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Magnetic Field Calculation of a Helical Undulator : Analytical and OPERA Model with “Tolerance” Parameter

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Advanced Photon Source

May 5-6, 2009

Third Special Workshop on Magnet Simulations
for Particle Accelerators in PAC09



U.S. Department
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On-Axis Field for Helical Undulators

(Helical) Solenoid W.R. Smythe (1939)

$$B_{tr} = \frac{\mu_0 I}{\lambda} \{kr_0 K_0(kr_0) + K_1(kr_0)\}$$

Static and Dynamic Electricity (McGraw-Hill, 1939), p. 272

Helical Undulator B.M. Kincaid (1977)

$$B_0 = 2B_{tr} \quad \text{J. Appl. Phys. 48, 2684 (1977)}$$

Helical Undulator with coil dimensions (a, b)

$$\mathbf{B}(kz - \phi) = B_0 \left\{ \hat{r} \cos(kz - \phi) + \hat{\phi} \sin(kz - \phi) \right\}$$

$$\mathbf{B}(x, y) = B_0 \left\{ \hat{x} \cos(kz) + \hat{y} \sin(kz) \right\}$$

$$B_0 = \frac{2\mu_0 j \lambda}{\pi} \sin\left(k \frac{a}{2}\right) \int_{r_0}^{r_0+b} \left\{ kr K_0(kr) + K_1(kr) \right\} \frac{dr}{\lambda} \quad (\text{Eq. } B_0)$$

Nucl. Instr & Meth. A 584, 266-272 (2008)

Compare (Eq. B_0) with OPERA model calculations

$$(k = 2\pi/\lambda)$$

K_n : modified Bessel functions

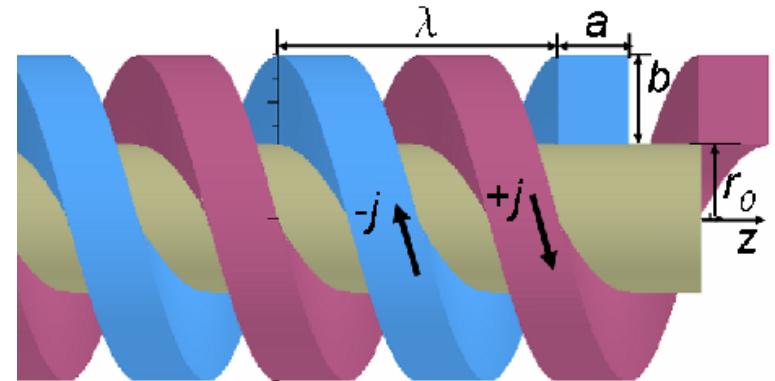
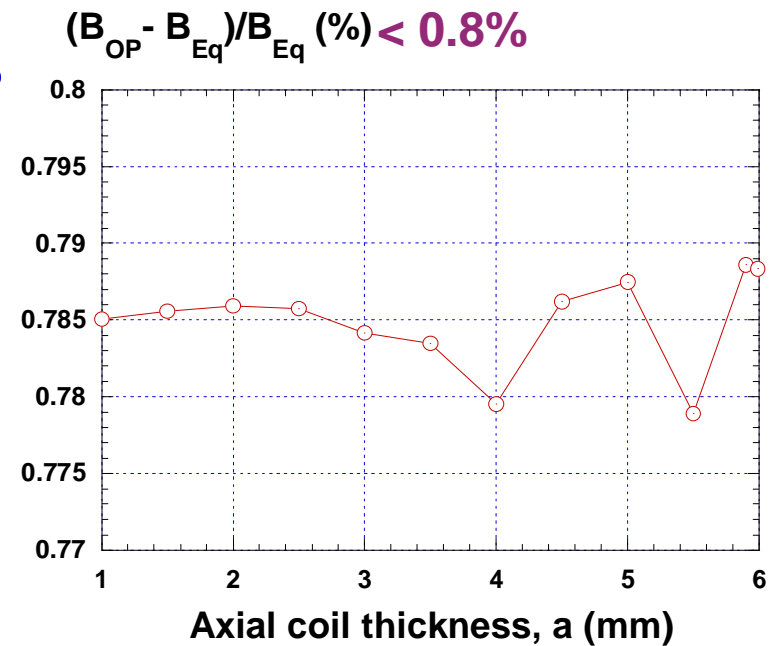
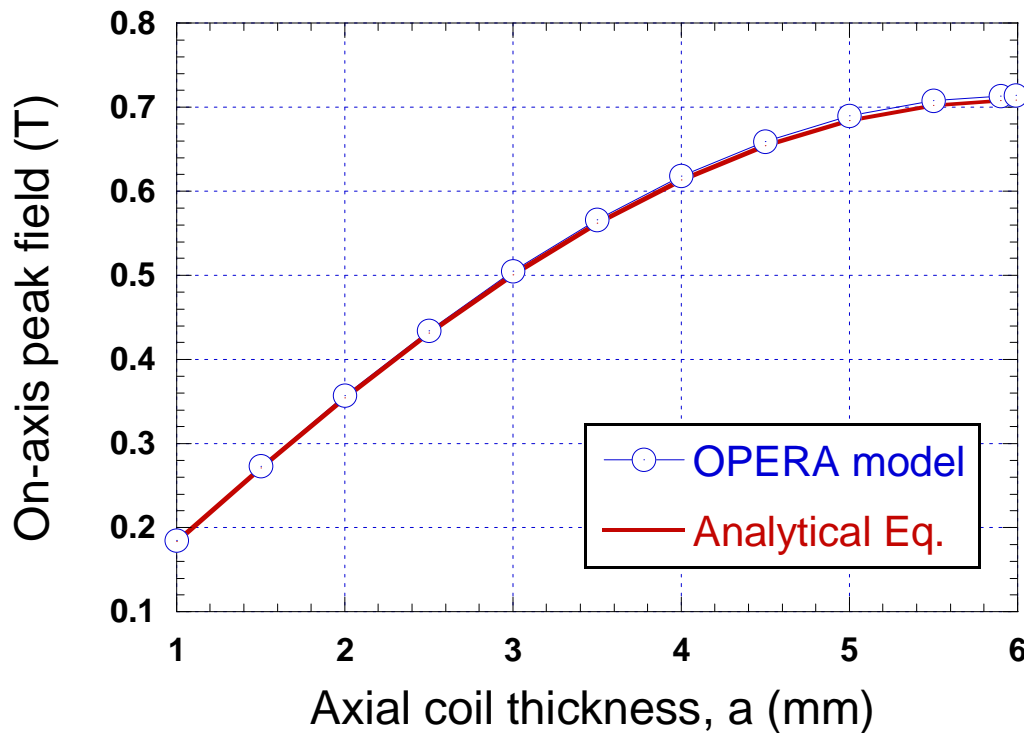


Figure 1 from Proc. 2007 PAC, p. 1136

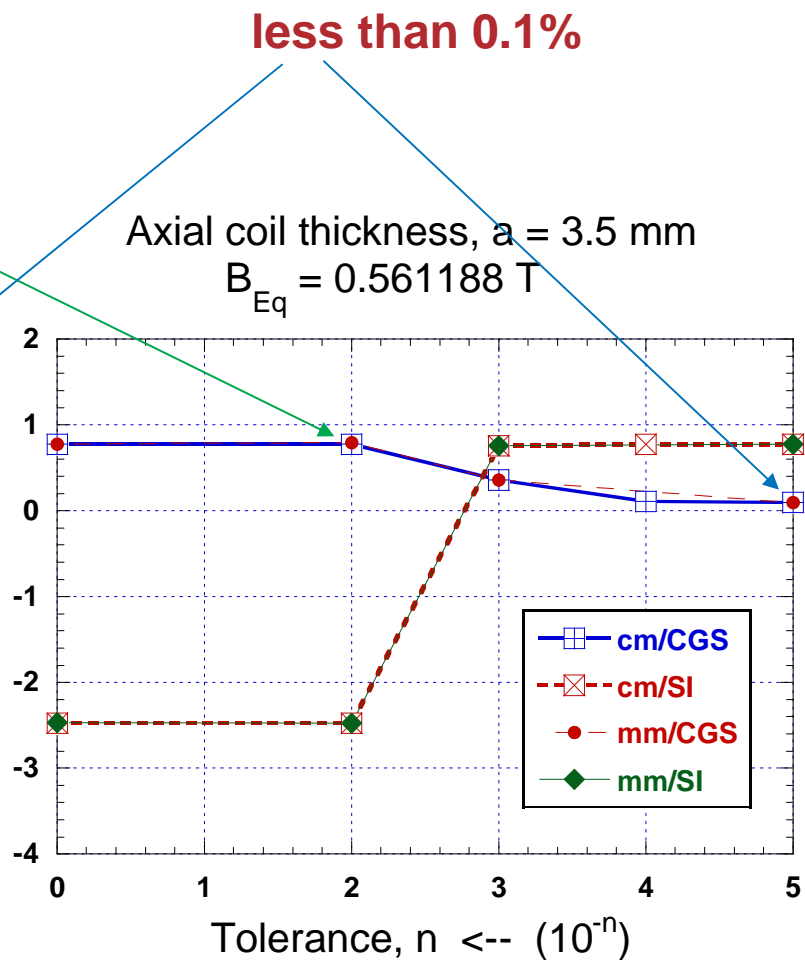
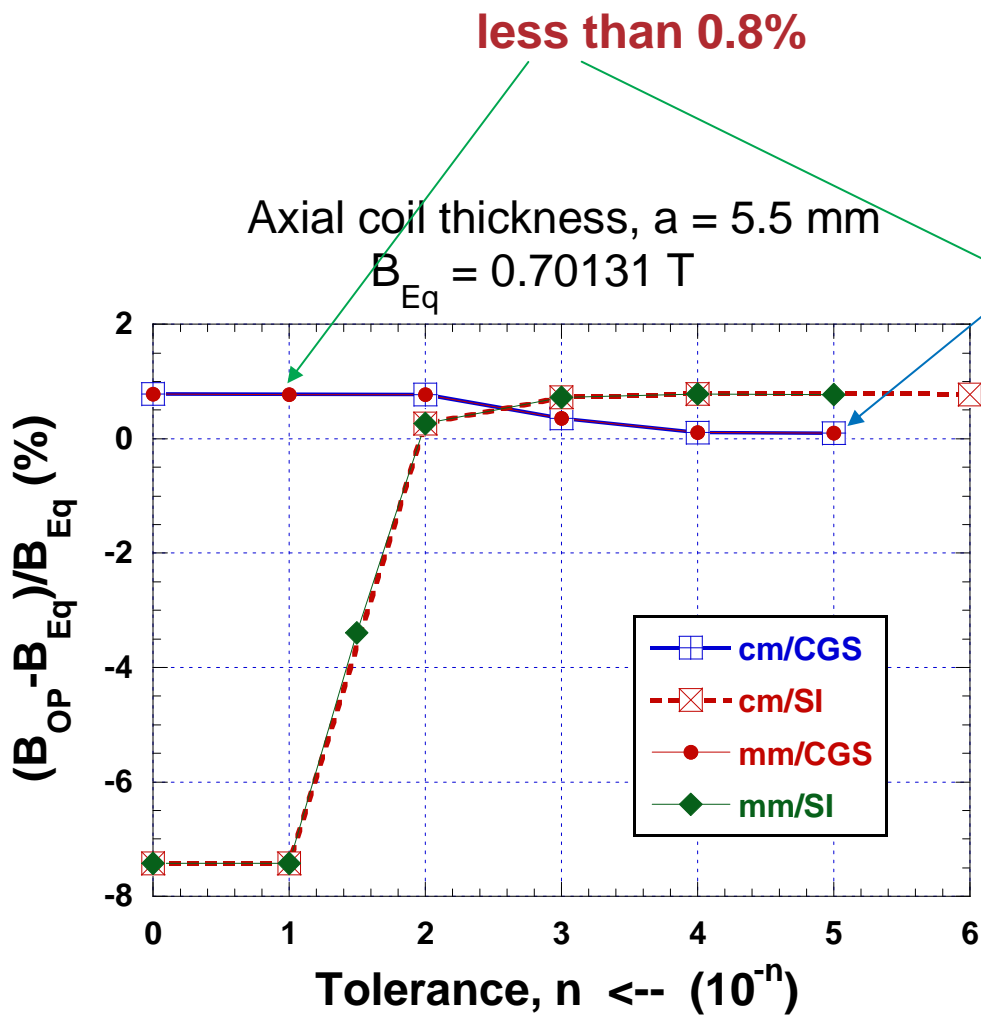
OPERA Model Calculations: cm, A/cm², CGS units, Tolerance = 1

Undulator period = 12.0 mm
Coil ID, 2 ro = 6.3 mm
Radial coil thickness, b = 4.0 mm

$$j(\text{eng}) = 1 \text{ kA/mm}^2$$



OPERA Model Calculations Depend on Tolerance Parameters



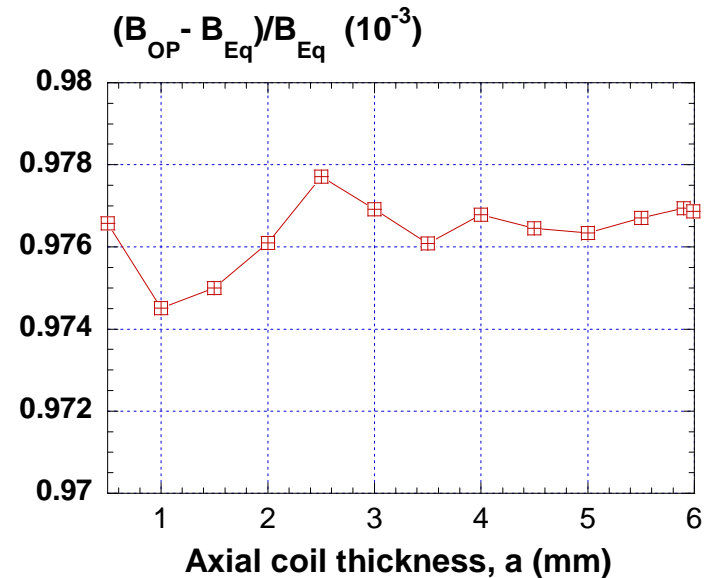
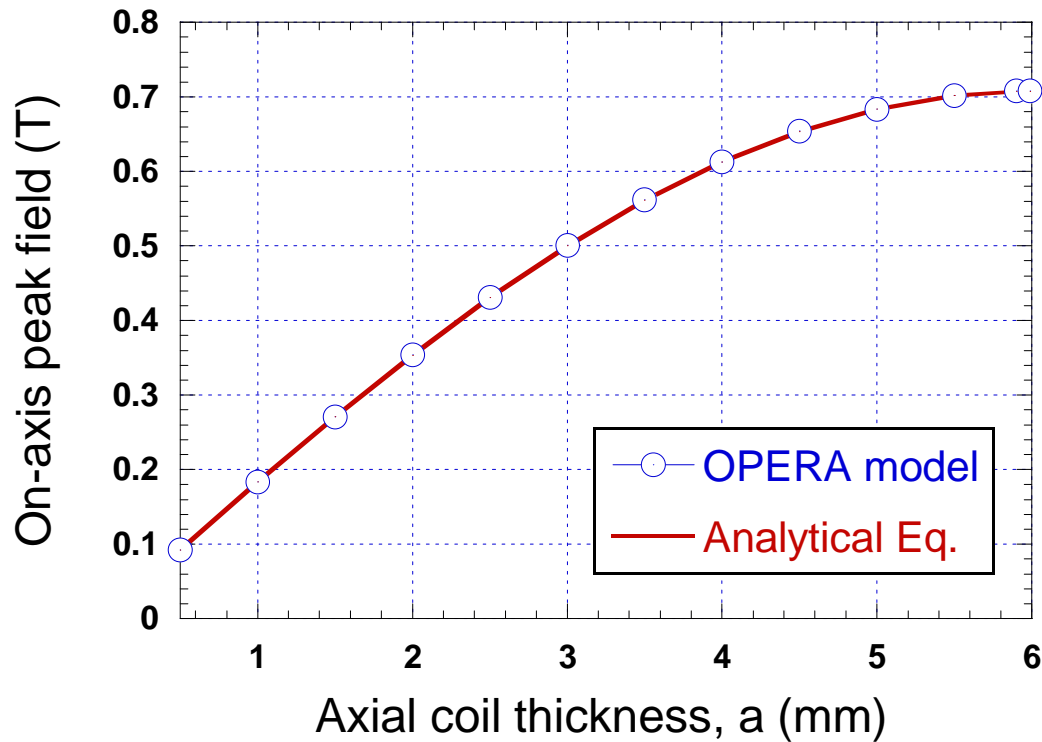
OPERA Model Calculations: cm, A/cm², CGS units, Tolerance = 10⁻⁵

Undulator period = 12.0 mm

Coil ID, 2 ro = 6.3 mm

Radial coil thickness, b = 3.84 mm

$j(\text{eng}) = 1 \text{ kA/mm}^2$



Summary

- Calculated on-axis fields of a helical undulator using the OPERA model of helical coils
- OPERA model agreed with a derived analytical formula within $(B_{OP} - B_{Eq})/B_{Eq} < 10^{-3}$ by using different units and the OPERA conductor Tolerance parameters
- Other methods available for easily modeling helical coils?