilde{U}(q_T) = -2\pi e^{K(q_T,\mu_H)}\frac{\omega_J}{q_T^2}(e^{\gamma_E})^{\omega_J}\frac{\Gamma\left[1+\frac{\omega_J}{2}\right]}{\Gamma\left[1-\frac{\omega_J}{2}\right]}$$

$$K(\mu_0,\mu) = -C_i\ln(z_{cut}^2)\frac{\Gamma_0}{2\beta_0}\left\{\ln r + \frac{\alpha_s(\mu_0)}{4\pi}\left(\frac{\Gamma_1}{\Gamma_0} - \frac{\beta_1}{\beta_0}\right)\right\} - \frac{\gamma_0}{2\beta_0}\ln r$$

$$r = \alpha_s(\mu)/\ \alpha_s(\mu_0)$$

$$\omega_J(\mu_L, z_{cut}^2) = \Gamma_{\rm cusp}(\alpha_s(\mu_L))\ln(z_{cut}^2)$$



- **N can be extracted by comparison with experiment**

How to access non-perturbative physics

Step II : Put in a model for non-perturbative physics. Consider one example

- **Separate out the perturbative and non -perturbative parts of the anomalous dimension using a smooth cut-off**

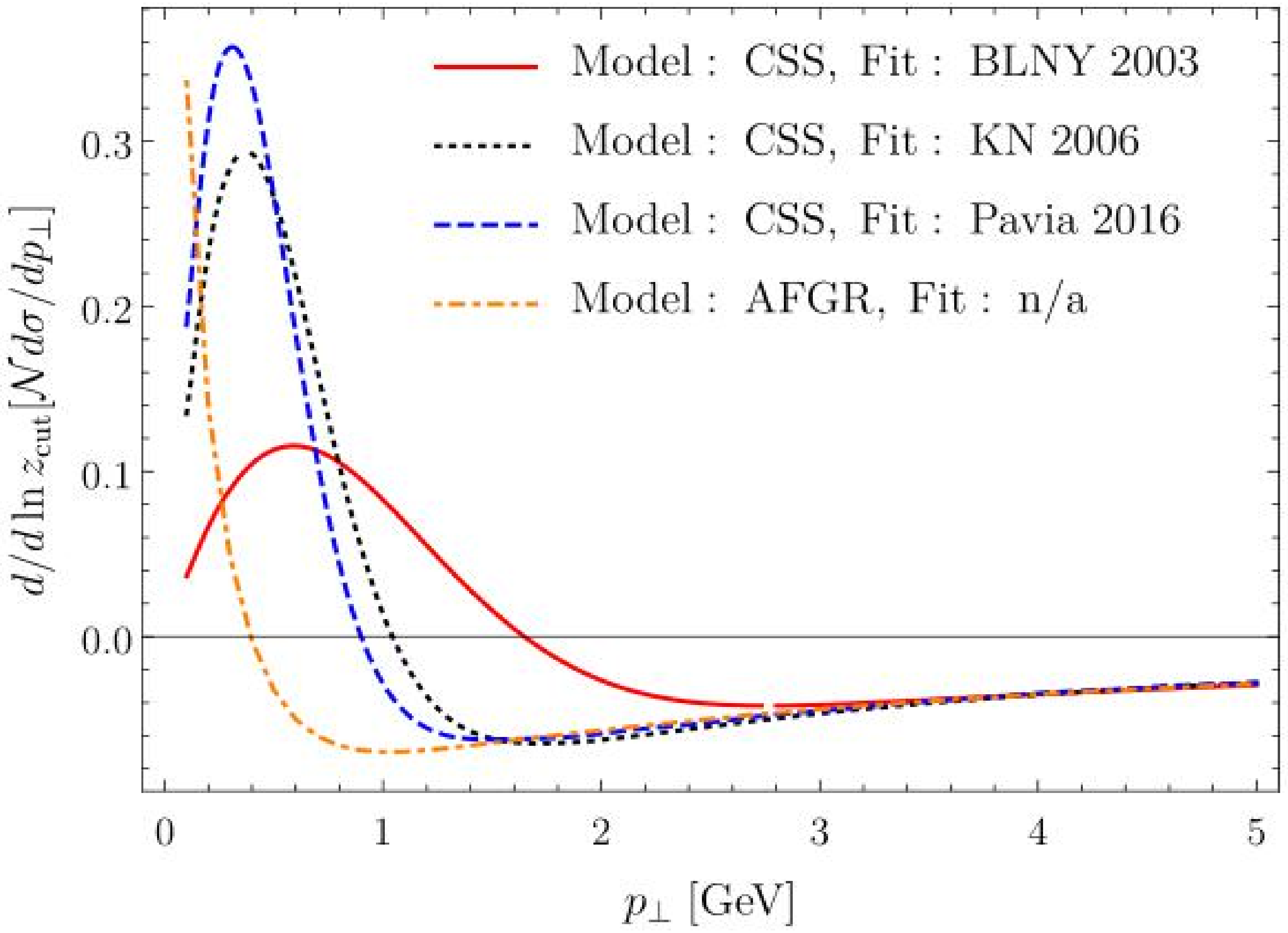
$$b_* = \frac{b}{\sqrt{1+(b/b_{max})^2}}, \qquad \mu_b = \frac{2\exp(\gamma_E)}{b_*}.$$

- **The non-perturbative input is in the function $g_k(b)$ which forms a part of the rapidity anomalous dimension**

$$\exp\Big[-\Big(\int_{\mu_b}^{\mu_H} d\ln(\mu')\,\Gamma_{\rm cusp}(\alpha_s(\mu')) + g_K(b)\Big)\ln(z_{\rm cut}) + \int_{\mu_L}^{\mu_H} d\ln(\mu')\gamma_i(\mu')\Big]$$

$$g_k(b) = \frac{1}{2}g_2 b^2$$

How to access non-perturbative physics



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