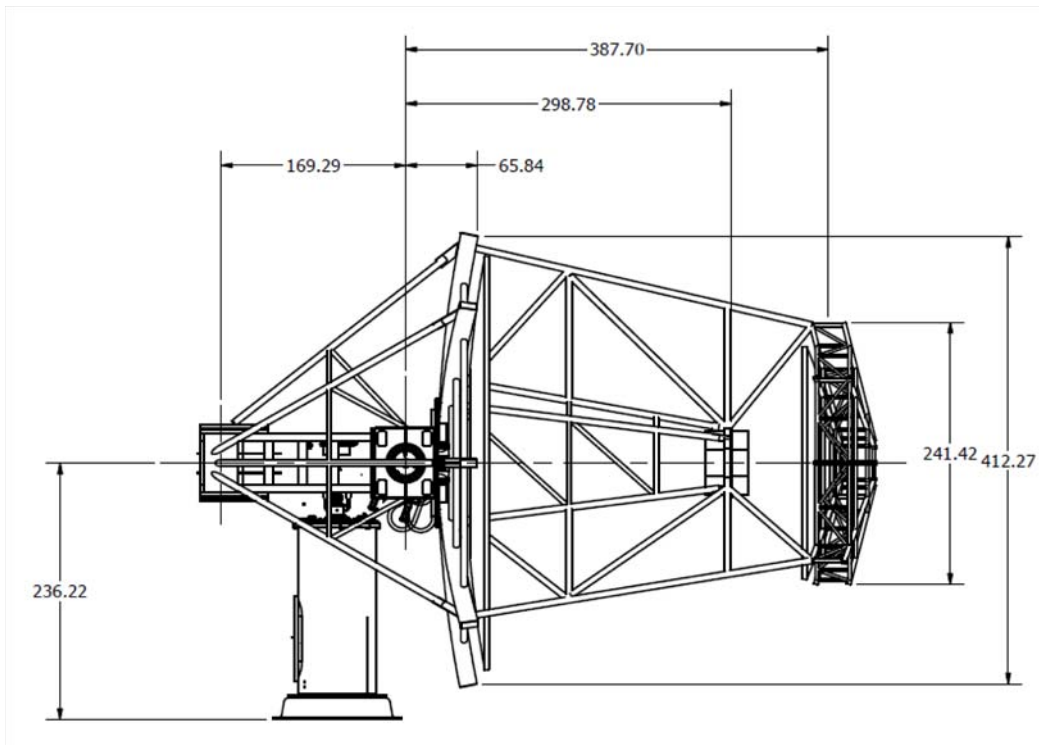


- 1.0 Define Material and Geometric Inputs used in the calculations
- 2.0 Calculation of the angular velocity and accelerations
- 3.0 Calculate the CounterWeight needed
- 4.0 Calculate approximate mass moments of inertia about the elevaton and azimuth axis
- 5.0 Calculate the wind torques and forces as a function of velocity, and elevation/azimuth angles
- 6.0 Calculate the wind torques at the park positions under maximum wind conditions
- 7.0 Calculate Foundation Strength



1.0 Inputs

Weight of components

$W_p := 7.1\text{ton}$	Weight of primary structure and mirrors
$W_s := 3.2\text{ton}$	Weight of secondary structure and mirrors
$W_c := 1\text{ton}$	Weight of Camera
$W_{\text{positioner}} := 12.6\text{ton}$	Weight of Positioner
$W_{\text{yoke}} := 6\text{ton}$	Weight of yoke

Number of Mirrors

$N_p := 48$	Number of Primary Mirrors
-------------	---------------------------

Ns := 24	Number of Secondary Mirrors
Define Geometry of Telescope	
Ls := 387in	Distance from elevation axis to secondary
Lc := 298in	Distance from elevation axis to camera
Lp := 65in	Distance from elevation axis to primary
Lcw := 169in	Distance from elevation axis to CW
Ds := 241in	Diameter of Secondary
Dp := 412in	Diameter of Primary
tp := 18in	Thickness of Primary Structure
ts := 36in	Thickness of Secondary Structure
Acamera := 0.6m	Length of camera along optical axis
Bcamera := 1.1m	Diameter of camera
HeightEL := 6m	Height of the elevation axis above the ground
Hcw := 48in	Height of CW

Wind Velocities and Park Positions

$V_{max} := 50 \frac{\text{km}}{\text{hr}}$	Max 10 minute averagewind speed
$V_{max} = 31.07 \cdot \text{mph}$	
$V_{op} := 36 \frac{\text{km}}{\text{hr}}$	Operational 10 minute average wind speed
$V_{op} = 22.37 \cdot \text{mph}$	
$V_{survival} := 120 \frac{\text{km}}{\text{hr}}$	Survival 10 minute average wind speed

$$V_{\text{survival}} = 74.56 \cdot \text{mph}$$

$$V_{\text{ibc}} := 90 \text{mph}$$

$$V_{\text{ibc}} = 144.84 \cdot \frac{\text{km}}{\text{hr}}$$

$$\alpha_{\text{park1}} := 15 \text{deg}$$

$$\alpha_{\text{park2}} := 35 \text{deg}$$

$$Z_o := 30 \text{ft}$$

Design wind speed of International Building Code IBC-2009 and ASCE7-05
Veritas was designed to 34m/s (76mph) based on the 1997 Uniform Building
Code which was the precursor to the IBC

Reference height for wind velocity

Bearing Friction Torques

$$T_{\text{frictionEL}} := 15.3 \text{kN}\cdot\text{m}$$

Bearing friction torque on elevation axis

$$T_{\text{frictionAZ}} := 21.4 \text{kN}\cdot\text{m}$$

Bearing friction torque on Azimuth axis

Seismic Loads

$$S_s := 0.276$$

$$S_1 := 0.075$$

From VERITAS foundation design documents

Soil Site Class D assumed

$$F_a := 1.6$$

$$F_v := 2.4$$

Tables 11.4-1 and 11.4-2 Site Coefficients

$$SDS := \frac{2}{3} \cdot F_a \cdot S_s$$

$$SDS = 0.29$$

$$SD1 := \frac{2}{3} \cdot F_v \cdot S_1$$

$$SD1 = 0.12$$

Design Spectral Response

$$T_o := 0.2 \cdot \frac{SD1}{SDS}$$

$$T_o = 0.08$$

$$T_s := \frac{SD1}{SDS}$$

$$T_s = 0.41$$

$$TL := 6$$

$$S_a := .4 \cdot SDS$$

$$S_a = 0.12$$

2.0 Calculate the angular velocity and accelerations needed to meet the CTA specifications

According to CTA Level B Sub-System Performance Requirements Section 4.7 the telescope must be able to reach any point in the sky above 30 deg elevation within 90 seconds.

Assumption that both axis can rotate simultaneously.

The elevation axis has a range of 0 deg to 91 deg so to meet the requirement it must rotate from 30 deg to 90 deg in 90 seconds. The azimuth axis has a range of +/-270 degree. To meet the requirement it must rotate a maximum of 360 deg in 90 seconds.

2.1 Azimuth Axis

$$\theta_{\max} := 180\text{deg}$$

$$\omega_{\max} := 4.0 \frac{\text{deg}}{\text{s}}$$

$$\alpha_{\text{Az}} := 0.1 \frac{\text{deg}}{\text{s}^2}$$

$$t_a := \frac{\omega_{\max}}{\alpha_{\text{Az}}} \quad t_a = 40.00 \text{ s} \quad \text{Time for acceleration to maximum angular velocity}$$

$$\theta_a := \frac{1}{2} \alpha_{\text{Az}} \cdot t_a^2 \quad \theta_a = 80.00 \cdot \text{deg}$$

Angle during acceleration period
Defined maximum angular motion

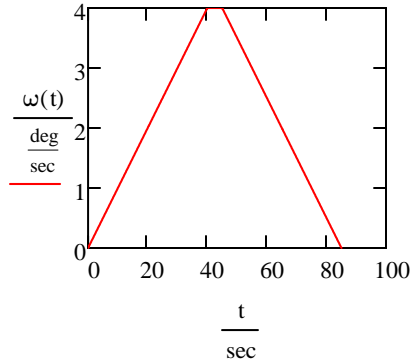
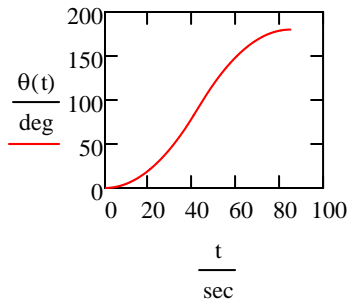
$$t_c := \frac{\theta_{\max} - 2 \cdot \theta_a}{\omega_{\max}} \quad t_c = 5.00 \text{ s} \quad \text{Time of constant angular velocity}$$

$$t_{\max} := 2 \cdot t_a + t_c \quad t_{\max} = 85.00 \text{ s} \quad \text{Time required to achieve } \theta_{\max}$$

$$t := 0.0\text{sec}, 0.1\text{sec} \dots t_{\max}$$

$$\omega(t) := \begin{cases} (\alpha_{\text{Az}} \cdot t) & \text{if } t \leq t_a \\ \omega_{\max} & \text{if } t > t_a \\ [\omega_{\max} - \alpha_{\text{Az}} \cdot (t - t_a - t_c)] & \text{if } t > t_a + t_c \end{cases}$$

$$\theta(t) := \begin{cases} \left(\frac{1}{2}\alpha Az \cdot t^2\right) & \text{if } t \leq t_a \\ [\theta_a + \omega_{\max} \cdot (t - t_a)] & \text{if } t \geq t_a \wedge t \leq t_c + t_a \\ \left[\omega_{\max} \cdot (t - t_c - t_a) - \frac{1}{2} \cdot \alpha Az \cdot (t - t_c - t_a)^2 + \theta_a + \omega_{\max} \cdot t_c \right] & \text{if } t > t_a + t_c \end{cases}$$



2.2 Elevation Axis

$$\theta_{\max} := 90 \text{deg}$$

$$\omega_{\max} := 2 \frac{\text{deg}}{\text{s}}$$

$$\alpha_{El} := 0.06 \frac{\text{deg}}{\text{s}^2}$$

$$t_a := \frac{\omega_{\max}}{\alpha_{El}} \quad t_a = 33.33 \text{ s} \quad \text{Time for acceleration to maximum angular velocity}$$

$$\theta_a := \frac{1}{2} \alpha_{El} \cdot t_a^2 \quad \theta_a = 33.33 \cdot \text{deg} \quad \text{Angle during acceleration period}$$

Defined maximum angular motion

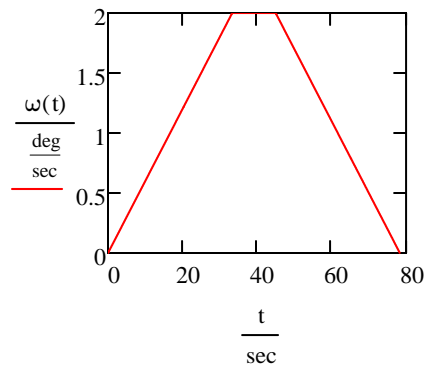
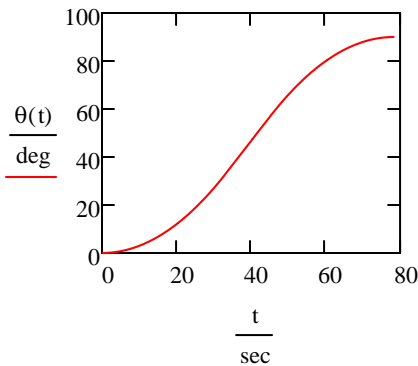
$$t_c := \frac{\theta_{\max} - 2 \cdot \theta_a}{\omega_{\max}} \quad t_c = 11.67 \text{ s} \quad \text{Time of constant angular velocity}$$

$$t_{\max} := 2 \cdot t_a + t_c \quad t_{\max} = 78.33 \text{ s} \quad \text{Time required to achieve } \theta_{\max}$$

$$t := 0.0 \text{sec}, 0.1 \text{sec} \dots t_{\max}$$

$$\omega(t) := \begin{cases} (\alpha_{El} \cdot t) & \text{if } t \leq t_a \\ \omega_{\max} & \text{if } t > t_a \\ [\omega_{\max} - \alpha_{El} \cdot (t - t_a - t_c)] & \text{if } t > t_a + t_c \end{cases}$$

$$\theta(t) := \begin{cases} \left(\frac{1}{2}\alpha EI \cdot t^2\right) & \text{if } t \leq t_a \\ [\theta_a + \omega_{\max} \cdot (t - t_a)] & \text{if } t \geq t_a \wedge t \leq t_c + t_a \\ \left[\omega_{\max} \cdot (t - t_c - t_a) - \frac{1}{2} \cdot \alpha EI \cdot (t - t_c - t_a)^2 + \theta_a + \omega_{\max} \cdot t_c \right] & \text{if } t > t_a + t_c \end{cases}$$



3.0 Calculate CW

$$W_{cw} := \frac{W_s \cdot L_s + W_c \cdot L_c + W_p \cdot L_p}{L_{cw}}$$

$$W_{cw} = 11.8 \cdot \text{ton}$$

Weight of counterweight

$$M_{cw} := W_{cw} \cdot g \cdot L_{cw}$$

$$M_{cw} = 333 \cdot \text{ft} \cdot \text{kip}$$

Moment of Counterweight

$$W_{\text{total}} := W_{cw} + W_s + W_c + W_p$$

Total weight of OSS

$$W_{\text{total}} = 23.1 \cdot \text{ton}$$

4.0 Calculate Inertias and Moments about elevation and Azimuth axis

$$I_s := W_s \cdot L_s^2$$

$$I_s = 280501.43 \text{ m}^{2.00} \cdot \text{kg}$$

Mass moment of inertia of secondary about elevation axis

$$M_s := W_s \cdot g \cdot L_s$$

$$M_s = 279840.82 \cdot \text{N} \cdot \text{m}$$

$$I_c := W_c \cdot L_c^2$$

$$I_c = 51975 \text{ m}^2 \cdot \text{kg}$$

Mass moment of inertia of camera about elevation axis

$$M_c := W_c \cdot g \cdot L_c$$

$$M_c = 67338.96 \cdot \text{N} \cdot \text{m}$$

$$I_p := W_p \cdot L_p^2$$

$$I_p = 17556.92 \text{ m}^{2.00} \cdot \text{kg}$$

Mass moment of inertia of primary about elevation axis

$$M_p := W_p \cdot g \cdot L_p$$

$$M_p = 104285.00 \cdot \text{N} \cdot \text{m}$$

$$I_{cw}(\text{LCW}) := \frac{W_s \cdot L_s + W_c \cdot L_c + W_p \cdot L_p}{\text{LCW}} \cdot \text{LCW}^2$$

$$I_{cw}(\text{Lcw}) = 197616.69 \cdot \text{kg} \cdot \text{m}^2$$

$$I_{\text{total}}(\text{LCW}) := I_s + I_c + I_p + I_{cw}(\text{LCW})$$

$$I_{\text{total}}(\text{Lcw}) = 547650 \text{ m}^2 \cdot \text{kg}$$

Calculate the Inertia Torques

$$\alpha_{\text{ElMin}} := 0.06 \frac{\text{deg}}{\text{s}^2}$$

Minimum Angular Velocity

$$\alpha_{\text{ElMax}} := .66 \frac{\text{deg}}{\text{s}^2}$$

Maximum Angular Velocity

$$\text{TorqueInertiaMinEl} := I_{\text{total}}(\text{Lcw}) \cdot \alpha_{\text{ElMin}}$$

$$\text{TorqueInertiaMinEl} = 423 \cdot \text{lbf} \cdot \text{ft}$$

$$\text{TorqueInertiaMaxEl} := I_{\text{total}}(\text{Lcw}) \cdot \alpha_{\text{ElMax}}$$

$$\text{TorqueInertiaMaxEl} = 4653 \cdot \text{lbf} \cdot \text{ft}$$

$$\text{TorqueInertiaMaxEl} = 6.31 \cdot \text{kN} \cdot \text{m}$$

Assume that the inertia about the azimuth axis is the same as the elevation axis

$$\alpha_{\text{AzMin}} := 0.1 \frac{\text{deg}}{\text{s}^2}$$

$$\alpha_{\text{AzMax}} := 2 \frac{\text{deg}}{\text{s}^2}$$

$$\text{TorqueInertiaMinAz} := \text{Itotal}(\text{Lcw}) \cdot \alpha_{\text{AzMin}}$$

$$\text{TorqueInertiaMinAz} = 705 \cdot \text{lbf} \cdot \text{ft}$$

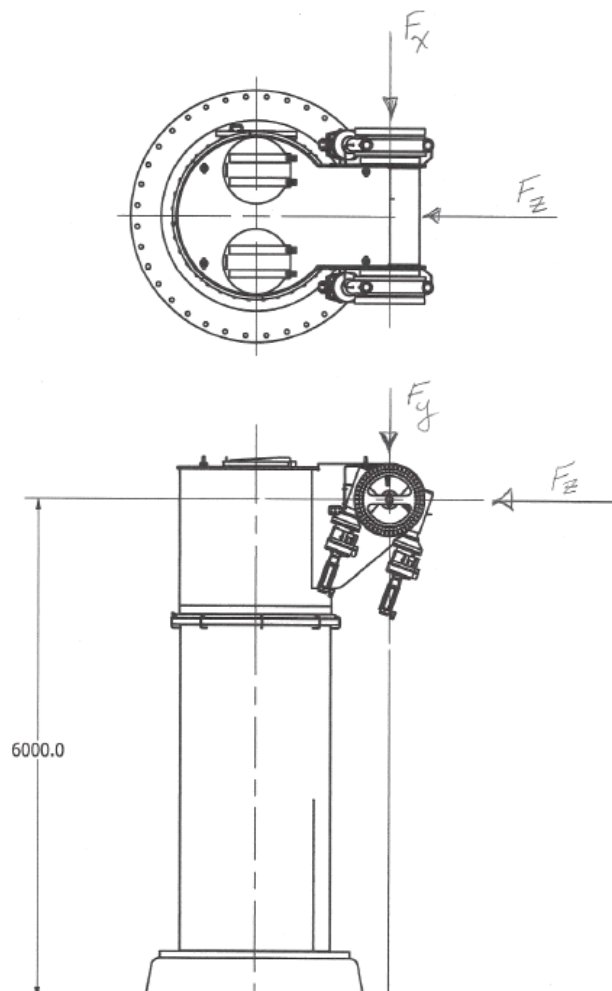
$$\text{TorqueInertiaMaxAz} := \text{Itotal}(\text{Lcw}) \cdot \alpha_{\text{AzMax}}$$

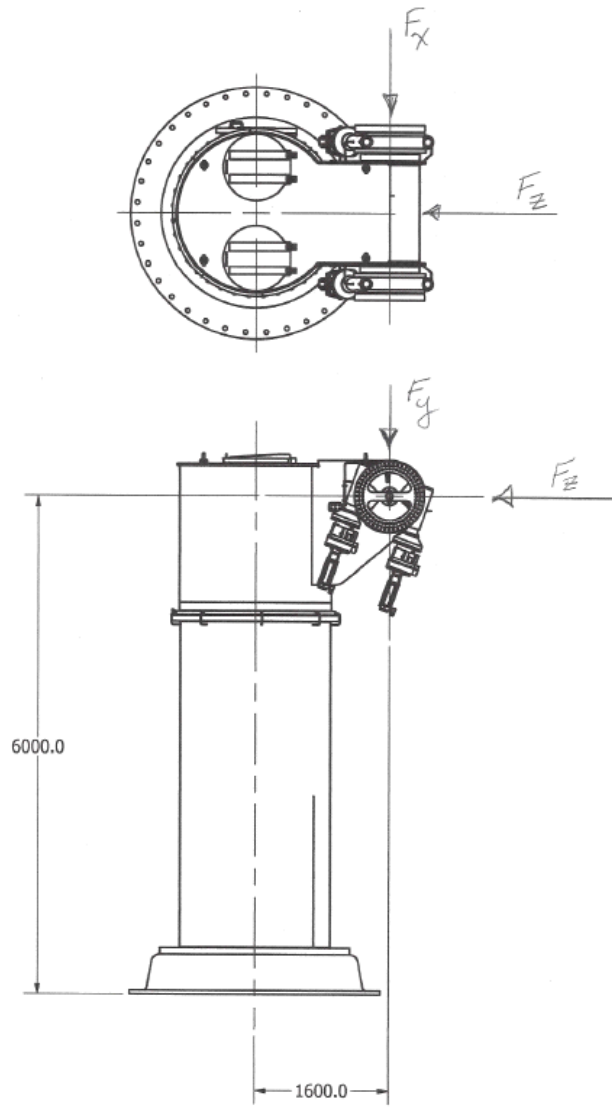
$$\text{TorqueInertiaMaxAz} = 14 \cdot \text{kip} \cdot \text{ft}$$

$$\text{TorqueInertiaMaxAz} = 19.12 \cdot \text{kN} \cdot \text{m}$$

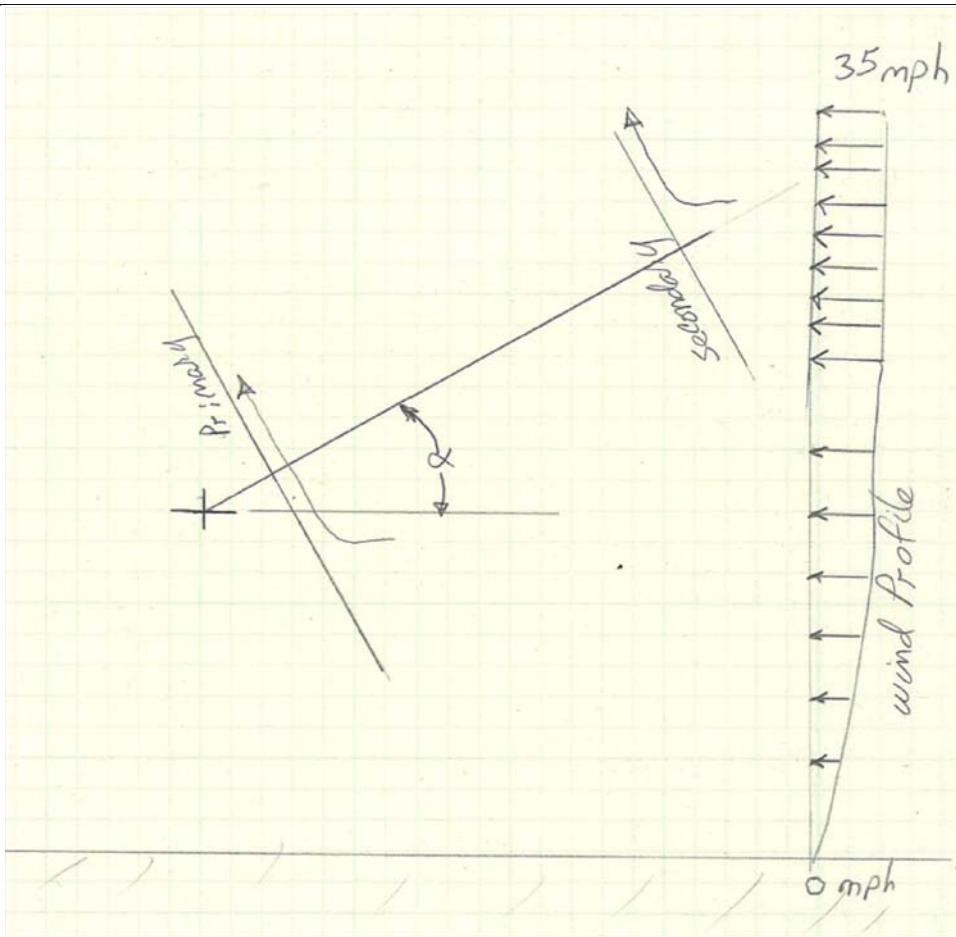
5.0 Calculate Torques and Forces Due to Wind

Assume each mirror is a flat plate at an angle to the wind





5.1 Define the Velocity Profile



Wind

Calculate the height to points on the primary/secondary as a function of the elevation angle and radial position on the mirror

$$h_{\text{Prim}}(\alpha, r_p) := \text{HeightEL} + L_p \cdot \sin(\alpha) + r_p \cdot \cos(\alpha)$$

$$h_{\text{Sec}}(\alpha, r_s) := \text{HeightEL} + L_s \cdot \sin(\alpha) + r_s \cdot \cos(\alpha)$$

**Calculate the wind torque assuming a smooth solid surface, no vortex shedding or drag
Ignore the curvature of the primary/secondary dishes and treat as flat plates**

$$\rho_{\text{air}} := 1.07 \frac{\text{kg}}{\text{m}^3} \quad \text{Density at sea level } 1.16$$

$$\alpha := 0\text{deg}..1\text{deg}..90\text{deg}$$

$$Z_{\text{ref}} := 30\text{ft}$$

$Z_o := .5ft$

Define the velocity profile.

$$\text{VelPrim}(\alpha, rp, \text{Velocity}) := \begin{cases} \text{Velocity} \cdot \frac{\ln\left(\frac{h\text{Prim}(\alpha, rp)}{Z_o}\right)}{\ln\left(\frac{Z_{\text{ref}}}{Z_o}\right)} & \text{if } h\text{Prim}(\alpha, rp) < Z_{\text{ref}} \\ \text{Velocity} & \text{otherwise} \end{cases}$$

$$\text{VelSec}(\alpha, rs, \text{Velocity}) := \begin{cases} \text{Velocity} \cdot \frac{\ln\left(\frac{h\text{Sec}(\alpha, rs)}{Z_o}\right)}{\ln\left(\frac{Z_{\text{ref}}}{Z_o}\right)} & \text{if } h\text{Sec}(\alpha, rs) < Z_{\text{ref}} \\ \text{Velocity} & \text{otherwise} \end{cases}$$

$$\text{VelH}(h, \text{Velocity}) := \begin{cases} \text{Velocity} \cdot \frac{\ln\left(\frac{h}{Z_o}\right)}{\ln\left(\frac{Z_{\text{ref}}}{Z_o}\right)} & \text{if } h < Z_{\text{ref}} \\ \text{Velocity} & \text{otherwise} \end{cases}$$

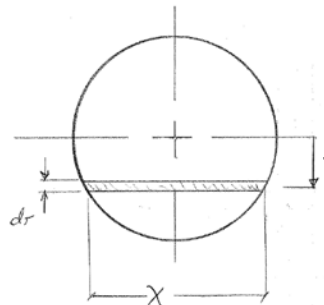
Calculate the vertical and horizontal force on the primary and secondary. Assume a conservative shape factor of 1.6

$C_d := 1.6$

Length of differential strip wind is acting on for integration

$$X_{\text{Prim}}(rp) := 2 \cdot \sqrt{\left(\frac{D_p}{2}\right)^2 - rp^2}$$

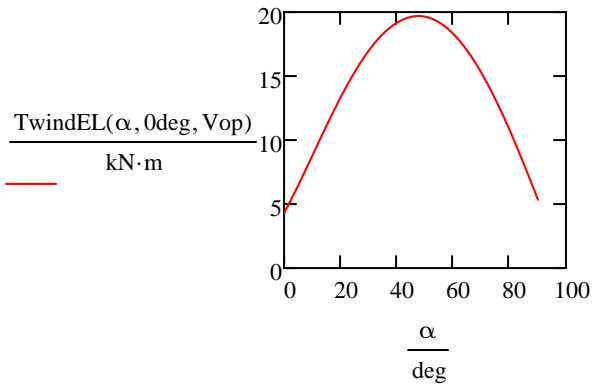
$$X_{\text{Sec}}(rs) := 2 \cdot \sqrt{\left(\frac{D_s}{2}\right)^2 - rs^2}$$



5.2 Elevation Axis Torques

Calculate the Wind Forces and Torques

$$\begin{aligned}
 \text{TwindEL}(\alpha, \beta, \text{Velocity}) := & \int_{-\frac{Dp}{2}}^{\frac{Dp}{2}} \frac{Cd \cdot \rho_{air} \cdot (\text{VelPrim}(\alpha, rp, \text{Velocity}) \cdot \cos(\beta))^2}{2} \cdot X\text{Prim}(rp) \cdot \cos(\alpha) \cdot (Lp \cdot \sin(\alpha) + rp \cdot \cos(\alpha)) \, drp \dots \\
 & + \int_{-\frac{Ds}{2}}^{\frac{Ds}{2}} \frac{Cd \cdot \rho_{air} \cdot (\text{VelSec}(\alpha, rs, \text{Velocity}) \cdot \cos(\beta))^2}{2} \cdot X\text{Sec}(rs) \cdot \cos(\alpha) \cdot (Ls \cdot \sin(\alpha) + rs \cdot \cos(\alpha)) \, drs \dots \\
 & + \frac{Cd \cdot \rho_{air} \cdot \left(\text{VelPrim} \left(\alpha, \frac{Dp}{2}, \text{Velocity} \right) \cdot \cos(\beta) \right)^2}{2} \cdot (tp \cdot Dp) \cdot \sin(\alpha) \cdot \left(Lp \cdot \sin(\alpha) - \frac{Dp}{2} \cdot \cos(\alpha) \right) \dots \\
 & + \frac{Cd \cdot \rho_{air} \cdot \left(\text{VelSec} \left(\alpha, \frac{Ds}{2}, \text{Velocity} \right) \cdot \cos(\beta) \right)^2}{2} \cdot (ts \cdot Ds) \cdot \sin(\alpha) \cdot \left(Ls \cdot \sin(\alpha) - \frac{Ds}{2} \cdot \cos(\alpha) \right)
 \end{aligned}$$



$\text{TVopEL}(\alpha, \beta) := \text{TwindEL}(\alpha, \beta, \text{Vop})$

$\alpha := 10\text{deg}$

$\beta := 0\text{deg}$

Given

$0\text{deg} < \alpha < 90\text{deg}$

$0\text{deg} < \beta < 90\text{deg}$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} := \text{Maximize}(\text{TVopEL}, \alpha, \beta)$$

$$\alpha_{\text{maxEL}} := \alpha \quad \alpha_{\text{maxEL}} = 47.64 \cdot \text{deg}$$

$$\beta_{\text{maxEL}} := \beta \quad \beta_{\text{maxEL}} = 0.00 \cdot \text{deg}$$

$$\text{TwindMaxEL}(\text{Velocity}) := \text{TwindEL}(\alpha_{\text{maxEL}}, \beta_{\text{maxEL}}, \text{Velocity})$$

$$\text{TwindMaxEL}(\text{Vop}) = 14.5 \cdot \text{kip} \cdot \text{ft} \quad \text{TwindMaxEL}(\text{Vop}) = 19.67 \cdot \text{kN} \cdot \text{m}$$

5.3 Calculate the wind torques about the Azimuth Axis

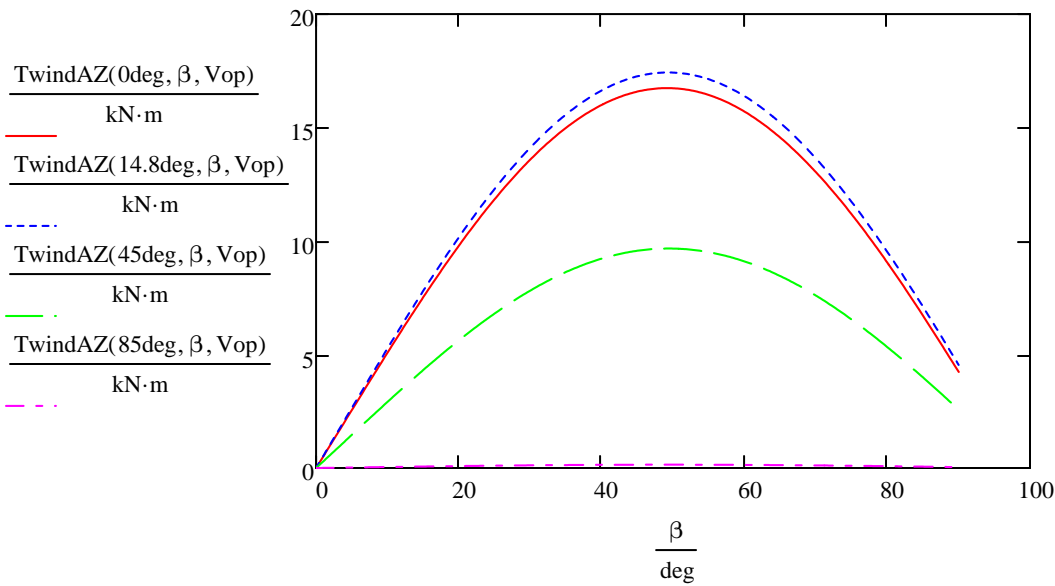
Calculate the height to points on the primary/secondary as a function of the elevation angle and radial position on the mirror

When $\beta=0\text{deg}$ then the wind is along the optical axis

$$\begin{aligned} \text{TwindAZ}(\alpha, \beta, \text{Velocity}) := & \int_{-\frac{D_p}{2}}^{\frac{D_p}{2}} \frac{\text{Cd} \cdot \rho_{\text{air}} \cdot \text{VelPrim}(\alpha, r_p, \text{Velocity})^2}{2} \cdot \text{XPrim}(r_p) \cdot \cos(\beta) \cdot \cos(\alpha) \cdot (L_p \cdot \cos(\alpha) - r_p \cdot \sin(\alpha)) \cdot \sin(\beta) \text{ drp} \dots \\ & + \int_{-\frac{D_s}{2}}^{\frac{D_s}{2}} \frac{\text{Cd} \cdot \rho_{\text{air}} \cdot \text{VelSec}(\alpha, r_s, \text{Velocity})^2}{2} \cdot \text{XSec}(r_s) \cdot \cos(\beta) \cdot \cos(\alpha) \cdot [(L_s \cdot \cos(\alpha) - r_s \cdot \sin(\alpha)) \cdot \sin(\beta)] \text{ drs} \dots \\ & + \int_{-\frac{D_p}{2}}^{\frac{D_p}{2}} \frac{\text{Cd} \cdot \rho_{\text{air}} \cdot \text{VelPrim}(\alpha, r_p, \text{Velocity})^2}{2} \cdot \text{tp} \cdot \sin(\beta) \cdot (L_p \cdot \cos(\alpha) - r_p \cdot \sin(\alpha)) \cdot \sin(\beta) \cdot \cos(\alpha) \text{ drp} \dots \\ & + \int_{-\frac{D_s}{2}}^{\frac{D_s}{2}} \frac{\text{Cd} \cdot \rho_{\text{air}} \cdot \text{VelSec}(\alpha, r_s, \text{Velocity})^2}{2} \cdot \text{ts} \cdot \sin(\beta) \cdot (L_s \cdot \cos(\alpha) - r_s \cdot \sin(\alpha)) \cdot \sin(\beta) \cdot \cos(\alpha) \text{ drs} \end{aligned}$$

$$\beta := 0.0\text{deg}, 0.2\text{deg} \dots 90\text{deg}$$

Azimuth Angle



$$TVopAZ(\alpha, \beta) := TwindAZ(\alpha, \beta, Vop)$$

$$\alpha := 10deg \quad \beta := 0deg$$

Given

$$0deg < \alpha < 90deg$$

$$0deg < \beta < 90deg$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} := \text{Maximize}(TVopAZ, \alpha, \beta)$$

$$\alpha_{maxAZ} := \alpha \quad \alpha_{maxAZ} = 10.13 \cdot deg$$

$$\beta_{maxAZ} := \beta \quad \beta_{maxAZ} = 49.27 \cdot deg$$

$$TwindMaxAZ(Velocity) := TwindAZ(\alpha_{maxAZ}, \beta_{maxAZ}, Velocity)$$

$$TwindMaxAZ(Vop) = 13.0 \cdot kip \cdot ft \quad TwindMaxAZ(Vop) = 17.61 \cdot kN \cdot m$$

5.4 Calculate the Force in the X Direction Due to Wind

$$\begin{aligned}
 F_{windX}(\alpha, \beta, Velocity) := & \int_{-\frac{Dp}{2}}^{\frac{Dp}{2}} \frac{Cd \cdot pair \cdot VelPrim(\alpha, rp, Velocity)^2}{2} \cdot XPrim(rp) \cdot \cos(\beta) \cdot \sin(\beta) \cdot \cos(\alpha) drp \dots \\
 & + \int_{-\frac{Ds}{2}}^{\frac{Ds}{2}} \frac{Cd \cdot pair \cdot VelSec(\alpha, rs, Velocity)^2}{2} \cdot XSec(rs) \cdot \cos(\beta) \cdot \sin(\beta) \cdot \cos(\alpha) drs \dots \\
 & + \int_{-\frac{Dp}{2}}^{\frac{Dp}{2}} \frac{Cd \cdot pair \cdot VelPrim(\alpha, rp, Velocity)^2}{2} \cdot tp \cdot \sin(\beta) \cdot \sin(\beta) \cdot \cos(\alpha) drp \dots \\
 & + \int_{-\frac{Ds}{2}}^{\frac{Ds}{2}} \frac{Cd \cdot pair \cdot VelSec(\alpha, rs, Velocity)^2}{2} \cdot ts \cdot \sin(\beta) \cdot \sin(\beta) \cdot \cos(\alpha) drs
 \end{aligned}$$

$$F_{pMirrorX}(\alpha, \beta, h, Velocity) := Cd \cdot pair \cdot \frac{VelH(h, Velocity)^2}{2} \cdot \frac{(\pi \cdot Dp^2)}{4Np} \cdot \cos(\beta) \cdot \cos(\alpha) \cdot \sin(\beta) + Cd \cdot pair \cdot \frac{VelH(h, Velocity)^2}{2} \cdot \frac{Dp \cdot tp}{Np} \cdot \sin(\beta)$$

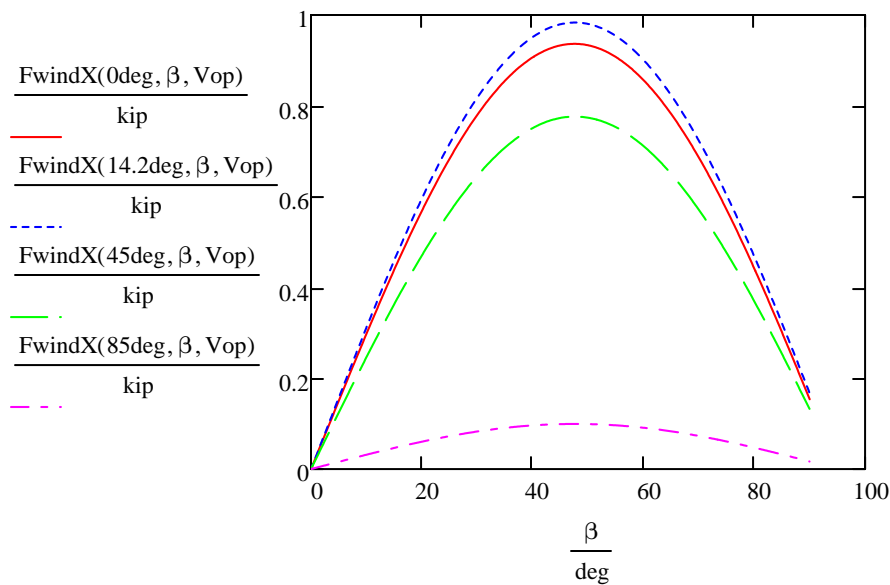
$$F_{sMirrorX}(\alpha, \beta, h, Velocity) := Cd \cdot pair \cdot \frac{VelH(h, Velocity)^2}{2} \cdot \frac{(\pi \cdot Ds^2)}{4Ns} \cdot \cos(\beta) \cdot \cos(\alpha) \cdot \sin(\beta) + Cd \cdot pair \cdot \frac{VelH(h, Velocity)^2}{2} \cdot \frac{Ds \cdot ts}{Ns} \cdot \sin(\beta)$$

$$F_{pwindX}(\alpha, \beta, Velocity) := \frac{1}{Np} \cdot \left(\int_{-\frac{Dp}{2}}^{\frac{Dp}{2}} \frac{Cd \cdot pair \cdot VelPrim(\alpha, rp, Velocity)^2}{2} \cdot XPrim(rp) \cdot \cos(\beta) \cdot \sin(\beta) \cdot \cos(\alpha) drp \dots \right. \\
 \left. + \int_{-\frac{Dp}{2}}^{\frac{Dp}{2}} \frac{Cd \cdot pair \cdot VelPrim(\alpha, rp, Velocity)^2}{2} \cdot tp \cdot \sin(\beta) \cdot \sin(\beta) \cdot \cos(\alpha) drp \right)$$

$$F_{windX}(\alpha, \beta, Velocity) := \frac{1}{N_s} \cdot \left(\int_{-\frac{D_s}{2}}^{\frac{D_s}{2}} \frac{Cd \cdot \rho_{air} \cdot VelSec(\alpha, rs, Velocity)^2}{2} \cdot XSec(rs) \cdot \cos(\beta) \cdot \sin(\beta) \cdot \cos(\alpha) \, drs \dots \right. \\ \left. + \int_{-\frac{D_s}{2}}^{\frac{D_s}{2}} \frac{Cd \cdot \rho_{air} \cdot VelSec(\alpha, rs, Velocity)^2}{2} \cdot ts \cdot \sin(\beta) \cdot \sin(\beta) \cdot \cos(\alpha) \, drs \right)$$

$\beta := 0.0deg, 0.2deg .. 90deg$

Azimuth Angle



$F_{VopX}(\alpha, \beta) := F_{windX}(\alpha, \beta, Vop)$

$\alpha := 10deg$

$\beta := 0deg$

Given

$0deg < \alpha < 90deg$

$0deg < \beta < 90deg$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} := \text{Maximize}(\text{FVopX}, \alpha, \beta)$$

$$\alpha_{\text{maxEL}} := \alpha \quad \alpha_{\text{maxEL}} = 14.19 \cdot \text{deg}$$

$$\beta_{\text{maxEL}} := \beta \quad \beta_{\text{maxEL}} = 47.66 \cdot \text{deg}$$

$$\text{FwindMaxX}(\text{Velocity}) := \text{FwindX}(\alpha_{\text{maxEL}}, \beta_{\text{maxEL}}, \text{Velocity})$$

$$\text{FwindMaxX}(\text{Vop}) = 1.0 \cdot \text{kip} \quad \text{FwindMaxX}(\text{Vop}) = 4.37 \cdot \text{kN}$$

5.5 Calculate the Force in the Z Direction Due to Wind

$$\begin{aligned} \text{FwindZ}(\alpha, \beta, \text{Velocity}) := & \int_{-\frac{Dp}{2}}^{\frac{Dp}{2}} \frac{\text{Cd} \cdot \text{pair} \cdot \text{VelPrim}(\alpha, \text{rp}, \text{Velocity})^2}{2} \cdot \text{XPrim}(\text{rp}) \cdot \cos(\beta) \cdot \cos(\beta) \cdot \cos(\alpha) \text{ drp} \dots \\ & + \int_{-\frac{Ds}{2}}^{\frac{Ds}{2}} \frac{\text{Cd} \cdot \text{pair} \cdot (\text{VelSec}(\alpha, \text{rs}, \text{Velocity}))^2}{2} \cdot \text{XSec}(\text{rs}) \cdot \cos(\alpha) \cdot \cos(\beta) \cdot \cos(\beta) \text{ drs} \dots \\ & + \frac{\text{Cd} \cdot \text{pair} \cdot \left(\text{VelPrim} \left(\alpha, \frac{Dp}{2}, \text{Velocity} \right) \right)^2}{2} \cdot (\text{tp} \cdot Dp) \cdot \sin(\alpha) \cdot \cos(\beta) \cdot \cos(\beta) \dots \\ & + \frac{\text{Cd} \cdot \text{pair} \cdot \left(\text{VelSec} \left(\alpha, \frac{Ds}{2}, \text{Velocity} \right) \right)^2}{2} \cdot (\text{ts} \cdot Ds) \cdot \sin(\alpha) \cdot \cos(\beta) \cdot \cos(\beta) \end{aligned}$$

$$\text{FpMirrorZ}(\alpha, \beta, h, \text{Velocity}) := \text{Cd} \cdot \text{pair} \cdot \frac{\text{VelH}(h, \text{Velocity})^2}{2} \cdot \frac{(\pi \cdot Dp^2)}{4Np} \cdot \cos(\beta) \cdot \cos(\alpha) \cdot \cos(\beta)$$

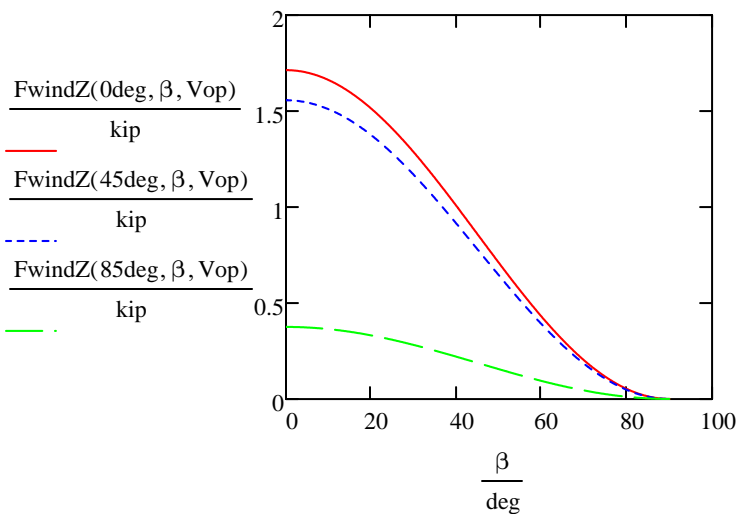
$$\text{FsMirrorZ}(\alpha, \beta, h, \text{Velocity}) := \text{Cd} \cdot \text{pair} \cdot \frac{\text{VelH}(h, \text{Velocity})^2}{2} \cdot \frac{(\pi \cdot Ds^2)}{4Ns} \cdot \cos(\beta) \cdot \cos(\alpha) \cdot \cos(\beta)$$

$$F_{pwindZ}(\alpha, \beta, Velocity) := \frac{1}{N_p} \cdot \left[\int_{-\frac{D_p}{2}}^{\frac{D_p}{2}} \frac{Cd \cdot \rho_{air} \cdot VelPrim(\alpha, r_p, Velocity)^2}{2} \cdot XPrim(r_p) \cdot \cos(\beta) \cdot \cos(\beta) \cdot \cos(\alpha) dr_p \dots \right. \\ \left. + \frac{Cd \cdot \rho_{air} \cdot \left(VelPrim\left(\alpha, \frac{D_p}{2}, Velocity\right) \right)^2}{2} \cdot (t_p \cdot D_p) \cdot \sin(\alpha) \cdot \cos(\beta) \cdot \cos(\beta) \right]$$

$$F_{swindZ}(\alpha, \beta, Velocity) := \frac{1}{N_s} \cdot \left[\int_{-\frac{D_s}{2}}^{\frac{D_s}{2}} \frac{Cd \cdot \rho_{air} \cdot (VelSec(\alpha, r_s, Velocity))^2}{2} \cdot XSec(r_s) \cdot \cos(\alpha) \cdot \cos(\beta) \cdot \cos(\beta) dr_s \dots \right. \\ \left. + \frac{Cd \cdot \rho_{air} \cdot \left(VelSec\left(\alpha, \frac{D_s}{2}, Velocity\right) \right)^2}{2} \cdot (t_s \cdot D_s) \cdot \sin(\alpha) \cdot \cos(\beta) \cdot \cos(\beta) \right]$$

$\beta := 0.0deg, 0.2deg .. 90deg$

Azimuth Angle



Calculate the Maximum Force in Z

Initial Guess

$$\alpha := 10\text{deg} \quad \beta := 10\text{deg}$$

$$\text{FwindMaxZ}(\alpha, \beta) := \text{FwindZ}(\alpha, \beta, \text{Vop})$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} := \text{Maximize}(\text{FwindMaxZ}, \alpha, \beta)$$

$$\alpha\text{FZmax} := \alpha \quad \alpha\text{FZmax} = 17.81 \cdot \text{deg}$$

$$\beta\text{FZmax} := \beta \quad \beta\text{FZmax} = -0.00 \cdot \text{deg}$$

$$\text{FwindMaxZ}(\text{Velocity}) := \text{FwindZ}(\alpha\text{FZmax}, \beta\text{FZmax}, \text{Velocity})$$

$$\text{FwindMaxZ}(\text{Vop}) = 1.8 \cdot \text{kip} \quad \text{FwindMaxZ}(\text{Vop}) = 8.22 \cdot \text{kN}$$

6.0 Calculate Torques and Forces For All Wind Velocities

6.1 Tracking Wind Velocity 36km/hr

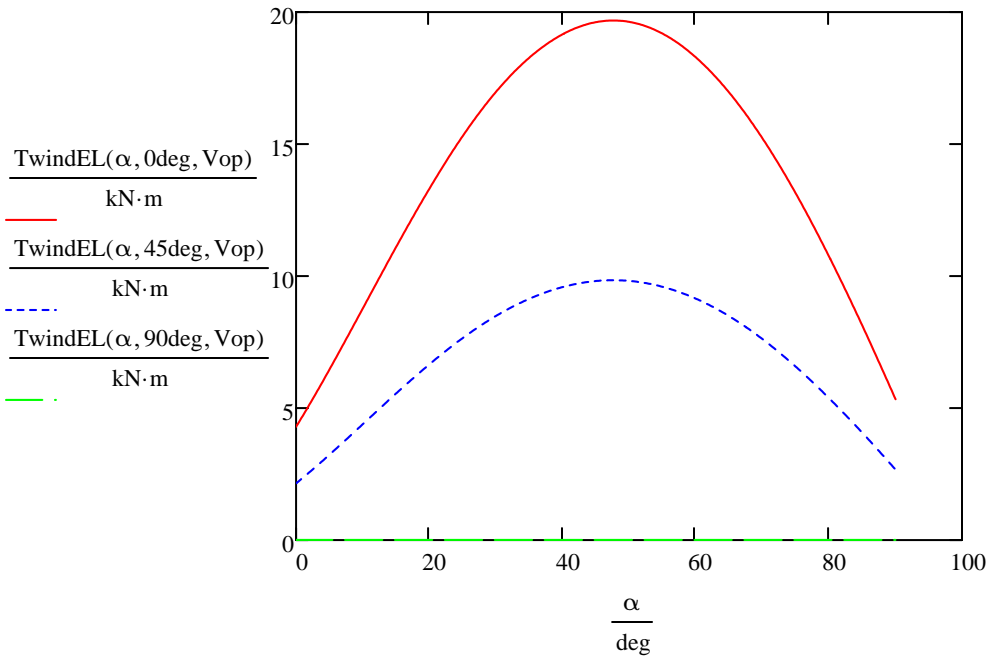
Elevation Torque

$$\text{TwindMaxEL}(\text{Vop}) = 19.67 \cdot \text{kN} \cdot \text{m} \quad \text{TwindMaxEL}(\text{Vop}) = 14.51 \cdot \text{kip} \cdot \text{ft}$$

$$\text{TmaxEL} := \text{TwindMaxEL}(\text{Vop}) + \text{TfrictionEL} + \text{TorqueInertiaMaxEl}$$

$$\text{TmaxEL} = 41.28 \cdot \text{kN} \cdot \text{m} \quad \text{TmaxEL} = 30.45 \cdot \text{kip} \cdot \text{ft}$$

$$\alpha := 0\text{deg}, 0.2\text{deg} .. 90\text{deg}$$



Azimuth Torque

$$T_{windMaxAZ}(Vop) = 17.61 \cdot \text{kN} \cdot \text{m}$$

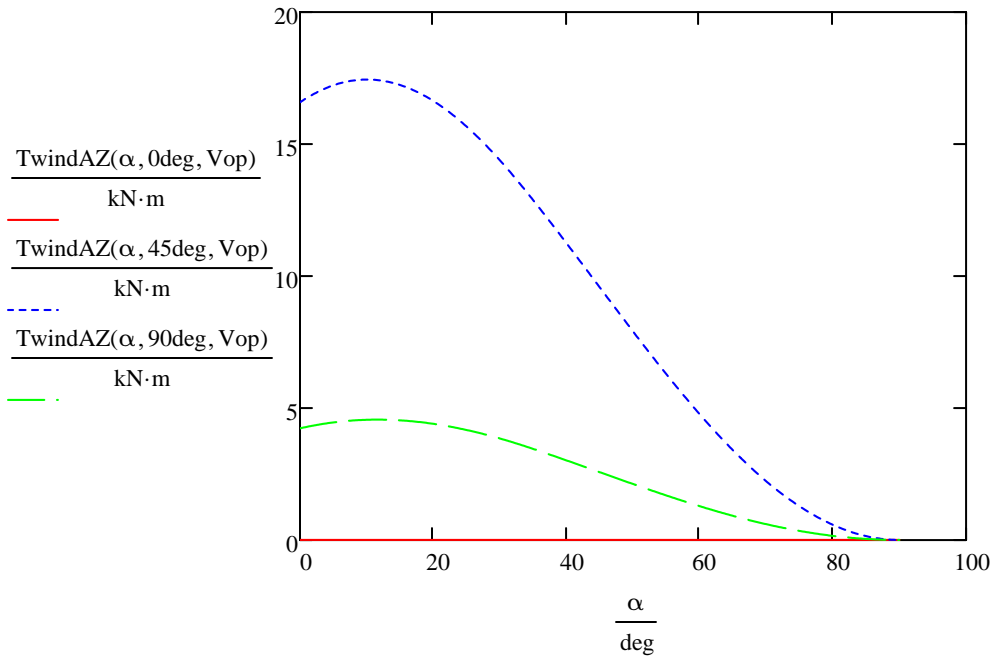
$$T_{windMaxAZ}(Vop) = 12.99 \cdot \text{kip} \cdot \text{ft}$$

$$T_{maxAZ} := T_{windMaxAZ}(Vop) + T_{frictionAZ} + T_{orqueInertiaMaxAz}$$

$$T_{maxAZ} = 58.12 \cdot \text{kN} \cdot \text{m}$$

$$T_{maxAZ} = 42.87 \cdot \text{kip} \cdot \text{ft}$$

$$\alpha := 0deg, 0.2deg .. 90deg$$



$$F_{windMaxX}(Vop) = 4374.18 \text{ N}$$

$$F_{windMaxX}(Vop) = 0.98 \cdot \text{kip}$$

$$F_{windMaxZ}(Vop) = 8215.08 \text{ N}$$

$$F_{windMaxZ}(Vibc) = 29.90 \cdot \text{kip}$$

$$y_n := -9\text{ft}$$

Primary mirror wind loads

$$F_{pMirrorX}(0\text{deg}, 0\text{deg}, y_n + 19.7\text{ft}, Vibc) = 0.00 \cdot \text{kip}$$

$$F_{pMirrorZ}(0\text{deg}, 0\text{deg}, y_n + 19.7\text{ft}, Vop) = 0.02 \cdot \text{kip}$$

$$F_{pwindX}(35\text{deg}, 90\text{deg}, Vibc) = 0.02 \cdot \text{kip}$$

$$F_{pwindZ}(0\text{deg}, 0\text{deg}, Vop) = 0.03 \cdot \text{kip}$$

Secondary mirror wind loads

$$F_{sMirrorX}(35\text{deg}, 90\text{deg}, y_n + 19.7\text{ft}, Vibc) = 0.03 \cdot \text{kip}$$

$$F_{sMirrorZ}(35\text{deg}, 90\text{deg}, y_n + 19.7\text{ft}, Vibc) = 0.00 \cdot \text{kip}$$

$$F_{swindX}(35\text{deg}, 90\text{deg}, Vibc) = 0.06 \cdot \text{kip}$$

$$F_{swindZ}(0\text{deg}, 0\text{deg}, Vop) = 0.02 \cdot \text{kip}$$

6.2 Slewing Wind Velocity 50km/hr

Elevation Torque

$$T_{windMaxEL}(V_{max}) = 37.95 \cdot \text{kN} \cdot \text{m}$$

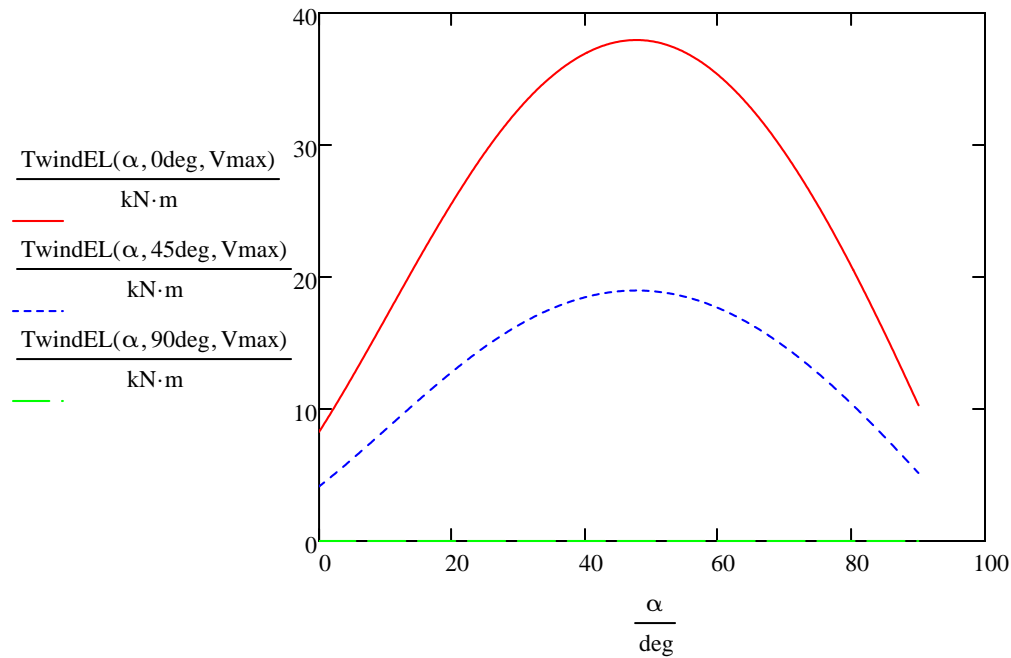
$$T_{windMaxEL}(V_{max}) = 27.99 \cdot \text{kip} \cdot \text{ft}$$

$$T_{maxEL} := T_{windMaxEL}(V_{max}) + T_{frictionEL} + \text{TorqueInertiaMaxEl}$$

$$T_{maxEL} = 59.56 \cdot \text{kN} \cdot \text{m}$$

$$T_{maxEL} = 43.93 \cdot \text{kip} \cdot \text{ft}$$

$$\alpha := 0\text{deg}, 0.2\text{deg} .. 90\text{deg}$$



Azimuth Torque

$$T_{windMaxAZ}(V_{max}) = 33.97 \cdot \text{kN} \cdot \text{m}$$

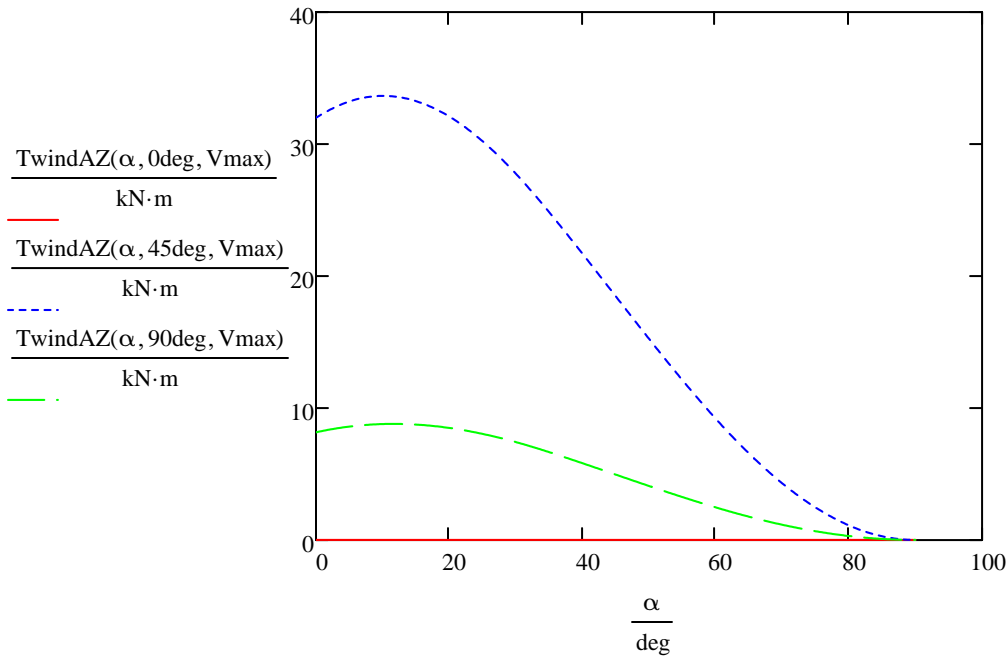
$$T_{windMaxAZ}(V_{max}) = 25.05 \cdot \text{kip} \cdot \text{ft}$$

$$T_{maxAZ} := T_{windMaxAZ}(V_{max}) + T_{frictionAZ} + \text{TorqueInertiaMaxAz}$$

$$T_{maxAZ} = 74.48 \cdot \text{kN} \cdot \text{m}$$

$$T_{maxAZ} = 54.94 \cdot \text{kip} \cdot \text{ft}$$

$$\alpha := 0\text{deg}, 0.2\text{deg} .. 90\text{deg}$$



$$F_{windMaxX}(V_{max}) = 8437.85 \text{ N}$$

$$F_{windMaxX}(V_{max}) = 1.90 \cdot \text{kip}$$

$$F_{windMaxZ}(V_{max}) = 15846.99 \text{ N}$$

$$F_{windMaxZ}(V_{max}) = 3.56 \cdot \text{kip}$$

6.3 CTA Survival Wind Velocity 120km/hr

Under survival wind conditions the telescope will not translate so the inertia and friction torques are not considered

Elevation Torque

$$T_{windMaxEL}(V_{survival}) = 218.61 \cdot \text{kN} \cdot \text{m}$$

$$T_{windMaxEL}(V_{survival}) = 161.24 \cdot \text{kip} \cdot \text{ft}$$

$$TEL_{park1} := T_{windEL}(\alpha_{park1}, 0deg, V_{survival})$$

Elevation Axis wind torque at the Park Position 1

$$TEL_{park1} = 123.17 \cdot \text{kN} \cdot \text{m}$$

$$TEL_{park1} = 90.84 \cdot \text{kip} \cdot \text{ft}$$

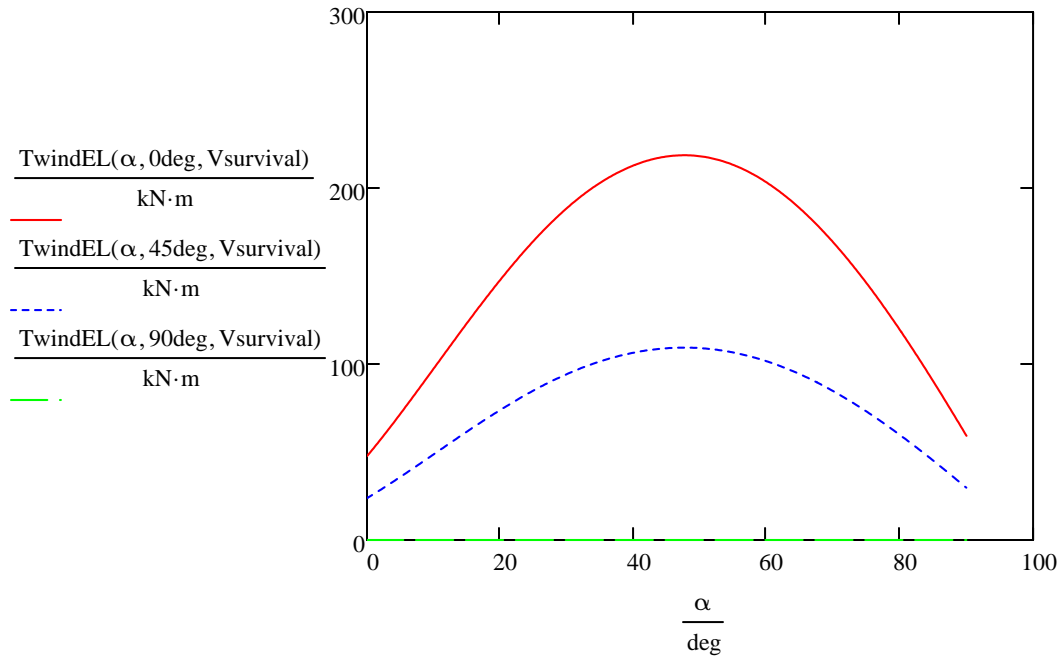
$$TEL_{park2} := T_{windEL}(\alpha_{park2}, 0deg, V_{survival})$$

Elevation Axis wind torque at the Park Position 2

$$TEL_{park2} = 202.92 \cdot \text{kN} \cdot \text{m}$$

$$TEL_{park2} = 149.67 \cdot \text{kip} \cdot \text{ft}$$

$\alpha := 0\text{deg}, 0.2\text{deg} .. 90\text{deg}$



Azimuth Torque

$$\text{TwindMaxAZ}(V_{\text{survival}}) = 195.64 \cdot \text{kN}\cdot\text{m}$$

$$\text{TwindMaxAZ}(V_{\text{survival}}) = 144.30 \cdot \text{kip}\cdot\text{ft}$$

$$\text{Tazpark1} := \text{TwindAZ}(\alpha_{\text{park1}}, 45\text{deg}, V_{\text{survival}})$$

Azimuth Axis wind torque at the Park Position 1

$$\text{Tazpark1} = 191.49 \cdot \text{kN}\cdot\text{m}$$

$$\text{Tazpark1} = 141.24 \cdot \text{kip}\cdot\text{ft}$$

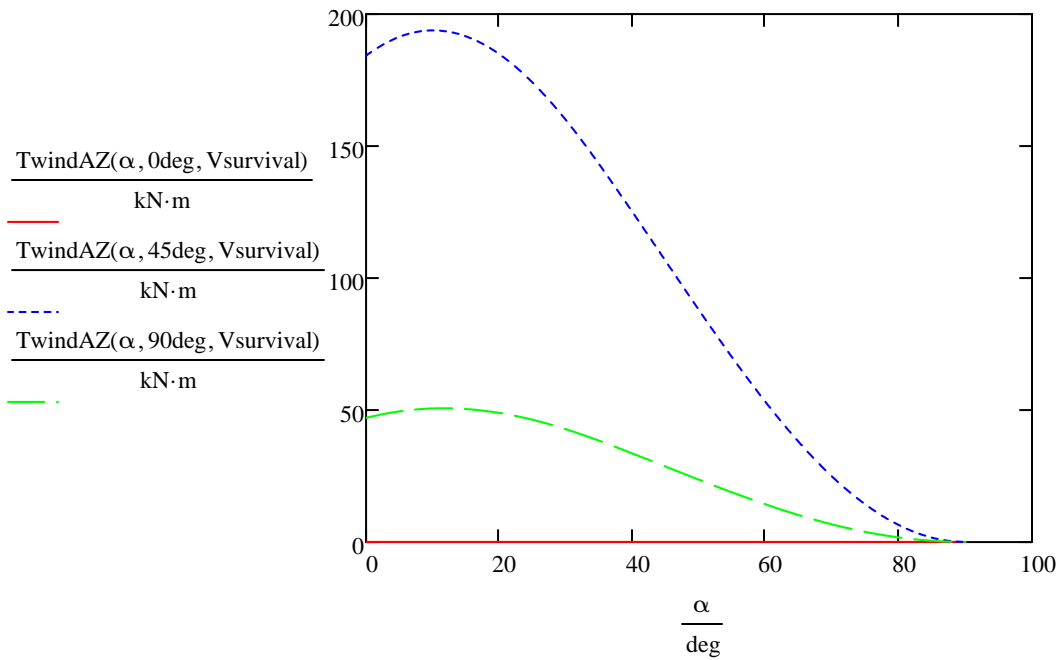
$$\text{Tazpark2} := \text{TwindEL}(\alpha_{\text{park2}}, 45\text{deg}, V_{\text{survival}})$$

Azimuth Axis wind torque at the Park Position 2

$$\text{Tazpark2} = 101.46 \cdot \text{kN}\cdot\text{m}$$

$$\text{Tazpark2} = 74.83 \cdot \text{kip}\cdot\text{ft}$$

$\alpha := 0\text{deg}, 0.2\text{deg} .. 90\text{deg}$



6.4 IBC Survival Wind Velocity 90mph

Under survival wind conditions the telescope will not translate so the inertia and friction torques are not considered

Elevation Torque

$$T_{windMaxEL}(V_{ibc}) = 318.48 \cdot \text{kN} \cdot \text{m}$$

$$T_{windMaxEL}(V_{ibc}) = 234.90 \cdot \text{kip} \cdot \text{ft}$$

$$TEL_{park1} := T_{windEL}(35deg, 0deg, V_{ibc})$$

Elevation Axis wind torque at the Park Position 1

$$TEL_{park1} = 295.63 \cdot \text{kN} \cdot \text{m}$$

$$TEL_{park1} = 218.05 \cdot \text{kip} \cdot \text{ft}$$

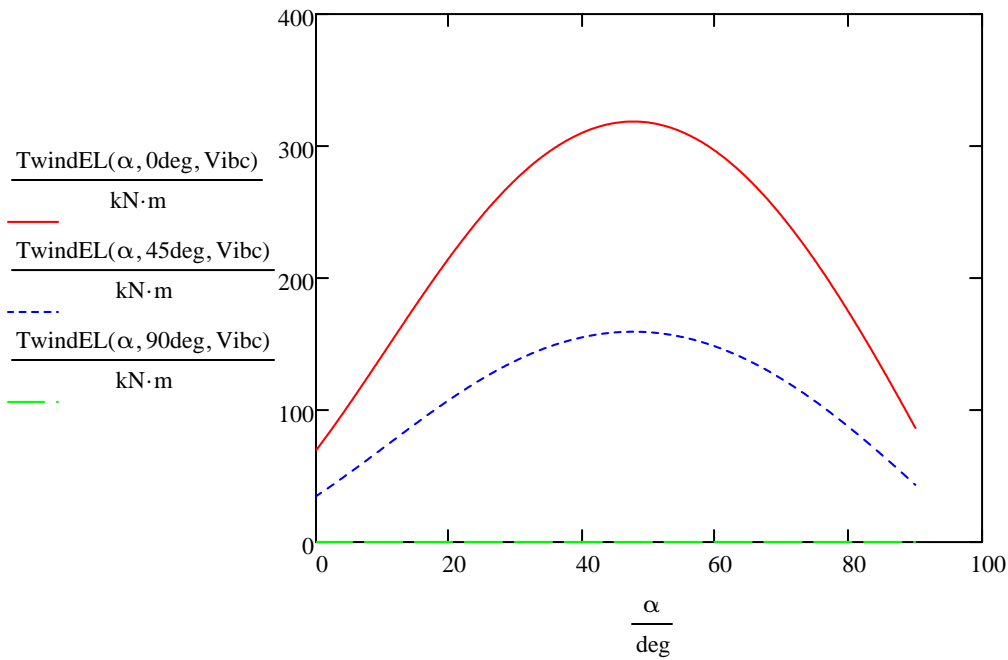
$$TEL_{park2} := T_{windEL}(\alpha_{park2}, 0deg, V_{ibc})$$

Elevation Axis wind torque at the Park Position 2

$$TEL_{park2} = 295.63 \cdot \text{kN} \cdot \text{m}$$

$$TEL_{park2} = 218.05 \cdot \text{kip} \cdot \text{ft}$$

$$\alpha := 0deg, 0.2deg .. 90deg$$



Azimuth Torque

$$\text{TwindMaxAZ}(\text{Vibc}) = 285.03 \cdot \text{kN} \cdot \text{m}$$

$$\text{TwindMaxAZ}(\text{Vibc}) = 210.23 \cdot \text{kip} \cdot \text{ft}$$

$$\text{T AZpark1} := \text{TwindAZ}(-15\text{deg}, 0\text{deg}, \text{Vibc})$$

Azimuth Axis wind torque at the Park Position 1

$$\text{T AZpark1} = 0.00 \cdot \text{kN} \cdot \text{m}$$

$$\text{T AZpark1} = 0.00 \cdot \text{kip} \cdot \text{ft}$$

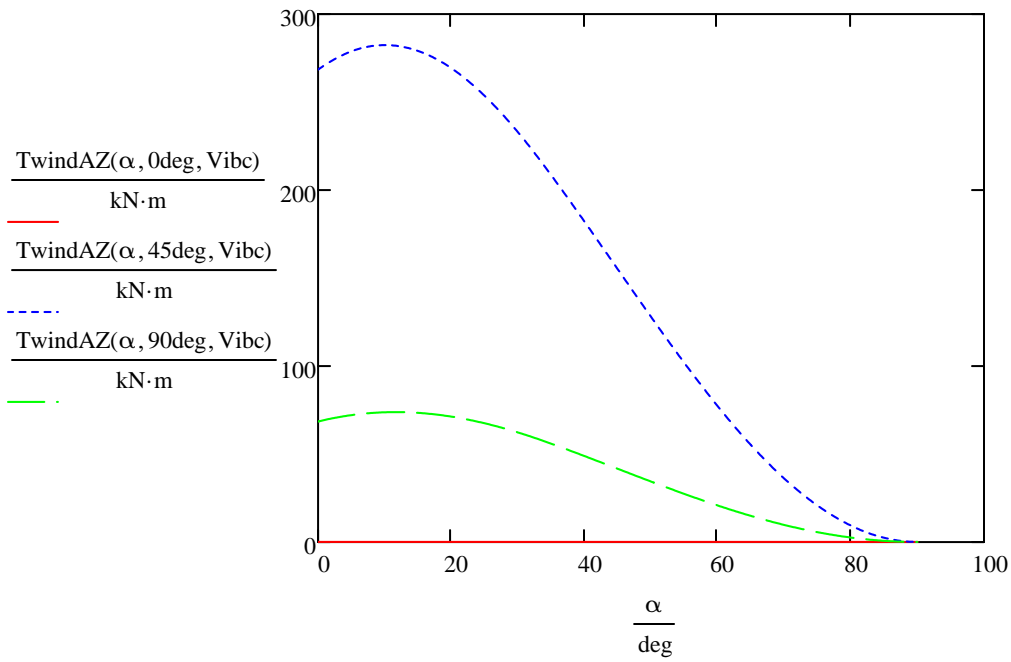
$$\text{T AZpark2} := \text{TwindEL}(\alpha_{\text{park2}}, 45\text{deg}, \text{Vibc})$$

Azimuth Axis wind torque at the Park Position 2

$$\text{T AZpark2} = 147.82 \cdot \text{kN} \cdot \text{m}$$

$$\text{T AZpark2} = 109.02 \cdot \text{kip} \cdot \text{ft}$$

$$\alpha := 0\text{deg}, 0.2\text{deg} .. 90\text{deg}$$



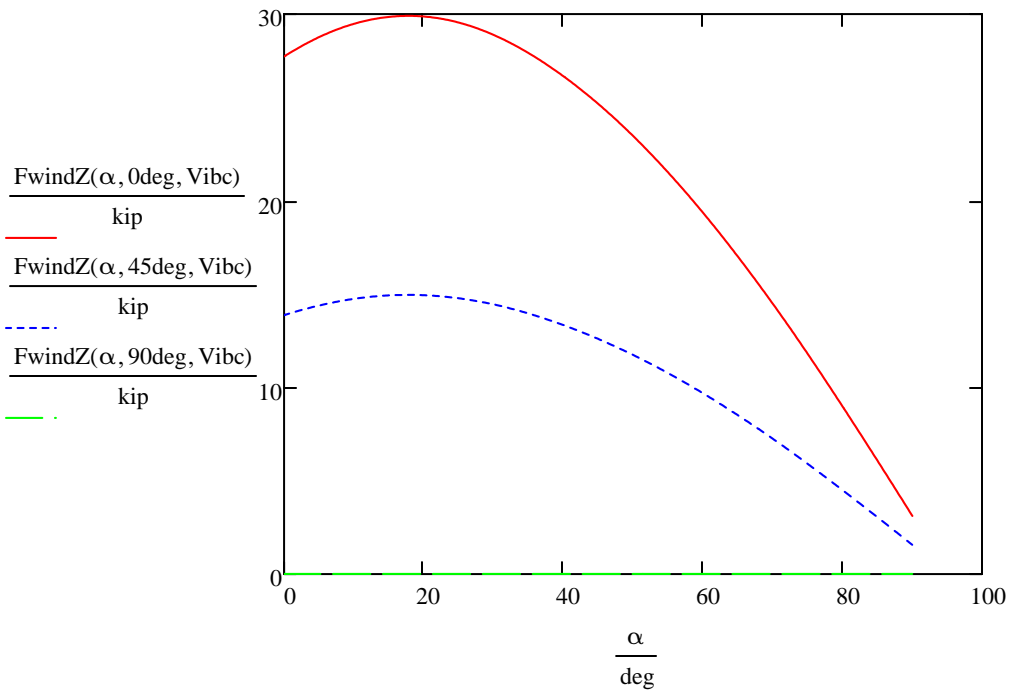
Force in Z direction

$F_{windZ}(35deg, 0deg, Vibc) = 124.13 \cdot kN$

$F_{windZ}(35deg, 0deg, Vibc) = 27.91 \cdot kip$

$F_{windMaxZ}(Vibc) = 132.98 \cdot kN$

$F_{windMaxZ}(Vibc) = 29.90 \cdot kip$



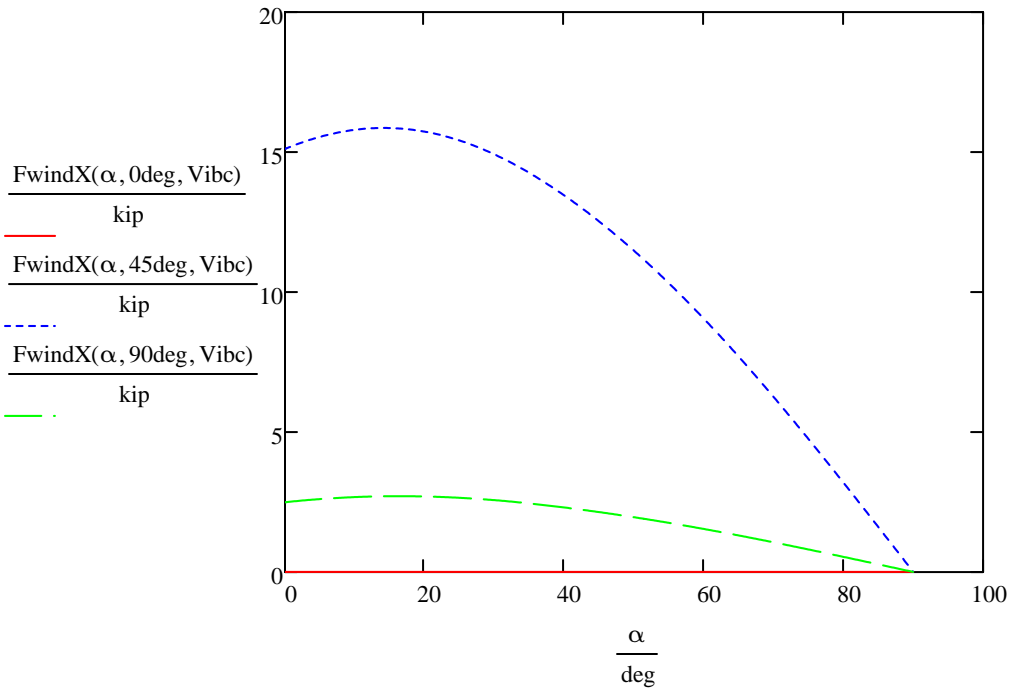
Force in X Direction

$F_{windX}(35deg, 47.6deg, Vibc) = 63.64 \cdot kN$

$F_{windX}(35deg, 47.6deg, Vibc) = 14.31 \cdot kip$

$F_{windMaxX}(Vibc) = 70.81 \cdot kN$

$F_{windMaxX}(Vibc) = 15.92 \cdot kip$



7.0 Calculate Concrete Foundation Strength at the 15degree park position

7.1 Inputs

$W_t := W_{total} + W_{positioner} + W_{yoke}$

Total Weight of OSS, Counterweight and Positioner

$W_t = 41.72 \cdot \text{ton}$

HeightEL = 6.00 m

Height of the elevation axis above the ground

Lelevation := 1.6m

Distance from elevation axis to centerline of tower/foundation

$$q_{soil} := 3000 \frac{\text{lb}}{\text{ft}^2}$$

Define Concrete and Foundation Properties

$\phi_f := 0.9$

$f_c := 4\text{ksi}$

Strength of concrete

$f_y := 60\text{ksi}$

Yield strength of steel reinforcement

$d_b := 25\text{mm}$

Diameter of reinforcement steel

$$A_b := \frac{\pi \cdot d_b^2}{4}$$

$Ab = 0.76 \cdot \text{in}^2$ Area of reinforcement bar

$\text{nbars} := 17$ Number of bars in cross section

$As := \text{nbars} \cdot Ab$ $As = 12.93 \cdot \text{in}^2$ Total steel area in cross section

$\text{dflange} := 2134\text{mm}$ Diameter of bolt circle on flange

$\text{dbf} := 1.25\text{in}$ Diameter of bolts on flange

$\text{nfbolts} := 24$ Number of flange bolts

$Df := 5\text{m}$ Diameter of foundation

$r_f := \frac{Df}{2}$

$\text{depth} := 800\text{mm}$ Depth of Foundation

$d := 800\text{mm} - 75\text{mm}$ Depth of steel

$d = 28.54 \cdot \text{in}$

$r_1 := \frac{\text{dflange}}{2}$

$A_f := \frac{\pi \cdot Df^2}{4}$ Bearing Area of foundation

$\text{PuFoundation} := 150 \frac{\text{lbf}}{\text{ft}^3} \cdot A_f \cdot \text{depth}$

$\text{PuFoundation} = 83.2 \cdot \text{kip}$

Define Concrete Breakout Strength of Bolts

$k_c := 24$ Cast in place bolts -- D5.2.1

$\phi_{bt} := 0.75$

$\text{hef} := 600\text{mm}$ Effective length of bolt in concrete

$$N_b := k_c \cdot \left(\frac{\text{hef}}{\text{in}} \right)^{1.5} \cdot \sqrt{\frac{\text{fc}}{\text{psi}}} \cdot \text{lbf}$$

$$N_b = 174.3 \cdot \text{kip}$$

7.2 Calculate the Maximum Moment on Foundation

Survival Wind Loading -- at -15degree elevation angle and 0degree Azimuth

$$F_z := F_{\text{windZ}}(15\text{deg}, 0\text{deg}, \text{Vibc})$$

$$F_z = 132.73 \cdot \text{kN} \quad F_z = 29.84 \cdot \text{kip}$$

$$F_y := W_t \cdot g$$

$$F_x := F_{\text{windX}}(15\text{deg}, 0\text{deg}, \text{Vibc})$$

$$F_x = 0.00 \cdot \text{kip}$$

$$M_x := T_{\text{windEL}}(15\text{deg}, 0\text{deg}, \text{Vibc})$$

$$M_x = 132.35 \cdot \text{kip} \cdot \text{ft} \quad \text{Moment about the elevation axis}$$

$$M_y := T_{\text{windAZ}}(15\text{deg}, 0\text{deg}, \text{Vibc})$$

$$M_y = 0.00 \cdot \text{kip} \cdot \text{ft} \quad \text{Moment about the azimuth axis}$$

$$M_u := 1.2W_{\text{total}} \cdot g \cdot \text{Lelevation} + 1.6F_z \cdot \text{HeightEL} + 1.6M_x + 1.2W_{\text{yoke}} \cdot g \cdot \text{Lelevation}$$

$$M_u = 1518.48 \cdot \text{kip} \cdot \text{ft} \quad M_u = 2058.79 \cdot \text{kN} \cdot \text{m}$$

$$M_u' := W_{\text{total}} \cdot g \cdot \text{Lelevation} + F_z \cdot \text{HeightEL} + M_x + W_{\text{yoke}} \cdot g \cdot \text{Lelevation}$$

$$M_u' = 1025.49 \cdot \text{kip} \cdot \text{ft}$$

$$P_u := 1.2 \cdot F_y + 1.2 \cdot P_{\text{uFoundation}}$$

$$P_u = 199.98 \cdot \text{kip} \quad P_u = 889.57 \cdot \text{kN}$$

$$P_u' := F_y + P_{\text{uFoundation}}$$

$$P_u' = 166.65 \cdot \text{kip}$$

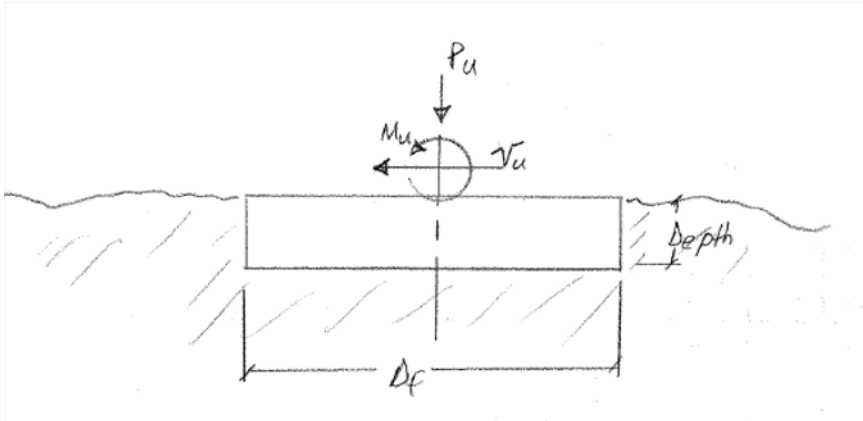
$$V_u := 1.6Fz$$

$$V_u' := Fz$$

$$e := \frac{M_u'}{P_u'}$$

$$e = 1.88 \text{ m}$$

$$e = 6.15 \cdot \text{ft}$$



7.3 Foundation Strength

7.3.1 Soil Pressure

Assume that soil is in compression under the entire foundation -- solve for the soil pressure

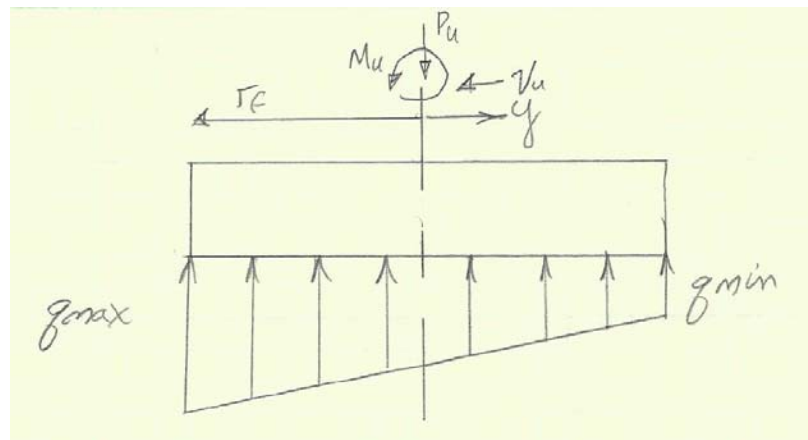
$$I_f := \frac{\pi \cdot D_f^4}{64}$$

$$q_{\max} := \frac{P_u'}{A_f} + \frac{M_u' \cdot \frac{D_f}{2}}{I_f}$$

$$q_{\max} = 3.15 \cdot \frac{\text{kip}}{\text{ft}^2}$$

$$q_{\min} := \frac{P_u'}{A_f} - \frac{M_u' \cdot \frac{D_f}{2}}{I_f}$$

$$q_{\min} = -1577.76 \cdot \frac{\text{lbf}}{\text{ft}^2}$$



q_{min} is negative which is not possible -- assume a soil pressure distribution shown below. Solve for max soil pressure.

$$e := \frac{Mu'}{Pu'}$$

$$e = 1.88 \text{ m}$$

$$e = 6.15 \text{ ft}$$

Initial Guess of max soil pressure and distribution

$$q_{max} := 1 \frac{\text{kip}}{\text{ft}^2}$$

$$a := 1 \text{ m}$$

Given

$$Pu' = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} dy$$

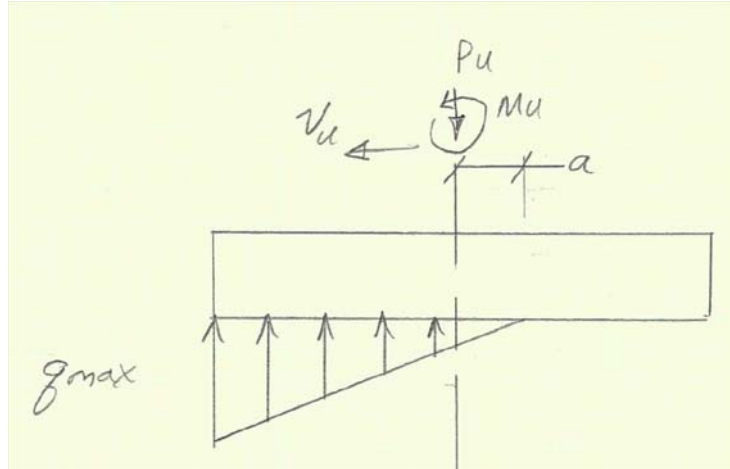
$$Mu' = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot y dy$$

$$\left(\begin{matrix} q_{max} \\ a \end{matrix} \right) := \text{Find}(q_{max}, a)$$

$$q_{max} = 7.65 \frac{\text{kip}}{\text{ft}^2}$$

$$a = -1.01 \text{ m}$$

An assumed soil strength value of 3kips/ft² is listed on the foundation drawings but the actual value is not known



7.3.2 Calculate Strength of Concrete bending using factored values .

Check strength of concrete in bending at flange mounting plate

$$r_d := r_1$$

Radius where max bending moment is checked.

Recalculate the soil pressure using P_u and M_u with load factors applied

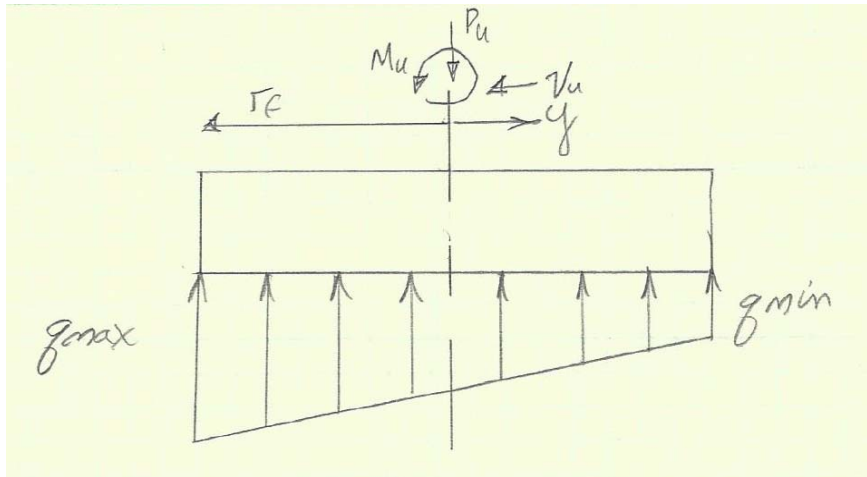
Calculate max soil pressure

$$q_{max} := \frac{P_u}{A_f} + \frac{M_u \cdot \frac{D_f}{2}}{I_f}$$

$$q_{max} = 4.5 \cdot \frac{\text{kip}}{\text{ft}^2}$$

$$q_{min} := \frac{P_u}{A_f} - \frac{M_u \cdot \frac{D_f}{2}}{I_f}$$

$$q_{min} = -2.56 \cdot \frac{\text{kip}}{\text{ft}^2}$$



q_{min} is negative which is not possible - assume a soil pressure distribution shown below. Solve for max soil pressure..

$$e := \frac{M_u}{P_u}$$

$$e = 2.31 \text{ m}$$

$$e = 7.59 \cdot \text{ft}$$

Initial Guess of soil pressure

$$q_{max} := 18 \frac{\text{kip}}{\text{ft}^2}$$

$$a := -1 \text{ m}$$

Given

$$P_u = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} dy$$

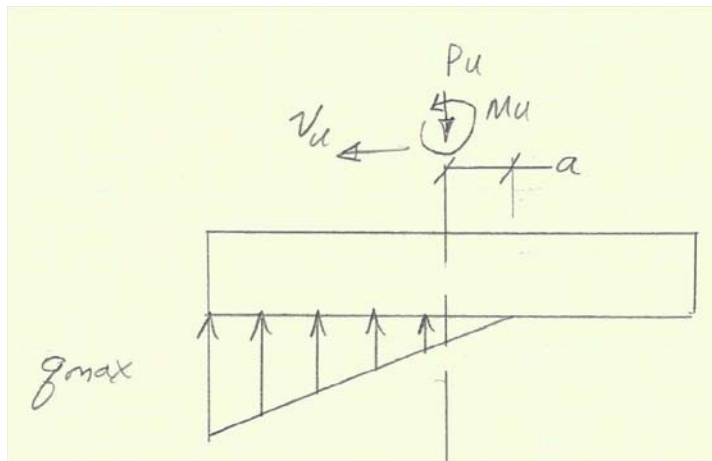
$$M_u = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot y dy$$

$$\left(\begin{matrix} q_{max} \\ a \end{matrix} \right) := \text{Find}(q_{max}, a)$$

$$q_{max} = 55.23 \cdot \frac{\text{kip}}{\text{ft}^2}$$

Maximum soil pressure

$$a = -2.06 \text{ m}$$



Use the soil pressure calculated with load factors to find the moment in the concrete

$$\text{Muconcrete} := \int_{rd}^{rf} (q_{\max}) \cdot \left(\frac{a+y}{rf+a} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot (y - rd) \, dy$$

Muconcrete = -2035.09 · ft · kip

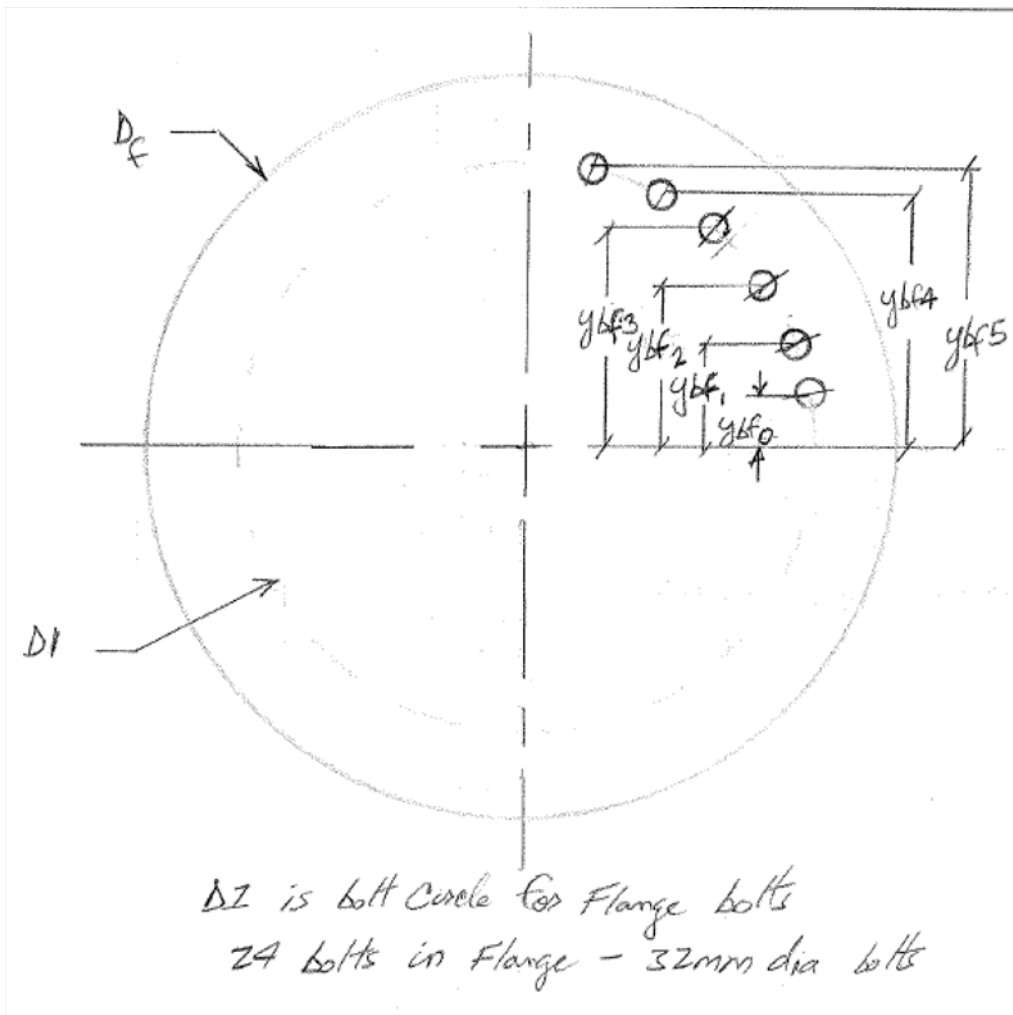
Internal moment in concrete

$$\phi M_n := \phi_f \cdot f_y \cdot A_s \cdot \left(d - \frac{A_s \cdot f_y}{2 \cdot 0.85 \cdot f_c \cdot D_f} \right)$$

$\phi M_n = 1627.63 \cdot \text{ft} \cdot \text{kip}$

Greater than $\phi \text{Muconcrete}$ so ok

7.3.3 Calculate the Forces in the Flange Bolts



Calculate distance to bolts

$$ybf_0 := \frac{dflange}{2} \cdot \sin\left(\frac{15deg}{2}\right)$$

$$ybf_1 := \frac{dflange}{2} \cdot \sin\left(15deg + \frac{15deg}{2}\right)$$

$$ybf_2 := \frac{dflange}{2} \cdot \sin\left(30deg + \frac{15deg}{2}\right)$$

$$ybf_3 := \frac{dflange}{2} \cdot \sin\left(45deg + \frac{15deg}{2}\right)$$

$$ybf_4 := \frac{dflange}{2} \cdot \sin\left(60deg + \frac{15deg}{2}\right)$$

$$ybf_5 := \frac{dflange}{2} \cdot \sin\left(75deg + \frac{15deg}{2}\right)$$

$$ybf = \begin{pmatrix} 139.27 \\ 408.32 \\ 649.55 \\ 846.51 \\ 985.78 \\ 1057.87 \end{pmatrix} \cdot \text{mm} \qquad ybf = \begin{pmatrix} 5.48 \\ 16.08 \\ 25.57 \\ 33.33 \\ 38.81 \\ 41.65 \end{pmatrix} \cdot \text{in}$$

$$F_{bolt} := \frac{Mu}{\sum_{i=0}^5 \left[4 \left(ybf_i \cdot \frac{ybf_i}{ybf_5} \right) \right]}$$

Factored forces in bolt

Fbolt = 35.84·kip

dbf = 31.75·mm Diameter of flange bolts

$$Abf := \frac{\pi \cdot dbf^2}{4} \qquad Abf = 1.23 \cdot \text{in}^2 \qquad \text{Cross sectional area of flange bolts}$$

fybf := 36ksi Yield strength of flange bolt material -- specified on drawing but assumed.

$$\phi_b := 0.75$$

Strength reduction factor for bolts

$$F_{bfStress} := \frac{F_{bolt}}{A_{bf}}$$

$$F_{bfStress} = 29.20 \cdot \text{ksi}$$

$$\phi_{Fnt} := \phi_b \cdot f_{ybf}$$

$$\phi_{Fnt} = 27.00 \cdot \text{ksi}$$

Allowable bolt stress in tension

$$F_{sbolt} := \frac{V_u}{n_{bolts} \cdot A_{bf}}$$

$$F_{sbolt} = 1.62 \cdot \text{ksi}$$

Shear stress per bolt

$$\phi_{Fns} := \phi_b \cdot .533 \cdot f_{ybf}$$

$$\phi_{Fns} = 14.39 \cdot \text{ksi}$$

Allowable bolt stress in Shear

$$\frac{F_{bfStress}}{\phi_{Fnt}} + \frac{F_{sbolt}}{\phi_{Fns}} = 1.19$$

Greater than 1 so not ok.

7.3.4 Calculate Overturning and Sliding Safety Factors

$$\frac{P_u \cdot r_f}{M_u + V_u \cdot \text{depth}} = 1.24$$

Less than 1.5 so not ok

Check sliding assuming a coefficient of friction of 0.5

$$\frac{P_u \cdot 0.5}{V_u} = 2.79$$

Greater than 1.5 so ok

8. Foundation Strength at 35 degree Park Position and no support on secondary

8.1 Calculate the Maximum Moment on Foundation

Survival Wind Loading -- at 35degree elevation angle and 0degree Azimuth

$$F_z := F_{windZ}(35\text{deg}, 0\text{deg}, V_{ibc})$$

$$F_z = 124.13 \cdot \text{kN}$$

$$F_z = 27.91 \cdot \text{kip}$$

$$F_y := W_t \cdot g$$

$$F_x := F_{windX}(35deg, 0deg, Vibc)$$

$$F_x = 0.00 \cdot kip$$

$$M_x := T_{windEL}(35deg, 0deg, Vibc)$$

$$M_x = 218.05 \cdot kip \cdot ft \quad \text{Moment about the elevation axis}$$

$$M_y := T_{windAZ}(35deg, 0deg, Vibc)$$

$$M_y = 0.00 \cdot kip \cdot ft \quad \text{Moment about the azimuth axis}$$

$$M_u := 1.2W_{total} \cdot g \cdot Lelevation + 1.6F_z \cdot HeightEL + 1.6M_x + 1.2W_{yoke} \cdot g \cdot Lelevation$$

$$M_u = 1594.70 \cdot kip \cdot ft \quad M_u = 2162.12 \cdot kN \cdot m$$

$$M_u' := W_{total} \cdot g \cdot Lelevation + F_z \cdot HeightEL + M_x + W_{yoke} \cdot g \cdot Lelevation$$

$$M_u' = 1073.12 \cdot kip \cdot ft$$

$$P_u := 1.2 \cdot F_y + 1.2 \cdot P_{uFoundation}$$

$$P_u = 199.98 \cdot kip \quad P_u = 889.57 \cdot kN$$

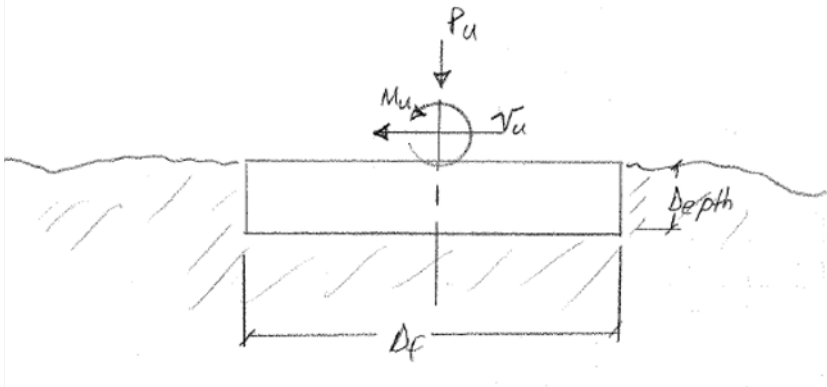
$$P_u' := F_y + P_{uFoundation}$$

$$P_u' = 166.65 \cdot kip$$

$$V_u := 1.6F_z$$

$$V_u' := F_z$$

$$e := \frac{M_u'}{P_u'} \quad e = 1.96 \text{ m} \quad e = 6.44 \text{ ft}$$



8.2 Soil Pressure

Assume that soil is in compression under the entire foundation -- solve for the soil pressure

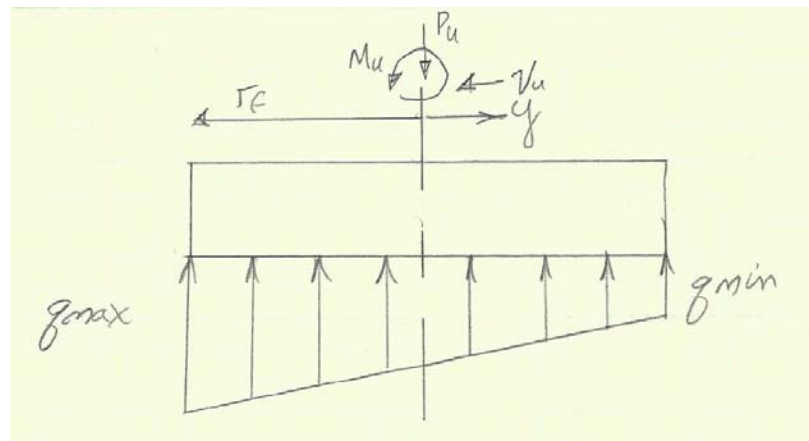
$$I_f := \frac{\pi \cdot D_f^4}{64}$$

$$q_{\max} := \frac{P_u'}{A_f} + \frac{M_u' \cdot \frac{D_f}{2}}{I_f}$$

$$q_{\max} = 3.26 \cdot \frac{\text{kip}}{\text{ft}^2}$$

$$q_{\min} := \frac{P_u'}{A_f} - \frac{M_u' \cdot \frac{D_f}{2}}{I_f}$$

$$q_{\min} = -1687.67 \cdot \frac{\text{lbf}}{\text{ft}^2}$$



q_{\min} is negative which is not possible -- assume a soil pressure distribution shown below. Solve for max soil pressure.

$$e := \frac{M_u'}{P_u'}$$

$$e = 1.96 \text{ m}$$

$$e = 6.44 \text{ ft}$$

Initial Guess of max soil pressure and distribution

$$q_{max} := 1 \frac{\text{kip}}{\text{ft}^2} \quad a := 1 \text{ m}$$

Given

$$Pu' = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} dy$$

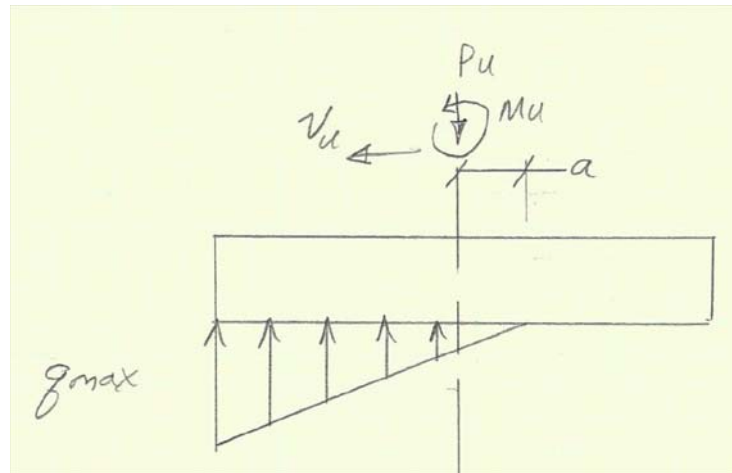
$$Mu' = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot y dy$$

$$\left(\begin{matrix} q_{max} \\ a \end{matrix} \right) := \text{Find}(q_{max}, a)$$

$$q_{max} = 9.54 \cdot \frac{\text{kip}}{\text{ft}^2}$$

$$a = -1.22 \text{ m}$$

An assumed soil strength value of 3kips/ft² is listed on the foundation drawings but the actual value is not known



Calculate soil pressure using equivalent square foundation

$$\text{side} := \sqrt{\frac{\pi \cdot Df^2}{4}}$$

$$\text{side} = 4.43 \text{ m}$$

Dimension of square foundation

$$e := \frac{Mu'}{Pu'}$$

$$e = 1.96 \text{ m}$$

$$\frac{\text{side}}{6} = 0.74 \text{ m}$$

eccentricity greater than side/6

$$q_{max} := \frac{2 \cdot Pu'}{3 \cdot \text{side} \cdot \left(\frac{\text{side}}{2} - e \right)}$$

$$q_{max} = 9.21 \cdot \text{ksf}$$

8.3. Calculate Strength of Concrete bending using factored values .

Check strength of concrete in bending at flange mounting plate

$$rd := r1$$

Radius where max bending moment is checked.

Recalculate the soil pressure using P_u and M_u with load factors applied

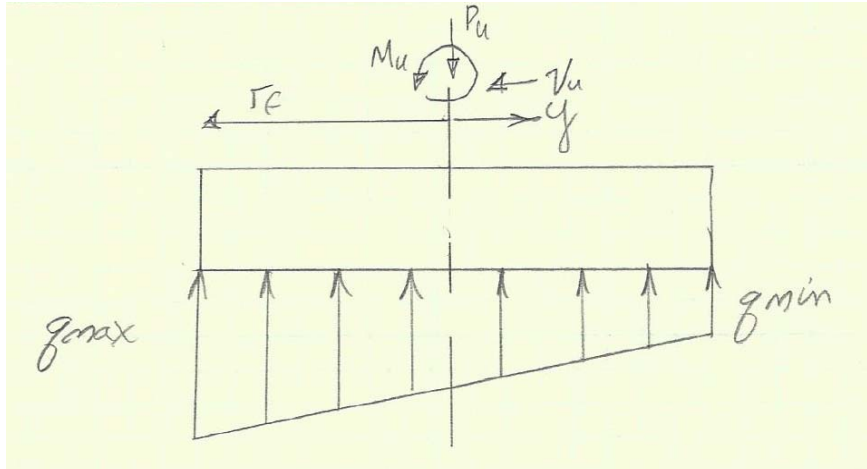
Calculate max soil pressure

$$q_{max} := \frac{P_u}{A_f} + \frac{M_u \cdot \frac{D_f}{2}}{I_f}$$

$$q_{max} = 4.6 \cdot \frac{\text{kip}}{\text{ft}^2}$$

$$q_{min} := \frac{P_u}{A_f} - \frac{M_u \cdot \frac{D_f}{2}}{I_f}$$

$$q_{min} = -2.73 \cdot \frac{\text{kip}}{\text{ft}^2}$$



q_{min} is negative which is not possible - assume a soil pressure distribution shown below. Solve for max soil pressure..

$$e := \frac{M_u}{P_u}$$

$$e = 2.43 \text{ m}$$

$$e = 7.97 \cdot \text{ft}$$

Initial Guess of soil pressure

$$q_{max} := 18 \cdot \frac{\text{kip}}{\text{ft}^2}$$

$$a := -1 \text{ m}$$

Given

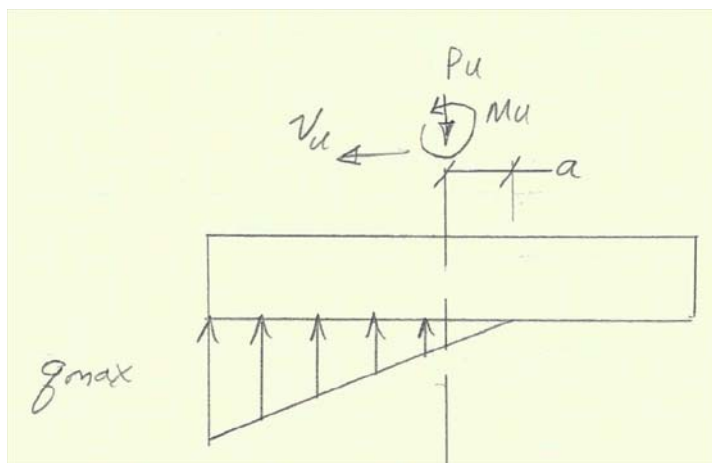
$$P_u = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \, dy$$

$$M_u = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot y \, dy$$

$$\left(\begin{matrix} q_{max} \\ a \end{matrix} \right) := \text{Find}(q_{max}, a)$$

$$q_{max} = 239.64 \cdot \frac{\text{kip}}{\text{ft}^2}$$

Maximum soil pressure



$a = -2.34 \text{ m}$

$rd := |a|$

Use the soil pressure calculated with load factors to find the moment in the concrete

$$\text{Muconcrete} := \int_{rd}^{rf} (q_{\text{max}}) \cdot \left(\frac{a + y}{a + rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot (y - rd) \, dy$$

Muconcrete = 61.00·ft·kip

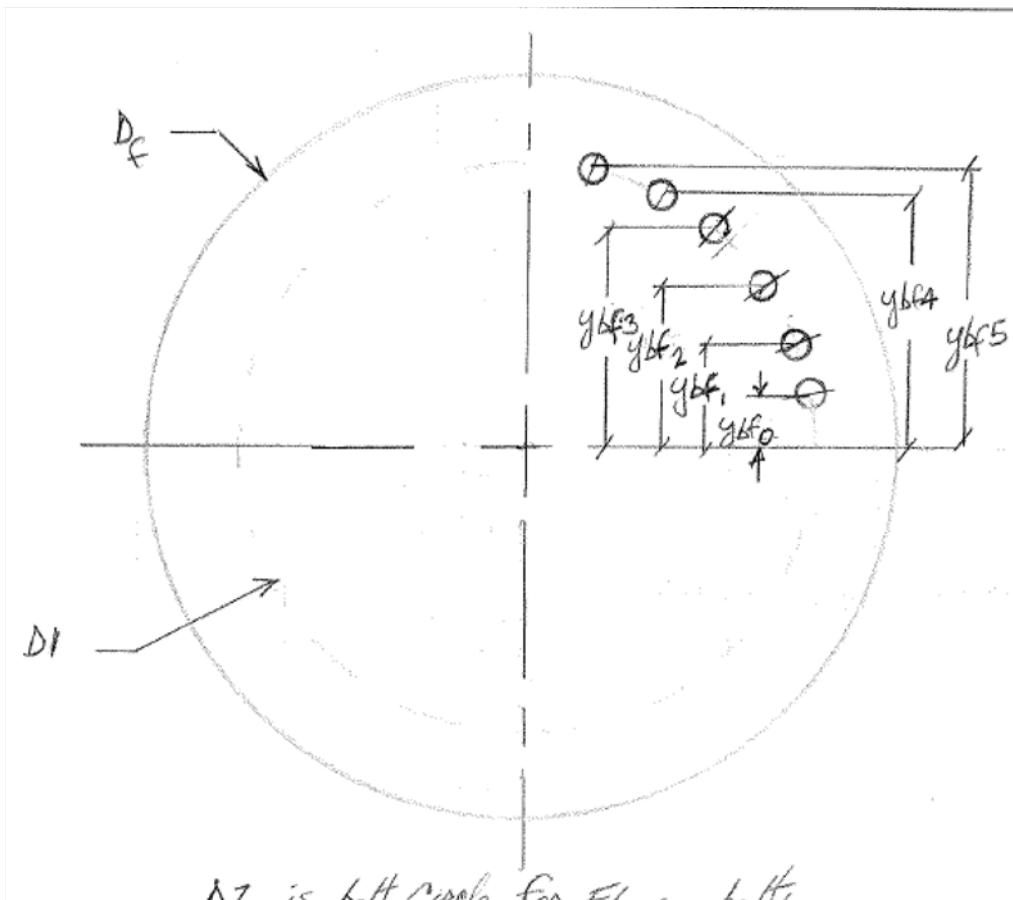
Internal moment in concrete

$$\phi M_n := \phi_f \cdot f_y \cdot A_s \cdot \left(d - \frac{A_s \cdot f_y}{2 \cdot 0.85 \cdot f_c \cdot D_f} \right)$$

$\phi M_n = 1627.63 \cdot \text{ft} \cdot \text{kip}$

Greater than $\phi \text{Muconcrete}$ so ok

8.4 Calculate the Forces in the Flange Bolts



24 bolts in Flange - 32mm dia bolts

Calculate distance to bolts

$$ybf_0 := \frac{dflange}{2} \cdot \sin\left(\frac{15deg}{2}\right)$$

$$ybf_1 := \frac{dflange}{2} \cdot \sin\left(15deg + \frac{15deg}{2}\right)$$

$$ybf_2 := \frac{dflange}{2} \cdot \sin\left(30deg + \frac{15deg}{2}\right)$$

$$ybf_3 := \frac{dflange}{2} \cdot \sin\left(45deg + \frac{15deg}{2}\right)$$

$$ybf_4 := \frac{dflange}{2} \cdot \sin\left(60deg + \frac{15deg}{2}\right)$$

$$ybf_5 := \frac{dflange}{2} \cdot \sin\left(75deg + \frac{15deg}{2}\right)$$

$$ybf = \begin{pmatrix} 139.27 \\ 408.32 \\ 649.55 \\ 846.51 \\ 985.78 \\ 1057.87 \end{pmatrix} \cdot \text{mm} \qquad ybf = \begin{pmatrix} 5.48 \\ 16.08 \\ 25.57 \\ 33.33 \\ 38.81 \\ 41.65 \end{pmatrix} \cdot \text{in}$$

$$F_{bolt} := \frac{Mu}{\sum_{i=0}^5 \left[4 \left(ybf_i \cdot \frac{ybf_i}{ybf_5} \right) \right]} - \frac{Pu}{24} \qquad \text{Factored forces in bolt}$$

$$F_{bolt} = 29.30 \cdot \text{kip}$$

$$dbf = 31.75 \cdot \text{mm} \qquad \text{Diameter of flange bolts}$$

$$Abf := \frac{\pi \cdot dbf^2}{4} \qquad Abf = 1.23 \cdot \text{in}^2 \qquad \text{Cross sectional area of flange bolts}$$

$$f_{ybf} := 36\text{ksi}$$

Yield strength of flange bolt material -- specified on drawing but assumed.

$$\phi_b := 0.75$$

Strength reduction factor for bolts

$$F_{bfStress} := \frac{F_{bolt}}{A_{bf}}$$

$$F_{bfStress} = 23.88\text{-ksi}$$

$$\phi_{Fnt} := \phi_b \cdot f_{ybf}$$

$$\phi_{Fnt} = 27.00\text{-ksi}$$

Allowable bolt stress in tension

$$F_{sbolt} := \frac{V_u}{n_{fbolts} \cdot A_{bf}}$$

$$F_{sbolt} = 1.52\text{-ksi}$$

Shear stress per bolt

$$\phi_{Fns} := \phi_b \cdot .533 \cdot f_{ybf}$$

$$\phi_{Fns} = 14.39\text{-ksi}$$

Allowable bolt stress in Shear

$$\frac{F_{bfStress}}{\phi_{Fnt}} + \frac{F_{sbolt}}{\phi_{Fns}} = 0.99$$

Greater than 1 so not ok.

8.5 Calculate Overturning and Sliding Safety Factors

$$\frac{P_u \cdot r_f}{M_u + V_u \cdot \text{depth}} = 1.19$$

Less than 1.5 so not ok

Check sliding assuming a coefficient of friction of 0.5

$$\frac{P_u \cdot 0.5}{V_u} = 2.99$$

Greater than 1.5 so ok

9. Foundation Strength at 35 degree Park Position and no support on secondary Adding Concrete Blocks to increase P_u

Concrete blocks 3x3x3

$$W_{block} := 4\text{kip}$$

Nblock := 10

9.1 Calculate the Maximum Moment on Foundation

Survival Wind Loading -- at 35degree elevation angle and 0degree Azimuth

$F_z := F_{windZ}(35deg, 0deg, V_{ibc})$

$F_z = 124.13 \cdot kN$ $F_z = 27.91 \cdot kip$

$F_y := W_t \cdot g$

$F_x := F_{windX}(35deg, 0deg, V_{ibc})$

$F_x = 0.00 \cdot kip$

$M_x := T_{windEL}(35deg, 0deg, V_{ibc})$

$M_x = 218.05 \cdot kip \cdot ft$ Moment about the elevation axis

$M_y := T_{windAZ}(35deg, 0deg, V_{ibc})$

$M_y = 0.00 \cdot kip \cdot ft$ Moment about the azimuth axis

$M_u := 1.2W_{total} \cdot g \cdot Lelevation + 1.6F_z \cdot HeightEL + 1.6M_x + 1.2W_{yoke} \cdot g \cdot Lelevation$

$M_u = 1594.70 \cdot kip \cdot ft$ $M_u = 2162.12 \cdot kN \cdot m$

$M_u' := W_{total} \cdot g \cdot Lelevation + F_z \cdot HeightEL + M_x + W_{yoke} \cdot g \cdot Lelevation$

$M_u' = 1073.12 \cdot kip \cdot ft$

$P_u := 1.2 \cdot F_y + 1.2 \cdot P_{uFoundation} + 1.2 \cdot N_{block} \cdot W_{block}$

$P_u = 247.98 \cdot kip$ $P_u = 1103.08 \cdot kN$

$P_u' := F_y + P_{uFoundation} + N_{block} \cdot W_{block}$

$P_u' = 206.65 \cdot kip$

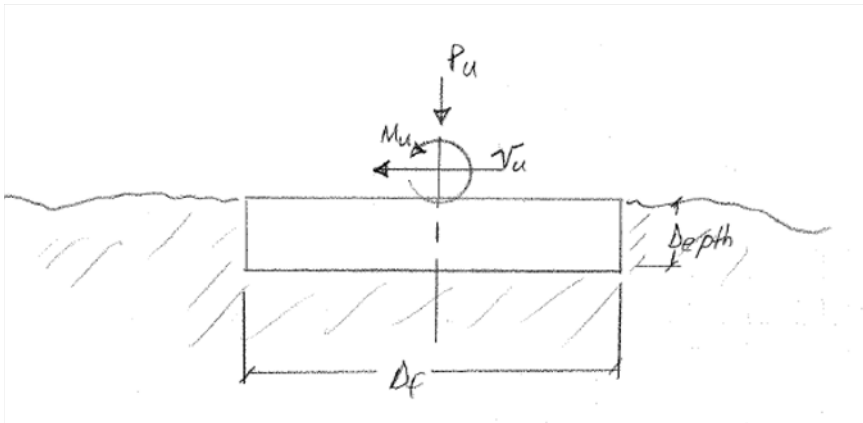
$V_u := 1.6F_z$

$$Vu' := Fz$$

$$e := \frac{Mu'}{Pu'}$$

$$e = 1.58 \text{ m}$$

$$e = 5.19 \text{ ft}$$



9.2 Soil Pressure

Assume that soil is in compression under the entire foundation -- solve for the soil pressure

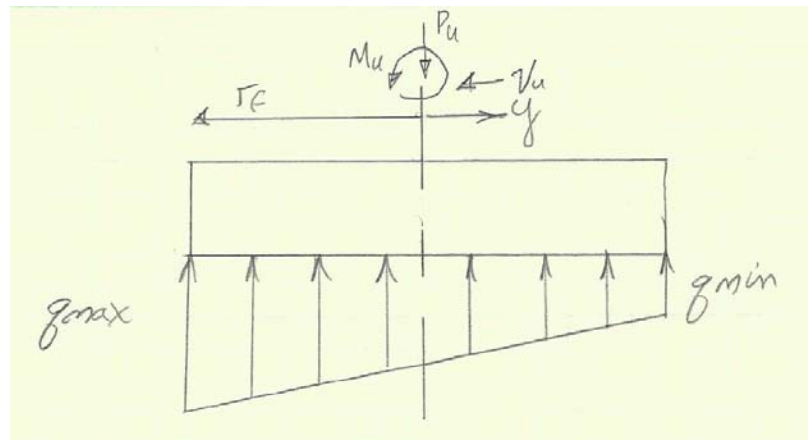
$$I_f := \frac{\pi \cdot Df^4}{64}$$

$$q_{max} := \frac{Pu'}{Af} + \frac{Mu' \cdot \frac{Df}{2}}{I_f}$$

$$q_{max} = 3.45 \cdot \frac{\text{kip}}{\text{ft}^2}$$

$$q_{min} := \frac{Pu'}{Af} - \frac{Mu' \cdot \frac{Df}{2}}{I_f}$$

$$q_{min} = -1498.41 \cdot \frac{\text{lbf}}{\text{ft}^2}$$



q_{min} is negative which is not possible -- assume a soil pressure distribution shown below. Solve for max soil pressure.

$$e := \frac{Mu'}{Pu'} \quad e = 1.58 \text{ m} \quad e = 5.19 \text{ ft}$$

Initial Guess of max soil pressure and distribution

$$q_{\max} := 4 \frac{\text{kip}}{\text{ft}^2} \quad a := 1 \text{ m}$$

Given

$$Pu' = \int_{-a}^{rf} q_{\max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} dy$$

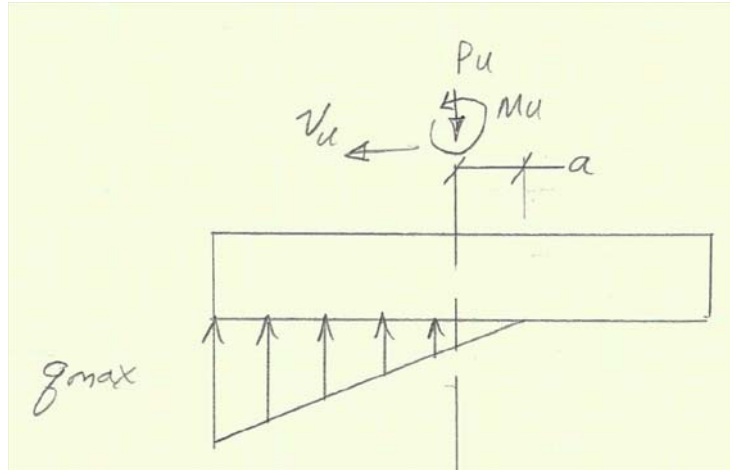
$$Mu' = \int_{-a}^{rf} q_{\max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot y dy$$

$$\left(\begin{matrix} q_{\max} \\ a \end{matrix} \right) := \text{Find}(q_{\max}, a)$$

$$q_{\max} = 5.43 \cdot \frac{\text{kip}}{\text{ft}^2}$$

An assumed soil strength value of 3kips/ft² is listed on the foundation drawings but the actual value is not known

$$a = -0.28 \text{ m}$$



Calculate soil pressure using equivalent square foundation

$$\text{side} := \sqrt{\frac{\pi \cdot Df^2}{4}} \quad \text{side} = 4.43 \text{ m} \quad \text{Dimension of square foundation}$$

$$e := \frac{Mu'}{Pu'} \quad e = 1.58 \text{ m}$$

$$\frac{\text{side}}{6} = 0.74 \text{ m} \quad \text{eccentricity greater than side/6}$$

$$q_{\max} := \frac{2 \cdot Pu'}{3 \cdot \text{side} \cdot \left(\frac{\text{side}}{2} - e \right)}$$

$$q_{\max} = 4.56 \cdot \text{ksf}$$

9.3. Calculate Strength of Concrete bending using factored values .

Check strength of concrete in bending at flange mounting plate

$r_d := r_1$ Radius where max bending moment is checked.

Recalculate the soil pressure using P_u and M_u with load factors applied

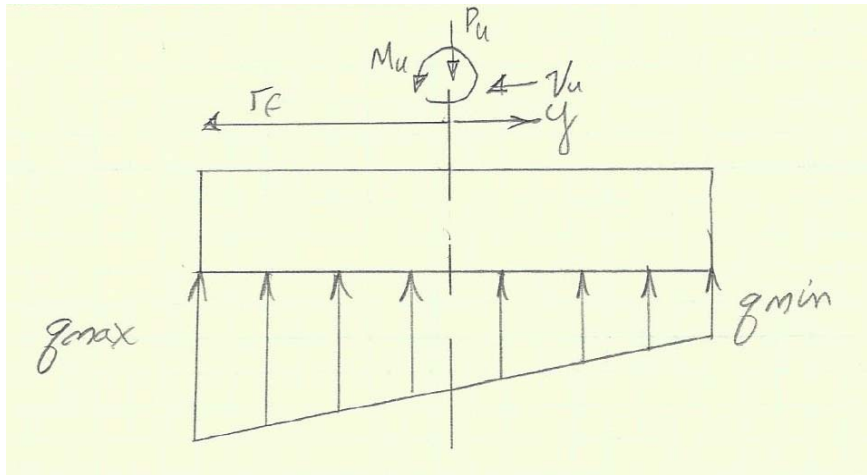
Calculate max soil pressure

$$q_{max} := \frac{P_u}{A_f} + \frac{M_u \cdot \frac{D_f}{2}}{I_f}$$

$$q_{max} = 4.9 \cdot \frac{\text{kip}}{\text{ft}^2}$$

$$q_{min} := \frac{P_u}{A_f} - \frac{M_u \cdot \frac{D_f}{2}}{I_f}$$

$$q_{min} = -2.51 \cdot \frac{\text{kip}}{\text{ft}^2}$$



q_{min} is negative which is not possible - assume a soil pressure distribution shown below. Solve for max soil pressure..

$$e := \frac{M_u}{P_u}$$

$$e = 1.96 \text{ m}$$

$$e = 6.43 \cdot \text{ft}$$

Initial Guess of soil pressure

$$q_{\max} := 18 \frac{\text{kip}}{\text{ft}^2} \quad a := -1\text{m}$$

Given

$$P_u = \int_{-a}^{rf} q_{\max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \, dy$$

$$M_u = \int_{-a}^{rf} q_{\max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot y \, dy$$

$$\left(\begin{matrix} q_{\max} \\ a \end{matrix} \right) := \text{Find}(q_{\max}, a)$$

$$q_{\max} = 14.09 \cdot \frac{\text{kip}}{\text{ft}^2} \quad \text{Maximum soil pressure}$$

$$a = -1.22 \text{ m}$$

$$rd := |a|$$

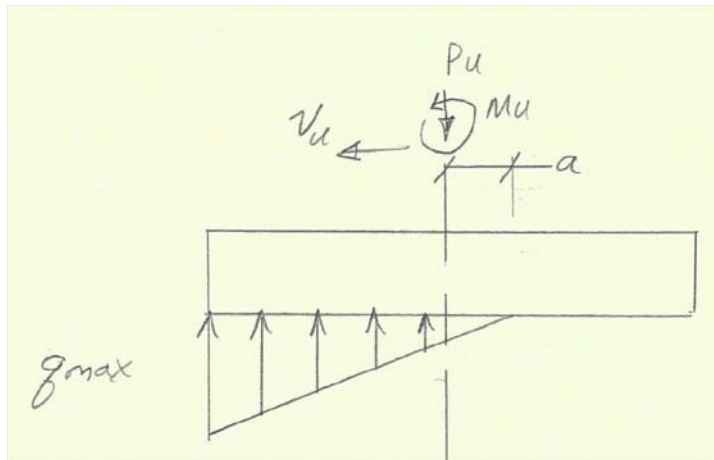
Use the soil pressure calculated with load factors to find the moment in the concrete

$$M_{\text{concrete}} := \int_{rd}^{rf} (q_{\max}) \cdot \left(\frac{a+y}{rf+a} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot (y - rd) \, dy$$

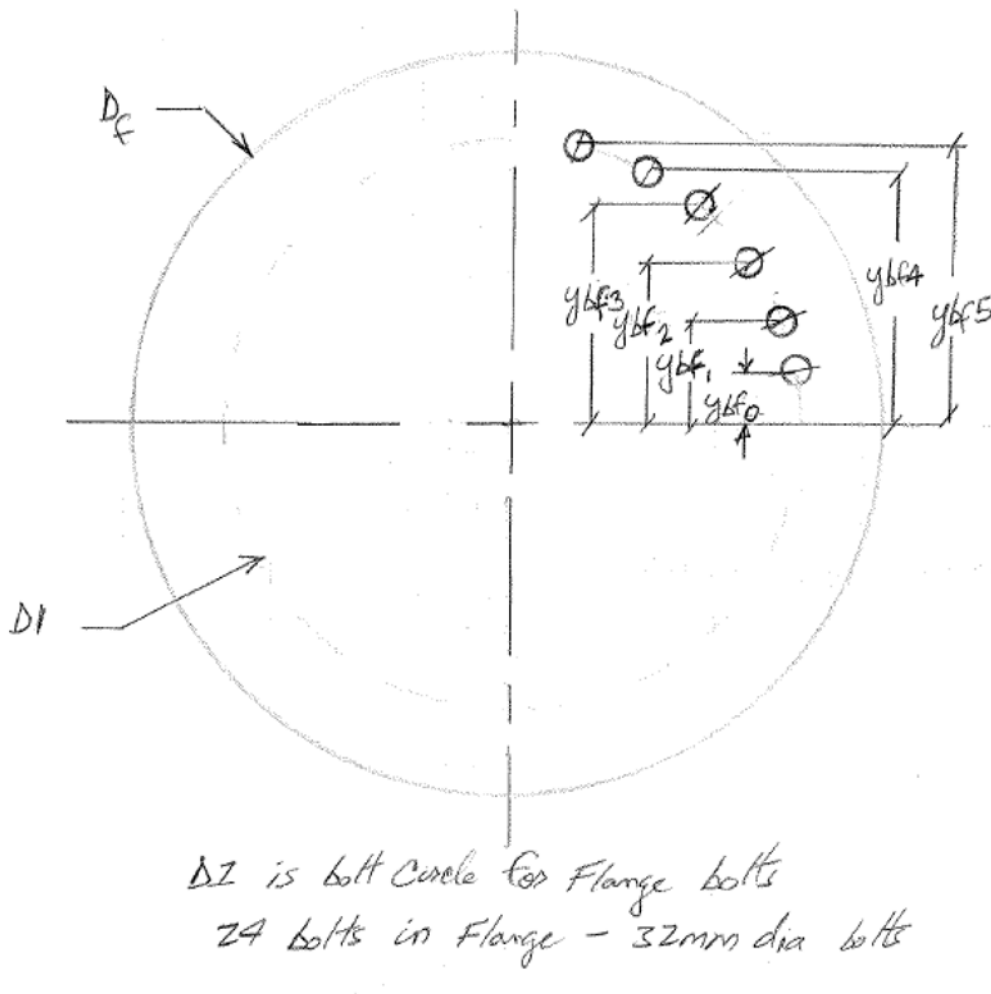
$$M_{\text{concrete}} = 604.93 \cdot \text{ft} \cdot \text{kip} \quad \text{Internal moment in concrete}$$

$$\phi M_n := \phi \cdot f_y \cdot A_s \cdot \left(d - \frac{A_s \cdot f_y}{2 \cdot 0.85 \cdot f_c \cdot D_f} \right)$$

$$\phi M_n = 1627.63 \cdot \text{ft} \cdot \text{kip} \quad \text{Greater than } \phi M_{\text{concrete}} \text{ so ok}$$



9.4 Calculate the Forces in the Flange Bolts



Calculate distance to bolts

$$ybf_0 := \frac{dflange}{2} \cdot \sin\left(\frac{15deg}{2}\right)$$

$$ybf_1 := \frac{dflange}{2} \cdot \sin\left(15deg + \frac{15deg}{2}\right)$$

$$ybf_2 := \frac{dflange}{2} \cdot \sin\left(30deg + \frac{15deg}{2}\right)$$

$$ybf_3 := \frac{dflange}{2} \cdot \sin\left(45deg + \frac{15deg}{2}\right)$$

$$ybf_4 := \frac{dflange}{2} \cdot \sin\left(60deg + \frac{15deg}{2}\right)$$

$$y_{bf_5} := \frac{d_{flange}}{2} \cdot \sin\left(75\text{deg} + \frac{15\text{deg}}{2}\right)$$

$$y_{bf} = \begin{pmatrix} 139.27 \\ 408.32 \\ 649.55 \\ 846.51 \\ 985.78 \\ 1057.87 \end{pmatrix} \cdot \text{mm} \quad y_{bf} = \begin{pmatrix} 5.48 \\ 16.08 \\ 25.57 \\ 33.33 \\ 38.81 \\ 41.65 \end{pmatrix} \cdot \text{in}$$

$$F_{bolt} := \frac{M_u}{\sum_{i=0}^5 \left[4 \left(y_{bf_i} \cdot \frac{y_{bf_i}}{y_{bf_5}} \right) \right]} - \frac{F_y}{24} \quad \text{Factored forces in bolt}$$

$$F_{bolt} = 34.16 \cdot \text{kip}$$

$$d_{bf} = 31.75 \cdot \text{mm} \quad \text{Diameter of flange bolts}$$

$$A_{bf} := \frac{\pi \cdot d_{bf}^2}{4} \quad A_{bf} = 1.23 \cdot \text{in}^2 \quad \text{Cross sectional area of flange bolts}$$

$$f_{ybf} := 36 \text{ksi} \quad \text{Yield strength of flange bolt material -- specified on drawing but assumed.}$$

$$\phi_b := 0.75 \quad \text{Strength reduction factor for bolts}$$

$$F_{bfStress} := \frac{F_{bolt}}{A_{bf}}$$

$$F_{bfStress} = 27.84 \cdot \text{ksi}$$

$$\phi_{Fnt} := \phi_b \cdot f_{ybf} \quad \phi_{Fnt} = 27.00 \cdot \text{ksi} \quad \text{Allowable bolt stress in tension}$$

$$F_{sbolt} := \frac{V_u}{n_{fbolts} \cdot A_{bf}} \quad F_{sbolt} = 1.52 \cdot \text{ksi} \quad \text{Shear stress per bolt}$$

$$\phi_{Fns} := \phi_b \cdot .533 \cdot f_{ybf} \quad \phi_{Fns} = 14.39 \cdot \text{ksi} \quad \text{Allowable bolt stress in Shear}$$

$$\frac{F_{bfStress}}{\phi_{Fnt}} + \frac{F_{sbolt}}{\phi_{Fns}} = 1.14 \quad \text{Greater than 1 so not ok.}$$

9.5 Calculate Overturning and Sliding Safety Factors

$$\frac{P_u' \cdot r_f}{M_u' + V_u' \cdot \text{depth}} = 1.48 \quad \text{Greater than 1.5 so ok}$$

Check sliding assuming a coefficient of friction of 0.5

$$\frac{P_u' \cdot 0.5}{V_u'} = 3.70 \quad \text{Greater than 1.5 so ok}$$

10. Foundation Strength at 35 degree Park Position and no support on secondary Determine the size of a new foundation needed and adding Hilti Anchor Bolts

$D_f := 6\text{m}$ Diameter of foundation

$$r_f := \frac{D_f}{2}$$

$$A_f := \frac{\pi \cdot D_f^2}{4} \quad \text{Bearing Area of foundation}$$

$$P_u\text{Foundation} := 150 \frac{\text{lb}}{\text{ft}^3} \cdot A_f \cdot \text{depth}$$

$$P_u\text{Foundation} = 119.8 \cdot \text{kip}$$

10.1 Calculate the Maximum Moment on Foundation

Survival Wind Loading -- at 35degree elevation angle and 0degree Azimuth

$$Fz := FwindZ(35deg, 0deg, Vibc)$$

$$Fz = 124.13 \cdot kN \quad Fz = 27.91 \cdot kip$$

$$Fy := Wt \cdot g$$

$$Fx := FwindX(35deg, 0deg, Vibc)$$

$$Fx = 0.00 \cdot kip$$

$$Mx := TwindEL(35deg, 0deg, Vibc)$$

$$Mx = 218.05 \cdot kip \cdot ft \quad \text{Moment about the elevation axis}$$

$$My := TwindAZ(35deg, 0deg, Vibc)$$

$$My = 0.00 \cdot kip \cdot ft \quad \text{Moment about the azimuth axis}$$

$$Mu := 1.2Wtotal \cdot g \cdot Lelevation + 1.6Fz \cdot HeightEL + 1.6Mx + 1.2Wyoke \cdot g \cdot Lelevation$$

$$Mu = 1594.70 \cdot kip \cdot ft \quad Mu = 2162.12 \cdot kN \cdot m$$

$$Mu' := Wtotal \cdot g \cdot Lelevation + Fz \cdot HeightEL + Mx + Wyoke \cdot g \cdot Lelevation$$

$$Mu' = 1073.12 \cdot kip \cdot ft$$

$$Pu := 1.2 \cdot Fy + 1.2 \cdot PuFoundation$$

$$Pu = 243.92 \cdot kip \quad Pu = 1084.99 \cdot kN$$

$$Pu' := Fy + PuFoundation$$

$$Pu' = 203.26 \cdot kip$$

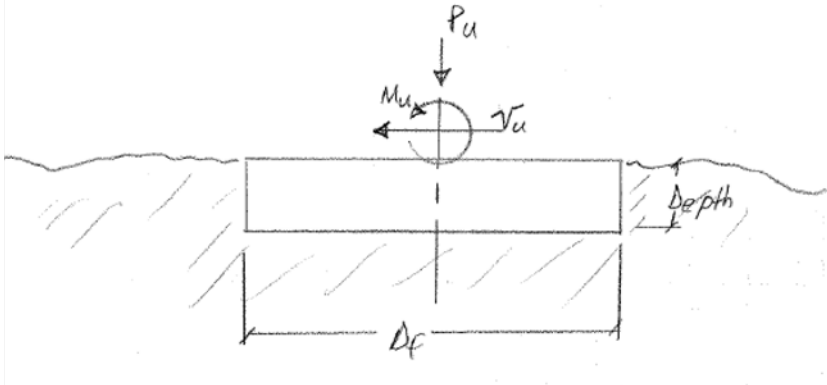
$$Vu := 1.6Fz$$

$$Vu' := Fz$$

$$e := \frac{Mu'}{Pu'}$$

$$e = 1.61 \text{ m}$$

$$e = 5.28 \cdot ft$$



10.2 Soil Pressure

Assume that soil is in compression under the entire foundation -- solve for the soil pressure

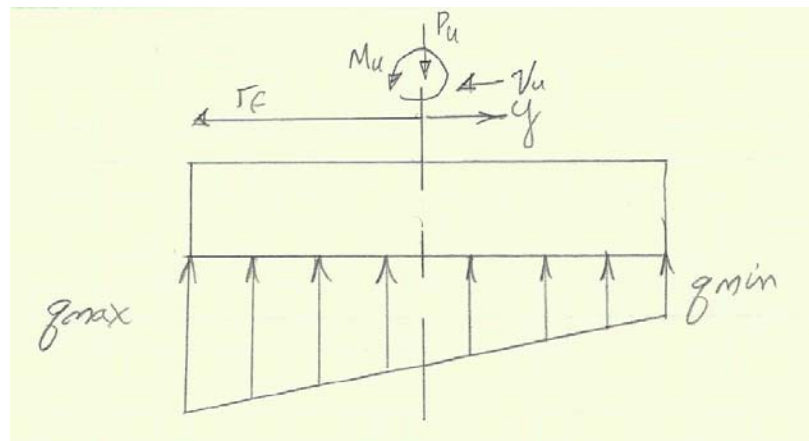
$$I_f := \frac{\pi \cdot D_f^4}{64}$$

$$q_{\max} := \frac{P_u'}{A_f} + \frac{M_u' \cdot \frac{D_f}{2}}{I_f}$$

$$q_{\max} = 2.10 \cdot \frac{\text{kip}}{\text{ft}^2}$$

$$q_{\min} := \frac{P_u'}{A_f} - \frac{M_u' \cdot \frac{D_f}{2}}{I_f}$$

$$q_{\min} = -765.10 \cdot \frac{\text{lbf}}{\text{ft}^2}$$



q_{\min} is negative which is not possible -- assume a soil pressure distribution shown below. Solve for max soil pressure.

$$e := \frac{M_u'}{P_u'}$$

$$e = 1.61 \text{ m}$$

$$e = 5.28 \text{ ft}$$

Initial Guess of max soil pressure and distribution

$$q_{max} := 1 \frac{\text{kip}}{\text{ft}^2} \quad a := 1 \text{ m}$$

Given

$$Pu' = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} dy$$

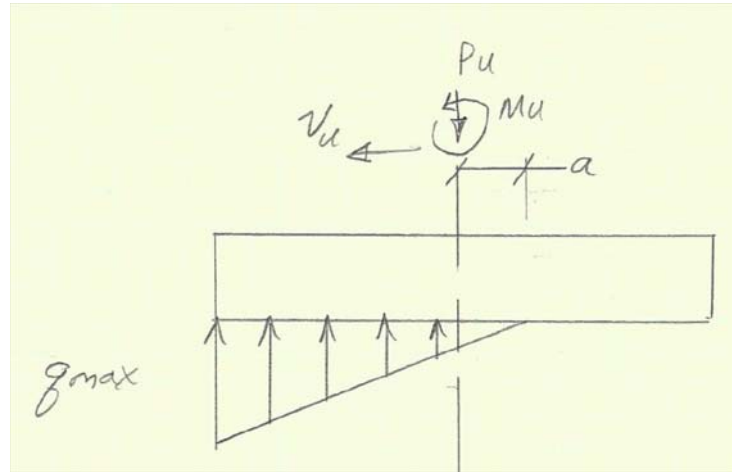
$$Mu' = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot y dy$$

$$\left(\begin{matrix} q_{max} \\ a \end{matrix} \right) := \text{Find}(q_{max}, a)$$

$$q_{max} = 2.65 \cdot \frac{\text{kip}}{\text{ft}^2}$$

$$a = 0.41 \text{ m}$$

An assumed soil strength value of 3kips/ft² is listed on the foundation drawings but the actual value is not known



Calculate soil pressure using equivalent square foundation

$$\text{side} := \sqrt{\frac{\pi \cdot Df^2}{4}}$$

$$\text{side} = 5.32 \text{ m}$$

Dimension of square foundation

$$e := \frac{Mu'}{Pu'}$$

$$e = 1.61 \text{ m}$$

$$\frac{\text{side}}{6} = 0.89 \text{ m}$$

eccentricity greater than side/6

$$q_{max} := \frac{2 \cdot Pu'}{3 \cdot \text{side} \cdot \left(\frac{\text{side}}{2} - e \right)}$$

$$q_{max} = 2.26 \cdot \text{ksf}$$

10.3. Calculate Strength of Concrete bending using factored values .

Check strength of concrete in bending at flange mounting plate

$$rd := r1$$

Radius where max bending moment is checked.

Recalculate the soil pressure using P_u and M_u with load factors applied

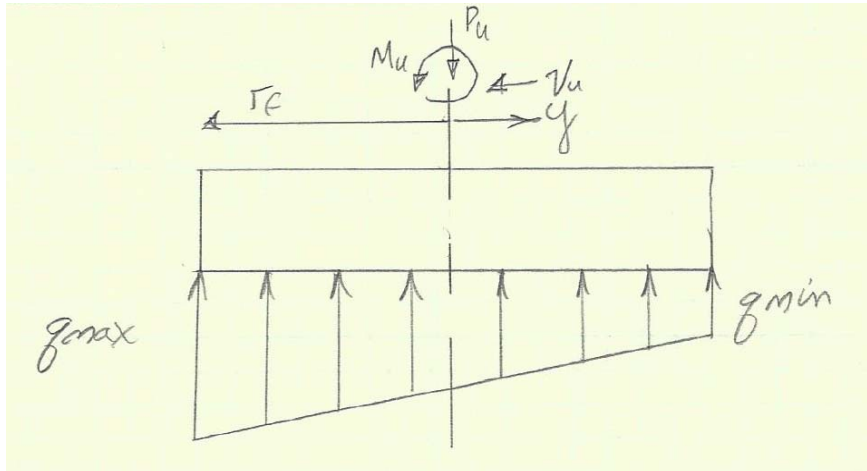
Calculate max soil pressure

$$q_{max} := \frac{P_u}{A_f} + \frac{M_u \cdot \frac{D_f}{2}}{I_f}$$

$$q_{max} = 2.9 \cdot \frac{\text{kip}}{\text{ft}^2}$$

$$q_{min} := \frac{P_u}{A_f} - \frac{M_u \cdot \frac{D_f}{2}}{I_f}$$

$$q_{min} = -1.33 \cdot \frac{\text{kip}}{\text{ft}^2}$$



q_{min} is negative which is not possible - assume a soil pressure distribution shown below. Solve for max soil pressure..

$$e := \frac{M_u}{P_u}$$

$$e = 1.99 \text{ m}$$

$$e = 6.54 \cdot \text{ft}$$

Initial Guess of soil pressure

$$q_{max} := 18 \cdot \frac{\text{kip}}{\text{ft}^2}$$

$$a := -1 \text{ m}$$

Given

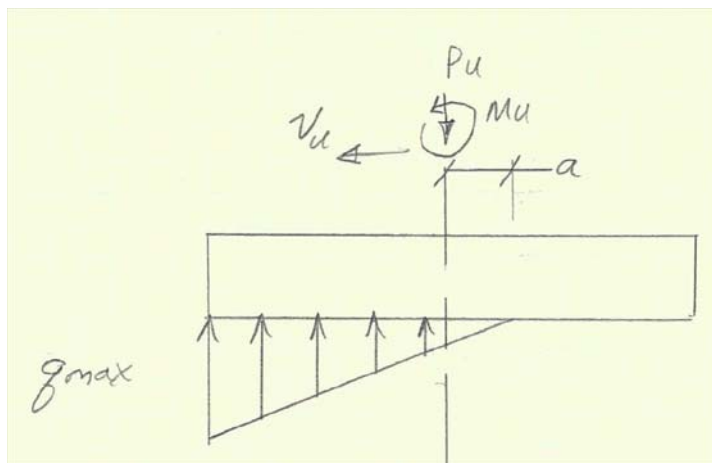
$$P_u = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \, dy$$

$$M_u = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot y \, dy$$

$$\left(\begin{matrix} q_{max} \\ a \end{matrix} \right) := \text{Find}(q_{max}, a)$$

$$q_{max} = 5.06 \cdot \frac{\text{kip}}{\text{ft}^2}$$

Maximum soil pressure



$$a = -0.57 \text{ m}$$

$$rd := |a|$$

Use the soil pressure calculated with load factors to find the moment in the concrete

$$Mu_{concrete} := \int_{rd}^{rf} (q_{max}) \cdot \left(\frac{a+y}{rf+a} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot (y - rd) \, dy$$

$$Mu_{concrete} = 1135.76 \cdot \text{ft} \cdot \text{kip} \quad \text{Internal moment in concrete}$$

$$\phi Mn := \phi f \cdot fy \cdot As \cdot \left(d - \frac{As \cdot fy}{2 \cdot 0.85 \cdot fc \cdot Df} \right)$$

$$\phi Mn = 1633.26 \cdot \text{ft} \cdot \text{kip} \quad \text{Greater than } Mu_{concrete} \text{ so ok}$$

Calculate the shear strength of the concrete

$$Vu_{concrete} := \int_{rd}^{rf} (q_{max}) \cdot \left(\frac{a+y}{rf+a} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \, dy$$

$$Vu_{concrete} = 243.92 \cdot \text{kip}$$

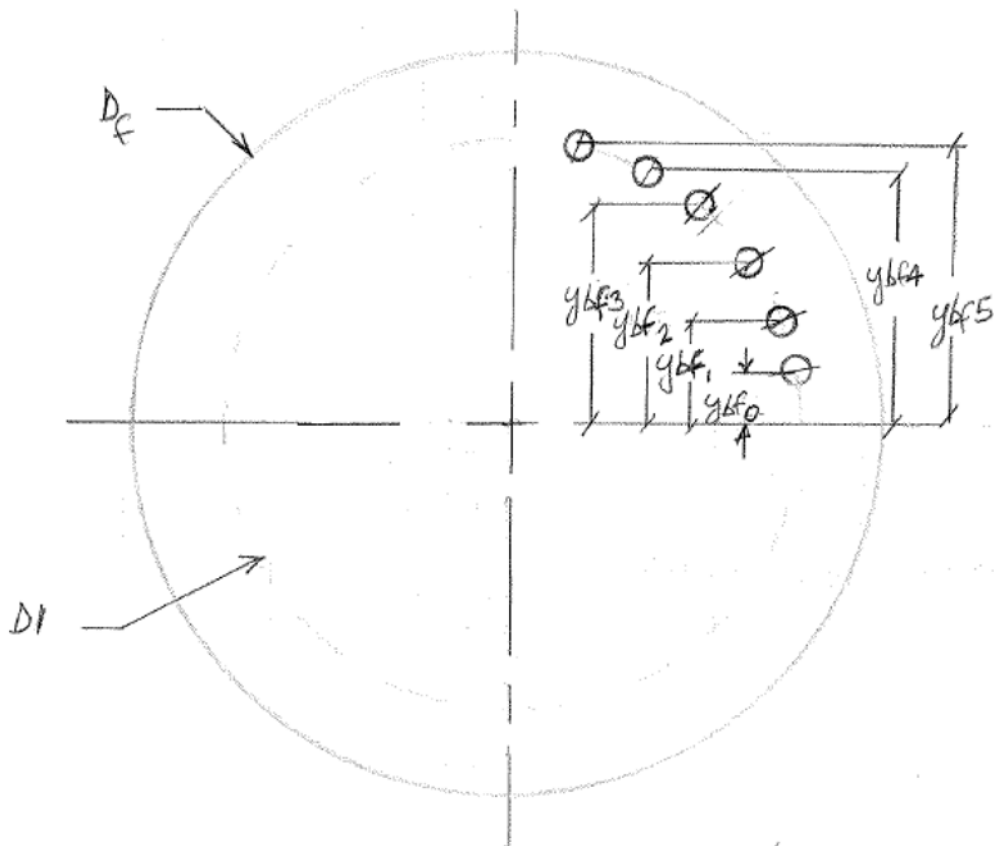
$$\phi_{shear} := 0.75 \quad \text{Strength reduction factor for shear}$$

$$\phi Vn := \phi_{shear} \cdot 2 \cdot \sqrt{\frac{fc}{\text{psi}}} \cdot \frac{Df \cdot \text{depth}}{\text{in}^2} \cdot \text{lbf}$$

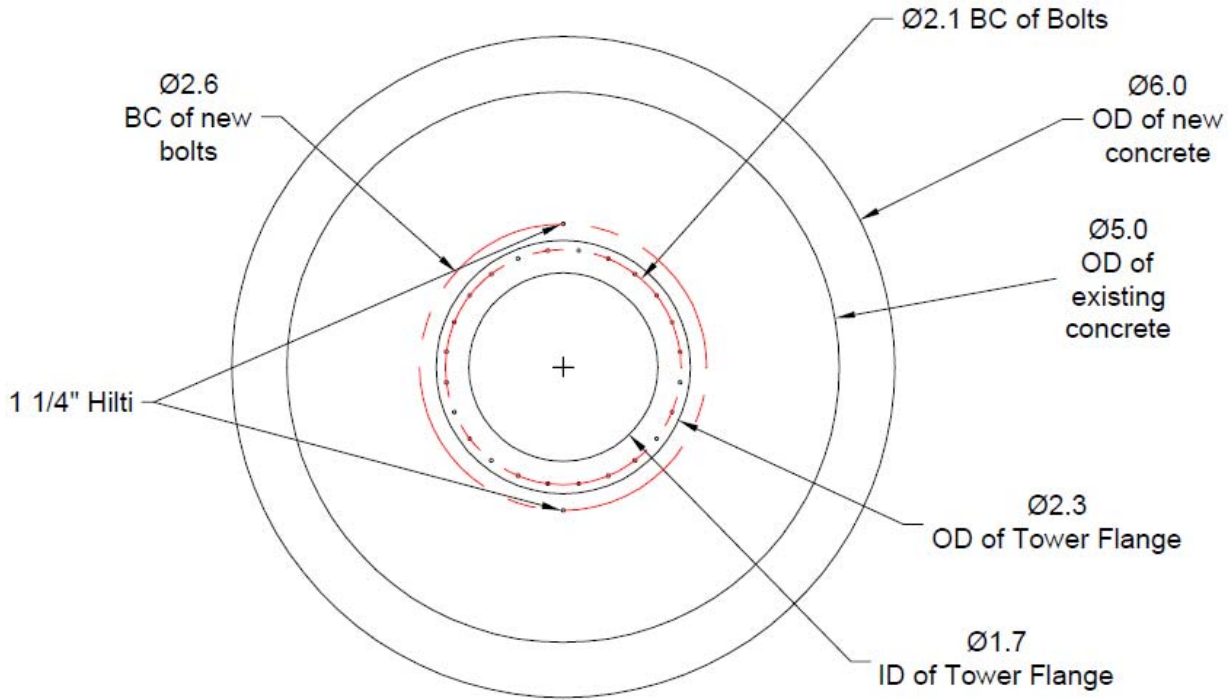
$$\phi Vn = 705.82 \cdot \text{kip}$$

10.4 Calculate the Forces in the Flange Bolts

Add a Hilti anchor that is directly south and north and therefore in-line with the park position and can resist the maximum moment. The Hilti anchor will be the same diameter as the existing bolts.



*D1 is bolt circle for Flange bolts
24 bolts in Flange - 32mm dia bolts*



Df = 6.00 m

Df = 236.22·in

Diameter of foundation

dflange = 2.13 m

dflange = 2.13 m

Diameter of bolt circle with existing anchor bolts

dHilti := 2.6m

Calculate distance to bolts

$$ybf_0 := \frac{dflange}{2} \cdot \sin\left(\frac{15deg}{2}\right)$$

$$ybf_1 := \frac{dflange}{2} \cdot \sin\left(15deg + \frac{15deg}{2}\right)$$

$$ybf_2 := \frac{dflange}{2} \cdot \sin\left(30deg + \frac{15deg}{2}\right)$$

$$ybf_3 := \frac{dflange}{2} \cdot \sin\left(45deg + \frac{15deg}{2}\right)$$

$$ybf_4 := \frac{dflange}{2} \cdot \sin\left(60deg + \frac{15deg}{2}\right)$$

$$ybf_5 := \frac{dflange}{2} \cdot \sin\left(75deg + \frac{15deg}{2}\right)$$

$$ybf_6 := \frac{dHilti}{2} \quad \text{Distnace from NA to Hilti anchor}$$

$$ybf = \begin{pmatrix} 139.27 \\ 408.32 \\ 649.55 \\ 846.51 \\ 985.78 \\ 1057.87 \\ 1300.00 \end{pmatrix} \cdot \text{mm} \quad ybf = \begin{pmatrix} 5.48 \\ 16.08 \\ 25.57 \\ 33.33 \\ 38.81 \\ 41.65 \\ 51.18 \end{pmatrix} \cdot \text{in}$$

$$Fbolt := \frac{\mu}{\sum_{i=0}^5 \left[4 \left(ybf_i \cdot \frac{ybf_i}{ybf_5} \right) + 2 \cdot ybf_6 \cdot \frac{ybf_6}{ybf_5} \right]} - \frac{Pu}{24} \quad \text{Factored forces in bolt}$$

$$Fbolt = 20.01 \cdot \text{kip} \quad \text{Maximum bolt force in existing anchor bolts}$$

$$Fhilti := Fbolt \cdot \frac{ybf_6}{ybf_5} \quad Fhilti = 24.59 \cdot \text{kip}$$

$$dbf = 31.75 \cdot \text{mm} \quad \text{Diameter of flange bolts}$$

$$Abf := \frac{\pi \cdot dbf^2}{4} \quad Abf = 1.23 \cdot \text{in}^2 \quad \text{Cross sectional area of flange bolts}$$

$$fybf := 36 \text{ksi} \quad \text{Yield strength of flange bolt material -- specified on drawing but assumed.}$$

$$\phi_b := 0.75 \quad \text{Strength reduction factor for bolts}$$

$$FbfStress := \frac{Fbolt}{Abf}$$

$$FbfStress = 16.30 \cdot \text{ksi}$$

$$\phi F_{nt} := \phi_b \cdot f_y b f \quad \phi F_{nt} = 27.00 \cdot \text{ksi} \quad \text{Allowable bolt stress in tension}$$

$$F_{sbolt} := \frac{V_u}{n_{fbolts} \cdot A_{bf}} \quad F_{sbolt} = 1.52 \cdot \text{ksi} \quad \text{Shear stress per bolt}$$

$$\phi F_{ns} := \phi_b \cdot .533 \cdot f_y b f \quad \phi F_{ns} = 14.39 \cdot \text{ksi} \quad \text{Allowable bolt stress in Shear}$$

$$\frac{F_{bfStress}}{\phi F_{nt}} + \frac{F_{sbolt}}{\phi F_{ns}} = 0.71 \quad \text{Less than 1 so ok.}$$

10.4.2 Calculate the bolt spacing needed for Hilit Anchor

$$\frac{\pi \cdot d_{flange}}{24} = 11.00 \cdot \text{in} \quad \text{Current spacing between anchor bolts}$$

10.5 Calculate Overturning and Sliding Safety Factors

$$\frac{P_u \cdot r_f}{M_u + V_u \cdot \text{depth}} = 1.75 \quad \text{Greater than 1.5 so ok}$$

Check sliding assuming a coefficient of friction of 0.5

$$\frac{P_u \cdot 0.5}{V_u} = 3.64 \quad \text{Greater than 1.5 so ok}$$

10.6 Calculate the connection between the existing foundation concrete and the new concrete

$$d_{bar} := \frac{3}{4} \text{in} \quad \text{Diameter of reinforcement bar at joint}$$

Use two bars

$$A_v := 2 \frac{\pi \cdot d_{bar}^2}{4} \quad \text{Area of shear reinforcement bars}$$

fybar := 60ksi

Yield strength of bars -- Grade 60

φshear := 0.75

Strength reduction factor for shear

Df = 6.00 m

Df = 19.69·ft

New concrete foundation diameter

Dold := 5m

Dold = 16.40·ft

Existing concrete foundation diameter

$$Rold := \frac{Dold}{2}$$

$$V_{new} := depth \cdot \frac{\pi}{4} \cdot (Df^2 - Dold^2)$$

Volume of new concrete needed

$$V_{new} = 9.04 \cdot yd^3$$

$$V_{new} = 6.91 \cdot m^3$$

$$W_{newConcrete} := V_{new} \cdot 150 \frac{lb}{ft^3}$$

$$W_{newConcrete} = 36.61 \cdot kip$$

Weight of new concrete

Calculate the shear at interface -- first calculate the distribution of the factored soil pressure on the bottom of the foundation

rd := 1m

Radius where shear is checked

Initial Guess of soil pressure

$$q_{max} := 5 \frac{kip}{ft^2}$$

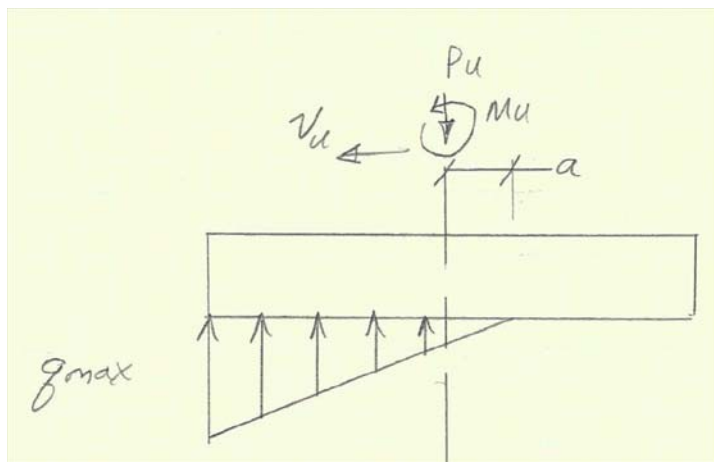
$$a := -1m$$

Given

$$P_u = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot dy$$

$$M_u = \int_{-a}^{rf} q_{max} \cdot \left(\frac{a+y}{a+rf} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot y \cdot dy$$

$$\left(\begin{matrix} q_{max} \\ a \end{matrix} \right) := \text{Find}(q_{max}, a)$$



$$q_{\max} = 5.06 \cdot \frac{\text{kip}}{\text{ft}^2} \quad \text{Maximum soil pressure}$$

$$a = -0.57 \text{ m}$$

Use the soil pressure calculated with load factors to find the shear in the concrete

$$rd := |a|$$

$$V_{\text{joint}} := \int_{rd}^{\text{Rold}} (q_{\max}) \cdot \left(\frac{a+y}{rf+a} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} \cdot \left(1 - \frac{\sqrt{\text{Rold}^2 - y^2}}{\sqrt{rf^2 - y^2}} \right) dy + \int_{rd}^{rf} (q_{\max}) \cdot \left(\frac{a+y}{rf+a} \right) \cdot 2 \cdot \sqrt{rf^2 - y^2} dy$$

$$V_{\text{joint}} = 308.42 \cdot \text{kip} \quad \text{Total shear force carried by the new concrete and transferred across joint}$$

Carry shear across joint by shear friction.

μ is 0.6 for concrete placed against hardened concrete that is not intentionally roughened

$$\mu := 0.6$$

$$\phi V_n := \phi_{\text{shear}} \cdot \mu \cdot A_v \cdot f_y$$

$$\phi V_n = 23.86 \cdot \text{kip} \quad \text{Shear strength of shear friction joint}$$

$$N_{\text{shear}} := \frac{V_{\text{joint}}}{\phi V_n} \quad N_{\text{shear}} = 12.93 \quad \text{Number of radial rebars needed for shear transfer from new concrete to old.}$$

$$\text{Use 15 bars} \quad N_{\text{shear}} := 15$$

$$\frac{180 \text{ deg}}{N_{\text{shear}}} = 12.00 \cdot \text{deg} \quad \text{Angle spacing between bars.}$$

$$\frac{\pi \text{ Dold}}{2} \cdot \frac{12 \text{ deg}}{180 \text{ deg}} = 20.61 \cdot \text{in} \quad \text{Spacing around the circumference of bars}$$

Development length of radial bars

$$l_d := \frac{\frac{f_y}{\text{psi}}}{25 \cdot \sqrt{\frac{f_c}{\text{psi}}}} \cdot d_{\text{bar}}$$

$$l_d = 28.46 \cdot \text{in}$$

$$\frac{D_f - D_{old}}{2} = 19.69 \cdot \text{in}$$

Greater than the thickness $(D_f - D_{old})/2$ of new concrete so use hooked bar

$$L_{dh} := \frac{0.02 \cdot f_y}{\frac{\text{psi}}{\sqrt{\frac{f_c}{\text{psi}}}}} \cdot d_{\text{bar}}$$

$$L_{dh} = 14.23 \cdot \text{in}$$

Calculate the shear strength of the concrete and max shear in new concrete

$$V_{\text{unew}} := \int_{R_{old}}^{r_f} (q_{\text{max}}) \cdot \left(\frac{a + y}{r_f + a} \right) \cdot 2 \cdot \sqrt{r_f^2 - y^2} \, dy$$

$$V_{\text{unew}} = 53.71 \cdot \text{kip}$$

$$\phi_{\text{shear}} := 0.75$$

Strength reduction factor for shear

$$\phi V_n := \phi_{\text{shear}} \cdot 2 \cdot \sqrt{\frac{f_c}{\text{psi}}} \cdot \frac{(D_f - D_{old}) \cdot \text{depth}}{2 \cdot \text{in}^2} \cdot \text{lbf}$$

$$\phi V_n = 58.82 \cdot \text{kip}$$

Calculate minimum circumferential reinforcement

$$A_{\text{min}} := 0.0018 \cdot \text{depth} \cdot \frac{(D_f - D_{old})}{2}$$

$$A_{min} = 1.12 \cdot \text{in}^2$$

Use #4 bars -- 3 on top and 3 on bottom

$$\text{spacing} := \frac{\frac{D_f - D_{old}}{2} - 6\text{in}}{2}$$

$$\text{spacing} = 6.84 \cdot \text{in}$$

$$d1 := D_{old} + 6\text{in} \quad d1 = 5.15 \text{ m} \quad d1 = 202.85 \cdot \text{in}$$

$$d2 := d1 + 2\text{spacing} \quad d2 = 5.50 \text{ m} \quad d2 = 216.54 \cdot \text{in}$$

$$d3 := d2 + 2\text{spacing} \quad d3 = 5.85 \text{ m} \quad d3 = 230.22 \cdot \text{in}$$

$$r \cdot \cos(\alpha) \cdot \sin(\beta)$$

$$\cos(\alpha) \cdot \sin(\beta)$$