FFAG TRACKING STUDIES USING CYCLOPS

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FFAG DESIGN TOOLS

FFAG designs have generally been developed using synchrotron lattice codes – or adaptations of them – perhaps because their designers have mostly come from a synchrotron background.

But synchrotron codes are poorly adapted for use in accelerators with fixed magnetic fields:

- The central orbit is a spiral – rather than a fixed-radius ring - with the equilibrium-orbit (E.O.) radius depending on energy;
- A wide radial region of magnetic field must be characterized.

As a result, special arrangements must be made to deal with momentum-dependent effects accurately.

ORBIT-TRACKING TOOLS

Méot et al[1]. have avoided these problems by using ZGOUBI:
– an orbit tracking code originally developed for the study and tuning of mass spectrometers and beam lines.

Here, we report studies made with the cyclotron orbit code CYCLOPS[2], which tracks particles through magnetic fields specified on a polar grid and determines the equilibrium orbits and their optical properties.

This has the advantages of:

- Being designed for multi-sector machines with wide aperture magnets
- Simultaneous computation of orbit properties at all energies
- Capability of tracking through measured magnetic fields
- Availability of the sister code GOBLIN for accelerated-orbit studies.

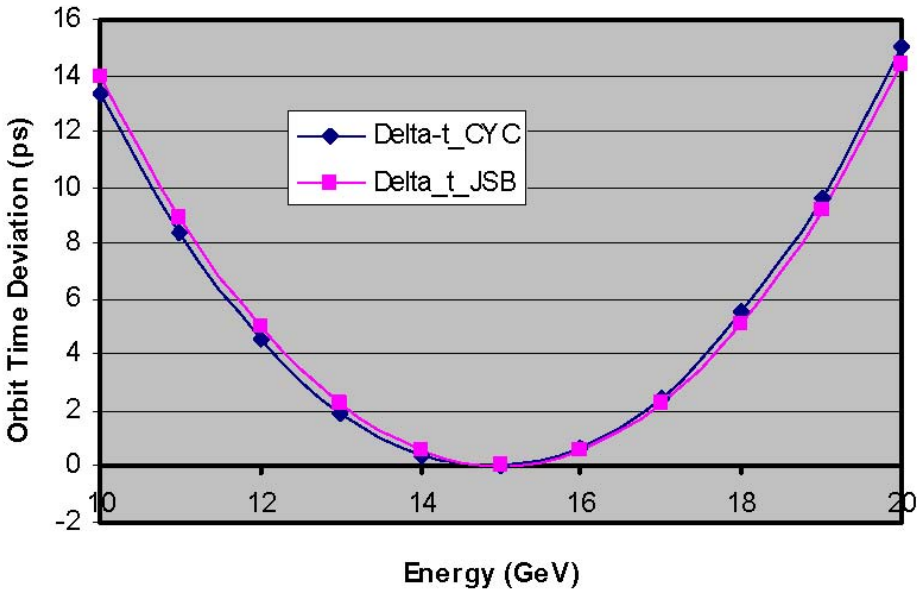
Some initial results were reported[3] at Cyclotrons'07 and FFAG'08.

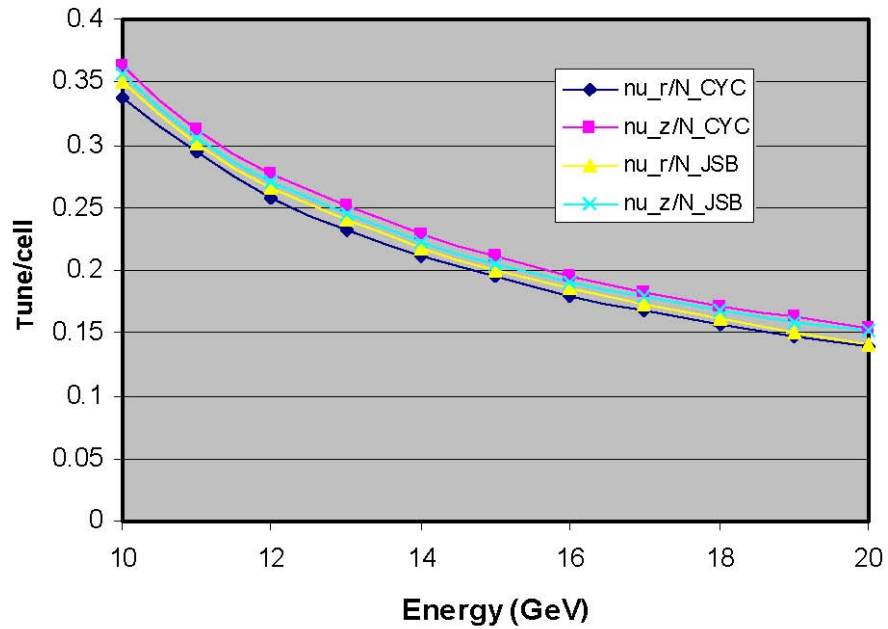
1. F. Lemuet, F. Méot, G. Rees, Proc. PAC'05, 2693 (2005).
2. M.M. Gordon, Part. Accel. **16**, 39 (1984).
3. M.K. Craddock, Y.-N. Rao, Cyclotrons'07, 370 (2007).

TEST RUNS WITH THE FODO-2 LATTICE

FODO-2 was one of several linear non-scaling FFAG lattices developed by Scott Berg in 2003-4 for 10-20 GeV muons:

- D is a positive-bending, and F a negative-bending, sector magnet.



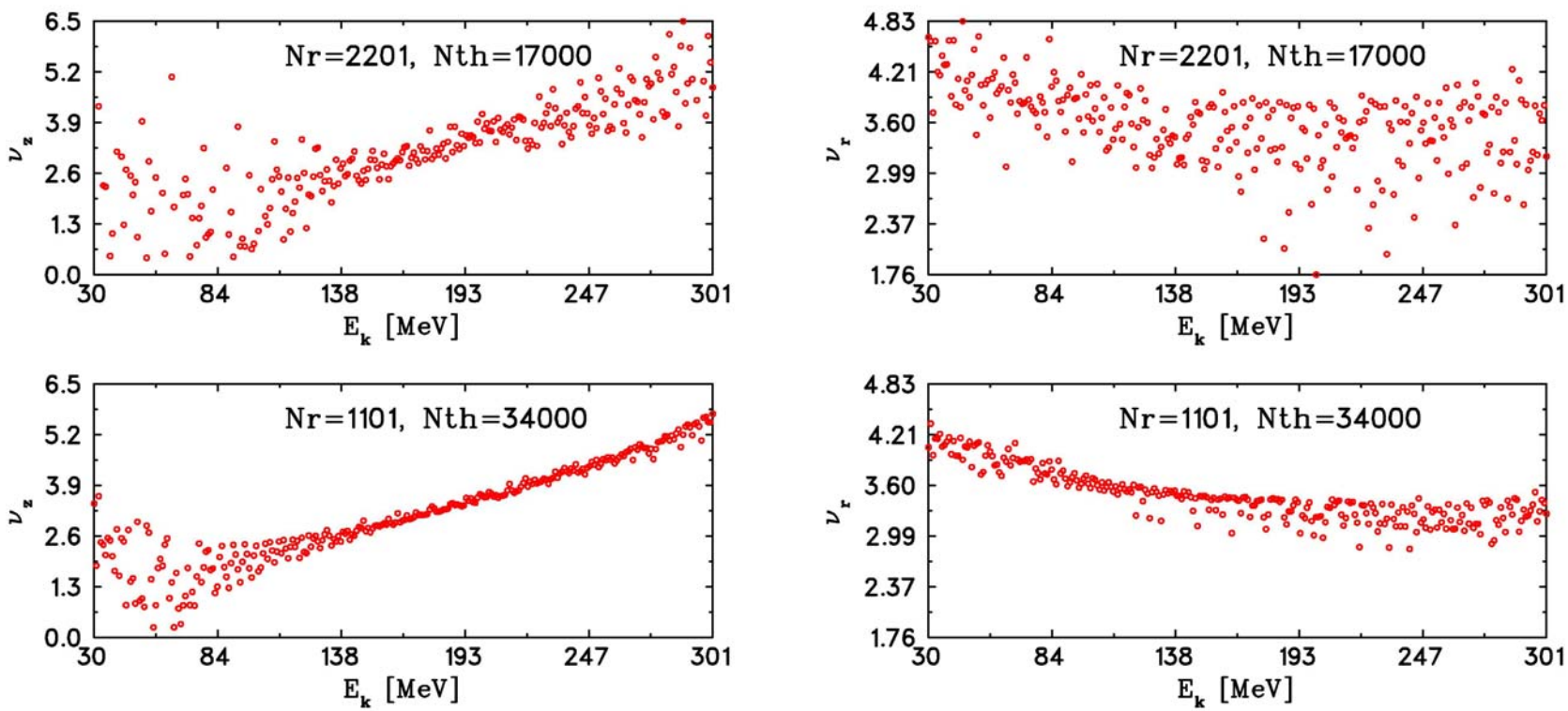


- In order to get good E.O. solutions with the hard magnet edges, a very fine field grid was required (800 θ values, 400 r values).
- The agreement between the CYCLOPS results (CYC) and Berg's (JSB) was very satisfactory.

JOHNSTONE-KOSCIELNIAK MEDICAL FFAG (1)

In 2007 Carol Johnstone & Shane Koscielniak developed an LNS FFAG, using a FODO lattice, for cancer therapy with 18-400 MeV/u carbon ions[4]. This used edge- as well as gradient-focusing to minimize the tune variation.

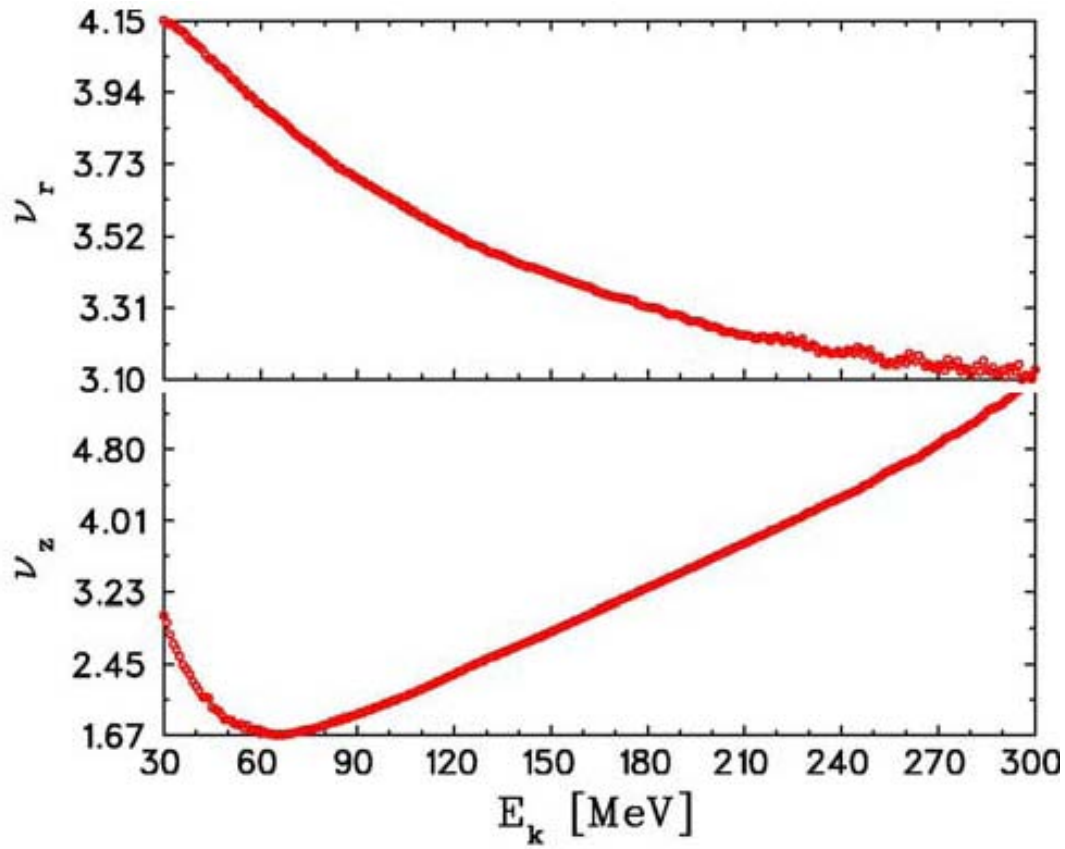
But **non-radial** hard magnet edges are tricky to model with a polar grid – and lead to noisy results from CYCLOPS – even with 37 million grid points!



4. C. Johnstone, S.R. Koscielniak, *Proc. PAC'07,* 2951-3 (2007).

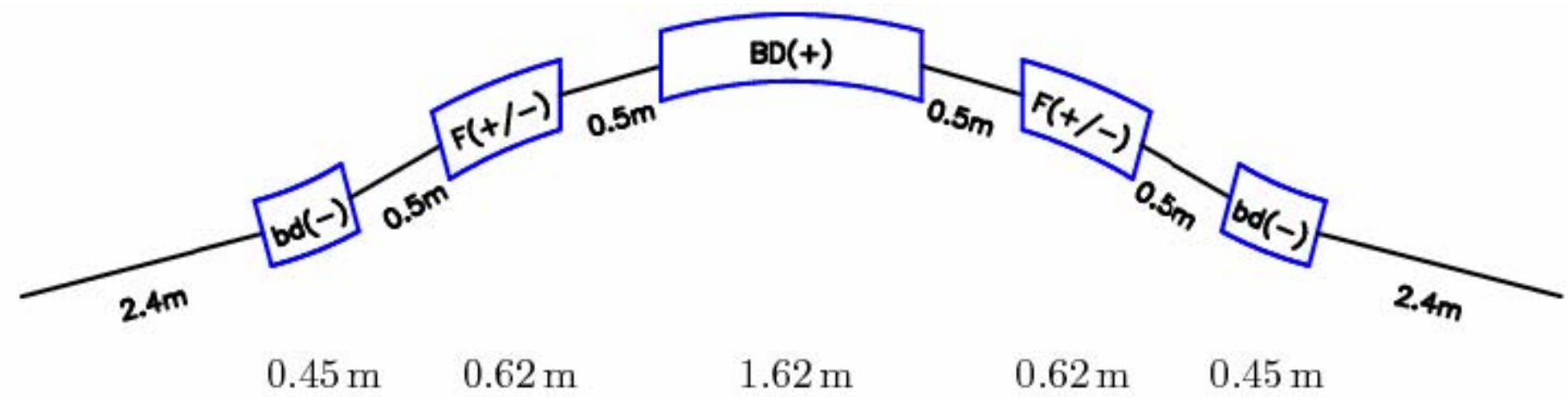
JOHNSTONE-KOSCIELNIAK MEDICAL FFAG (2)

The brute-force method of reducing the mesh size was clearly inadequate.
But smoothing the hard field edges with a steep sinusoidal fall-off
– proved to be a simple but effective technique
- gave tunes that vary almost perfectly smoothly with energy.



REES'S ISOCHRONOUS IFFAG

G.H. Rees[1,5] has designed several FFAGs using novel 5-magnet "pumplet" cells, in which variations in field gradient and sign enable each magnet's function to vary with radius – providing great flexibility.



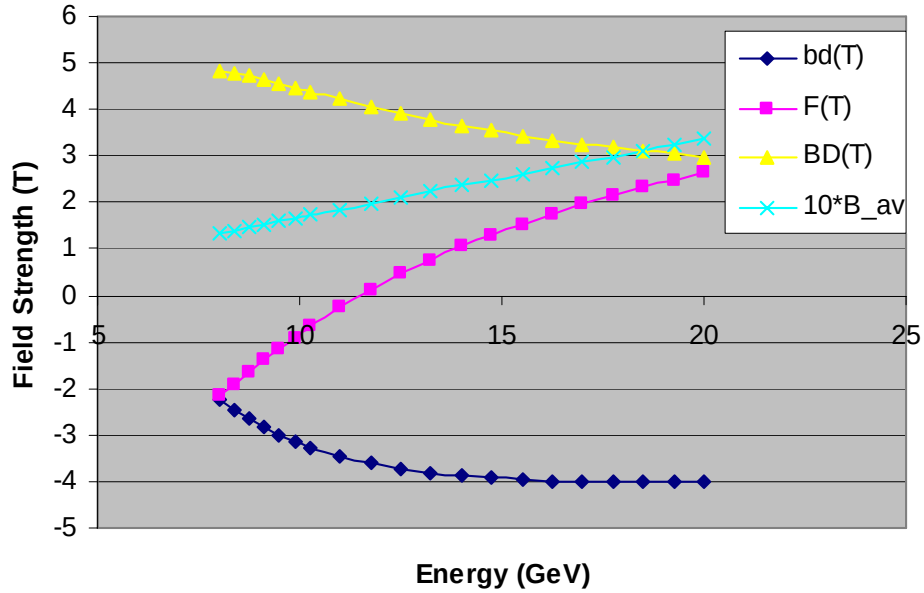
- The example shown is an isochronous design (IFFAG) for accelerating muons from 8-20 GeV in 16 turns.
- This is remarkable in achieving both isochronism and vertical focusing at highly relativistic energies ($77 \leq \gamma \leq 190$) without invoking spiral magnet edge focusing [recall isochronous $\Delta\nu_z^2 = -(r/B_{av})(dB_{av}/dr) = -\beta^2\gamma^2$].
- Highest energy spiral-sector isochronous cyclotron design had $\gamma \leq 15$.

5. G.H. Rees, *FFAG'04* (2004); *FFAG'05* (2005); *ICFA-Beam Dynamics Newsletter* **43**, 74 (2007)

IFFAG FIELDS & TUNES

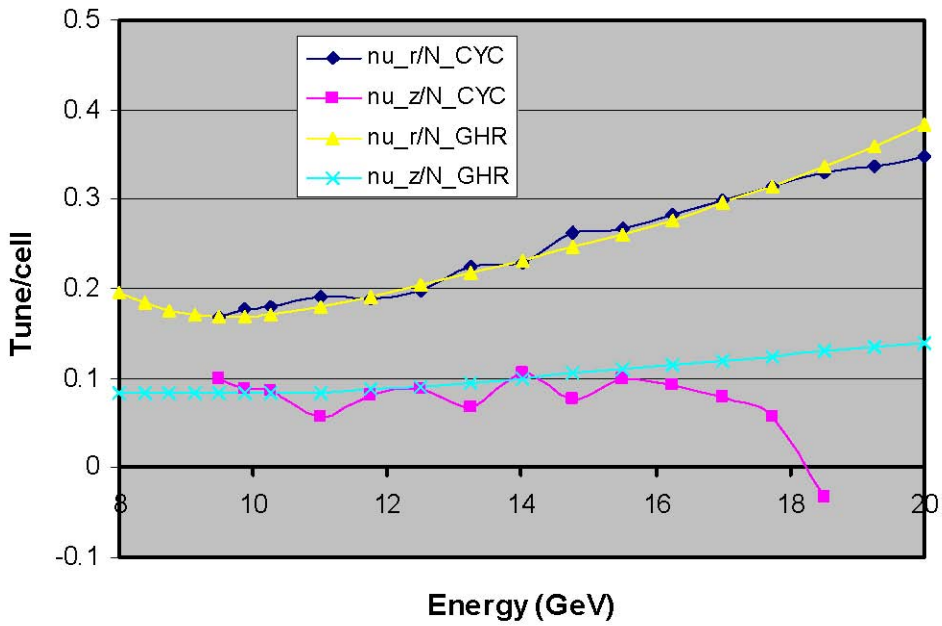
For the **fields**, note how:

- F reverses sign at ~11 GeV
- bd focusing vanishes at high E
- BD focusing vanishes at low E
- B_{av} rises linearly with E
- The vertical defocusing associated with rising B_{av} is offset by **strong AG focusing**.



For the **tunes** our initial results[3] (♦■) were in general agreement with Rees's (▲ x) – except above 17 GeV - but rather noisy.

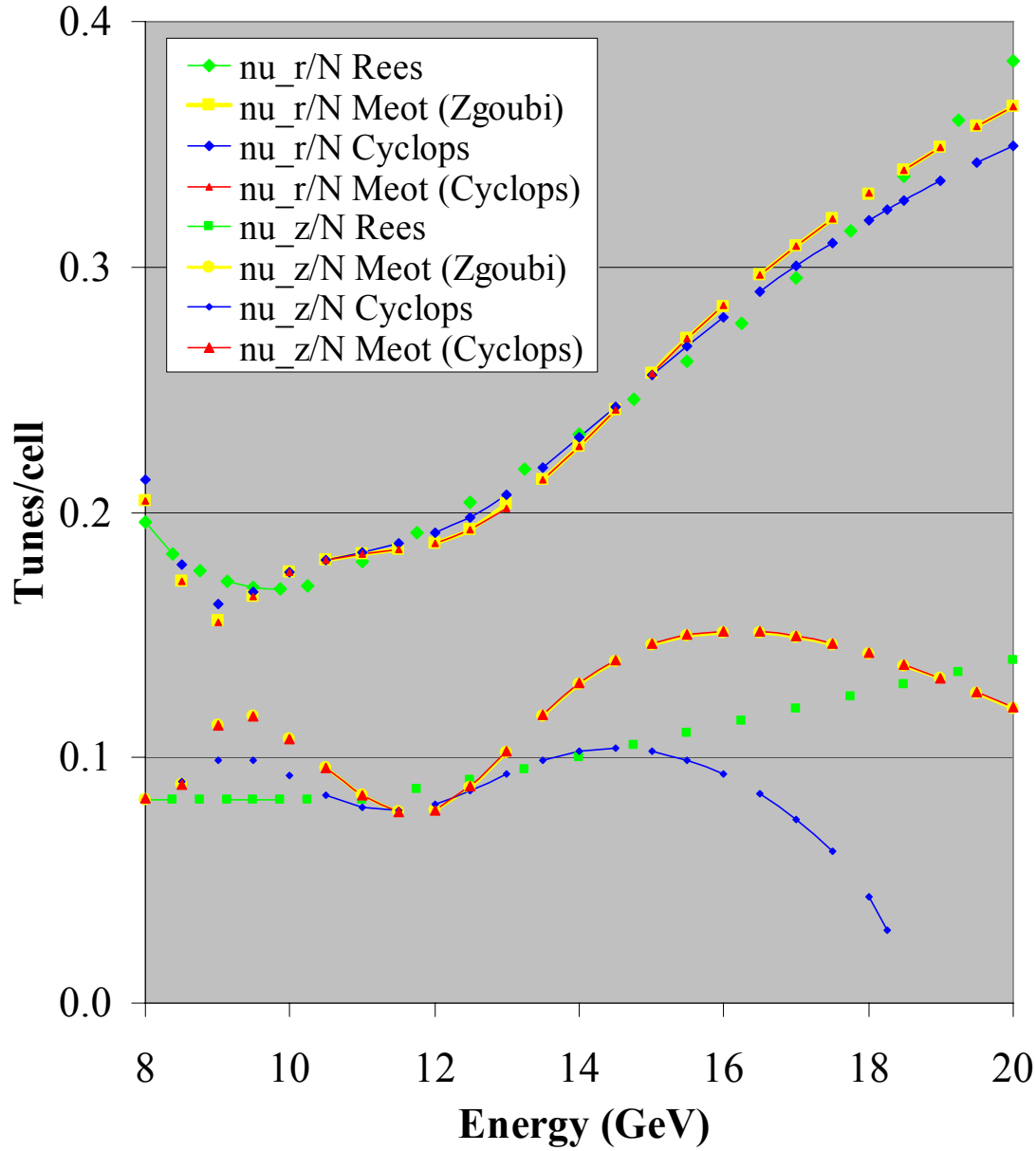
As before, field smoothing is needed to track through the non-radial hard edges accurately.



IFFAG WITH SOFTENED MAGNET EDGES

Steep sinusoidal edges remove the noise in the tune data, but not the drop-off in ν_z above 15 GeV (-◆-).

Méot's tracking results using ZGOUBI (-■-) agree with Rees's predictions (-■-) at 8, 11 & 20 GeV – but only after slight adjustments in the magnet positions and field profiles. For this adjusted configuration CYCLOPS (-▲-) gives results identical to those from ZGOUBI.



RADIAL-SECTOR CYCLOTRONS WITH REVERSE BENDS

The IFFAG is essentially an isochronous ring cyclotron with an unusually complicated magnet arrangement – 5 magnets/cell rather than 1.

An isochronous cyclotron's top energy is limited by vertical focusing:

$$\nu_z^2 \approx -\beta^2\gamma^2 + F^2(1 + 2\tan^2\varepsilon)$$

where ε is spiral angle and the magnetic "flutter" (mean square deviation)

$$F^2 \equiv \left\langle \left(B(\theta)/B_{av} - 1\right)^2 \right\rangle .$$

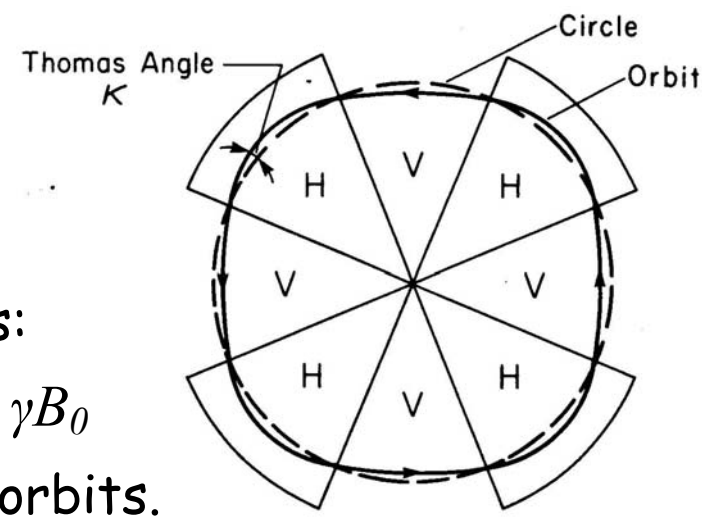
How high an energy could a radial-sector cyclotron reach by simply **converting the low-field "valley" to a reverse bend** - maximizing F^2 and introducing AG focusing?

We assume:

- N radial sectors (hill fraction h)
- Hard-edge magnets with $B \leq 5\mathrm{T}$
- No drift spaces
- Equal and opposite hill and valley fields:

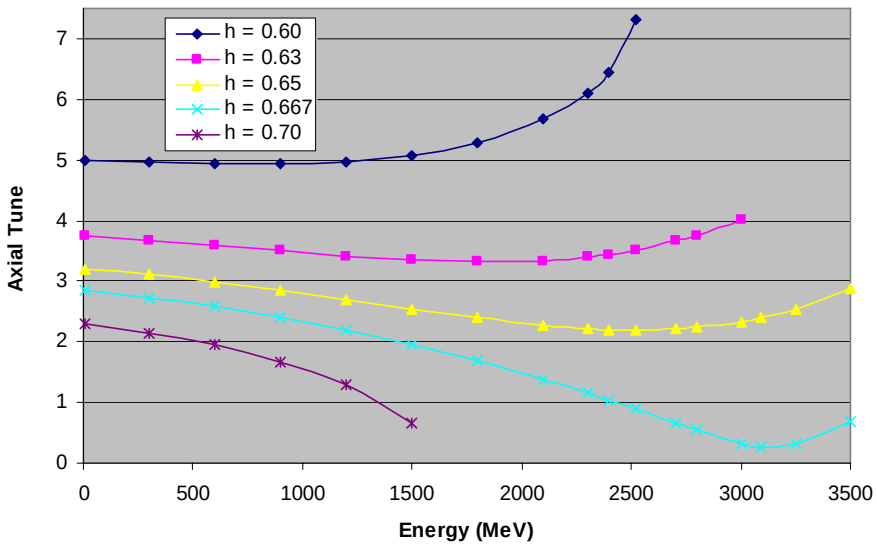
$$B_h = -B_v = B(r) = \gamma B_0$$

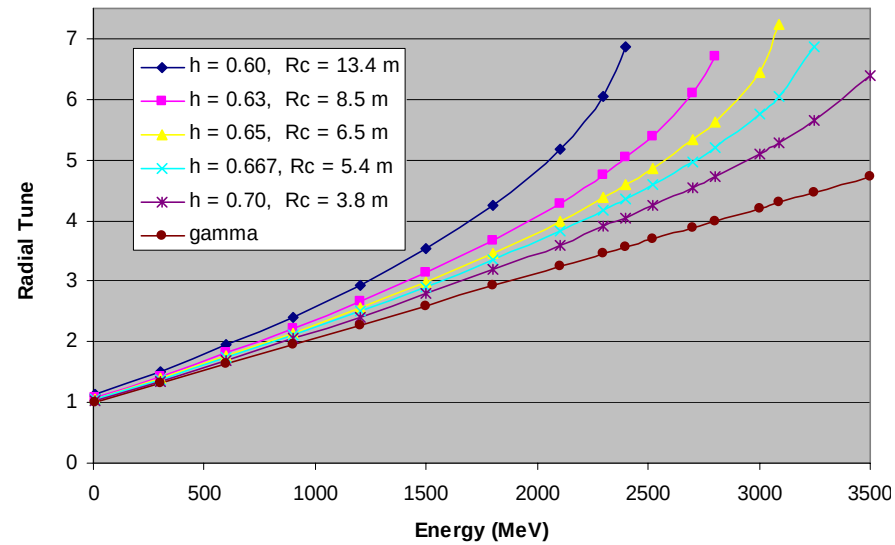
- Field contours following the scalloped orbits.



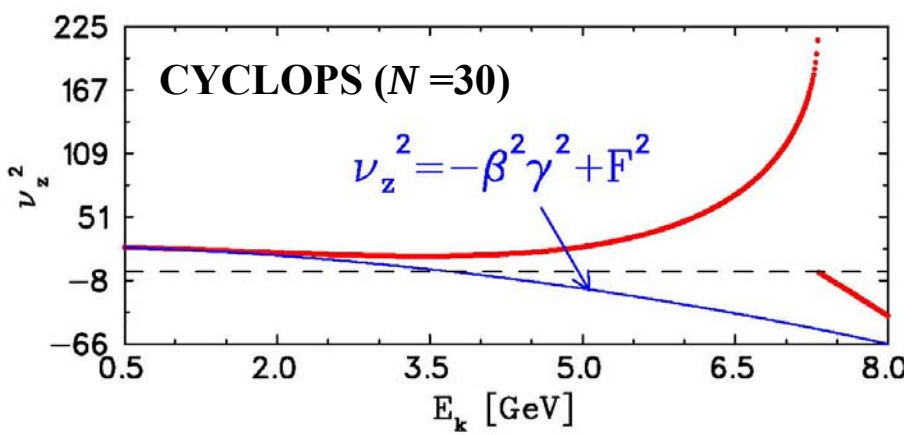
REVERSE-BEND CYCLOTRONS – SIMULATIONS

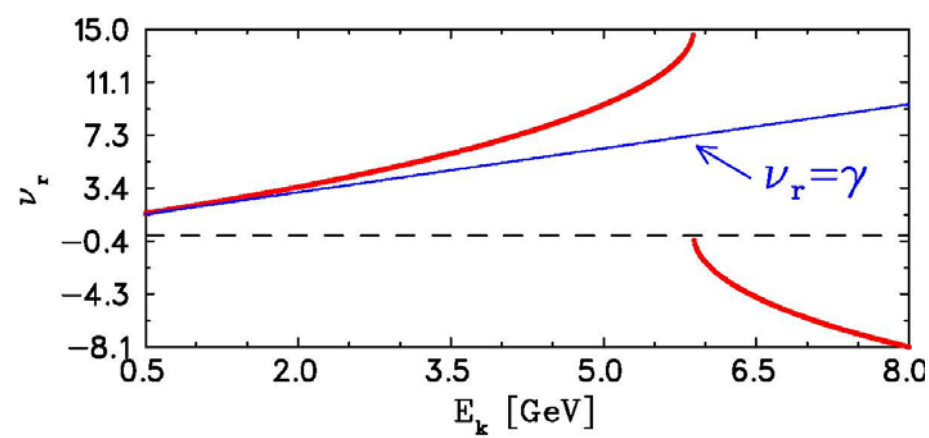
For $N = 15$ the $N/2$ stopband is a limiting factor: either we widen the hills - reducing the radius and both tunes (say $h = 0.65$, $E = 3$ GeV, $R_c = 6.5$ m),



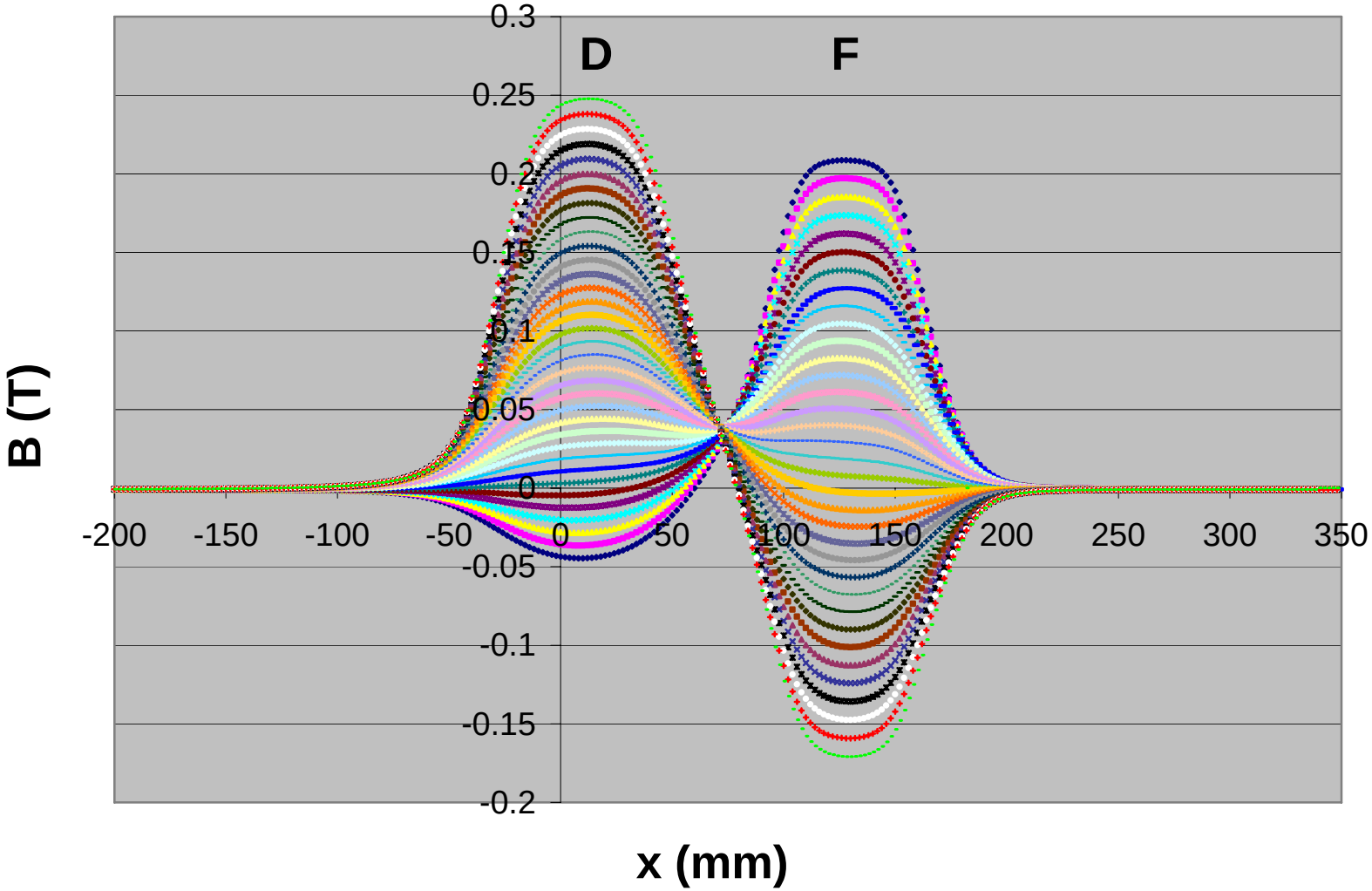


or increase the number of sectors (say to $N = 30$, with $h = 0.6$)
- a more effective way of repelling $\nu_r = N/2$ ($E = 6$ GeV, but $R_c = 14.9$ m).



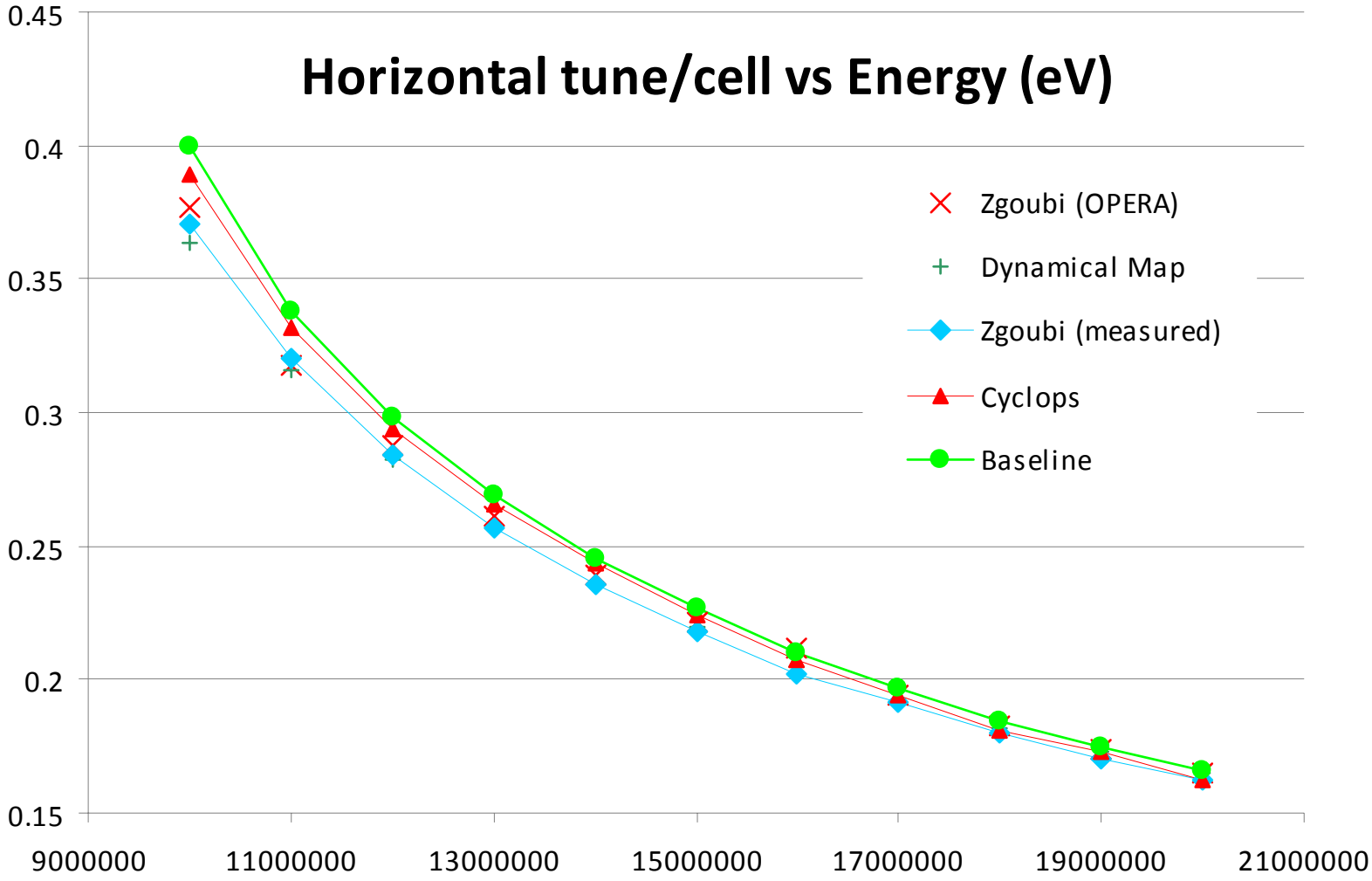


EMMA



For EMMA, Carl Beard kindly provided field measurements on a 2-mm Cartesian grid, which were then converted to a polar grid for CYCLOPS.

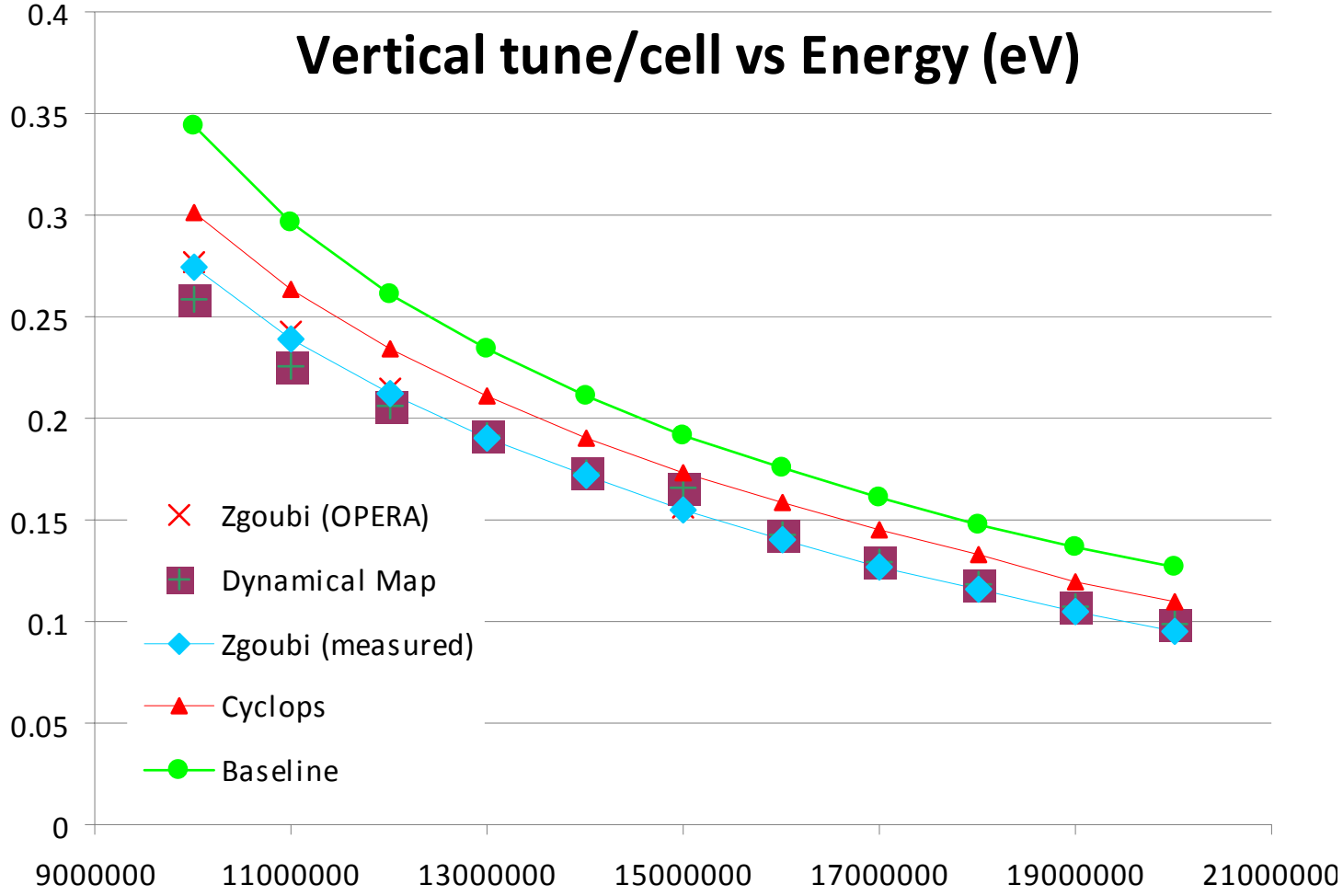
EMMA – HORIZONTAL TUNE



CYCLOPS results agree well with Scott Berg's "Baseline" data,

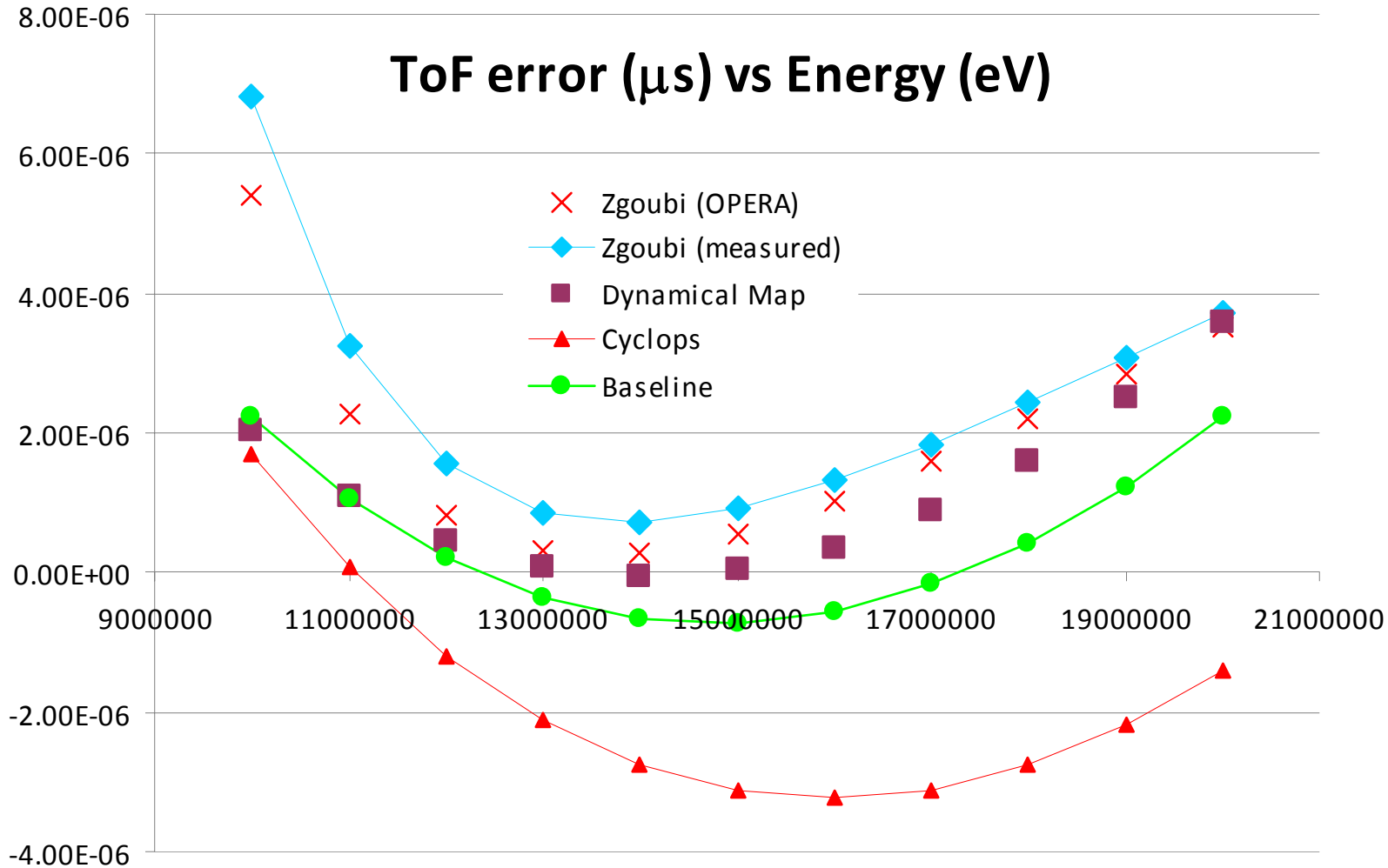
- less well with ZGOUBI runs (courtesy of Yoel Giboudot).

EMMA – VERTICAL TUNE



Here CYCLOPS, ZGOUBI and the Baseline all agree on the trend -- but disagree on the amplitude.

EMMA – TIME OF FLIGHT



N.B. CYCLOPS, ZGOUBI & Baseline all assume different reference frequencies

- so the vertical displacements are probably not significant
- but the displacements in energy of the minima are significant.

SUMMARY

The cyclotron equilibrium orbit code CYCLOPS is:

- designed for **energy-dependent E.O.s** in **wide-aperture magnets**
- ideal for finding E.O. properties in **measured magnetic fields**
- less friendly with hard-edge magnets – edge softening needed.

CYCLOPS has been applied to several FFAGs and AG cyclotrons:

- LNS FODO-2 for 10-20 GeV muons
- LNS edge-focusing FODO medical FFAG for 18-400 MeV/u C ions
- NLNS isochronous pumplet IFFAG for 8-20 GeV muons
- Reverse-bend ring cyclotrons (3 GeV @ R_c = 6.5 m, 6 GeV @ R_c = 15 m)
- LNS EMMA for 10-20 MeV electrons

Projects under way:

- CYCLOPS: Carol Johnstone's isochronous 18-150 MeV FFAG
- GOBLIN: accelerated orbits in EMMA.