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Magnetic Field Calculation of a Helical Undulator : Analytical and OPERA Model with "Tolerance" Parameter

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On-Axis Field for Helical Undulators

(Helical) Solenoid W.R. Smythe (1939)

($k = 2\pi/\lambda$)

K_n: *modified Bessel functions*

$$B_{tr} = \frac{\mu_0 I}{\lambda}\{kr_0 K_0(kr_0) + K_1(kr_0)\}$$

Static and Dynamic Electricity (McGraw-Hill, 1939), p. 272

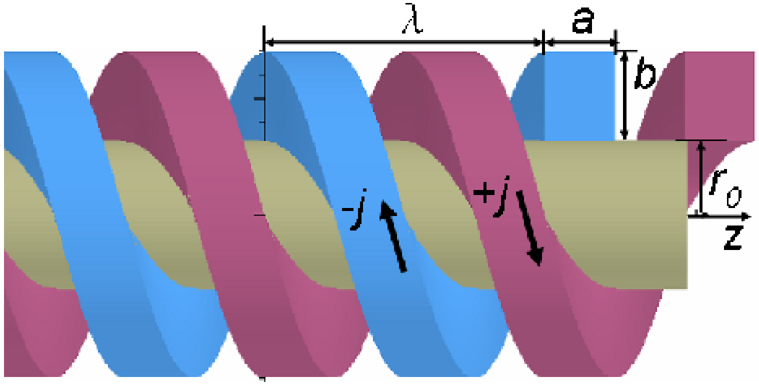


Figure 1 from Proc. 2007 PAC, p. 1136

Helical Undulator B.M. Kincaid (1977)

$$B_0 = 2B_{tr}$$

J. Appl. Phys. 48, 2684 (1977)

Helical Undulator with coil dimensions (a, b)

$$\mathbf{B}(kz-\phi) = B_0\left\{\hat{r}\cos(kz-\phi) + \hat{\phi}\sin(kz-\phi)\right\}$$

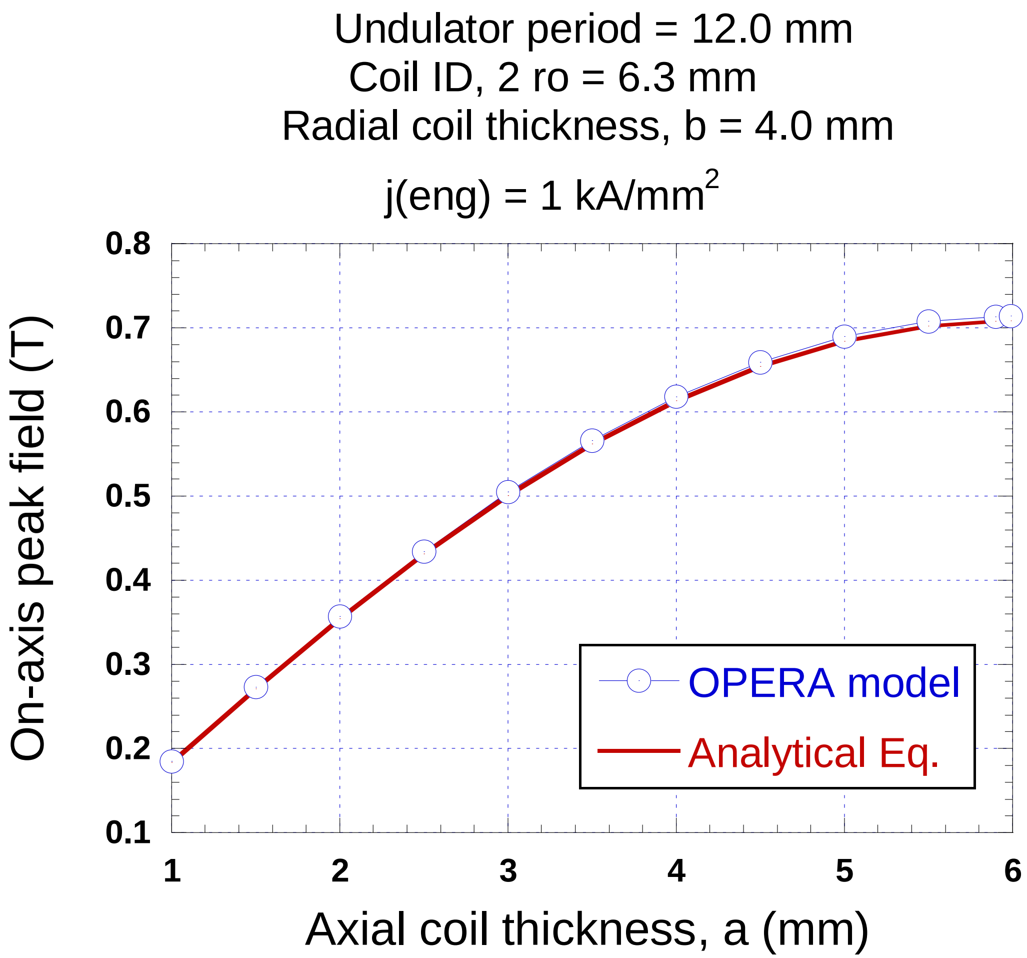
$$\mathbf{B}(x,y) = B_0\left\{\hat{x}\cos(kz) + \hat{y}\sin(kz)\right\}$$

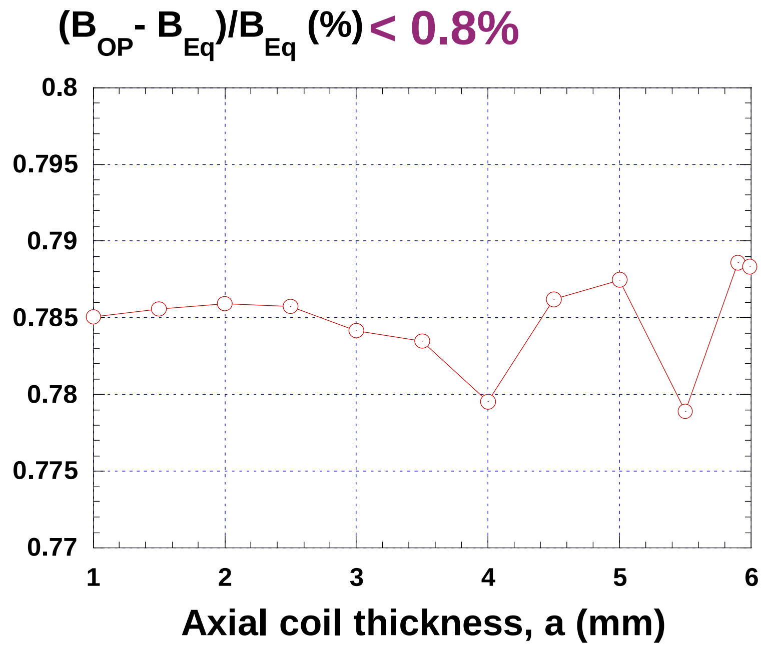
$$B_0 = \frac{2\mu_0 j\lambda}{\pi}\sin\left(k\frac{a}{2}\right)\int_{r_0}^{r_0+b}\{krK_0(kr) + K_1(kr)\}\frac{dr}{\lambda} \qquad (Eq.\ B_0)$$

Nucl. Instr & Meth. A 584, 266-272 (2008)

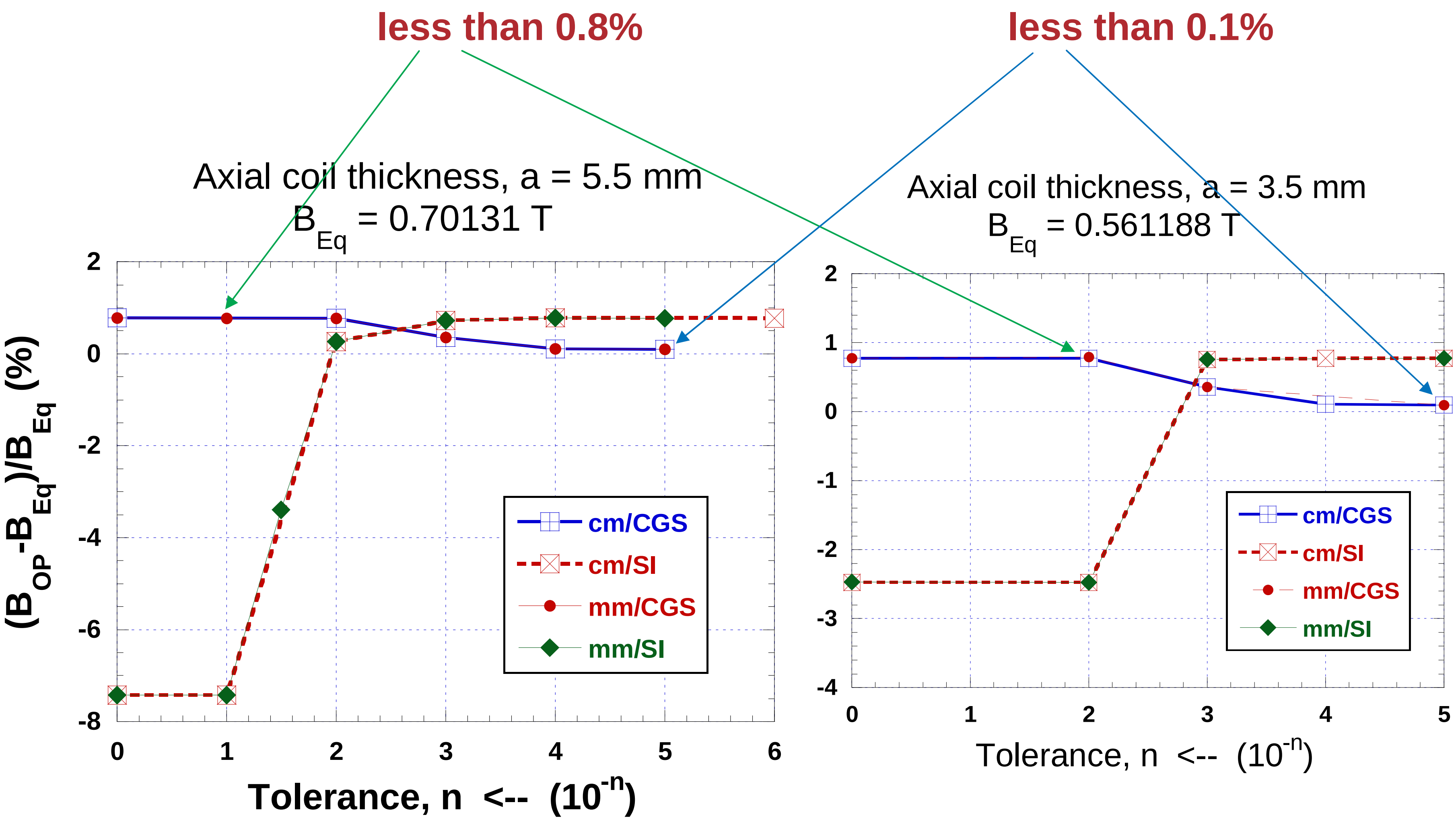
Compare (*Eq. B_0*) with OPERA model calculations

OPERA Model Calculations: cm, A/cm2, CGS units, Tolerance = 1





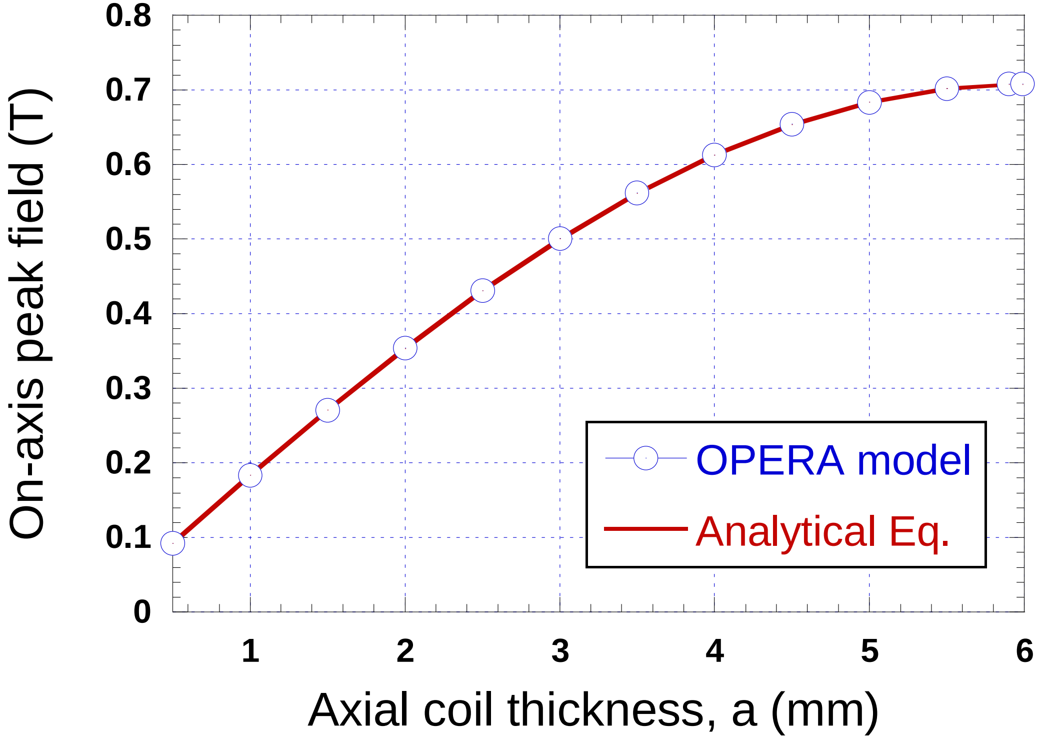
OPERA Model Calculations Depend on Tolerance Parameters

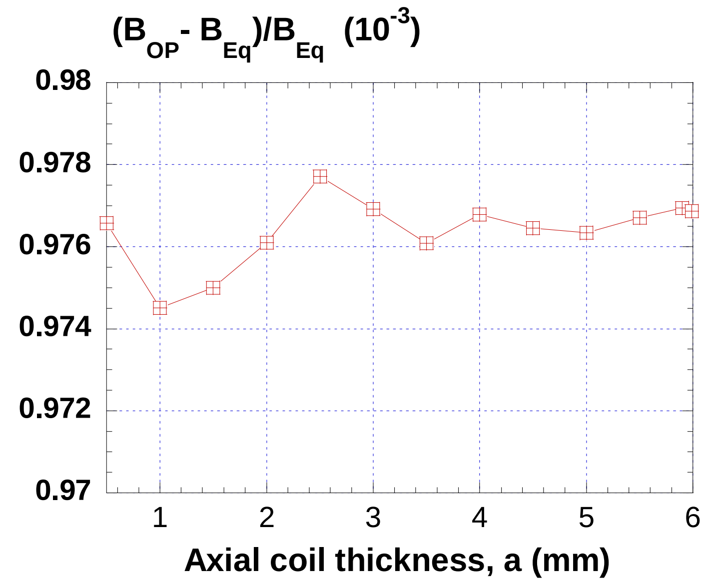


OPERA Model Calculations: cm, A/cm^2, CGS units, Tolerance = 10^{-5}

Undulator period = 12.0 mm
Coil ID, 2 ro = 6.3 mm
Radial coil thickness, b = 3.84 mm

j(eng) = 1 kA/mm^2





Summary

- Calculated on-axis fields of a helical undulator using the OPERA model of helical coils

- OPERA model agreed with a derived analytical formula within $(B_{OP} - B_{Eq})/B_{Eq} < 10^{-3}$ by using different units and the OPERA conductor Tolerance parameters

- Other methods available for easily modeling helical coils?