Quantum limits to force detection & quantum backaction evasion

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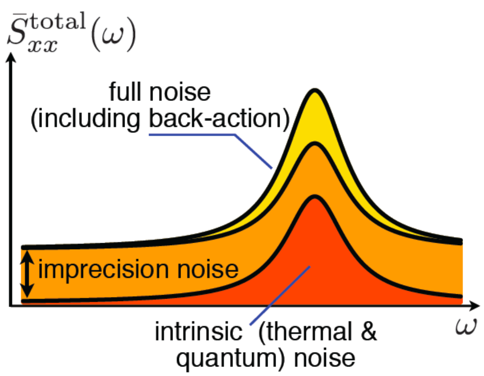

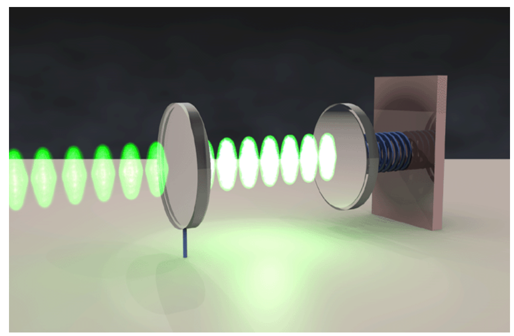

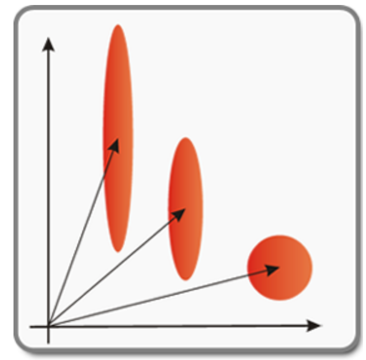

- Quantum limit on continuous position detection
  - AC, Marquardt, Girvin, Devoret and Schoelkopf, RMP 82, 1155 (2010)
- One & two-mode backaction evasion
  - Force detection with no quantum limits?

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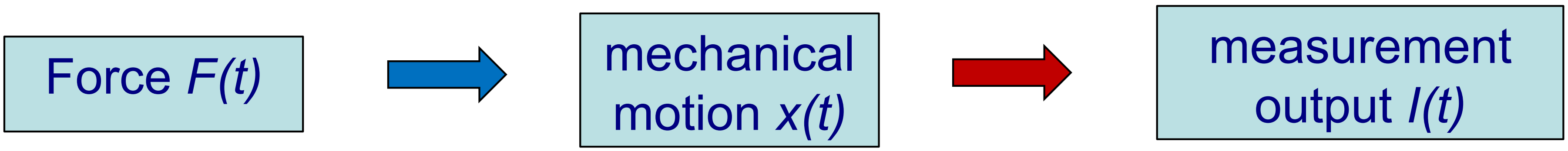
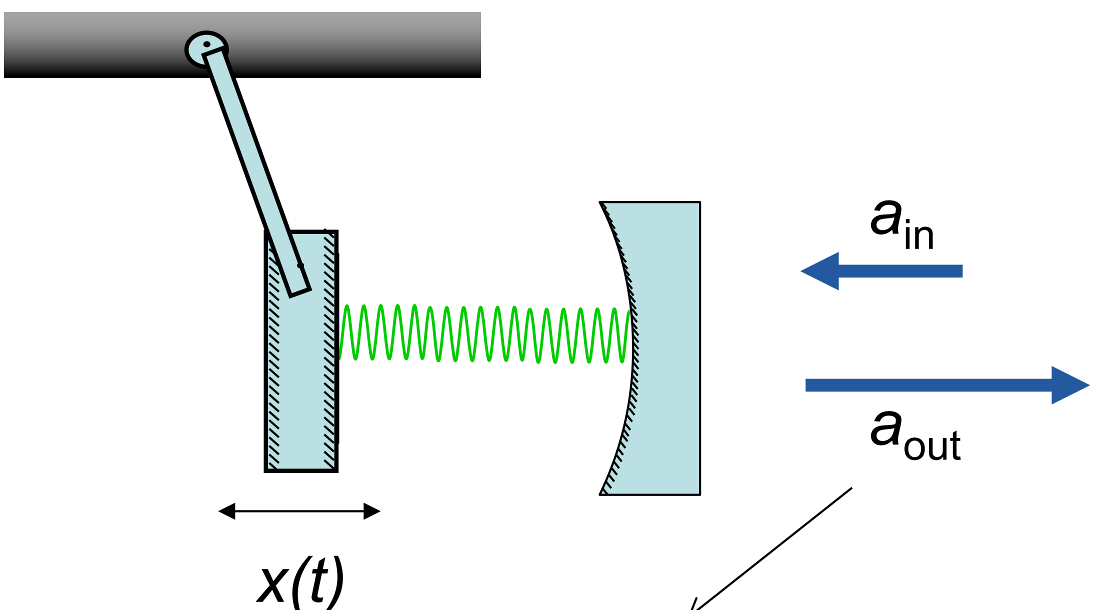
Generic force detection



- **Issue: quantum limits on monitoring *x(t)*….**
- Example: Fabry-Pérot cavity with a moveable end mirror….
  - Cavity resonance frequency depends on mirror position

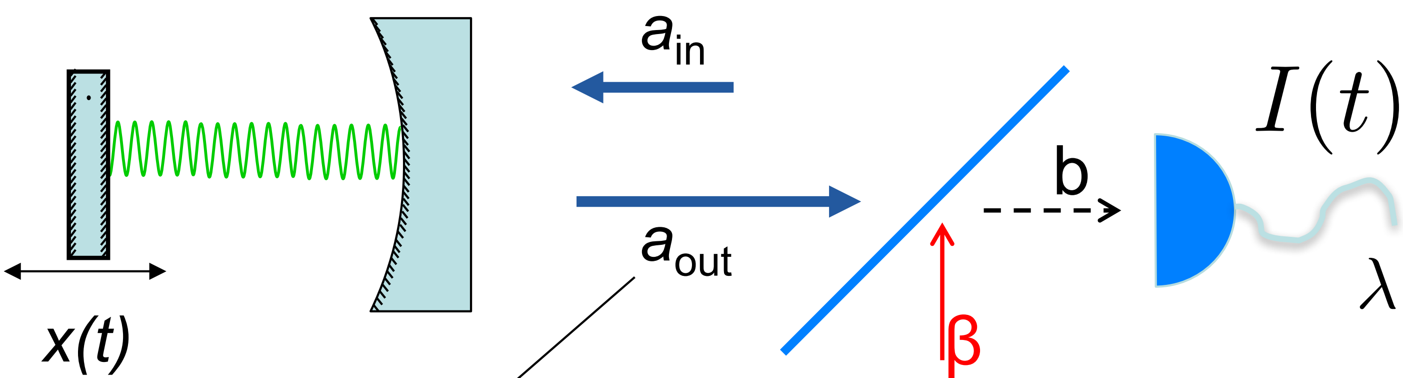


- Measure *x(t)* via phase of reflected light

$$a_{\rm out} = e^{i(\theta_0 + \theta' x)} a_{\rm in} \simeq e^{i\theta_0} a_{\rm in}(1 + i\theta' x)$$

Homodyne measurement

- Mechanical motion written on phase quadrature of output light



$$I(t) = \lambda x(t) + \xi(t)$$

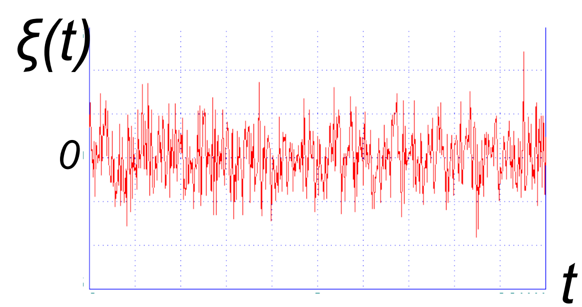
$$\lambda \sim (d\omega_c/dx)\,|a_{\rm in}|$$

$$a_{\rm out} = e^{i(\theta_0 + \theta' x)} a_{\rm in} \simeq e^{i\theta_0} a_{\rm in}(1 + i\theta' x)$$

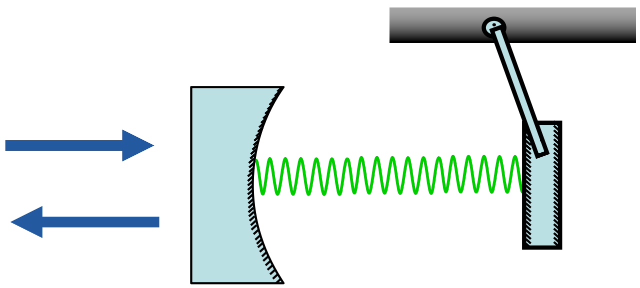
- Measure phase quadrature via homodyne interferometry

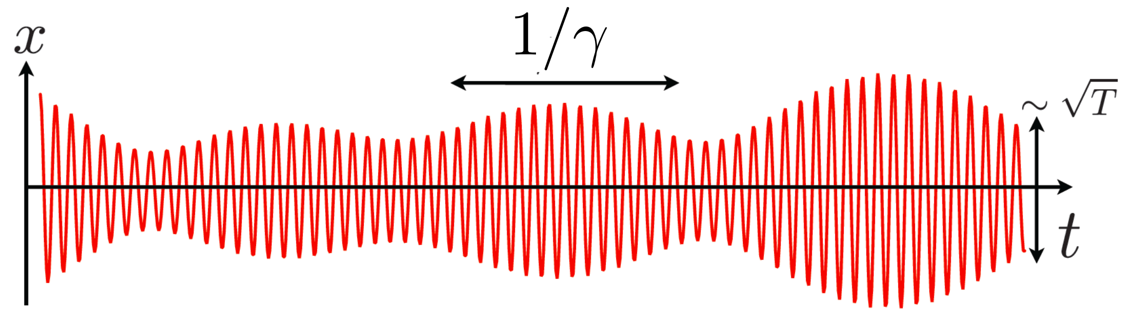
$$I \propto |a_{\rm out} + \beta|^2 - |\beta|^2$$

- Shot noise in $I(t)$…
  - Will take time to infer a change in phase
  - Will take time to infer mechanical position



Weak continuous measurement



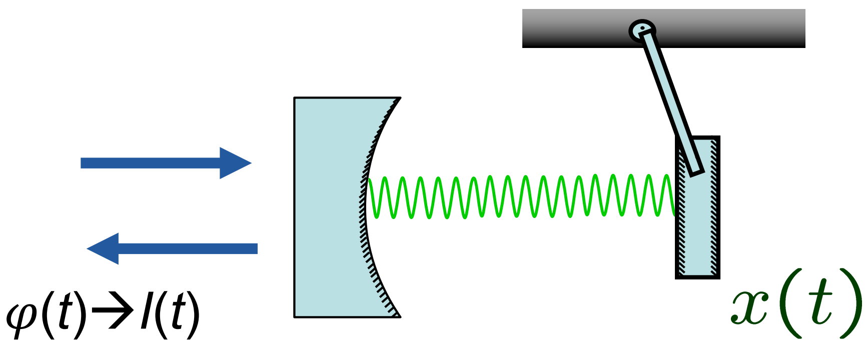


- Measurement is weak
  - **Not trying to measure instantaneous position x(t)!**

- Instead, try to get information over time-scales >> 1 / ω_M
  - i.e. try to measure the slowly varying **quadrature amplitudes**
  - Goal: sensitivity near the zero-point level…
  - **Problem: quadratures don't commute with one another….**

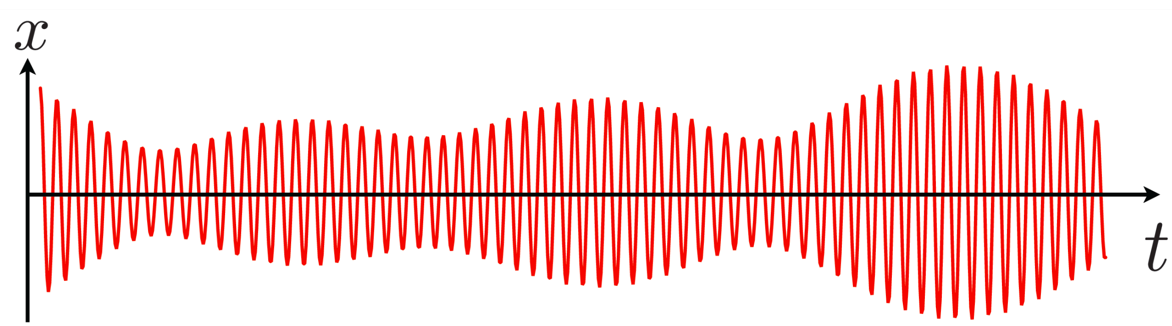
$$\hat{x}(t) \propto \hat{X}(t) \cos \omega_{\mathrm{M}} t + \hat{Y}(t) \sin \omega_{\mathrm{M}} t \qquad \left[\hat{X}, \hat{Y}\right] = i$$

Added noise of the measurement

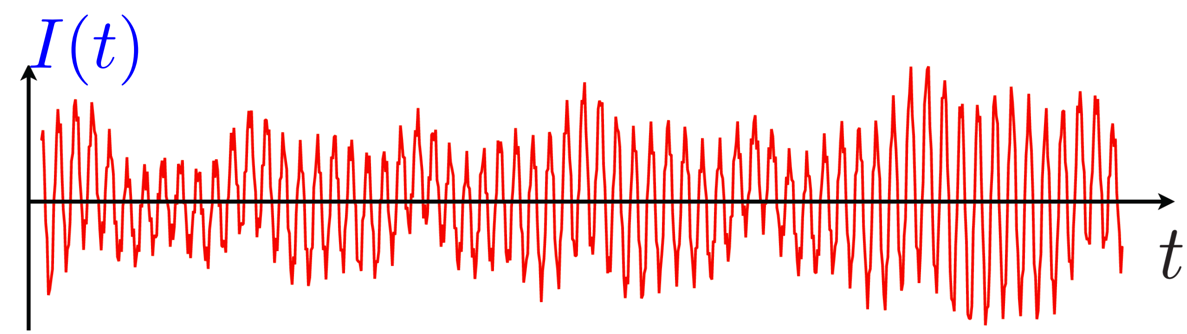


$$I(t) = \lambda\, x(t) + \delta I_0(t)$$

- Intrinsic behaviour of the mechanics:



- What we see in the measurement:

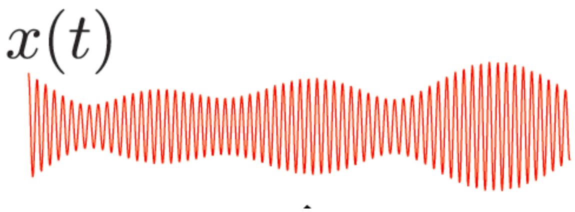


$$\begin{aligned} I(t) &= \lambda\, x(t) + \delta I_0(t) \\ &= \lambda \left[x_0(t) + \delta x_{\mathrm{add}}(t) \right] \end{aligned}$$

How small can we make the total added noise?

Quantum noise spectral densities

- First issue: how do we quantify the size of the noise?
  - Noise spectral density… size of noise at each ω



$$x[\omega] = \frac{1}{\sqrt{\tau}} \int_0^\tau dt e^{i\omega t} x(t)$$

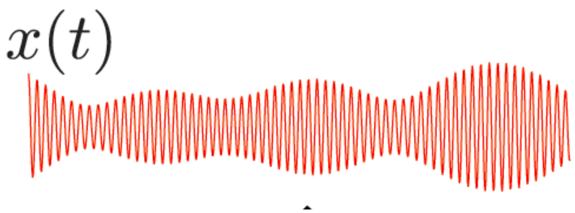
$$\langle |x[\omega]|^2 \rangle \equiv S_{xx}[\omega] = \int dt e^{i\omega t} \langle x(t) x(0) \rangle$$

- How do we think about this quantum mechanically?

$$\bar{S}_{xx}[\omega] = \frac{1}{2} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle \{ \hat{x}(t), \hat{x}(0) \} \rangle$$

- Plays the role of a classical noise spectral density
- Analogous spectral densities characterize detector…
- QM: uncertainty-principle constraints on noise
  - these have no classical analogue

Fluctuation dissipation theorem



$$\bar{S}_{xx}[\omega] = \frac{1}{2}\int_{-\infty}^{\infty} dt e^{i\omega t}\langle\{\hat{x}(t),\hat{x}(0)\}\rangle$$

- Size of position noise in thermal equilibrium?
  - Response of position to a force

$$x[\omega] = -\chi_{xx}[\omega]\, F[\omega] \qquad \chi_{xx}[\omega] = \frac{1}{m(\omega^2-\omega_M^2)+im\gamma\omega}$$

  - Fluctuation-dissipation theorem

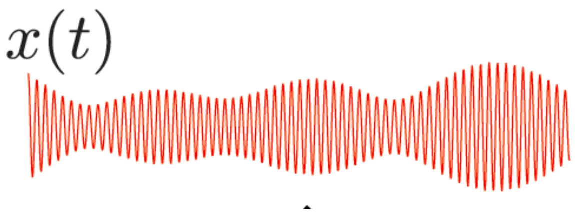
$$\bar{S}_{xx,\mathrm{eq}}[\omega,T] = \left(-\frac{\mathrm{Im}\ \chi_{xx}[\omega]}{\omega}\right)\left(\hbar\omega\coth\left(\frac{\hbar\omega}{2k_BT}\right)\right)$$

dissipation

$$= 1 + 2n_B[\omega]$$

$$\rightarrow 2k_BT$$

Fluctuation dissipation theorem



$$\bar{S}_{xx}[\omega] = \frac{1}{2}\int_{-\infty}^{\infty} dt e^{i\omega t} \langle \{\hat{x}(t), \hat{x}(0)\} \rangle$$

- Size of position noise in thermal equilibrium?

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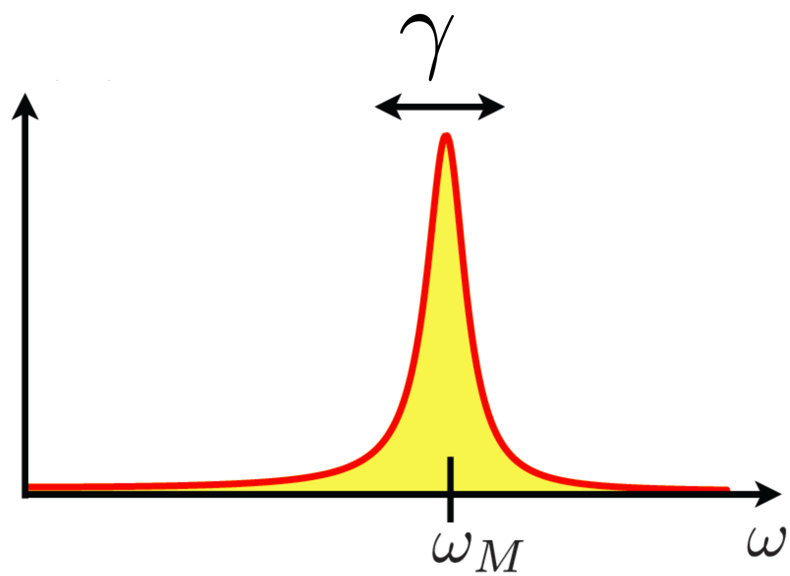
$$\bar{S}_{xx,\mathrm{eq}}[\omega, T] = \left(-\frac{\mathrm{Im}\ \chi_{xx}[\omega]}{\omega}\right)\left(\hbar\omega \coth\left(\frac{\hbar\omega}{2k_B T}\right)\right)$$

- At zero temperature?

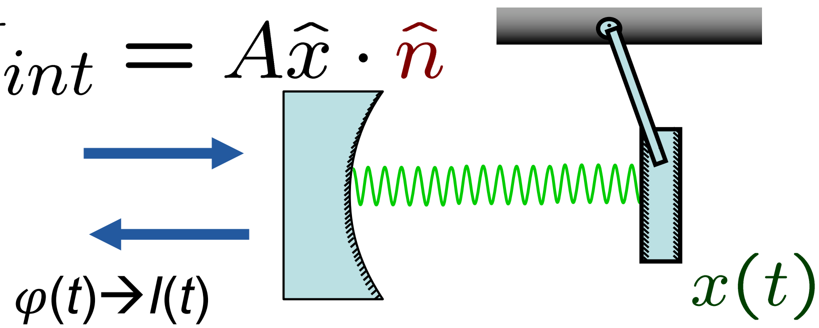
$$\bar{S}_{xx,\mathrm{eq}}[\omega, T=0] = \hbar\left|\mathrm{Im}\ \chi_{xx}[\omega]\right|$$

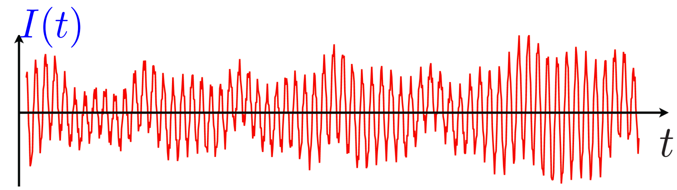


- "Size" of zero-point fluctuations at a given frequency…

Towards the quantum limit

$$H_{int} = A\hat{x} \cdot \hat{n}$$





$$I(t) = \lambda\, x(t) + \delta I_0(t)$$
$$= \lambda\left[x_0(t) + \delta x_{\rm add}(t)\right]$$

- How small can we make the added noise?

$$\delta x_{\rm add}(t) = \frac{\delta I_0(t)}{\lambda} + \delta x_{\rm BA}(t)$$

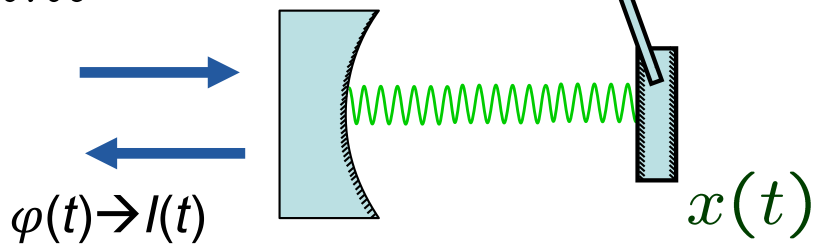
"Intrinsic" output noise (imprecision):

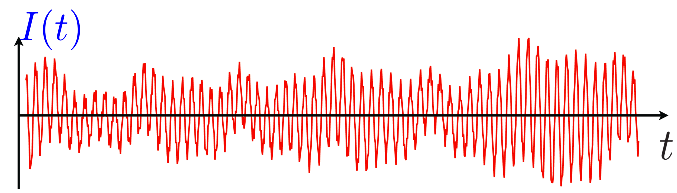
- Present even without coupling to oscillator (e.g. shot noise)
- Reduce **by *increasing* coupling strength** and/or laser power

$$\lambda = \frac{dI}{dx} \sim \frac{A\sqrt{\bar{n}}}{\kappa}$$

Towards the quantum limit

$$H_{int} = A\hat{x} \cdot \hat{n}$$





$$\begin{aligned} I(t) &= \lambda\, x(t) + \delta I_0(t) \\ &= \lambda\left[x_0(t) + \delta x_{\mathrm{add}}(t)\right] \end{aligned}$$

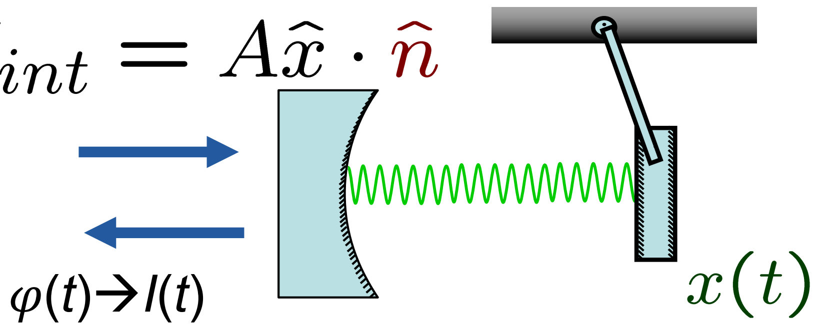
$$\delta x_{\mathrm{add}}(t) = \frac{\delta I_0(t)}{\lambda} + \delta x_{\mathrm{BA}}(t)$$

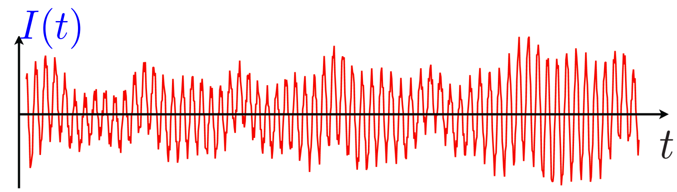
Back-action noise:

- Measuring *x* ***must*** disturb *p*
  - Leads to extra uncertainty in *x* at later times
  - Due to backaction force of detector (e.g. cavity photon number)
- Suppress by ***decreasing*** **coupling strength** / laser power…

Amplifier quantum limit

$$H_{int} = A\hat{x} \cdot \hat{n}$$





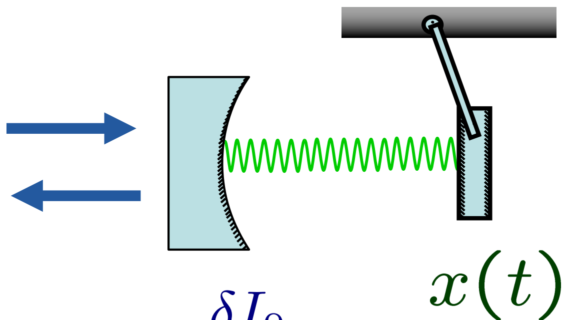
$$I(t) = \lambda\, x(t) + \delta I_0(t)$$
$$= \lambda \left[x_0(t) + \delta x_{\rm add}(t)\right]$$

- How small can we make the added noise?

$$\delta x_{\rm add}(t) = \frac{\delta I_0(t)}{\lambda} + \delta x_{\rm BA}(t)$$

- Quantum Limit
  - If our detector has a "large gain", then $\delta x_{add}(t)$ *cannot be arbitrarily small*
  - The *smallest* it can be (at each frequency) is the size of the oscillator zero-point motion…

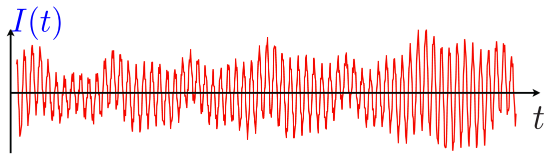
Precise statement of the QL



$$\delta x_{\mathrm{add}} = \frac{\delta I_0}{\lambda} + \delta x_{\mathrm{BA}}$$

$$\langle \hat{I}(t) \rangle = \lambda \langle x(t) \rangle$$

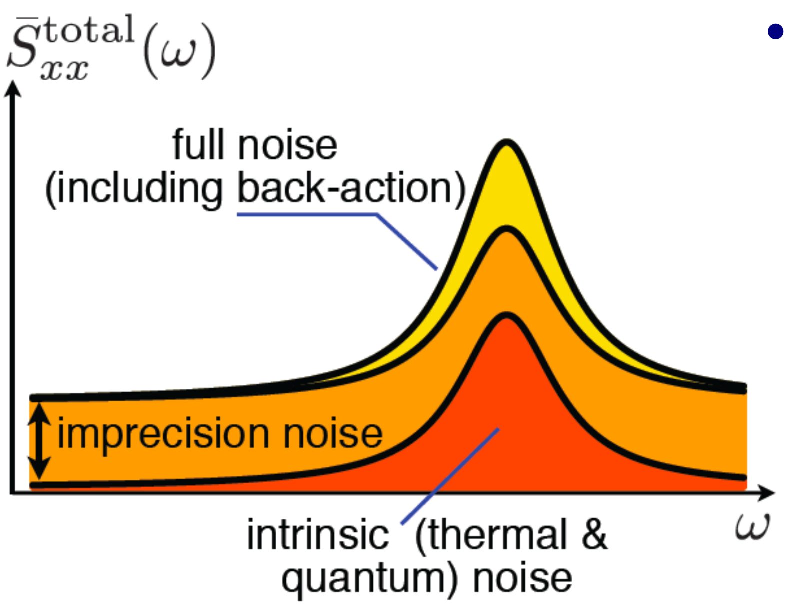
If there were no added noise:



$$\bar{S}_{II}[\omega] = \lambda^2 \bar{S}_{xx}[\omega]$$

Including noise added by detector:

$$\bar{S}_{II}[\omega] = \lambda^2 \left[\bar{S}_{xx}[\omega] + \bar{S}_{xx,\mathrm{BA}}[\omega] \right] + \bar{S}_{I_0 I_0}[\omega]$$



- Spectral density of the added noise

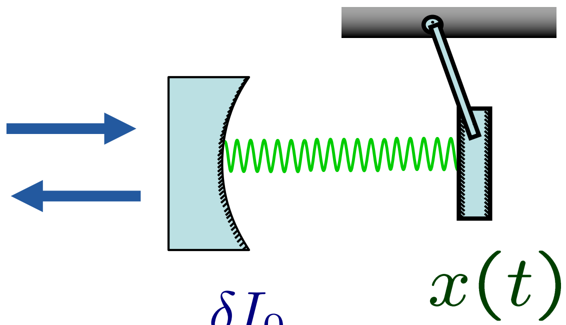
$$\bar{S}_{xx,\mathrm{add}}[\omega] = \frac{\bar{S}_{I_0 I_0}[\omega]}{\lambda^2} + \bar{S}_{xx,\mathrm{BA}}[\omega]$$

- Quantum limit

$$\bar{S}_{xx,\mathrm{add}}[\omega] \geq \bar{S}_{xx,\mathrm{eq}}[\omega, T \to 0]$$

↑ Spectral density of zero-point motion!

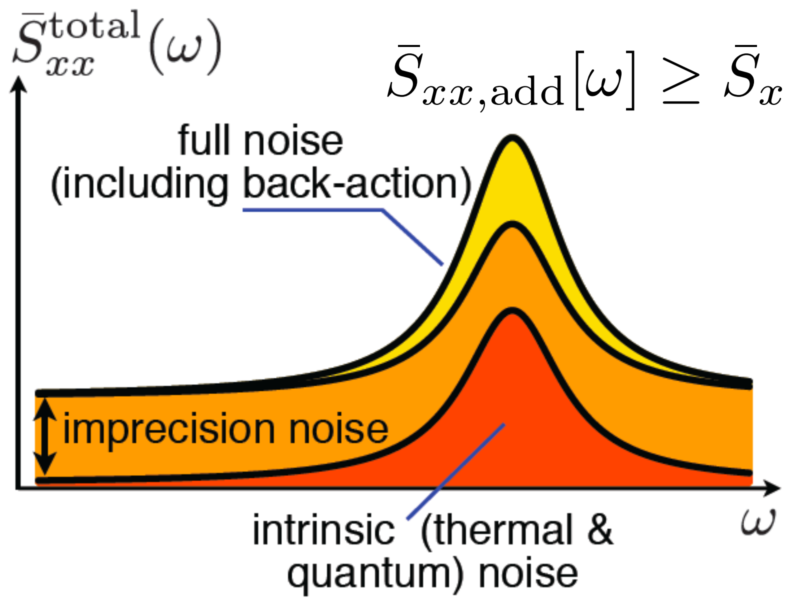
Precise statement of the QL



$$\delta x_{\rm add} = \frac{\delta I_0}{\lambda} + \delta x_{\rm BA}$$

$$\langle \hat{I}(t) \rangle = \lambda \langle x(t) \rangle$$

$$\bar{S}_{xx,\rm add}[\omega] \geq \bar{S}_{xx,\rm eq}[\omega, T \to 0]$$



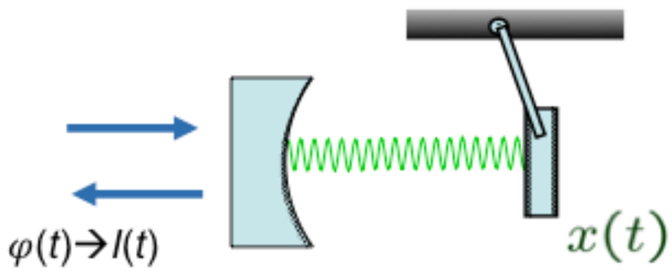
- Corresponding limit to force sensitivity?

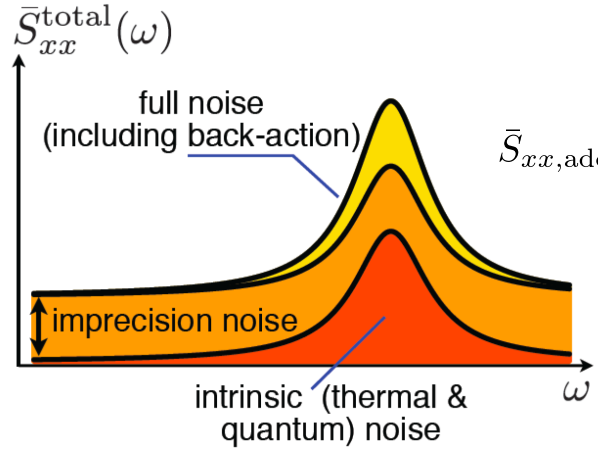
$$x[\omega] = -\chi_{xx}[\omega]\, F[\omega] \implies \bar{S}_{FF,\rm add}[\omega] \geq m\gamma\hbar|\omega|$$

- This ADDS to the intrinsic force fluctuations from the mechanical bath…

$$\bar{S}_{FF,\rm th}[\omega] = m\gamma\hbar\omega\left(1 + 2\bar{n}_{\rm th}[\omega]\right)$$
$$\simeq 2m\gamma k_{\rm B}T$$

Away from resonance?





$$\bar{S}_{xx,\mathrm{add}}[\omega] \geq \bar{S}_{xx,\mathrm{eq}}[\omega, T \to 0]$$

- $\omega \neq \omega_M$??
  - Achieving the true quantum limit requires correlated backaction and imprecision noises

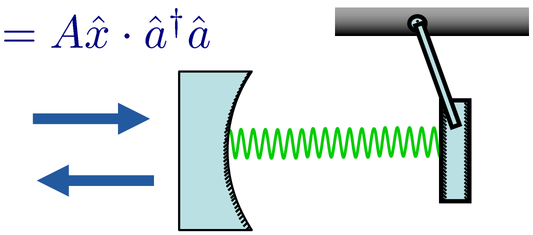
$$\delta x_{\mathrm{add}} = \frac{\delta I_0}{\lambda} + \delta x_{\mathrm{BA}}$$

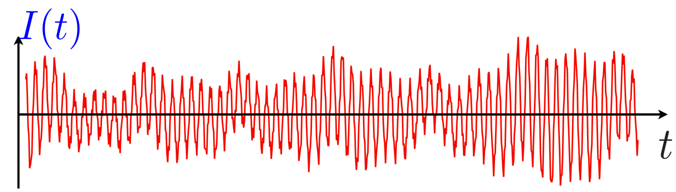
- Consider $\omega >> \omega_M$.
  - Quantum limit: $\bar{n}_{\mathrm{th}} \to \bar{n}_{\mathrm{th}} + 1/2$
  - No correlations: (“SQL”) $\bar{n}_{\mathrm{th}} \to \bar{n}_{\mathrm{th}} + (\omega/\gamma)$

- “Standard quantum limit” ≠ “quantum limit”!
- Tricks for correlation:
  - variational readout, input squeezing, nonlinearity….

Quantum Backaction Evasion

$$H_{\text{int}} = A\hat{x} \cdot \hat{a}^\dagger \hat{a}$$





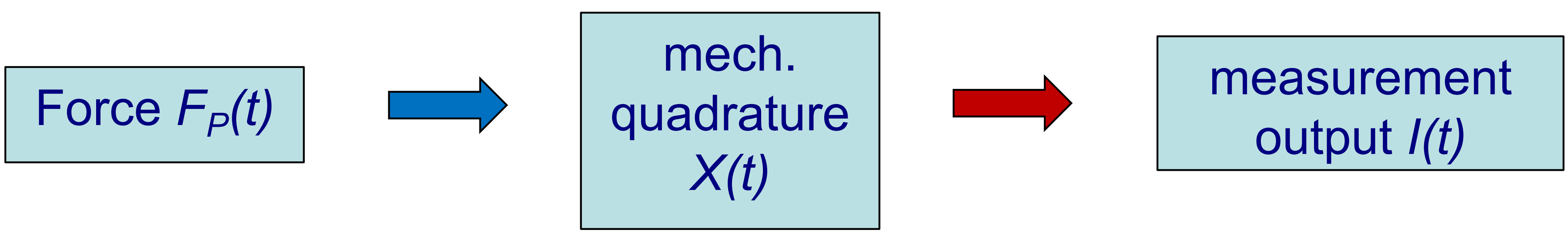
$$\delta x_{\text{add}}(t) = \frac{\delta I_0(t)}{\lambda} + \delta x_{\text{BA}}(t)$$

- Can we “beat” the quantum limit?
  - Change the rules of the game so that *backaction is irrelevant*
  - e.g. measure just a single mechanical quadrature

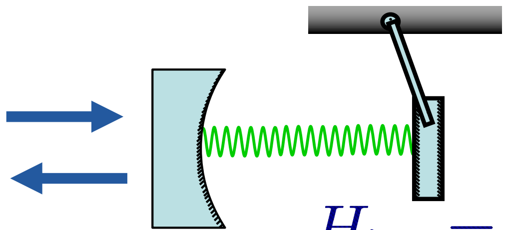
$$\hat{x}(t) \propto \hat{X}(t) \cos \omega_M t + \hat{Y}(t) \sin \omega_M t \qquad \left[\hat{X}, \hat{Y}\right] = i$$

- Measure *X(t)* only, backaction goes into *Y(t)*….
- Lets you measure a single force quadrature….

$$F(t) = F_X(t) \cos \omega t + \underline{F_P(t)} \sin \omega t$$



Double sideband scheme



$$H_{\text{int}} = A\hat{x} \cdot \hat{a}^\dagger \hat{a}$$

$$\hat{x}(t) \propto \hat{X}(t) \cos \omega_{\text{M}} t + \hat{Y}(t) \sin \omega_{\text{M}} t$$

- Measure just the "*X*" quadrature? (Braginsky et al, Science 80; Caves et al., RMP 80)
  - **Hard**: measure both *x* and *p* with time-dependent couplings

$$H_{\text{int}} \propto \hat{X} \cdot \hat{F} \propto \left[\cos\left(\omega_{\text{M}} t\right) \hat{x} - \sin\left(\omega_{\text{M}} t\right) \hat{p}\right] \cdot \hat{F}$$

  - **Easier**: use a time-dependent coupling to position

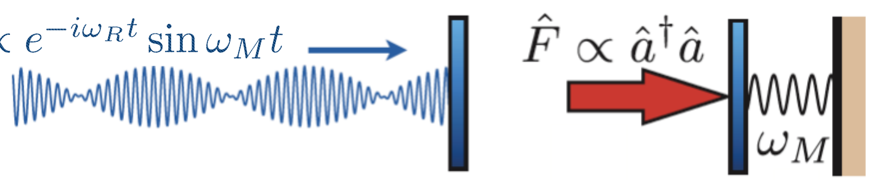
$$A \rightarrow A(t) \propto \cos \omega_{\text{M}} t$$

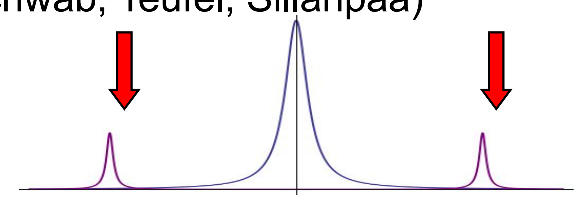
$$H_{\text{int}} \propto \hat{F} \cdot \left[\hat{X}(t)\left(1 + \cos 2\omega_{\text{M}} t\right) + \hat{Y}(t) \sin 2\omega_{\text{M}} t\right]$$

- Can realize with a cavity if $\omega_M >> \kappa$

(Quantum theory: AC, Marquardt and Jacobs, NJP 2007; Expts: Schwab, Teufel, Sillanpaa)

$$b_{\text{in}}(t) \propto e^{-i\omega_R t} \sin \omega_M t$$

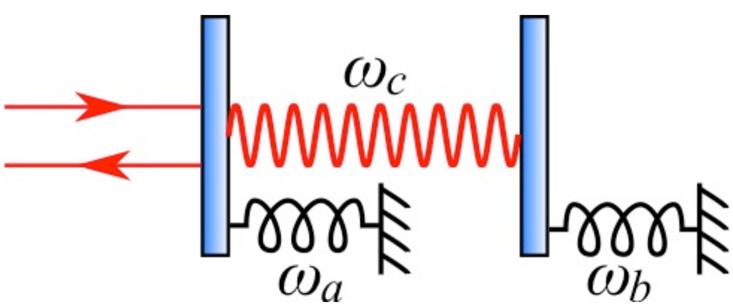




Can we do better?

$$F(t) = F_X(t)\cos\omega t + F_P(t)\sin\omega t$$

- Can we measure *both* force quadratures with no quantum limit?
  - Impossible if we encode the force in a single mechanical resonator
- Possible if you use two mechanical modes! (Caves & Tsang, PRL 2010)
  - No fundamental limit on continuous force detection
- General idea: use **joint quadratures** of two mechanical modes



$$X_\pm = \frac{X_a \pm X_b}{\sqrt{2}}$$

$$P_\pm = \frac{P_a \pm P_b}{\sqrt{2}}$$

- Two quadratures store measurement, two hold backaction…

Can we do better?

$$F(t) = F_X(t)\cos\omega t + F_P(t)\sin\omega t$$

- Can we measure *both* force quadratures with no quantum limit?
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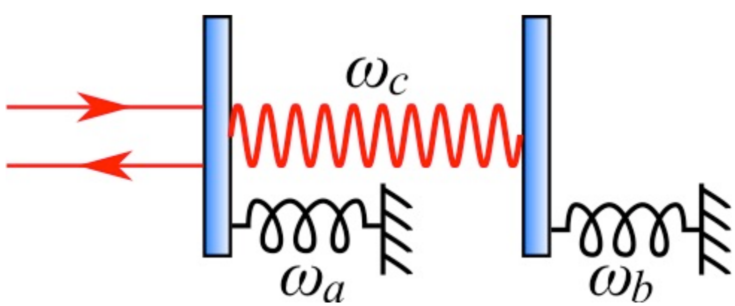
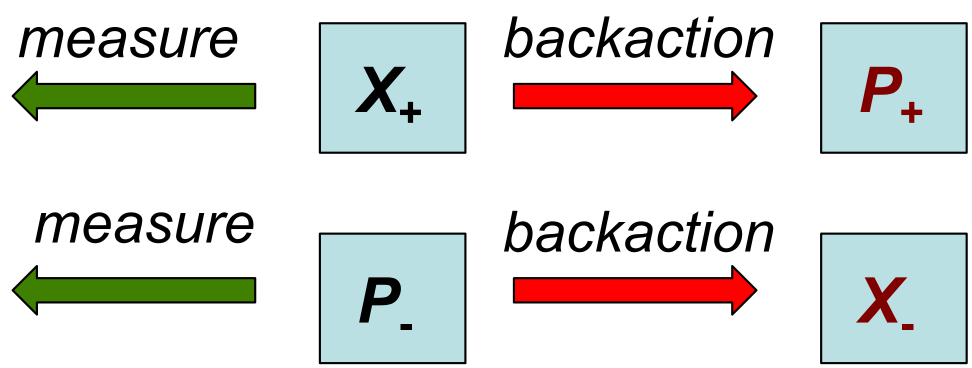
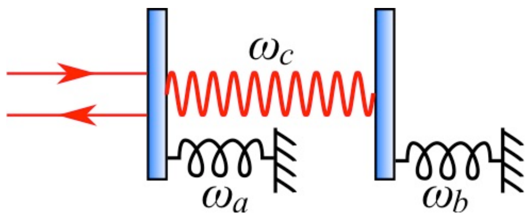
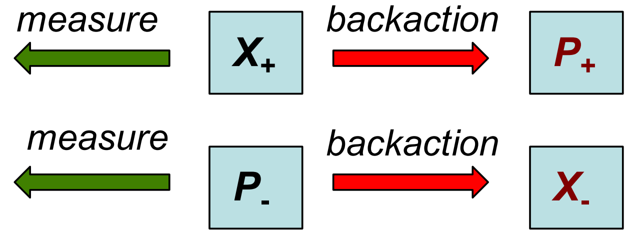




- Two quadratures store measurement, two hold backaction…

Implementation?





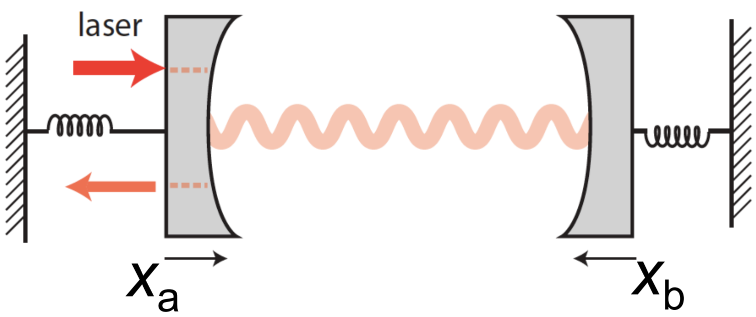
- Hard:
  - "Good" and "bad" quadratures need to be dynamically isolated
  - Read-out **only** the "good" collective quadratures…
- Nice trick (Hammerer et al, PRL 2009; Caves & Tsang PRX 2012; Koopmans 1931, …):

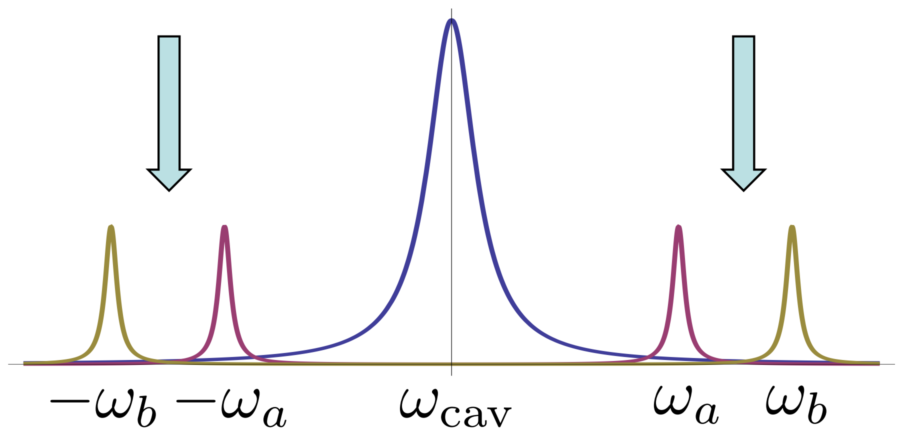
$$\hat{\mathcal{H}} = \Omega\left(\hat{X}_+\hat{X}_- + \hat{P}_+\hat{P}_-\right) \quad \Longrightarrow \quad \frac{d}{dt}X_+ = \Omega P_-, \quad \frac{d}{dt}P_- = -\Omega X_+$$

- X_+,P_- act dynamically like x,p of an oscillator, **but they commute**
- "Quantum mechanics free subsystem"
- Equivalent description
  - "negative mass oscillator" $H_M = \Omega(a^\dagger a - b^\dagger b)$

Optomechanical implementation

- Modified two-tone driving scheme (Woolley and AC, PRA 2013)



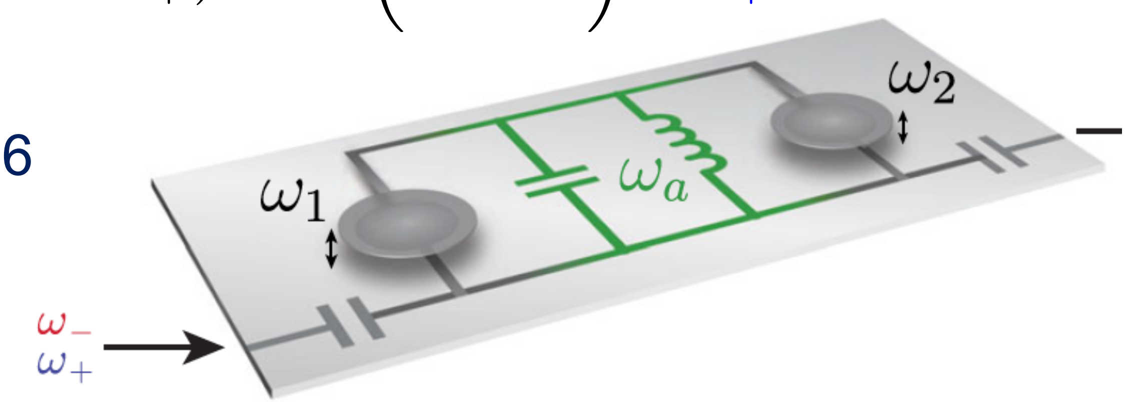


- Drive cavity in-between the two mechanical sidebands…
- In rotating frame, get positive and negative frequencies!

$$\omega_b - \omega_a = 2\Omega$$

$$H = \Omega \left(X_+ X_- + P_- P_+ \right) + G \left(\hat{d} + \hat{d}^\dagger \right) \cdot X_+$$

- Experiment (optomechanics): Ockeloen-Koppi et al, PRL 2016 (Sillanpaa group)
  - Quantum backaction suppressed by ~3 dB

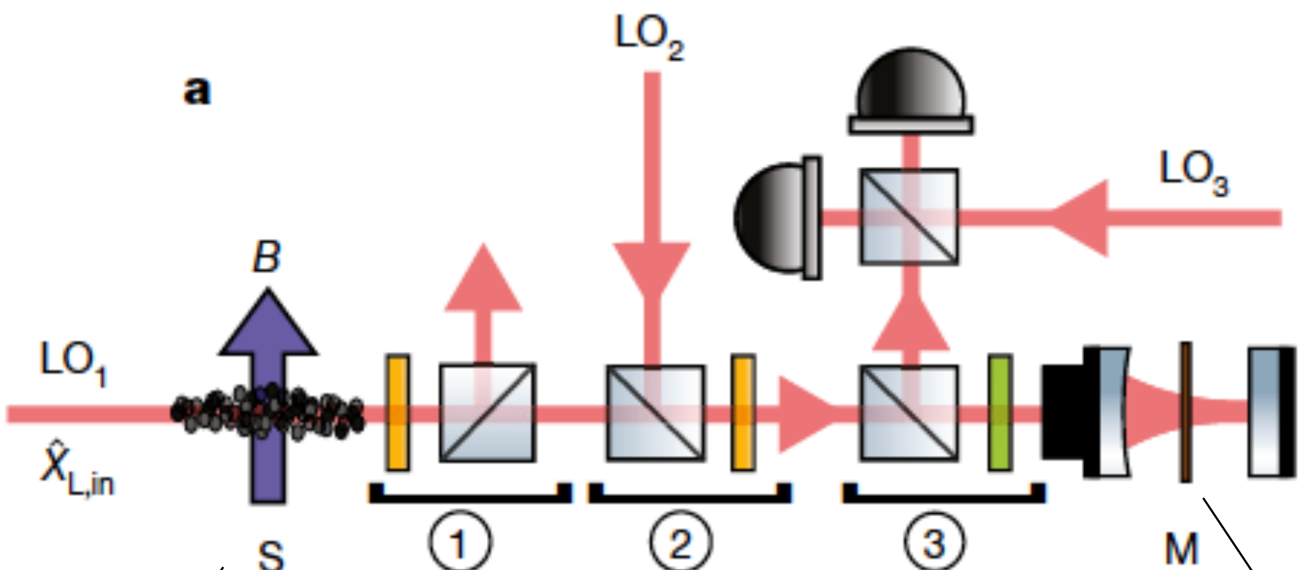


Atomic spin ensemble approach

- Use an atomic ensemble as the negative-frequency oscillator…

$$H_M = \Omega(a^\dagger a - b^\dagger b)$$

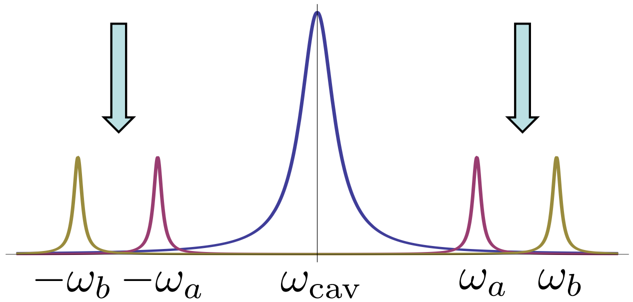
- Experiment: Moller et al, Nature 2017 (Polzik group)
  - Backaction suppressed by ~3 dB

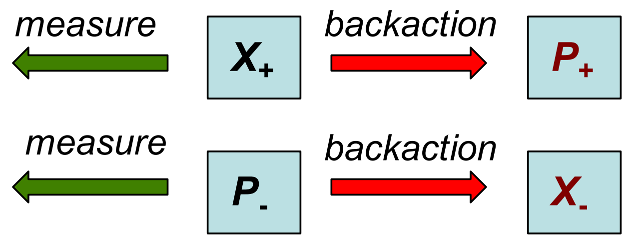


atomic spin ensemble

vibrating SiN membrane

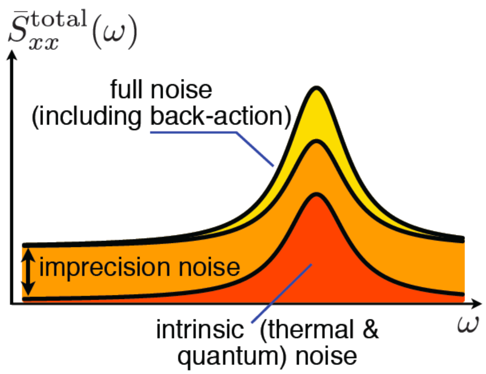
Caveats

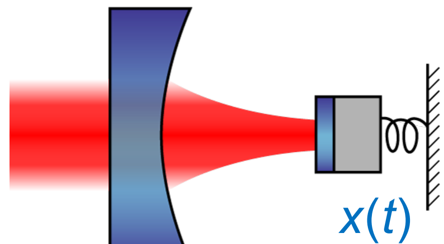


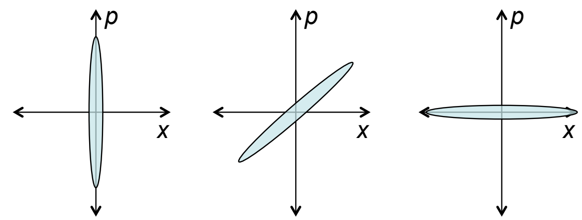


- Evade backaction, but suffer extra noise due to dissipation of second mode…. (Woolley and AC, PRA 2013)
- Can still beat "conventional" quantum limits:
  - On resonance: $$\bar{n}_{\rm add}[\omega_a] \rightarrow 0$$
  - Detuned from resonance: $$\bar{n}_{\rm add}[\omega_a + \Delta] = \frac{1}{2\sqrt{2}}$$

Summary







- Quantum limit on continuous position detection
- Backaction evading techniques
  - Two-mode BAE ➔ beat the conventional quantum limit
  - Other applications:
    - Preparation of non-classical states (PRA 2013, 2014, Science 2015)
    - Squeezing-enhanced dispersive measurement (PRL 2015)

- References:
  - AC, Marquardt, Girvin, Schoelkopf & Devoret, RMP 2010
  - AC, "Quantum measurement & quantum optomechanics", Les Houches 2015 Lecture notes (Oxford 2018?)