The Photon PDF

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in collaboration with P. Nason, G. Salam and G. Zanderighi

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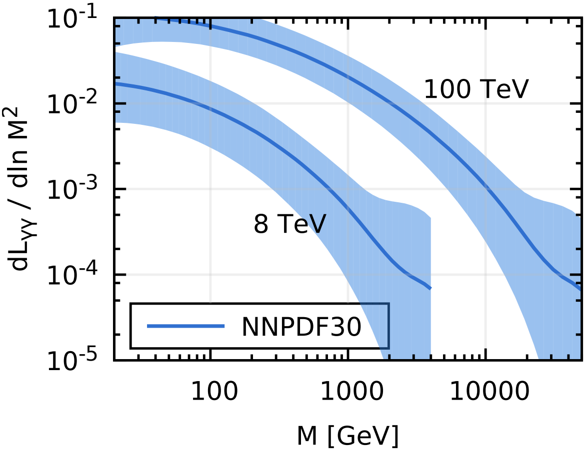
UCLA

Outline

- Introduction and Motivation
- Results for Photon PDF and uncertainties
- Derivation of formula
- Additional Results

AM, Nason, Salam, Zanderighi: PRL 117 (2016) 242002, 1708.01256

The $\gamma\gamma$ Luminosity from NNPDF30 — $S(750)$

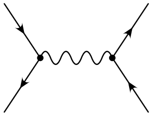


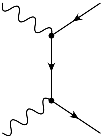
NNPDF30_nnlo_as_0118_qed

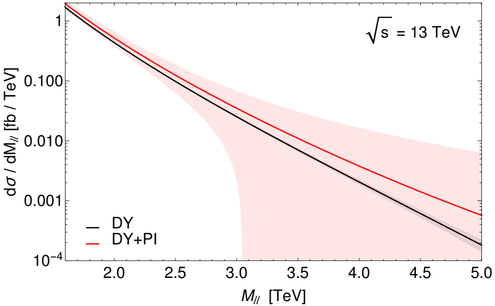
Drell-Yan: Photon Induced Contribution

E. Accomando et al. arXiv:1606.06646

Drell-Yan (DY) $q\bar{q} \to \ell^+\ell^-$. Photon-induced Drell-Yan (PI)







Drell-Yan: Photon Induced Contribution

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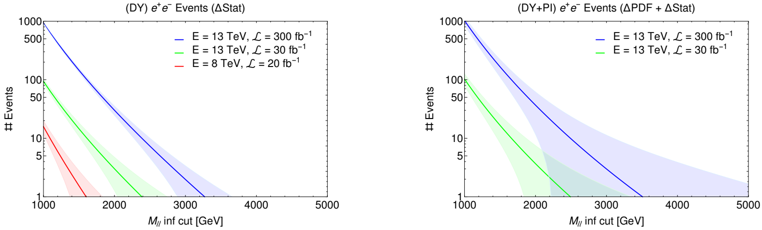


`NNPD23QED`.

The Photon PDF error is much larger than the statistical error.

Also needed for VBF and associated Higgs production, tops, di-bosons, di-photons, EW corrections.

A better photon PDF is needed.

Photon PDF

$f_{\gamma/p}(x,\mu)$ Probability to find a photon with momentum fraction x in a proton, in the $\overline{\text{MS}}$ scheme

L is a large log, $L \sim \log Q^2/m_p^2$. Expect

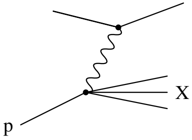
$$f_{\gamma/p} \sim \alpha L$$

Scheme dependence changes the order α (non-log) piece.

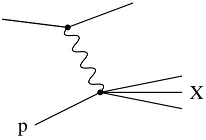
Previous Results

- Equivalent Photon Approximation (Fermi-Weizsäcker-Williams): Used for elastic scattering calculation, and to model photon as radiated from quarks above some mass scale. [EPA gets the αL term but not the full α term]
- Fit to data (mostly Drell-Yan)
- u, d quarks known to few percent, s to about 10 percent. γ has large uncertainty.

Photon PDF from DIS



Photon PDF



DIS

So the photon PDF should be given in terms of F_2 and F_L, up to kinematic factors.

Then some algebra . . .

$$x\, f_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi \underbrace{\alpha(\mu^2)}_{1/\alpha(\mu^2)}} \int_x^1 \frac{dz}{z} \Bigg\{ \int_{\frac{x^2 m_p^2}{1-z}}^{\overbrace{\frac{\mu^2}{1-z}}^{\text{Note limit}}} \overbrace{\frac{dQ^2}{Q^2}}^{L} \alpha^2_{\text{phys}}(Q^2)$$

$$\left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2\left(\frac{x}{z}, Q^2\right) - z^2 F_L\left(\frac{x}{z}, Q^2\right) \right]$$

$$\underbrace{-\alpha^2(\mu^2) z^2 F_2\left(\frac{x}{z}, \mu^2\right)}_{\overline{\text{MS}}\ \text{"conversion term"}} \Bigg\} + \underbrace{\mathcal{O}(\alpha^2, \alpha\alpha_s)}_{\text{no } L},$$

$$z p_{\gamma q}(z) = 1 + (1-z)^2 = 2 - 2z + z^2$$

include all $\alpha L(\alpha_s L)^n$, $\alpha(\alpha_s L)^n$, $\alpha^2 L^2(\alpha_s L)^n$ terms

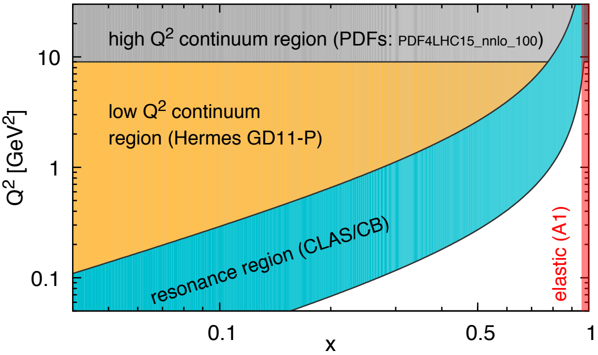
We have a formula for the photon PDF

i.e. an expression given by an $=$ sign with a controlled error of known parameteric form

To obtain the photon PDF, evaluate the integral using measured structure functions.

Overall α, so uncertainties get multiplied by α

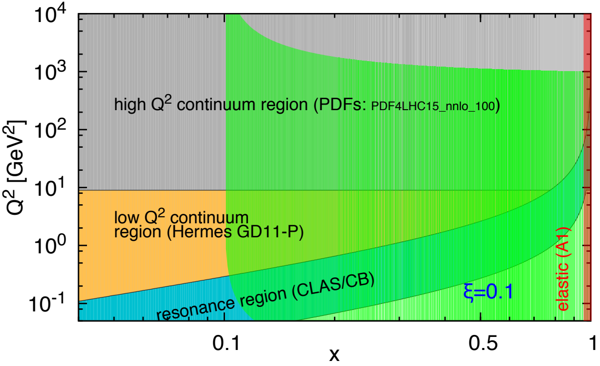
x, Q^2 plane and experimental inputs



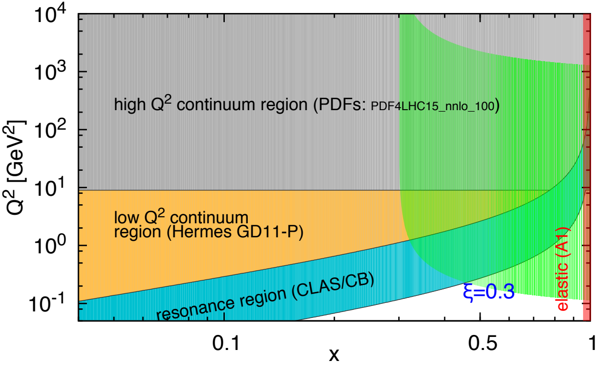
White region contributes at order α from states with invariant mass between $m_{\rm p}$ and $m_{\rm p} + m_\pi$.

Use fits to the data in the various regions.

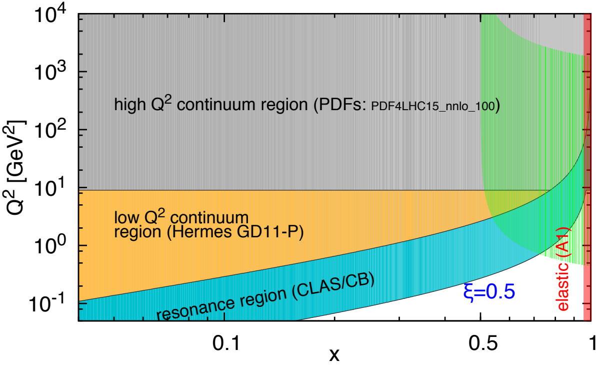
x, Q^2 Integration Region for $\mu = 30$ GeV



x, Q^2 Integration Region for $\mu = 30$ GeV



x, Q^2 Integration Region for $\mu = 30$ GeV



Elastic Form Factors: Dipole Form

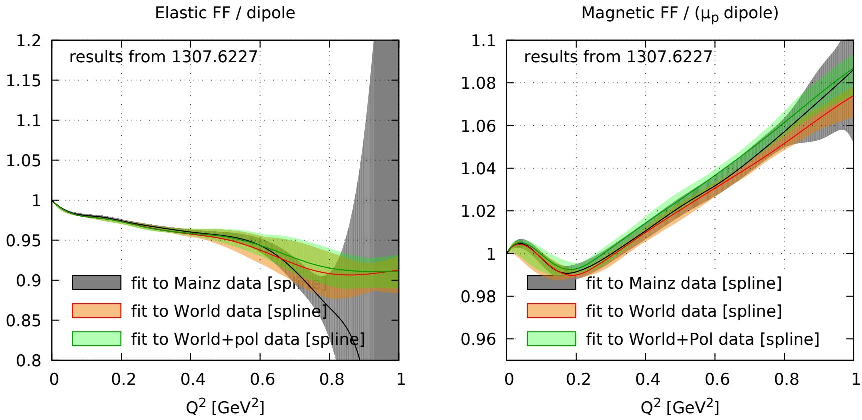
$$G_{\text{dipole}}(Q^2) = \frac{G(0)}{\left(1 + \frac{Q^2}{M_{\text{d}}^2}\right)^2}$$

$$M_{\text{d}} = 0.84\,\text{GeV}$$

$$M_{\text{d}} \neq M_\rho = 0.77\,\text{GeV}$$

note this is not VMD

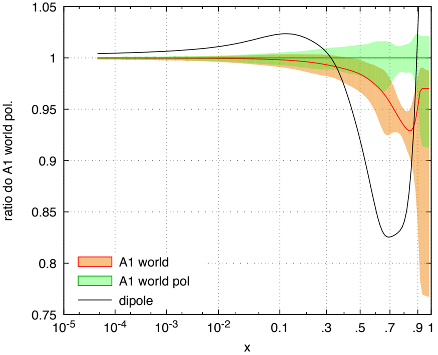
Elastic Form Factors



J. Bernauer et al. (A1 Collaboration) PRC 90 (2014) 015206

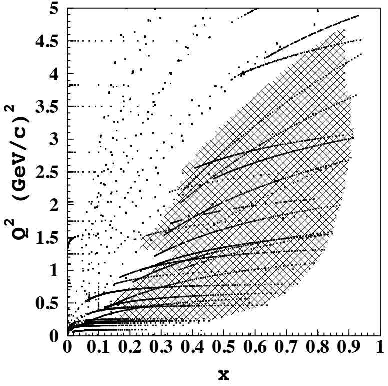
Elastic Contribution to f_γ

r_p, TPE e^- vs e^+



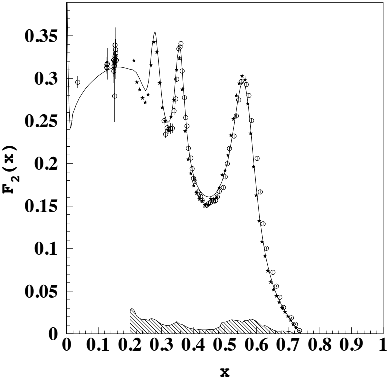
Resonance Region

Christy and Bosted, PRC 81 (2010) 055213
M. Osipenko et al. (CLAS Collaboration), PRD 67 (2003) 092001



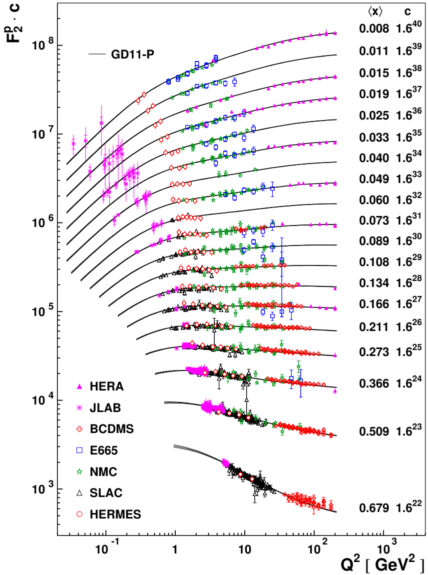
CLAS at $Q^2 = 0.775\,\text{GeV}^2$

$\Delta(1232)$, etc.



S. Simula — CLAS fit code.

Low Q^2 Continuum



HERMES GD11-P Fit

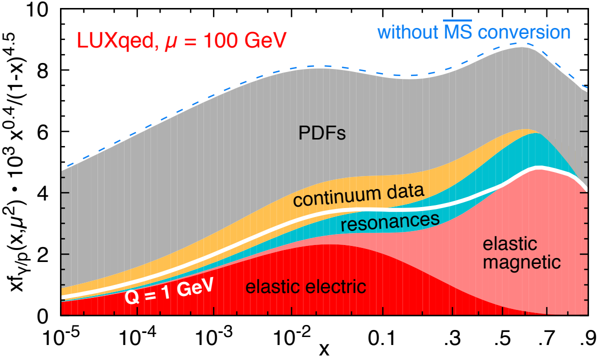
High Q^2 Continuum

Region where Q^2 dependence of $F_{2,L}(x, Q^2)$ is given by QCD perturbation theory. Can reliably use PDF fits to parton distributions and QCD coefficient functions. Automatically includes RGE constraints.

$$F_2(x, Q^2) = \sum_a C_{2,a}(Q^2/\mu^2, x) \otimes f_{a/p}(x, \mu^2)$$

Use the NNLO results for C and `PDF4LHC_nnlo_100`

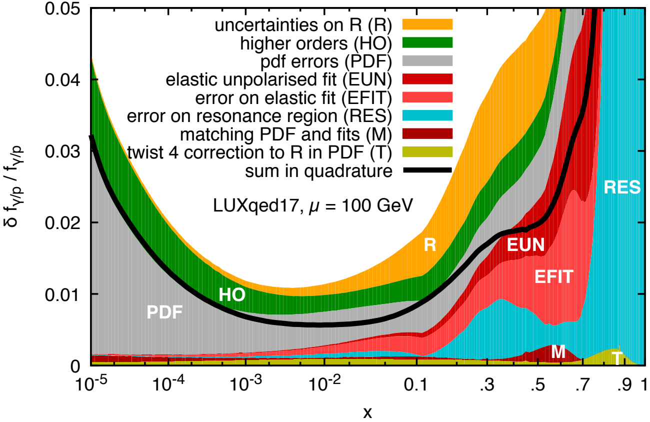
Contributions to Photon PDF



Elastic $\mu_p^2 = 7.8$

`LUXqed17_plus_PDF4LHC15_nnlo_100` on LHAPDF

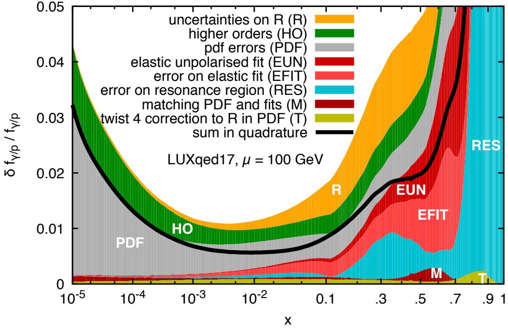
Uncertainties



Errors from various sources, stacked vertically

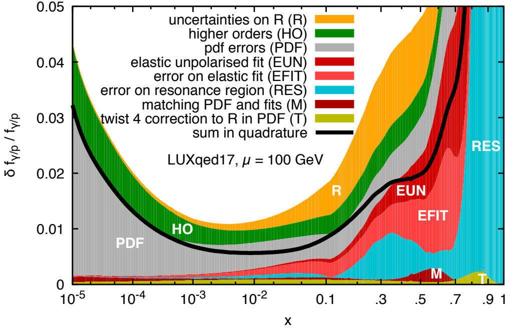
Final error given by summing the pieces in quadrature.
1–2% over most of the x range.

Uncertainties



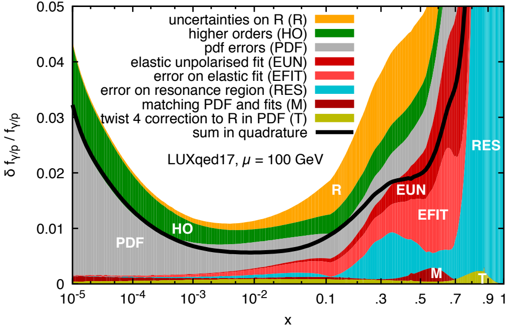
EFIT: The uncertainty on the elastic contribution that comes from the uncertainty on the A1 world polarised form factor fits.

Uncertainties



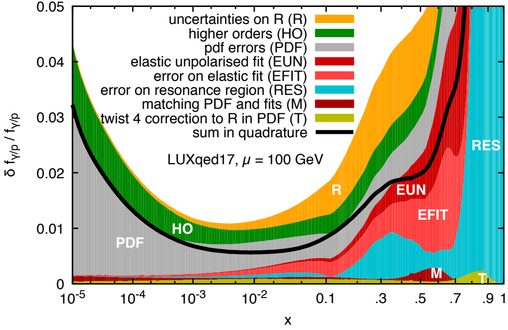
EUN: The uncertainty that comes from replacing the A1 world polarised fit (which includes a two-photon-exchange correction) with just the world unpolarised data (which does not).

Uncertainties



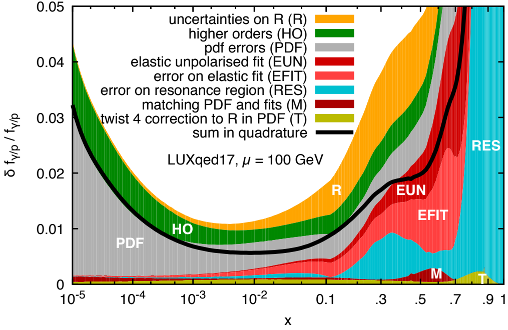
RES: Difference between Christy-Bosted and CLAS fits

Uncertainties



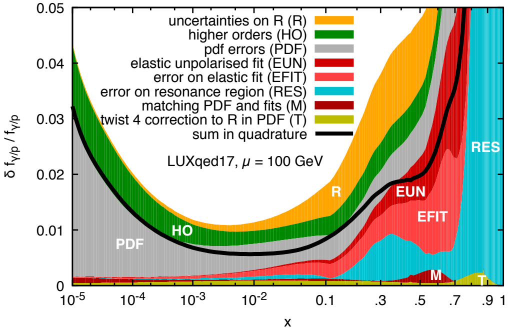
R: $\pm 50\%$ uncertainty on $R = \sigma_L/\sigma_T$ in the low-Q^2 continuum and resonance regions.

Uncertainties



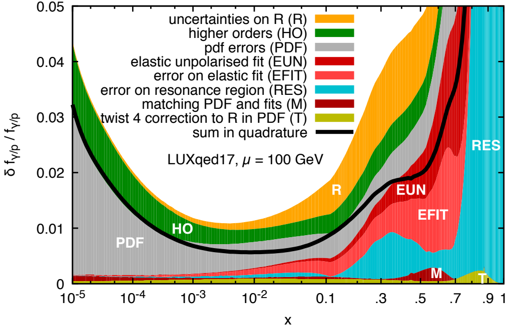
HO: Neglected higher order terms, estimated by replacing $\mu^2/(1-z)$ by μ^2 in the upper limit of the integral, and making a compensating change in the $\overline{\text{MS}}$ conversion term.

Uncertainties



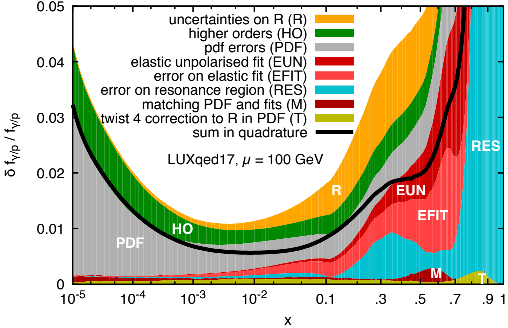
PDF: Standard PDF uncertainties

Uncertainties



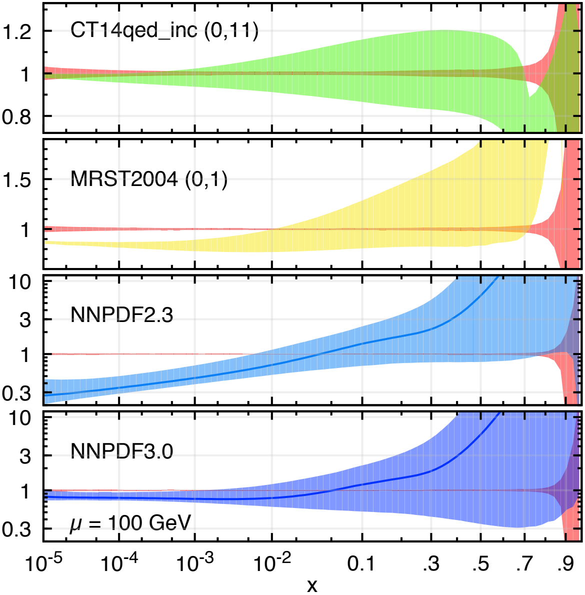
M: Changing the transition between CLAS/CB fits and HERMES from 9 to 5 GeV^2.

Uncertainties



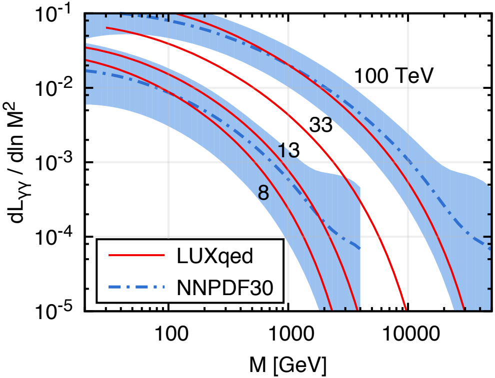
T: A potential twist-4 contribution to F_L given by multiplying F_L by $1 + 5.5\text{GeV}^2/Q^2$ for $Q^2 > 9\,\text{GeV}^2$. Cooper-Sarkar et al. arXiv:1605.08577

Comparison with Previous Results



- Note vertical scale: this is ratio (not percentage)
- `NNPDF30` extends `NNPDF23` with $\alpha(\alpha_s L)^n$ in running.

$\gamma\gamma$ Luminosity

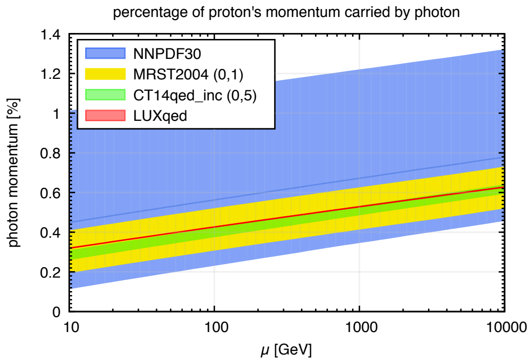


$pp \to HW^+(\to \ell^+\nu) + X$ at $\sqrt{s} = 13$ TeV: 91.2 ± 1.8 fb non-γ using HAWK

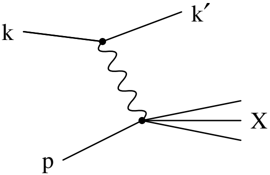
$5.5^{+4.3}_{-2.9}$ fb with `NNPDF30` 4.4 ± 0.1 fb with `LUXqed`

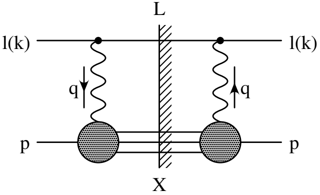
Error reduced by factor of ~ 40

γ Momentum Fraction



Deep Inelastic Scattering (DIS) Structure Functions





cross section given by hadronic tensor

$$W_{\mu\nu}(p,q) = \frac{1}{4\pi}\int \mathrm{d}^4 z\; e^{iq\cdot z}\langle p,s|\,[j_\mu(x), j_\nu(0)]\,|p,s\rangle\,.$$

Structure Functions

$$W_{\mu\nu}(p,q) = F_1\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{F_2}{p\cdot q}\left(p_\mu - \frac{p\cdot q\, q_\mu}{q^2}\right)\left(p_\nu - \frac{p\cdot q\, q_\nu}{q^2}\right)$$

and $F_{1,2}$ depend on

$$Q^2 = -q^2 \qquad\qquad x_{\rm bj} = \frac{Q^2}{2p\cdot q}$$

Bjorken scaling: $F_i(x_{\rm bj}, Q^2)$ independent of Q^2. QCD shows this is violated by $\ln Q^2$ terms due to calculable anomalous dimensions at large Q^2.

Longitudinal structure function:

$$F_L(x_{\rm bj}, Q^2) = \left(1 + \frac{4x_{\rm bj}^2 m_{\rm p}^2}{Q^2}\right) F_2(x_{\rm bj}, Q^2) - 2x_{\rm bj} F_1(x_{\rm bj}, Q^2).$$

In QCD, at large Q^2, F_L is $\mathcal{O}(\alpha_s)$.

Will use F_2 and F_L instead of F_2 and F_1.

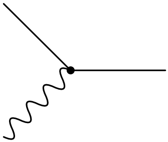
In the structure function $\sum_X$ is everything, including $X = p$, the elastic part.

Derivation: BSM probe + Factorization

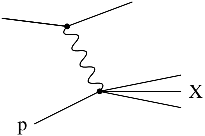
A massless neutral lepton l and a massive neutral lepton L with mass $M \gg m_{\rm p}$, with a transition magnetic moment interaction

$$\mathcal{L} = \frac{e}{\Lambda} \overline{L} \sigma^{\mu\nu} F_{\mu\nu} l + \text{h.c.}\,.$$

and work to lowest order in $1/\Lambda$.



$$\widehat{\sigma}(l + \gamma \to L + X) = \sigma_0 M^2 \delta(s - M^2) \qquad \sigma_0 = \frac{4\pi e^2}{\Lambda^2}$$



Include EM corrections: No photon couplings to the lepton line. The vacuum polarization bubbles give

$$\frac{1}{1-\Pi(q^2,\mu)}$$

$$\overline{\alpha}^{-1}(M_Z) = 127.94 \qquad \alpha_{\rm ph}^{-1}(0) = 137.036 \qquad \text{ratio} = 0.93$$

Renormalization of the the EM current: Collins, AM, Wise PRD73 (2006) 105019

Define

$$e_{\rm ph}(q^2) = \frac{e^2(\mu)}{1-\Pi(q^2,\mu)} \implies \frac{1}{1-\Pi(q^2,\mu)} = \frac{\alpha_{\rm ph}(q^2)}{\alpha(\mu^2)}$$

$\alpha_{\rm ph}$ does not depend on μ, only on q^2.

Compute

$$\sigma(l+p\to L+X) = \int {\rm d}x_{\rm bj} \int {\rm d}Q^2 \, \frac{{\rm d}\sigma}{{\rm d}x_{\rm bj}\,{\rm d}Q^2}$$

Function of

$$\xi = M^2/(2p\cdot k)$$

Hadron side gives $W_{\mu\nu}$

Lepton side gives

$$L^{\mu\nu} = \frac{1}{2} \text{Tr } \not{k} \left[\not{q}, \gamma^{\mu}, \right] (\not{k}' + M) \left[\gamma^{\nu}, \not{q} \right]$$

Compute $W_{\mu\nu} L^{\mu\nu}$ and integrate over phase space in x_{bj}, Q^2 plane

Total Cross Section

$$\sigma(l+p \to L+X) = \frac{1}{2\pi\alpha(\mu)}\sigma_0 \int_x^{1-\frac{2x\,m_{\rm p}}{M}} \frac{dz}{z} \int_{Q^2_{\rm min}}^{Q^2_{\rm max}} \frac{dQ^2}{Q^2}\alpha^2_{\rm ph}(q^2)$$

$$\left[\left(-z^2 - \frac{z^2 Q^2}{2M^2} + \frac{z^2 Q^4}{2M^4}\right) F_L(x/z, Q^2)\right.$$

$$\left. + \left(2 - 2z + z^2 + \frac{2x^2 m_{\rm p}^2}{Q^2} + \frac{z^2 Q^2}{M^2} - \frac{2zQ^2}{M^2} - \frac{2x^2 Q^2 m_{\rm p}^2}{M^4}\right) F_2(x/z, Q^2)\right].$$

$$Q^2_{\rm max} \to Q^2_{\uparrow} = \frac{M^2(1-z)}{z}$$

$$Q^2_{\rm min} \to Q^2_{\downarrow} = \frac{m_{\rm p}^2 x^2}{1-z}$$

neglecting power corrections, i.e. expanding limits in $m_{\rm p}$.

Take the green terms and define, with μ of order M

$$x f^{\rm PF}_{\gamma/p}(x,\mu) = \frac{1}{2\pi\alpha(\mu)} \int_x^1 \frac{dz}{z} \int_{\frac{m_{\rm p}^2 x^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2_{\rm ph}(q^2) \left[-z^2 F_L(x/z,Q^2) + \left(2 - 2z + z^2 + \frac{2x^2 m_{\rm p}^2}{Q^2}\right) F_2(x/z,Q^2) \right].$$

$$\sigma(l+p \to L+X) = \sigma_0 x f^{\rm PF}_{\gamma/p}(x,\mu) + \frac{1}{2\pi\alpha(\mu)} \sigma_0 \int_x^1 \frac{dz}{z} \int_{\frac{\mu^2}{1-z}}^{\frac{M^2(1-z)}{z}} \frac{dQ^2}{Q^2} \alpha^2_{\rm ph}(q^2) \left[-z^2 F_L(x/z,Q^2) + \left(2 - 2z + z^2 + \frac{2x^2 m_{\rm p}^2}{Q^2}\right) F_2(x/z,Q^2) \right].$$

$$+ \frac{1}{2\pi\alpha(\mu)} \sigma_0 \int_x^1 \frac{dz}{z} \int_{\frac{m_{\rm p}^2 x^2}{1-z}}^{\frac{M^2(1-z)}{z}} \frac{dQ^2}{Q^2} \alpha^2_{\rm ph}(q^2) \left[\left(-\frac{z^2 Q^2}{2M^2} + \frac{z^2 Q^4}{2M^4} \right) F_L(x/z,Q^2) + \left(\frac{z^2 Q^2}{M^2} - \frac{2zQ^2}{M^2} - \frac{2x^2 Q^2 m_{\rm p}^2}{M^4} \right) F_2(x/z,Q^2) \right].$$

In the remaining integrals:

1. Q^2 is large, $Q^2 \gg m_{\rm p}^2$
2. We can replace $F_i(x, Q^2)$ by $F_i(x, \mu^2)$ up to RGE corrections of order $\alpha_s(\mu), \alpha(\mu)$ with *no large logs*
3. F_L is order $\alpha_s(\mu)$ and can be dropped
4. $\alpha_{\rm ph}(q^2) \to \alpha(\mu)$ up to corrections of order $\alpha(\mu)$
5. The integrals are now elementary and can be done explicitly

$$\sigma(l + p \to L + X) = \sigma_0 x f^{\rm PF}_{\gamma/p}(x, \mu) + \frac{\alpha(\mu)}{2\pi}\sigma_0 \int_x^1 \frac{dz}{z} \Bigg[$$

$$z p_{\gamma q}(z) \left(\log \frac{M^2(1-z)^2}{z\mu^2}\right) - z^2 + 3z - 2\Bigg] F_2(x/z, \mu) + \mathcal{O}(\alpha^2, \alpha\alpha_s)\sigma_0$$

without any large logs.

$$z\, p_{\gamma q}(z) = 1 + (1-z)^2 \,.$$

Factorization

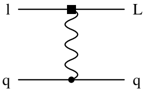
$$\sigma(l + p \to L + X) = \widehat{\sigma}(l + \gamma \to L + X) \otimes f_{\gamma/p} + \widehat{\sigma}(l + q \to L + X) \otimes f_{q/p} + \ldots$$

Partonic cross sections:

$$\widehat{\sigma}(l + \gamma \to L + X) = \sigma_0 M^2 \delta(s - M^2)$$
$$\widehat{\sigma}(l + \gamma \to L + X) \otimes f_{\gamma/p} = \sigma_0 \, \xi \, f_{\gamma/p}(\xi)$$

$$\sigma_0 \, \xi \, f_{\gamma/p}(\xi) = \sigma(l + p \to L + X) - \widehat{\sigma}(l + q \to L + X) \otimes f_{q/p} + \ldots$$

Factorization: Partonic rate



$$\sigma(l+q\to L+q) = \sigma_0 e_q^2 \frac{\alpha(\mu)}{2\pi}\Bigg[z\, p_{\gamma q}(z)\left(-\frac{1}{\epsilon_{\rm IR}} + \log\frac{M^2(1-z)^2}{z\mu^2}\right) + 3z - 2\Bigg]$$

$$\widehat{\sigma}(l+q\to L+q) = \sigma_0 e_q^2 \frac{\alpha(\mu)}{2\pi}\left[z\, p_{\gamma q}(z)\left(\log\frac{M^2(1-z)^2}{z\mu^2}\right) + 3z - 2\right]$$

From $1/\epsilon_{\rm IR}$:

$$P_{\gamma q} = e_q^2 \frac{\alpha(\mu)}{2\pi} p_{\gamma q}(z)$$

the log terms match up

Factorization

$$\sigma(l+p \to L+X) - \widehat{\sigma}(l+q \to L+X) \otimes f_{q/p} = \sigma_0 \xi\, f_\gamma(\xi,\mu)$$

$$F_2(x_{\rm bj},\mu^2) = \sum_q e_q^2\, x_{\rm bj}\, f_{q/p}(x_{\rm bj},\mu) + \mathcal{O}(\alpha_s,\alpha)$$

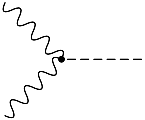
$$\left(-z^2+3z-2\right) - (3z-2) = -z^2$$

$$x\, f_{\gamma/p}(x,\mu) = x\, f^{\rm PF}_{\gamma/p}(x,\mu) + \frac{\alpha(\mu)}{2\pi}\int_x^1 \frac{dz}{z}\left(-z^2\right) F_2(x/z,\mu^2) + \mathcal{O}(\alpha^2(\mu),\alpha(\mu)\alpha_s(\mu))$$

This is the photon PDF in the $\overline{\rm MS}$ scheme

includes all terms of order $(\alpha L)^m\,(\alpha_s L)^n$, $\alpha\,(\alpha_s L)^n$

Scalar Production: Another BSM process



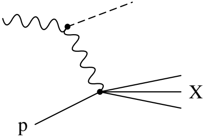
$$\mathcal{L} = \frac{e^2(\mu)\mu^\epsilon}{\Lambda} S\, F_{\mu\nu} F^{\mu\nu}$$

The spin-averaged $\gamma\gamma \to S$ cross section to lowest order is

$$\sigma(\gamma\gamma \to S) = \widehat{\sigma}(\gamma\gamma \to S) = \sigma_0 M^2 \delta(s - M^2)$$

$$\sigma_0 = \frac{\pi e^4}{2\Lambda^2} .$$

The cross section coefficient has again been called σ_0, so the formulæ can be easily compared with the $l \to L$ case.



$$\sigma(\gamma + p \to S + X) = \frac{1}{2\pi\alpha(\mu)}\sigma_0 \int_x^{1-\frac{2x\, m_p}{M}} \frac{dz}{z} \int_{Q^2_{\min}}^{Q^2_{\max}} \frac{dQ^2}{Q^2}\alpha^2_{\rm ph}(q^2)$$

$$\left[\left(-z^2 - \frac{2z^2Q^2}{M^2} - \frac{z^2Q^4}{M^4}\right)F_L(x/z, Q^2) + \right.$$

$$+\left(2 - 2z + z^2 + \frac{2x^2m_p^2}{Q^2} + \frac{2z^2Q^2}{M^2} - \frac{2zQ^2}{M^2} + \frac{4x^2m_p^2}{M^2}\right.$$

$$\left.\left. + \frac{z^2Q^4}{M^4} + \frac{2x^2Q^2m_p^2}{M^4}\right)F_2(x/z, Q^2)\right].$$

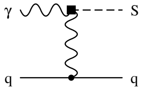
green terms same as before
blue terms are different

Integrate over phase space to get the total cross section:

$$\sigma(\gamma + p \to S + X) = \sigma_0 x f^{\mathrm{PF}}_{\gamma/p}(x,\mu) + \frac{\alpha(\mu)}{2\pi}\sigma_0 \int_x^1 \frac{dz}{z}\left[z p_{\gamma q}(z)\left(\log\frac{M^2(1-z)^2}{z\mu^2}\right) - \frac{3}{2}(1-z)^2\right] F_2(\mu^2, x/z)$$

first piece is same as before

second is different



$$\sigma(\gamma + q \to S + q) = \sigma_0 e_q^2 \frac{\alpha(\mu)}{2\pi}$$
$$\left[z p_{\gamma q}(z) \left(-\frac{1}{\epsilon_{\rm IR}} + \log \frac{M^2(1-z)^2}{z\mu^2} \right) - \frac{1}{2} z^2 + 3z - \frac{3}{2} \right]$$

$$\widehat{\sigma}(\gamma + q \to S + q) = \sigma_0 e_q^2 \frac{\alpha(\mu)}{2\pi} \left[z\, p_{\gamma q}(z) \left(\log \frac{M^2(1-z)^2}{z\mu^2} \right) - \frac{1}{2} z^2 + 3z - \frac{3}{2} \right]$$

first piece is same as before
second is different

$$\left[-\frac{3}{2}(1-z)^2\right] - \left[-\frac{1}{2}z^2 + 3z - \frac{3}{2}\right] = -z^2$$

Difference between σ and $\widehat{\sigma}$ is the same as before

Leads to the same expression for the photon PDF

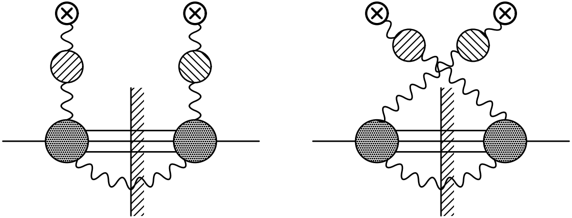
$$x\, f_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2_{\text{phys}}(Q^2) \right.$$
$$\left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2\left(\frac{x}{z}, Q^2\right) - z^2 F_L\left(\frac{x}{z}, Q^2\right) \right]$$
$$\left. - \alpha^2(\mu^2) z^2 F_2\left(\frac{x}{z}, \mu^2\right) \right\} + \mathcal{O}(\alpha^2, \alpha\alpha_s),$$

PDF Operators

Collins and Soper

$$f_\gamma(x,\mu^2) = -\frac{1}{4\pi x\, p^+}\int_{-\infty}^{\infty} dw\, e^{-ixwp^+}\, \langle p| F^{n\lambda}(wn) F^{n}{}_{\lambda}(0) + \text{h.c.}\, |p\rangle_c$$

no Wilson line since a $U(1)$ field



Define hadronic tensor in terms of one-photon-irreducible graphs.

$$f_\gamma(x,\mu^2) = -\frac{e^2(\mu^2)\,(\mathcal{S}\mu)^{2\epsilon}}{x\,p^+} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \left[2\pi\delta(q^+ + xp^+) + 2\pi\delta(q^+ - xp^+)\right]$$
$$\left[(n\cdot q)g^{\lambda\mu} - q^\lambda g^{n\mu}\right]\left[(n\cdot q)g^{\lambda\nu} - q^\lambda g^{n\nu}\right]$$
$$\frac{1}{\left(q^2\left[1-\Pi_D(q^2,\mu^2)\right]\right)^2}\left[W_{\mu\nu}^{(D)}(p,q) + W_{\nu\mu}^{(D)}(p,-q)\right]$$

Photon interacts with the proton via an electromagnetic current j^μ vertex.

$$f_\gamma(x,\mu^2) = \frac{8\pi}{x\alpha(\mu^2)\,(\mathcal{S}\mu)^{2\epsilon}}\frac{1}{(4\pi)^{D/2}}\frac{1}{\Gamma(D/2-1)}$$
$$\int_x^1 \frac{\mathrm{d}z}{z}\int_{\frac{m_p^2x^2}{1-z}}^{\infty}\frac{\mathrm{d}Q^2}{Q^2}\alpha^2_{\mathrm{ph,D}}(q^2)\left(Q^2(1-z)-x^2m_p^2\right)^{D/2-2}$$
$$\Bigg\{-z^2F_{L,D}(x/z,Q^2)+\left[2-2z+z^2+\frac{2m_p^2x^2}{Q^2}\right]F_{2,D}(x/z,Q^2)$$
$$-2\epsilon zxF_{1,D}(x/z,Q^2)\Bigg\}$$

This formula is exact.

Split the Q^2 integral into $m_p^2x^2/(1-z)\to\mu^2/(1-z)$ and $\mu^2/(1-z)\to\infty$

PF and conv terms

First part is finite. Can set $D \to 4$, and it gives $f_\gamma^{\rm PF}(x,\mu^2)$.
Introducing

$$s = \frac{Q^2(1-z)}{\mu^2}\,,$$

the second integral becomes

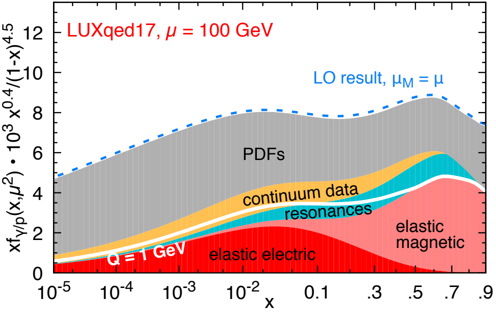
$$f_\gamma^{\rm con}(x,\mu^2) = \frac{(\mathcal{S}\mu)^{-2\epsilon}}{2\pi x\alpha(\mu^2)\mu^{2\epsilon}}\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\int_x^1\frac{{\rm d}z}{z}\int_1^\infty\frac{{\rm d}s}{s^{1+\epsilon}}$$
$$\alpha^2_{\rm ph,D}(-\mu^2 s/(1-z))\Big\{-z^2(1-\epsilon)F_{L,D}(x/z,\mu^2 s/(1-z))$$
$$+\left[2-2z+z^2-\epsilon z^2\right]F_{2,D}(x/z,\mu^2 s/(1-z))\Big\}$$

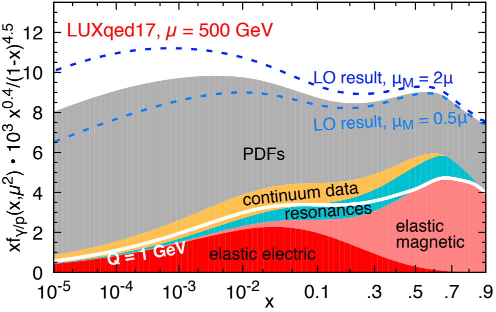
The factor of $(1-z)^{D/2-2}$ has cancelled.
Single scale integral with no large logs.

Expand in perturbation theory and evaluate:

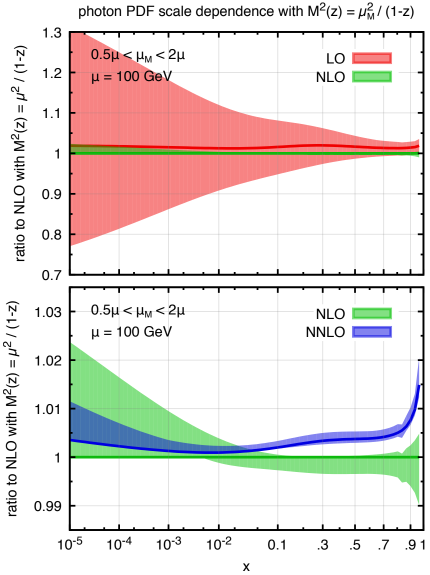
$$f_\gamma^{\rm con}(x,\mu^2) = \frac{\alpha(\mu^2)}{2\pi x}\int_x^1 \frac{{\rm d}z}{z}\left\{\frac{1}{\epsilon}\left[2-2z+z^2\right]F_2(x/z,\mu^2)\right\}$$
$$- z^2 F_2(x/z,\mu^2)\Bigg\},$$

Can extend this to higher order.


LUXqed17, μ = 100 GeV
LO result, μ_M = μ
PDFs
continuum data
resonances
Q = 1 GeV
elastic electric
elastic magnetic
xf_γ/p(x,μ²) • 10³ x^0.4/(1-x)^4.5
x
0
2
4
6
8
10
12
10^-5
10^-4
10^-3
10^-2
0.1
.3
.5
.7
.9


LUXqed17, μ = 500 GeV
LO result, μ_M = 2μ
LO result, μ_M = 0.5μ
PDFs
continuum data
resonances
Q = 1 GeV
elastic electric
elastic magnetic
xf_γ/p(x,μ²) • 10³ x^0.4/(1-x)^4.5
x
0
2
4
6
8
10
12
10^-5
10^-4
10^-3
10^-2
0.1
.3
.5
.7
.9

Scale Variation



μ Dependence

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} f_a = \sum_b P_{ab} \otimes f_b \,,$$

$$P_{ab} = \sum_{r,s} \left(\frac{\alpha_s}{2\pi}\right)^r \left(\frac{\alpha}{2\pi}\right)^s P_{ab}^{(r,s)}$$

- Since we have an expression for $f_\gamma(x, \mu^2)$, can compute μ derivative
- Derivative of $1/\alpha(\mu^2)$ gives $\beta_e\, f_\gamma$ with correct sign
- integrand at upper limit
- Derivative of $\overline{\mathrm{MS}}$ conversion term
- Write $F_{2,L}$ in terms of $C_{2,L,a} \otimes f_a$ using QCD pert. theory at μ.
- Compare both sides

Get the order α splitting functions

$$P_{\gamma q}^{(0,1)} = p_{\gamma q}(x) = \frac{1+(1-x)^2}{x}, \qquad P_{\gamma\gamma}^{(0,1)} = \beta^{(1,0)}\delta(1-x)$$

and the order $\alpha\alpha_s$ ones:

$$P_{\gamma\gamma}^{(1,1)} = \beta^{(1,1)}\delta(1-x)$$

$$P_{\gamma q}^{(1,1)} = C_F e_q^2 \Bigg[-3\ln(1-x)p_{\gamma q}(x) - \ln^2(1-x)p_{\gamma q}(x) + \left(2+\frac{7}{2}x\right)\ln x - \left(1-\frac{1}{2}x\right)\ln^2 x - 2x\ln(1-x) - \frac{7}{2}x - \frac{5}{2}\Bigg]$$

$$P_{\gamma g}^{(1,1)} = T_F\left(\sum e_q^2\right)\Bigg[-16+8x+\frac{20}{3}x^2+\frac{4}{3x}-(6+10x)\ln x - 2(1+x)\ln^2 x\Bigg]$$

agrees with de Florian et al Eur. Phys. J. C76 (2016) 282.

Two loop splitting functions from a one-loop calculation.

Other results

The f_γ PDF equation is *exact* to all orders in QED and QCD.

Have computed:

- The $\overline{\text{MS}}$ conversion term to one higher order
- f_γ to one higher order (NNLO)
- The 3-loop splitting functions $P_{\gamma i}$
- The photon TMDPDF
- Polarized photon PDF $f_{\Delta\gamma}(x, \mu^2)$ in terms of the polarized proton structure functions $g_{1,2}$.

Conclusions

- Have a formula for the photon PDF which is exact to all orders in QED and QCD
- Determine photon PDF with high precision (1–2%)
- Involves high and low Q^2 (including elastic) data
- Derived the two-loop $\alpha\alpha_s$ splitting functions using a formula with one-loop inputs.
- Available on LHAPDF
- This is a *calculation* of the photon PDF, with an error reduction of 40 relative to older results.
- Extended to one higher order