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FFAG 09

Tracking studies in EMMA with Dynamical Map

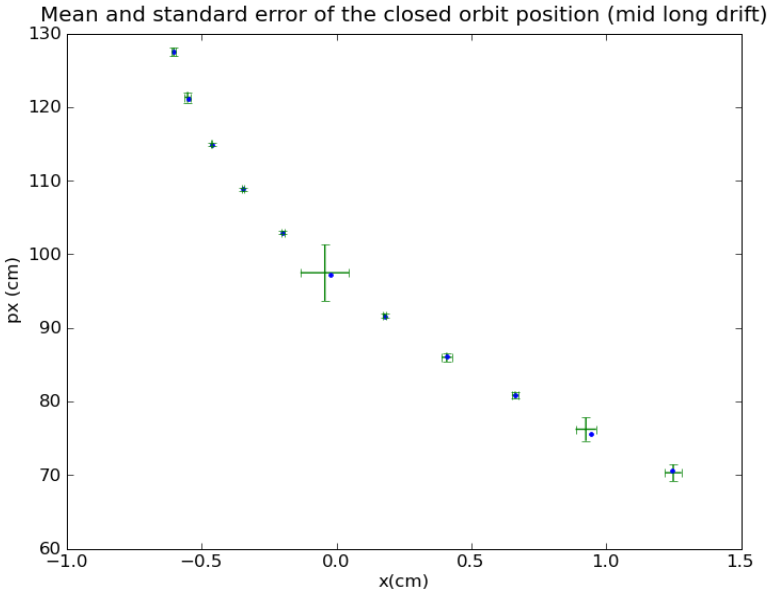
Contents

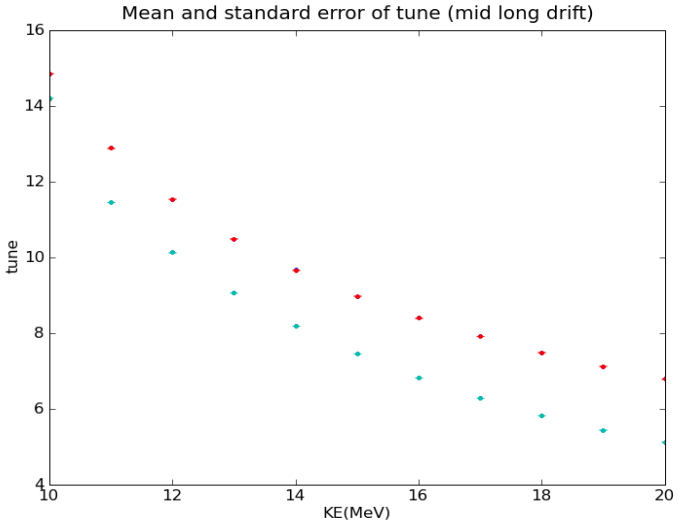
- Lattice representation
- Dynamical Map computation.
- Application to tracking results

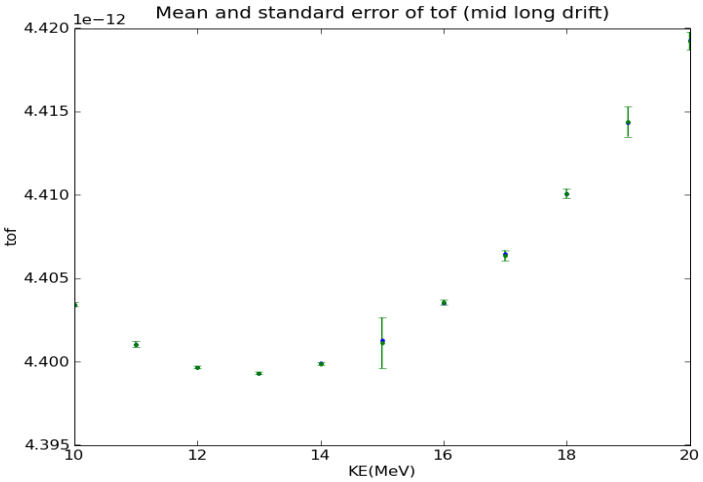
Philosophy of the study: Description of a lattice

- Hard egde model
- Hard edge model with Fringe Field
- Magnetic Field Map (e.g.OPERA)

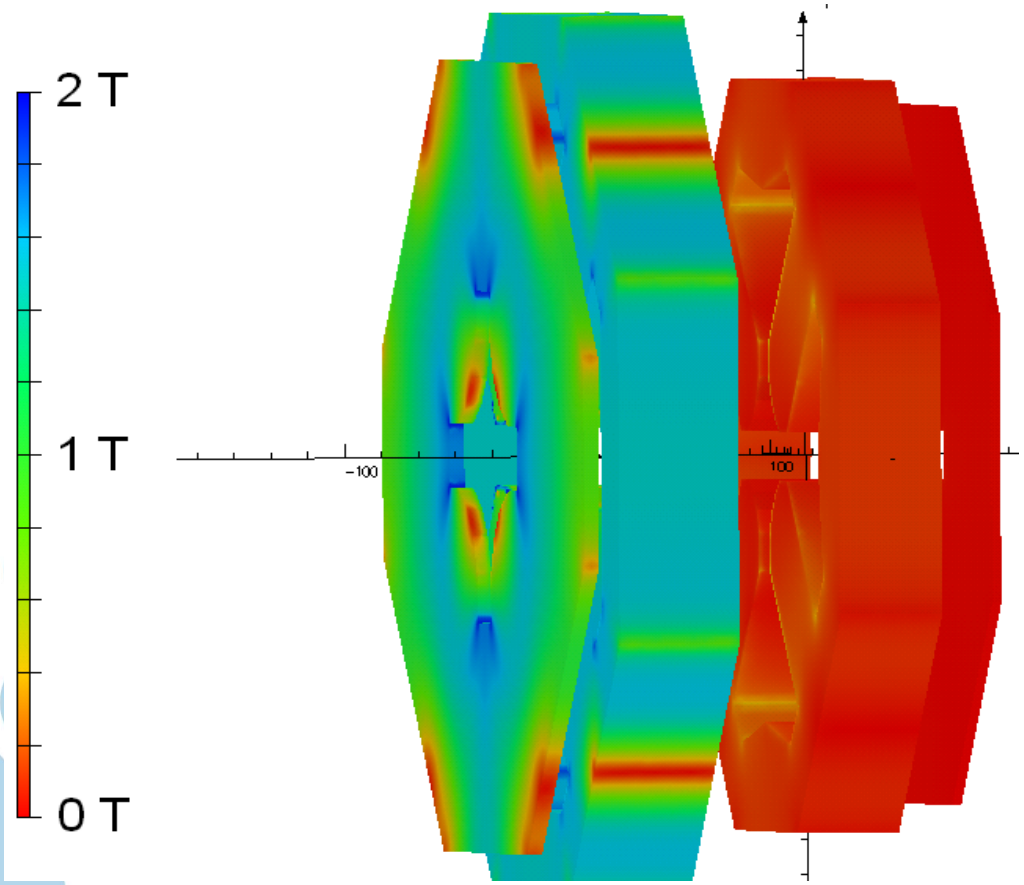
Tracking Example in Hard Edge Model : Random Sextuple Component Effect in EMMA magnets



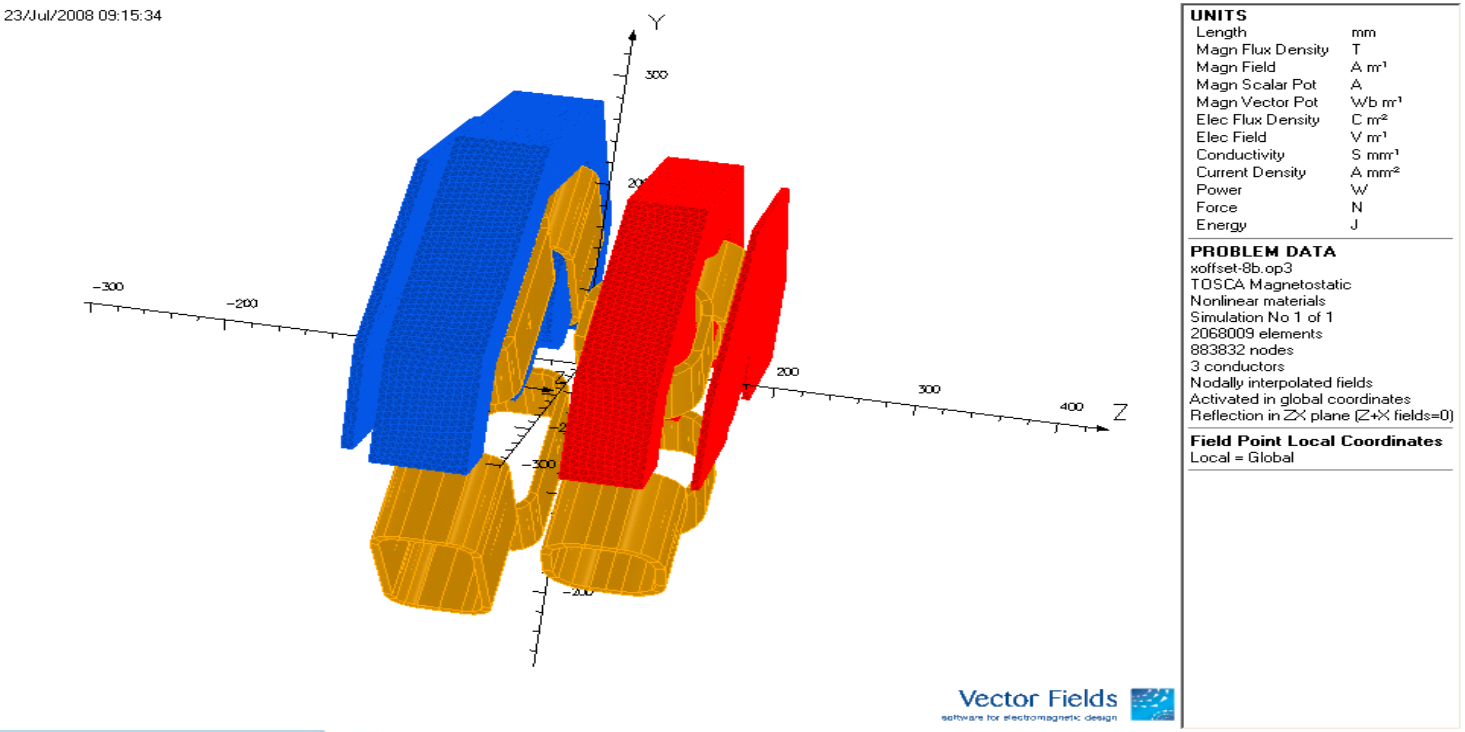




Quite Complex magnetic Field:



Magnetic Field Map Computation

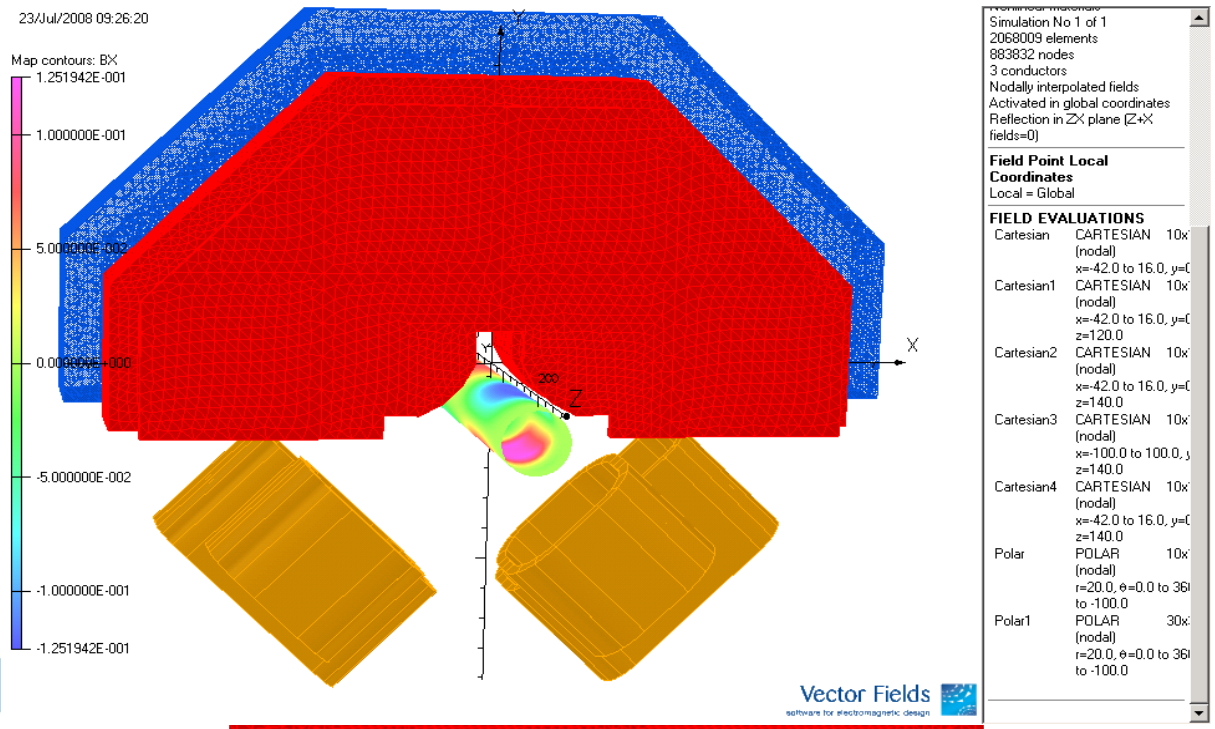


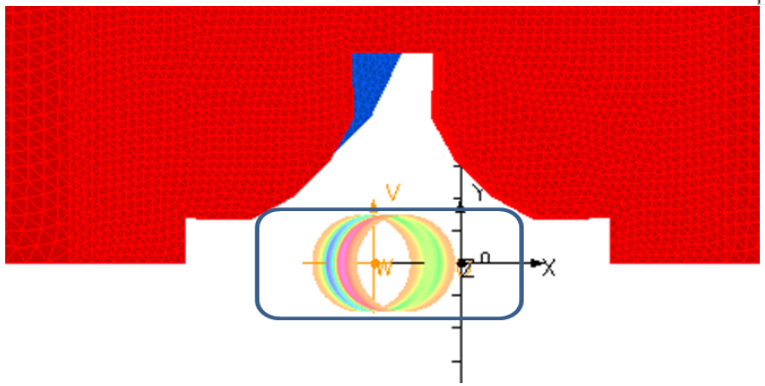
What do we do with this Field Map?

- Zgoubi will read the map, compute the derivative of the Field and track within for each particle for each cell.

- We can process this Field maps to output an analytical (Taylor expansion) solution for the particle dynamics.

Magnetic Field in Analytical Form





Potential Vector

Magnetic Field Values on the grid given by OPERA

Fast Fourier Transform

$$B_\rho = \sum a_{mn} I'_m(nk_z\rho)\sin(m\phi)\cos(nk_z z)$$

Cylindrical to cartesian conversion

$$B_y = \sum c_{mn}\sin(mk_x x)\cosh(k_y y)\cos(nk_z z)$$

Potential Vector
A (x,y,z)

Dynamical Map or Taylor Map (higher order transfer matrix)

- Symplectic Integrator : suitable for Hamiltonian defined problem and s-dependent magnetic field

Explicit Higher Order Symplectic Integrator for s-Dependent Magnetic Field, Y. Wu,¤ E. Forest,y and D. S. Robinz (2001)

$$\mathcal{M}_2(\Delta\sigma) = \exp(: -\frac{\Delta\sigma}{2} p_z :) \exp(: \frac{\Delta\sigma}{2} a_z :) \exp(: -\frac{\Delta\sigma}{2}(-\delta + \frac{p_x^2}{2(1+\delta)}) :)$$
$$\mathcal{A}_y \exp(: -\Delta\sigma \frac{p_y^2}{2(1+\delta)} :) \mathcal{A}_y^{-1} \exp(: -\frac{\Delta\sigma}{2}(-\delta + \frac{p_x^2}{2(1+\delta)}) :) \exp(: \frac{\Delta\sigma}{2} a_z :) \exp(: -\frac{\Delta\sigma}{2} p_z :).$$

- Simple Integrator : Runge Kutta method

Dynamical Map for an electron at 15 MeV

X

```
I  COEFFICIENT            ORDER EXPONENTS
1  -.1526960120987588E-01   0   0 0  0 0  0 0
2  0.3401128273867607       1   1 0  0 0  0 0
3  0.1642237907235073       1   0 1  0 0  0 0
4  0.1349437018669432E-01   1   0 0  0 0  0 1
```

Px

```
I  COEFFICIENT            ORDER EXPONENTS
1  -.1662658740250797       0   0 0  0 0  0 0
2  -7.346211164087502       1   1 0  0 0  0 0
3  -.6069240210898136       1   0 1  0 0  0 0
4  -.2793363824327076E-01   1   0 0  0 0  0 1
```

Y

```
I  COEFFICIENT            ORDER EXPONENTS
1   1.807979732539481       1   0 0  1 0  0 0
2  0.4618723192508760       1   0 0  0 1  0 0
```

Py

```
I  COEFFICIENT            ORDER EXPONENTS
1   9.629811976855628       1   0 0  1 0  0 0
2   3.013166294762159       1   0 0  0 1  0 0
```

z

```
I  COEFFICIENT            ORDER EXPONENTS
1  -.1018398440130244E-02   0   0 0  0 0  0 0
2  -.8963190423569946E-01   1   1 0  0 0  0 0
3  -.3602689454773938E-02   1   0 1  0 0  0 0
4   1.000000000000000       1   0 0  0 0  1 0
5  0.1767384323557966E-02   1   0 0  0 0  0 1
```

δ

```
I  COEFFICIENT            ORDER EXPONENTS
1   1.000000000000000       1   0 0  0 0  0 1
```

Xout=-.015+0.34 Xin+.16Pxin+.013δ

And so what ?

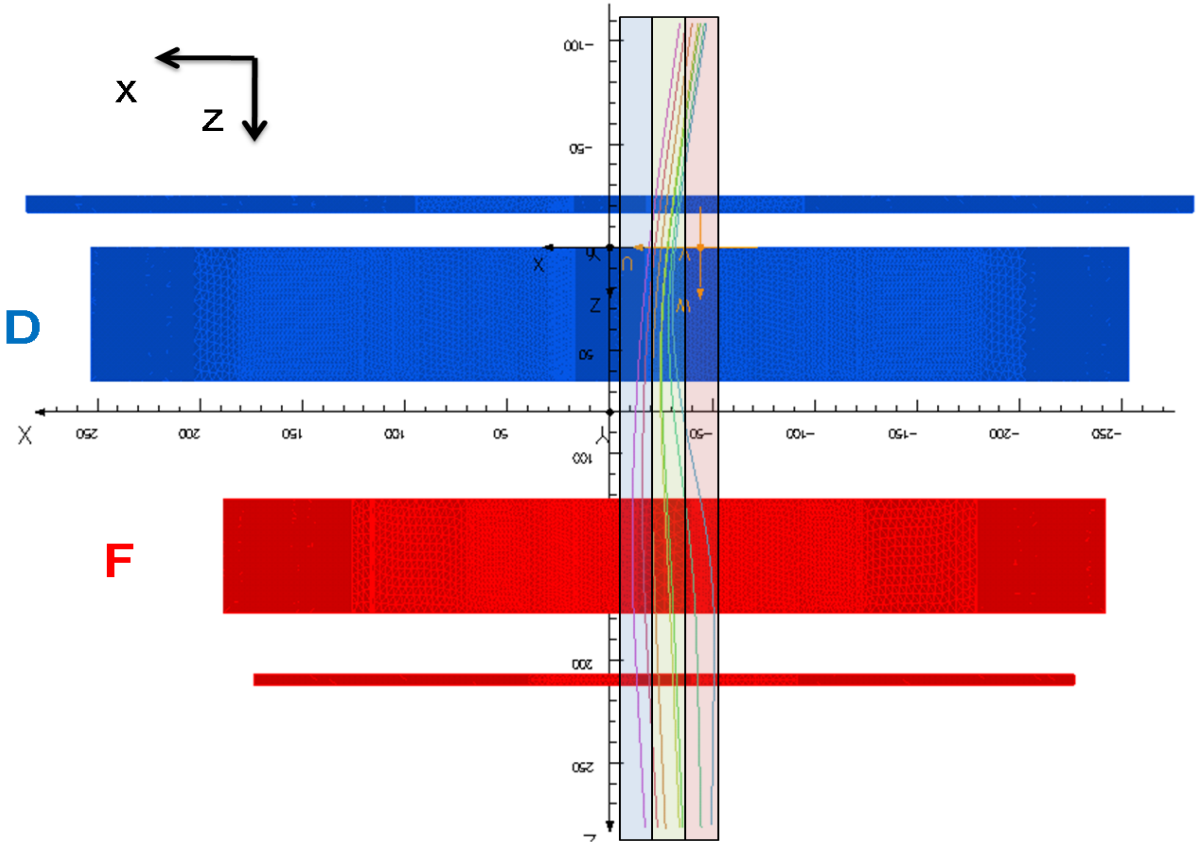
Dynamical Map are light (regarding memory) and really fast and easy to track in.

But they have a finite range of validity: Can they accurately simulate large excursions?

Are they really suitable for FFAG...?

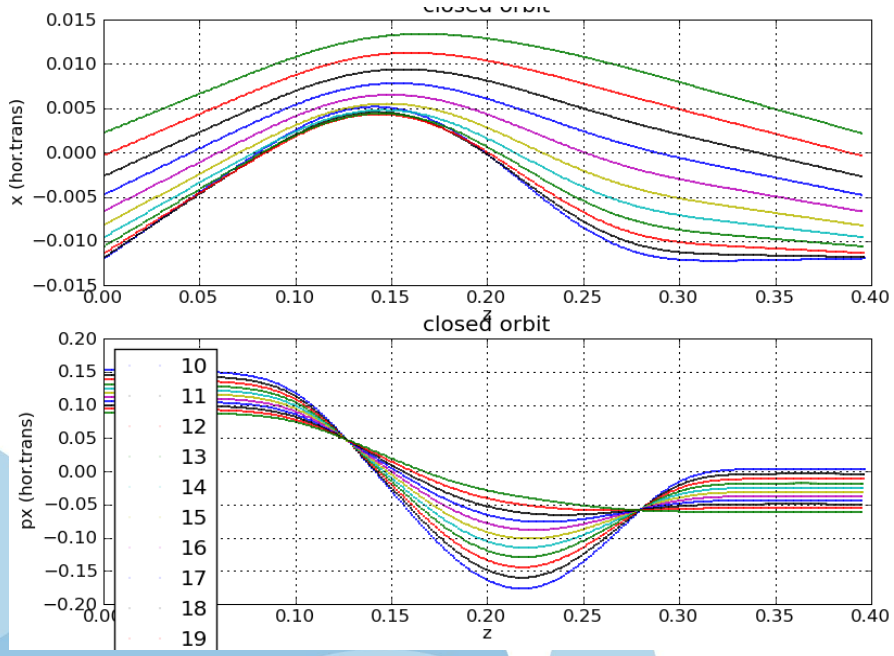
“Sampling” of the excursion

Computing of dynamical map for each energy.

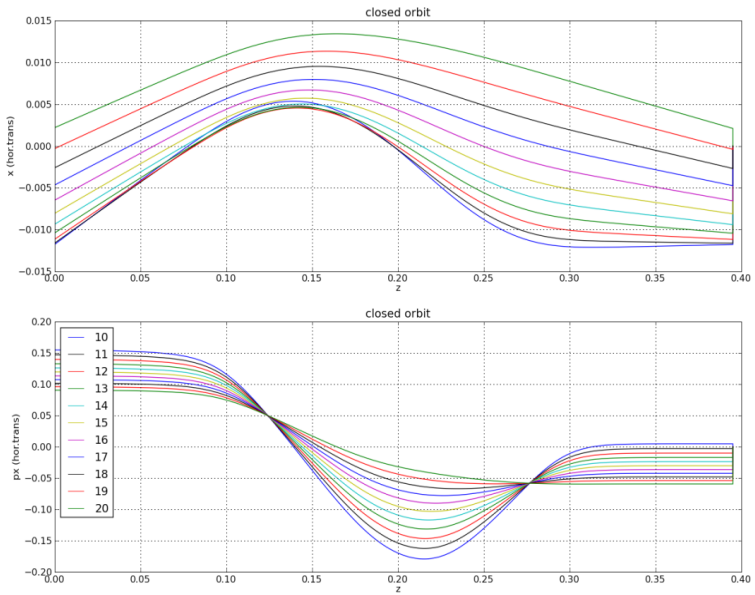


Closed Orbit calculation

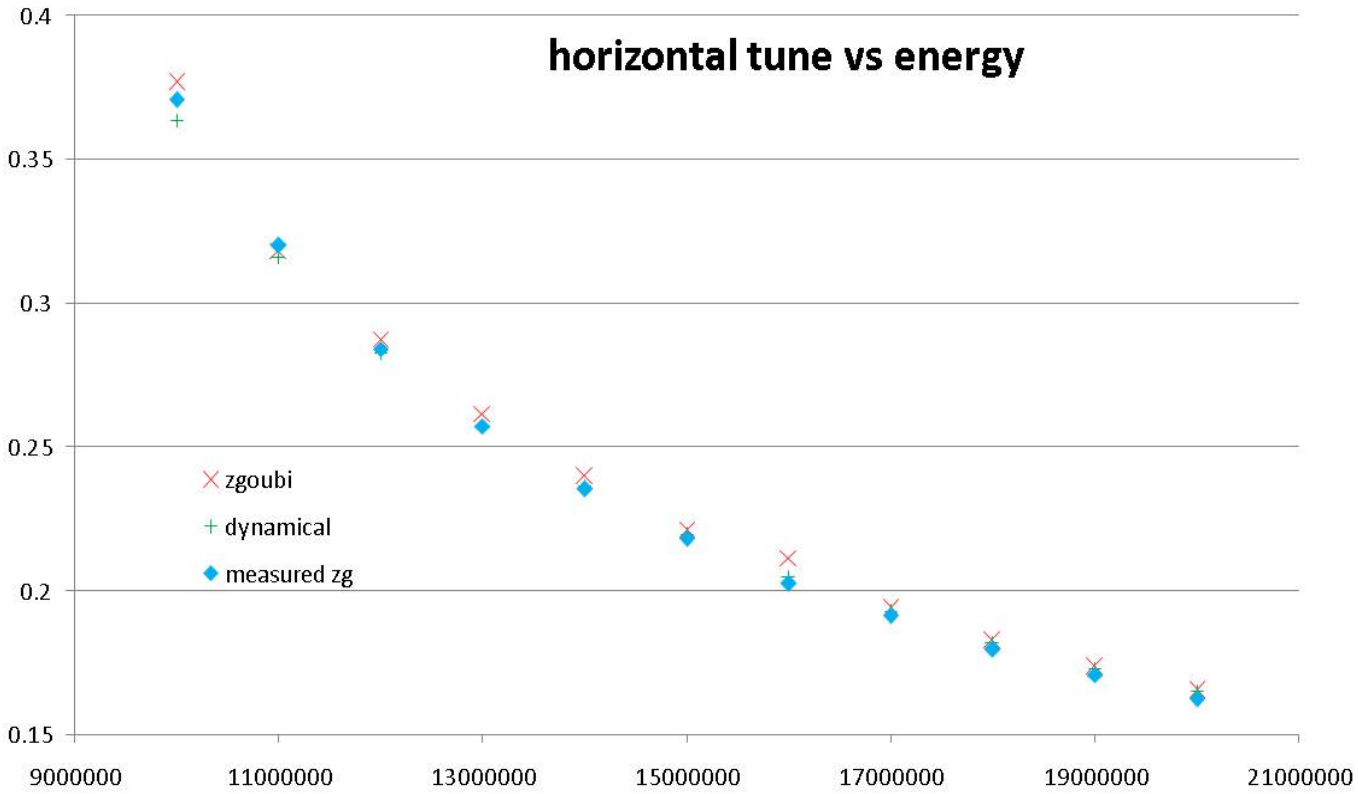
Zgoubi result



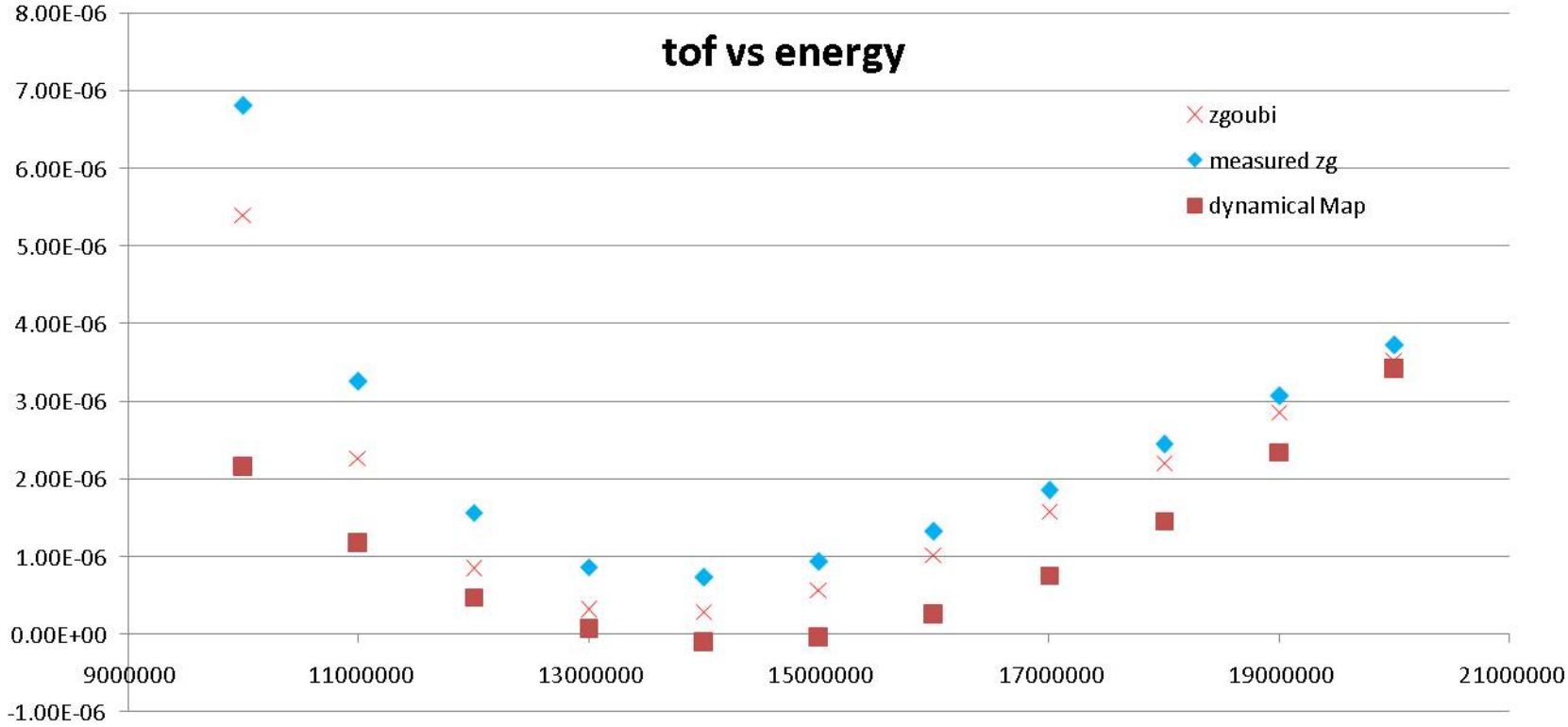
Dynamical Map result



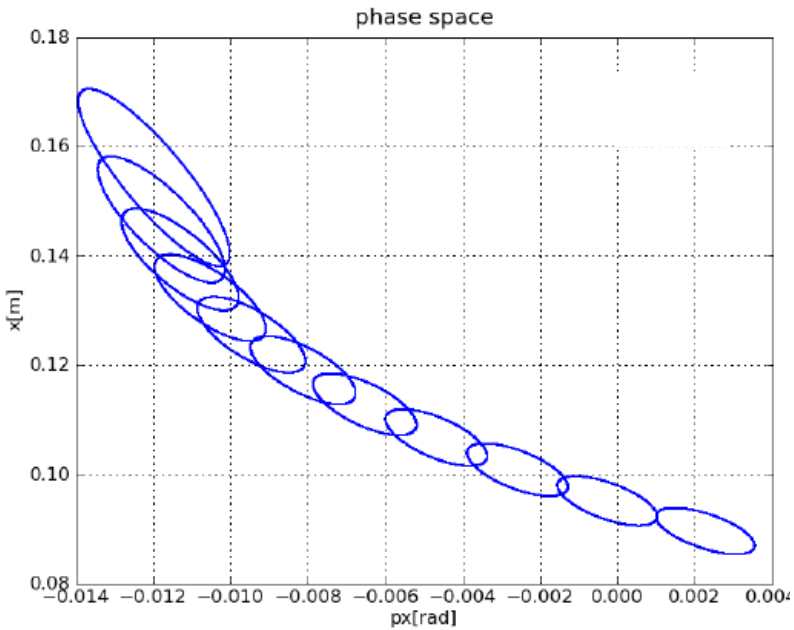
Tune and Time of Flight

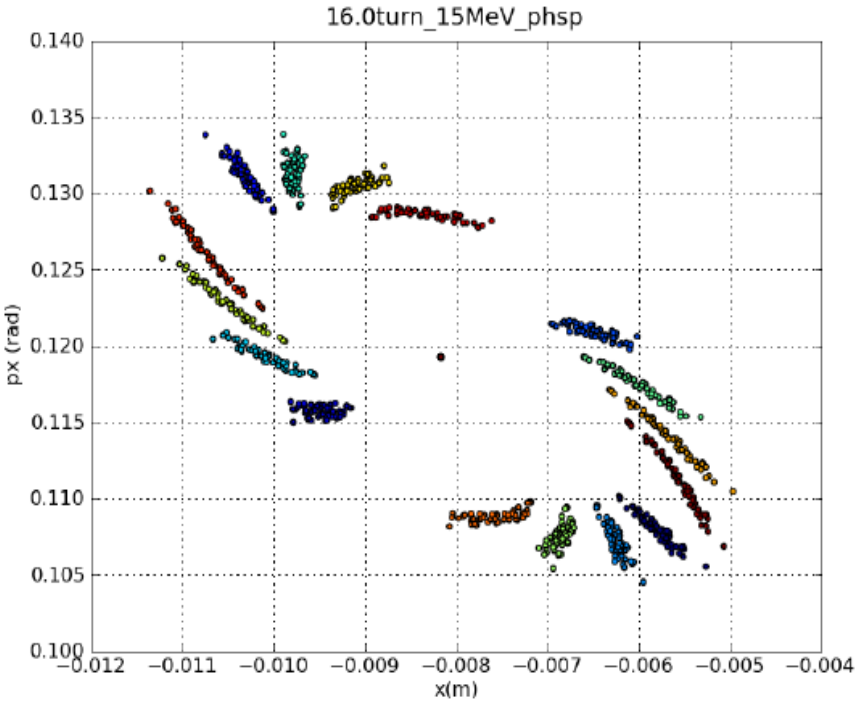


Tune and Time of Flight

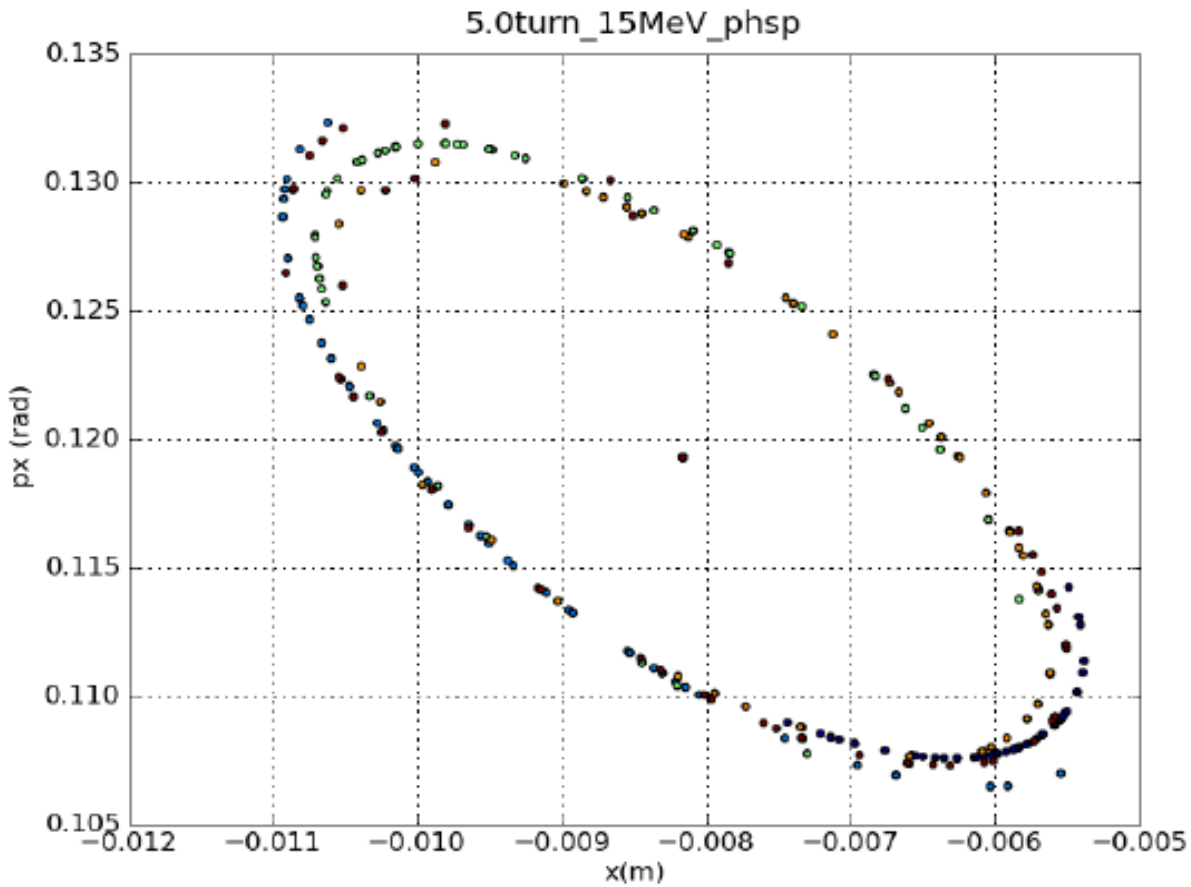


Betatron Motion in Dynamical Maps





Energy Deviation in Dynamical Maps



Small pause

What happens for the “in between” energies?

Dynamical Map for an electron at 15 MeV

X

```
I  COEFFICIENT            ORDER EXPONENTS
1  -.1526960120987588E-01   0   0 0  0 0  0 0
2  0.3401128273867607       1   1 0  0 0  0 0
3  0.1642237907235073       1   0 1  0 0  0 0
4  0.1349437018669432E-01   1   0 0  0 0  0 1
```

Px

```
I  COEFFICIENT            ORDER EXPONENTS
1  -.1662658740250797       0   0 0  0 0  0 0
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3  -.6069240210898136       1   0 1  0 0  0 0
4  -.2793363824327076E-01   1   0 0  0 0  0 1
```

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2  -.8963190423569946E-01   1   1 0  0 0  0 0
3  -.3602689454773938E-02   1   0 1  0 0  0 0
4   1.000000000000000       1   0 0  0 0  1 0
5  0.1767384323557966E-02   1   0 0  0 0  0 1
```

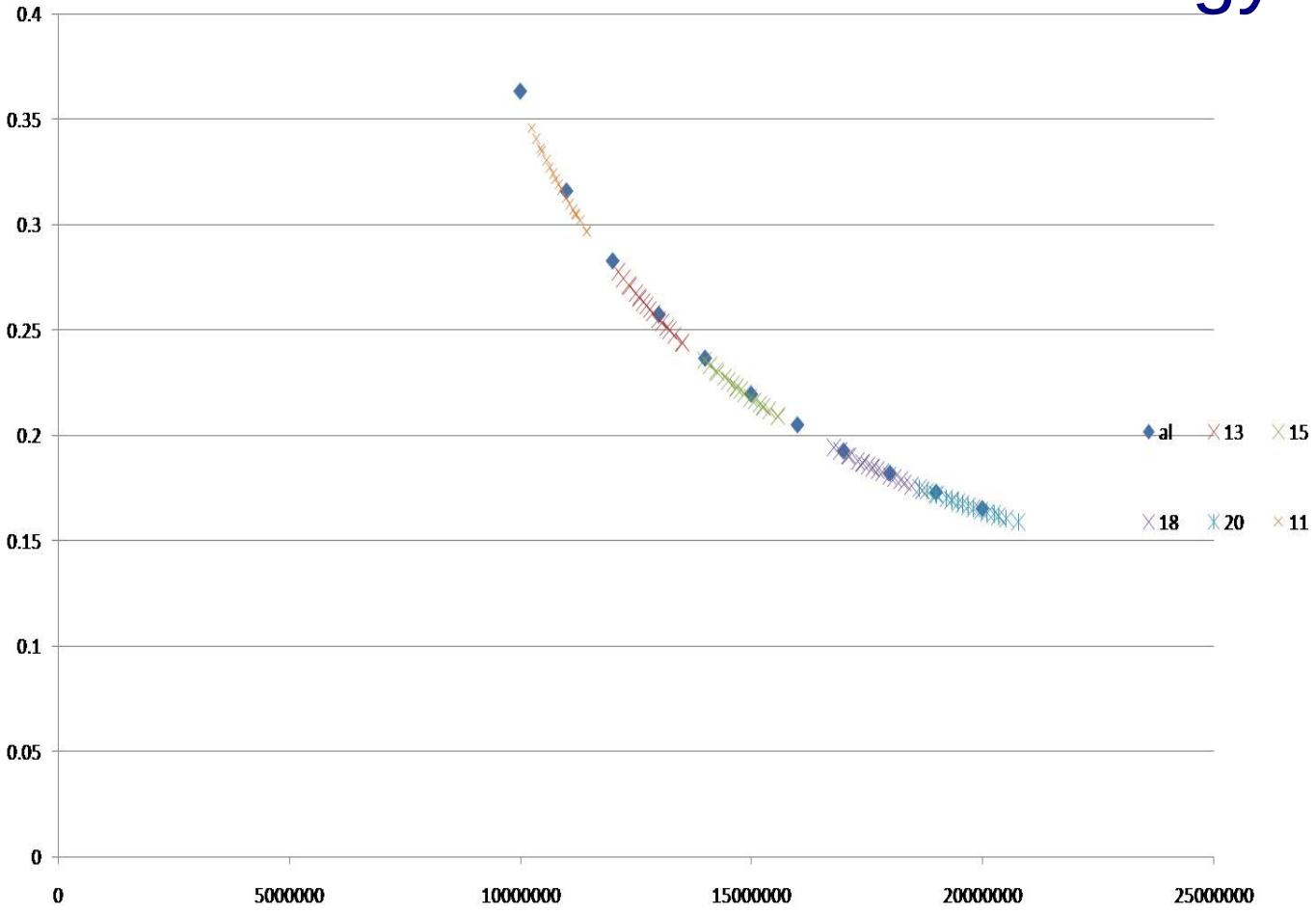
δ

```
I  COEFFICIENT            ORDER EXPONENTS
1   1.000000000000000       1   0 0  0 0  0 1
```

Xout=-.015+0.34 Xin+.16Pxin+.013δ

Tune with energy deviation in Dynamical Maps

Horizontal tune vs kinetic Energy

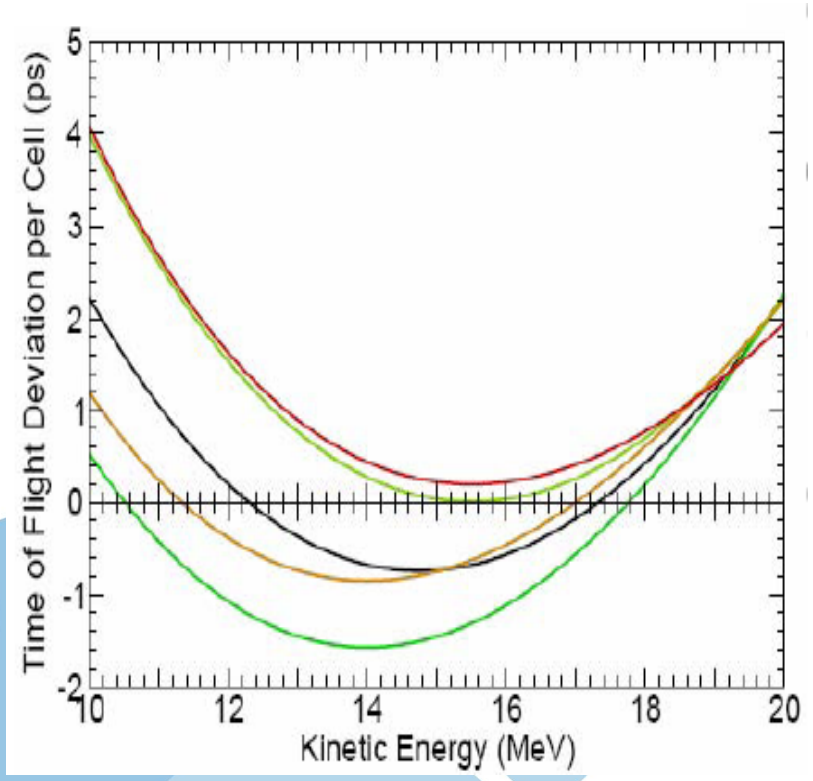


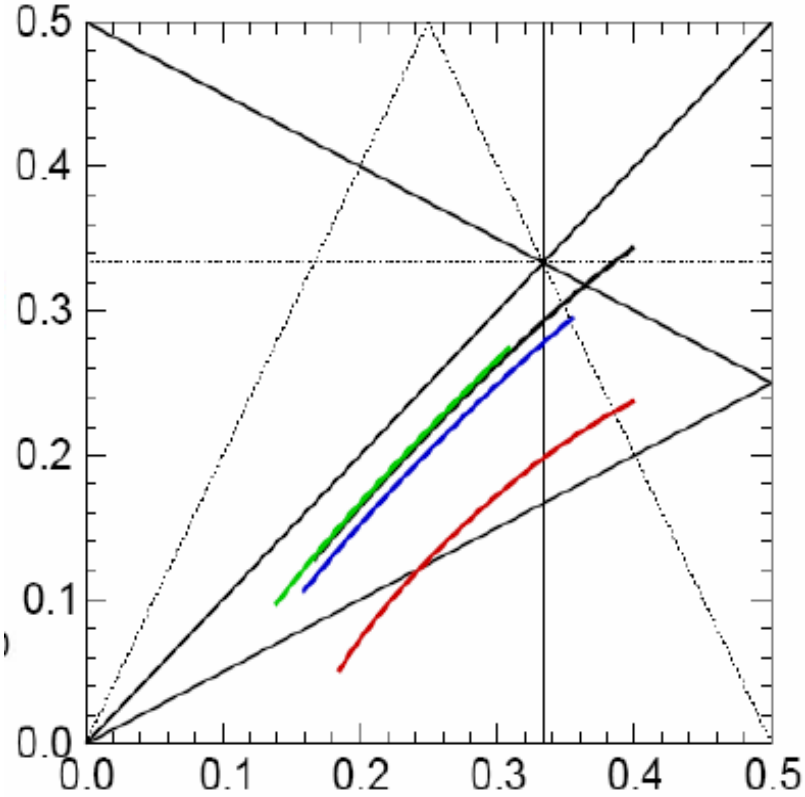
First conclusion:

- Each dynamical map computed for a different energy seems to be precise for a rather big energy deviation (compare to its reference energy).
- One has to check the time of flight behaviour.

So Let's use dynamical maps with its advantages...

EMMA degrees of Freedom



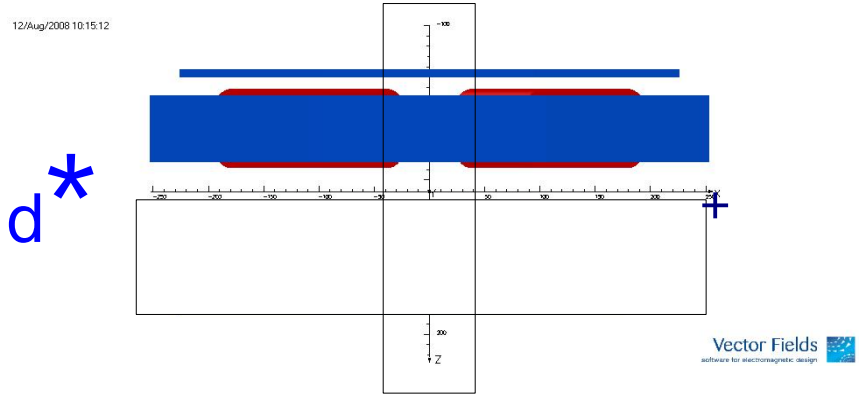


Vertical Tune per cell versus Horizontal Tune per cell with various lattice configuration

Four degrees of freedom

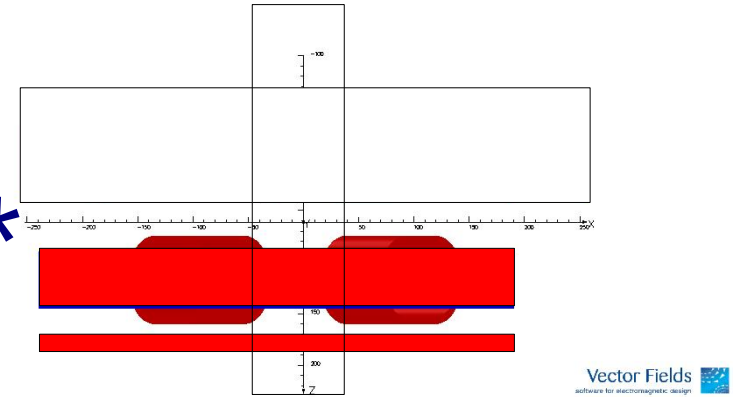
Let's compute them separetly...

α_d^*



X_d

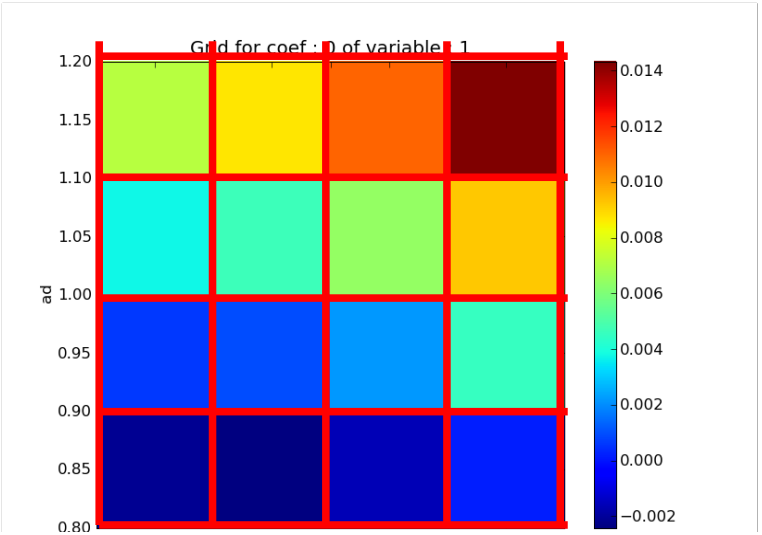
X_f

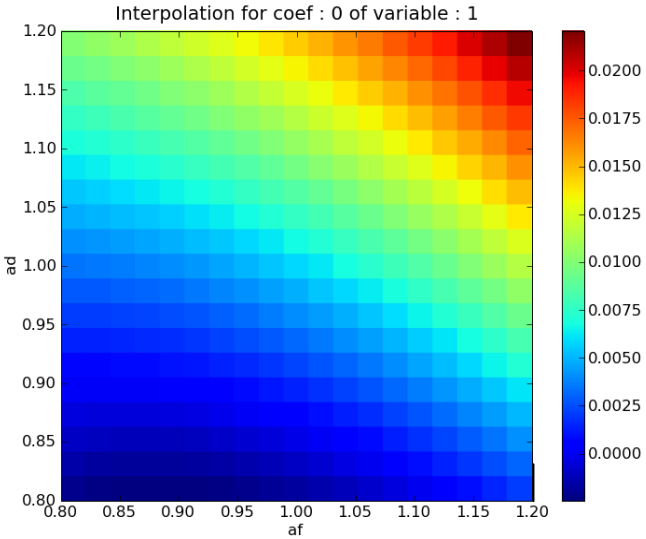


α_f^*

Interpolation of each coefficents of the dynamical map.

I COEFFICIENT ORDER EXPONENTS
1 0.960863315922732 1. 1. 0. 0. 0. 0.

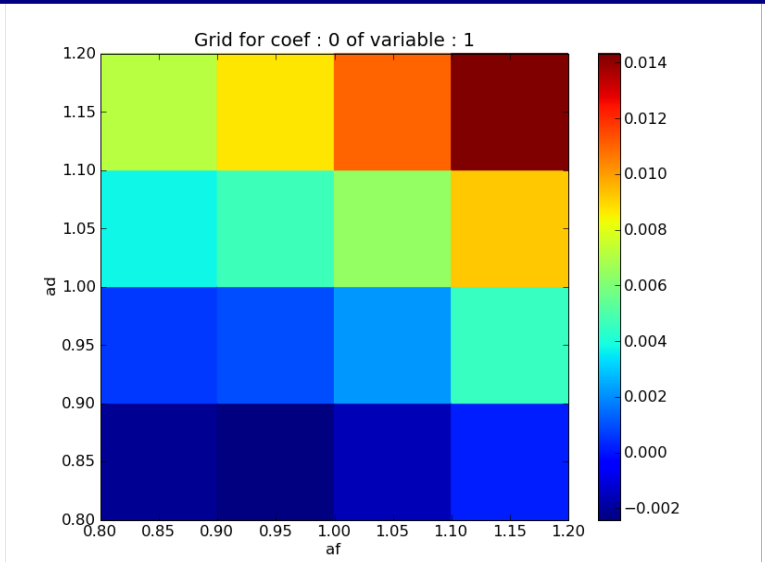




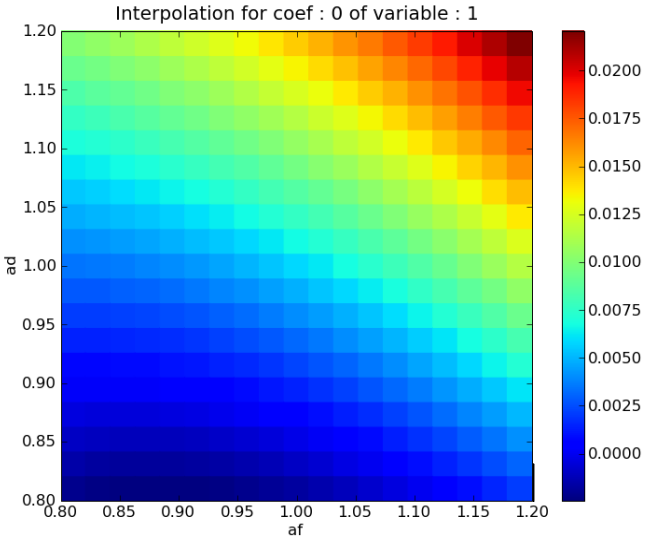
Interpolated map for af=1.2,ad=0.81

Interpolation of each coefficents of the dynamical map.

I COEFFICIENT ORDER EXPONENTS
1 0.960863315922732 1. 1. 0. 0. 0. 0.



Interpolated map for af=1.2,ad=0.81



Interpolation of each coefficents of the dynamical map.

I	COEFFICIENT	ORDER EXPONENTS					
1	0.960863315922732	1.	1.	0.	0.	0.	0.
2	0.24570344637953601	1.	0.	1.	0.	0.	0.

Interpolated map for af=1.2,ad=0.81

Interpolation of each coefficents of the dynamical map.

I	COEFFICIENT	ORDER EXPONENTS
1	0.960863315922732	1. 1. 0. 0. 0. 0.
2	0.24570344637953601	1. 0. 1. 0. 0. 0.
3	-0.0082587845378038482	1. 0. 0. 0. 0. 0.
4	4.902270671062646	2. 2. 0. 0. 0. 0.
5	1.7415825940854155	2. 1. 1. 0. 0. 0.
6	0.16919516739701174	2. 0. 2. 0. 0. 0.
7	2.0212048075863818	2. 0. 0. 2. 0. 0.
8	0.56936719743831266	2. 0. 0. 1. 1. 0.
9	0.86972871949682373	2. 1. 0. 0. 0. 0.
10	-0.0075926370412231648	2. 0. 1. 0. 0. 0.
11	0.056609896919403525	2. 0. 0. 0. 2. 0.
12	0.0041426149922234251	2. 0. 0. 0. 0. 0.

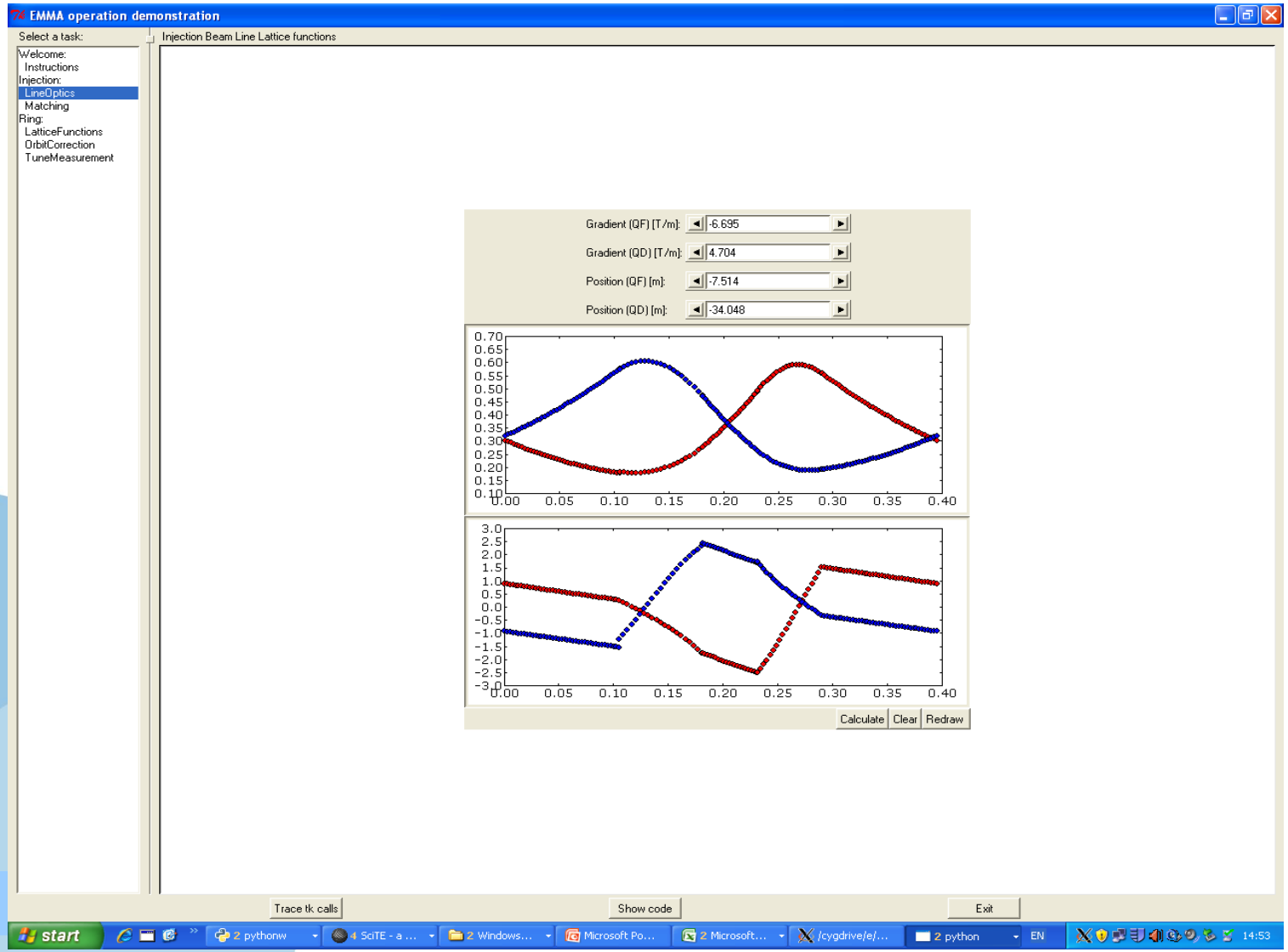
Interpolated map for af=1.2,ad=0.81

I	COEFFICIENT	ORDER EXPONENTS
1	0.2792904796322659E-15	0 0 0 0 0 0 0
2	0.9616633078760327	1 1 0 0 0 0 0
3	0.2458996365583791	1 0 1 0 0 0 0
4	-.8284614849871832E-02	1 0 0 0 0 0 1
5	4.889865507298404	2 2 0 0 0 0 0
6	1.736835738646354	2 1 1 0 0 0 0
7	0.1691314544470592	2 0 2 0 0 0 0
8	1.957123655631133	2 0 0 2 0 0 0
9	0.5408736211728222	2 0 0 1 1 0 0
10	0.8680681485716765	2 1 0 0 0 0 1
11	-.8088449008131963E-02	2 0 1 0 0 0 1
12	0.6092375459177638E-01	2 0 0 0 2 0 0
13	0.4192957225149813E-02	2 0 0 0 0 0 2

Integrated map for af=1.2,ad=0.81

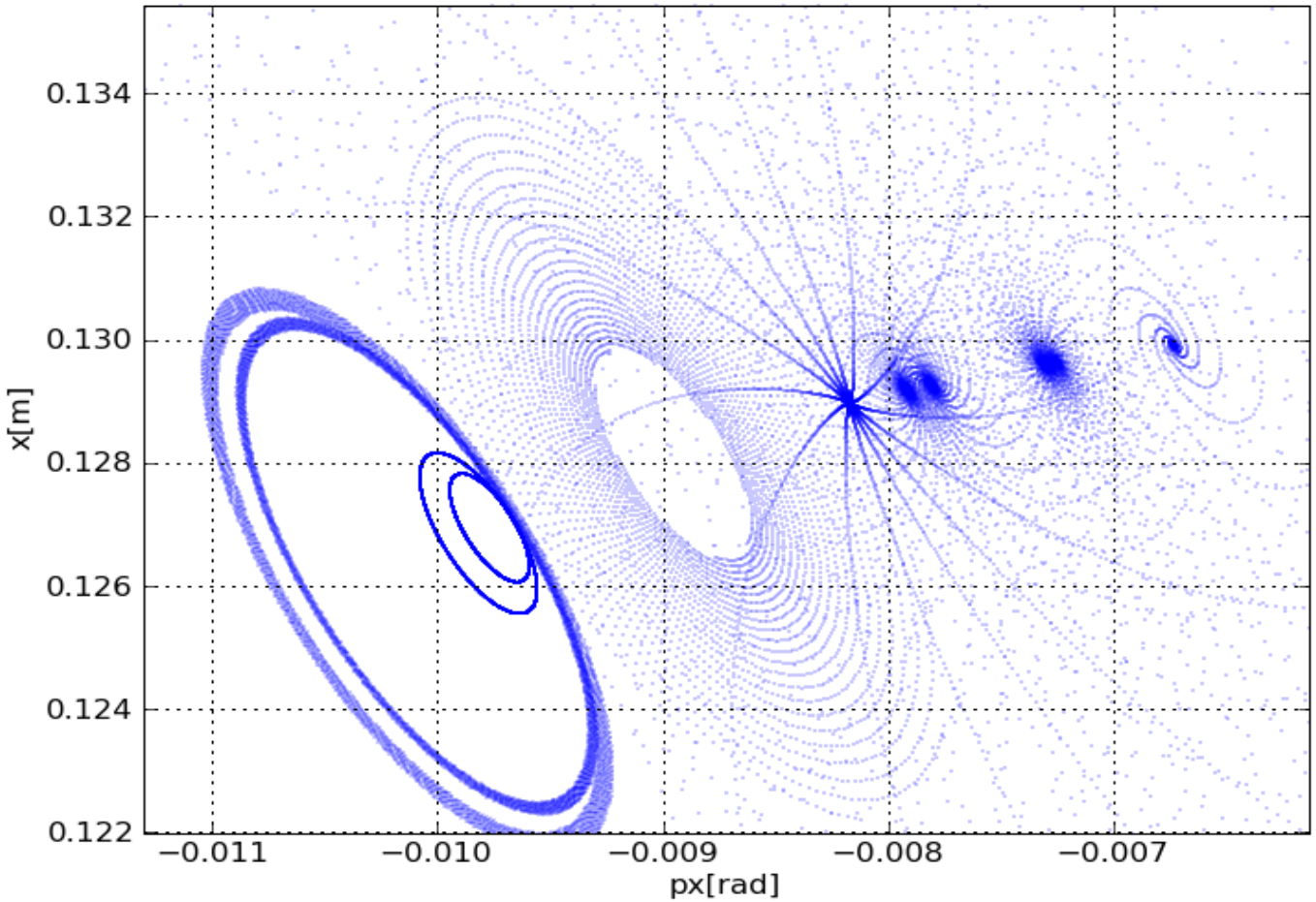
Within 5% agreement to the 3rd order.

EMMA on line model



Thanks to F.Meot, B.Shepherd and the EMMA team.

Thanks for your attention



EMMA tune measurement