Unfortunately, there is no current or planned photometry across the full southern hemisphere that could achieve our colour selection. The current best option, the SkyMapper survey (Keller et al. 2007) will only go as faint as $u = 20.7, g = 21.7$. Including these constraints (and re-examining the other photometric bands under similar limits) shows that a complex selection would be required to improve beyond a simple $J < 19.0$ sample. Hence, this is the one we present in our forecasts.

4. FISHER MATRIX FORECASTS ON THE GROWTH RATE

4.1. *Method*

We forecast the constraints on the growth rate using the Fisher matrix method of Howlett et al. (2017a), modelling the information contained in the two-point correlations between the density field measured using the galaxy redshifts and the velocity field from the SN Ia PVs. We have updated the Howlett et al. (2017a) models to account for the redshift dependence of the power spectra, growth rate and galaxy bias, but otherwise the method remains unchanged. As such, we present only a brief overview here and we refer the reader to Howlett et al. (2017a) for a more complete description.

For given parameters of interest $\boldsymbol{\lambda}$, we compute the corresponding elements of the Fisher Matrix $\mathbf{F}$, as

$$F_{ij} = \frac{\Omega_{sky}}{4\pi^2}\int_{r_{min}}^{r_{max}} r^2 dr \int_{k_{min}}^{k_{max}} k^2 dk \int_0^1 d\mu_\phi \\ \mathrm{Tr}\left[\mathbf{C}^{-1}(r,k,\mu_\phi)\frac{\partial \mathbf{C}(r,k,\mu_\phi)}{\partial\lambda_i}\mathbf{C}^{-1}(r,k,\mu_\phi)\frac{\partial \mathbf{C}(r,k,\mu_\phi)}{\partial\lambda_j}\right], \tag{6}$$

where Ω_{sky} is the sky coverage of the survey, r_{max} (r_{min}) are the comoving distances corresponding to the upper (lower) redshift limits of each redshift bin, and we set $k_{max} = 0.2h\,\mathrm{Mpc}^{-1}$ and $k_{min} = 2\pi/r_{max}$. μ_ϕ is the cosine of the angle ϕ between the k-vector and the observer's line-of-sight.

The covariance matrix, $\mathbf{C}$ consists of the anisotropic density-density, density-velocity and velocity-velocity power spectra $P_{\delta\delta}$, $P_{\delta v}$ and P_{vv} respectively, as well as the noise associated with each of these,

$$\mathbf{C}(r,k,\mu_\phi) = \begin{bmatrix} P_{\delta\delta}(r,k,\mu_\phi) + \frac{1}{\bar{n}_\delta(r)} & P_{\delta v}(r,k,\mu_\phi) \\ P_{\delta v}(r,k,\mu_\phi) & P_{vv}(r,k,\mu_\phi) + \frac{\sigma^2_{obs}(r)}{\bar{n}_v(r)} \end{bmatrix}. \tag{7}$$

The shot-noise in these measurements is inversely proportional to the galaxy number density $\bar{n}_\delta(r)$ for the density field, and to the average PV error divided by the SN-Ia number density $\sigma^2_{obs}(r)/\bar{n}_v(r)$ for the velocity field. The average PV error is given in terms of a fractional distance error α, and a contribution from random motions $\sigma_{obs,rand} = 300\,\mathrm{kms}^{-1}$,

$$\sigma^2_{obs}(r) = (\alpha H_0 r)^2 + \sigma^2_{obs,rand}. \tag{8}$$

Finally, we model the relevant power spectra using

$$P_{\delta\delta}(z(r),k,\mu_\phi) = \left(\frac{1}{\beta^2(z)} + \frac{2\mu_\phi^2}{\beta(z)} + \mu_\phi^4\right)(f(z)\sigma_8(z))^2 \\ D_g^2(k,\mu_\phi)\frac{P_{mm}(k,z)}{\sigma_8^2(z)}, \tag{9}$$

$$P_{\delta v}(z(r),k,\mu_\phi) = \frac{H(z)\mu_\phi}{k(1+z)}\left(\frac{1}{\beta(z)} + \mu_\phi^2\right)(f(z)\sigma_8(z))^2 \\ D_g(k,\mu_\phi)D_u(k)\frac{P_{m\theta}(k,z)}{\sigma_8^2(z)}, \tag{10}$$

$$P_{vv}(z(r),k,\mu_\phi) = \frac{H^2(z)\mu_\phi^2}{k^2(1+z)^2}(f(z)\sigma_8(z))^2 \\ D_u^2(k)\frac{P_{\theta\theta}(k,z)}{\sigma_8^2(z)}, \tag{11}$$

$$D_g(k,\mu_\phi) = \left[1 + \frac{(k\mu_\phi\sigma_\delta)^2}{2}\right]^{-1/2} \quad \text{and} \tag{12}$$

$$D_u(k) = \mathrm{sinc}(k\sigma_v). \tag{13}$$

We have written the above models in terms of the redshift corresponding to a given comoving distance $z(r)$ ($H(z)$ is the Hubble parameter at this redshift) and in a particular way to highlight the parameters of interest $\boldsymbol{\lambda} = \{f(z)\sigma_8(z), \beta(z), \sigma_\delta, \sigma_v\}$. The power spectra $P_{mm}(k,z)$, $P_{m\theta}(k,z)$ and $P_{\theta\theta}(k,z)$ are the real-space matter and velocity divergence auto- and cross-power spectra for the dark matter field and are computed using the implementation of two-loop Renormalised Perturbation Theory (Crocce & Scoccimarro 2006) found in the COPTER numerical package (Carlson et al. 2009).

The combination $f(z)\sigma_8(z)$ is the normalised growth rate that we present forecasts for in this work. We use this combination as both f and σ_8 are degenerate on linear scales, however their combination can still be used to constrain gravitational models even without explicit knowledge of σ_8 (Song & Percival 2009) and is what is typically measured using RSD and PV surveys. $\beta(z) = f(z)/b(z)$ is the ratio of the growth rate over the galaxy bias and here is treated as one of the nuisance parameters we marginalise over. We also marginalise over two additional nuisance parameters, σ_δ and σ_v, which characterise the non-linear damping of the density and velocity fields due to RSD. These are used as inputs to Lorentzian (for the density field) and sinc (for the velocity field) functions which reduce the power spectra on small scales but leave them unchanged on large scales. For these parameters we adopt the same values as used in Howlett et al. (2017a), $\sigma_\delta = 4.24\,h^{-1}\mathrm{Mpc}$ and $\sigma_v = 13.0\,h^{-1}\mathrm{Mpc}$, which were found to reproduce the effects of non-linear RSD in simulations (Koda et al. 2014).

The redshift dependence of the normalised growth rate and bias is included using $f(z)\sigma_8(z) = \Omega_m(z)^{0.55}\sigma_8(z = 0)D(z)$ and $b(z) = b(z = 0)D^{-1}(z)$, with $D(z)$ given by Eq. 1 for $a = 1/(1 + z)$ and normalised to unity at $z = 0$.