Scaling FFAG lattices for muon acceleration

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Motivations

Use the large transverse acceptance of scaling FFAG lattices

while using constant RF frequency acceleration to reach high accelerating gradient.
**Motivations**

**Use the large transverse acceptance of scaling FFAG lattices**

**while using constant RF frequency acceleration to reach high accelerating gradient.**

**Possible with either:**

**Harmonic number jump acceleration**

**Stationary bucket acceleration!**
I. LATTICE FOR HARMONIC NUMBER JUMP ACCELERATION.
   1- Reminder about harmonic number jump acceleration.
   2- Simultaneous $\mu^-$ and $\mu^+$ acceleration: need for double beam lattice.
   3- Issue of the excursion: need dispersion suppressors!
   4- Lattice example and 4D tracking results.

II. LATTICE FOR STATIONARY BUCKET ACCELERATION.
    1- Reminder about stationary bucket acceleration.
    2- Lattice details and tracking results.
Part I

Scaling FFAG lattices for Harmonic number jump acceleration
HARMONIC NUMBER JUMP ACCELERATION

To jump one harmonic every turn: \[ T_{i+1} - T_i = \frac{1}{f_{RF}} \]
To jump one harmonic every turn: 

$$T_{i+1} - T_i = \frac{1}{f_{RF}}$$

**Figure 1 - Revolution time as a function of particle energy in the case of a 3 to 10 GeV scaling FFAG ring, with k = 145 and average radius = 120 m.**

> Energy gain per turn must follow: 

$$\Delta E_i = \frac{1}{f_{RF} \cdot \left[ \frac{\Delta T}{\Delta E} \right] E_i}$$
Assuming that the initial number of harmonic \( h_0 \) is large we get\(^*\):\[
\frac{f_k}{f_0} \approx 1 - \frac{1}{h_0} \cdot \frac{k}{N}
\]

Every cavity working at a constant frequency \( f_k \) but the frequency has to be tuned to a slightly different value!

\( \mu^+ \) and \( \mu^- \) beams cannot be accelerated simultaneously if they circulated in opposite directions...

\(^*\)Look at the proceedings of PAC’09 for all details.
NEED FOR A DOUBLE BEAM LATTICE

A solution to circulate a particle and its antiparticle in the same direction in a scaling FFAG ring is to use a FD-symmetric lattice:

Figure 3 - Double beam FFAG lattice (k = 145). Closed orbits of $\mu^+$ and $\mu^-$ circulating in the same direction. Results are obtained from Runge-Kutta stepwise tracking in hard-edge field.
NEED FOR DISPERSION SUPPRESSOR INSERTIONS!

Harmonic jump condition: \( T_{i+1} - T_i = \frac{1}{f_{RF}} \)

In the same time: \( \frac{\Delta C_i}{\beta c} = T_{i+1} - T_i \)

In case of highly relativistic particles: \( \Delta R_i \approx \frac{c}{2\pi f_{RF}} = \frac{\lambda_{RF}}{2\pi} \)

average excursion = \( \lambda_{RF} \cdot \frac{N_{\text{turns}}}{2\pi} \) → Need for excursion reduced areas!
Dispersion suppressor with FFAG magnets

\[
\frac{2}{k_2 + 1} = \frac{1}{k_1 + 1} + \frac{1}{k_3 + 1}
\]
3 to 10 GeV muon double beam FFAG + excursion reduced areas

$B_{max}$ 3 T
Horizontal tune 23.52
Vertical tune 7.12

Figure 4 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 4 excursion reduced insertions.
3 to 10 GeV muon double beam FFAG + excursion reduced areas

Table 2 - Ring main cells parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean radius</td>
<td>120 m</td>
</tr>
<tr>
<td>Number of cells</td>
<td>4 × 11</td>
</tr>
<tr>
<td>Cell opening angle</td>
<td>4.5 deg.</td>
</tr>
<tr>
<td>Field index $k$</td>
<td>145</td>
</tr>
<tr>
<td>$B_{max}$</td>
<td>3 T</td>
</tr>
<tr>
<td>Horiz. phase adv. per cell</td>
<td>82.1 deg.</td>
</tr>
<tr>
<td>Vert. phase adv. per cell</td>
<td>31.8 deg.</td>
</tr>
</tbody>
</table>

Figure 4 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 4 excursion reduced insertions.
3 to 10 GeV Muon Double Beam FFAG + Excursion Reduced Areas

Table 2 - 1st Dispersion Supressor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean radius</td>
<td>120 m</td>
</tr>
<tr>
<td>Number of cells</td>
<td>4 × 4</td>
</tr>
<tr>
<td>Cell opening angle</td>
<td>4.3 deg.</td>
</tr>
<tr>
<td>Field index $k$</td>
<td>183.6</td>
</tr>
<tr>
<td>$B_{\text{max}}$</td>
<td>3 T</td>
</tr>
<tr>
<td>Horiz. phase adv. per cell</td>
<td>90 deg.</td>
</tr>
<tr>
<td>Vert. phase adv. per cell</td>
<td>27.6 deg.</td>
</tr>
</tbody>
</table>

Figure 4 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 4 excursion reduced insertions.
3 to 10 GeV muon double beam FFAG + excursion reduced areas

Table 2 - 1st dispersion suppressor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean radius</td>
<td>120 m</td>
</tr>
<tr>
<td>Number of cells</td>
<td>$4 \times 4$</td>
</tr>
<tr>
<td>Cell opening angle</td>
<td>3.34 deg.</td>
</tr>
<tr>
<td>Field index $k$</td>
<td>307.7</td>
</tr>
<tr>
<td>$B_{max}$</td>
<td>3 T</td>
</tr>
<tr>
<td>Horiz. phase adv. per cell</td>
<td>90 deg.</td>
</tr>
<tr>
<td>Vert. phase adv. per cell</td>
<td>20.4 deg.</td>
</tr>
</tbody>
</table>

Figure 4 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 4 excursion reduced insertions.
3 to 10 GeV muon double beam FFAG + excursion reduced areas

**Table 2 - 1st Dispersion Suppressor**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean radius</td>
<td>350 m</td>
</tr>
<tr>
<td>Number of cells</td>
<td>4 × 8</td>
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<tr>
<td>Cell opening angle</td>
<td>1.2425 deg.</td>
</tr>
<tr>
<td>Field index $k$</td>
<td>1168.6</td>
</tr>
<tr>
<td>$B_{\text{max}}$</td>
<td>3 T</td>
</tr>
<tr>
<td>Horiz. phase adv. per cell</td>
<td>64.6 deg.</td>
</tr>
<tr>
<td>Vert. phase adv. per cell</td>
<td>12.6 deg.</td>
</tr>
</tbody>
</table>

**Figure 4 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 4 excursion reduced insertions.**
3 to 10 GeV muon double beam FFAG + excursion reduced areas

Figure 5 - μ- closed orbits at 3, 6 and 10 GeV.
Figure 6 - $\mu^-$ (red) and $\mu^+$ (green) closed orbits at 3, 6 and 10 GeV.
Asymmetry between $\mu-$ and $\mu+$ behavior

$k=145$

$k=183.6$

$k=307.7$

$k=1168.6$
Asymmetry between $\mu^-$ and $\mu^+$ behavior

In dispersion suppressor cells: same amplitude but different beta function at the matching point!
μ-: matching done at the center of F magnets

Study of linear parameters using Runge-Kutta stepwise tracking in soft edge field model:

Figure 7 - μ- Tune variation between 3 and 10 GeV in the lattice with insertions (from stepwise tracking in a soft edge field model).
μ⁻: MATCHING DONE AT THE CENTER OF F MAGNETS

Study of linear parameters using Runge-Kutta stepwise tracking in soft edge field model:

Figure 8 - μ⁻: HORIZONTAL beta function at 6 GeV (quarter of a turn).

Figure 9 - μ⁻: VERTICAL beta function at 6 GeV (quarter a turn).
μ-: MATCHING DONE AT THE CENTER OF F MAGNETS

4D TRACKING RESULTS: RF FREQUENCY = 400 MHz, PEAK VOLTAGE 2GV/TURN.

Figure 10 - μ- : Longitudinal phase space showing a 6 turns acceleration cycle from 3 to 10 GeV with an initial beam 4D emittance of 0.2 eV.sec × 30 000 π.mm.mrad.

Figure 11 - μ- : Horizontal phase space showing the injected beam profile (red) and the same beam after a 6 turns acceleration cycle (green) with (4D emittance of 0.2 eV.sec × 30 000 π.mm.mrad).
μ+: matching done at the center of D magnets

Study of linear parameters using Runge-Kutta stepwise tracking in soft edge field model:

Figure 12 - μ+: Tune variation between 3 and 10 GeV in the lattice with insertions (from stepwise tracking in a soft edge field model).
μ⁺: MATCHING DONE AT THE CENTER OF D MAGNETS

STUDY OF LINEAR PARAMETERS USING RUNGE-KUTTA STEPWISE TRACKING IN SOFT EDGE FIELD MODEL:

**Figure 13** - μ⁺: **Horizontal beta function at 6 GeV (quarter of a turn).**

**Figure 14** - μ⁺: **Vertical beta function at 6 GeV (quarter a turn).**
μ+: matching done at the center of D magnets

Horizontal acceptance of about 40 000 π.mm.mrad 6 GeV.

Figure 15 - μ+: horizontal phase space showing 5 particles tracked over 100 turns (a fixed energy = 6 GeV).
μ⁺: Matching done at the center of D magnets

4D tracking results:

Tried 3 to 10 GeV acceleration cycle
(with RF frequency = 400 Hz, peak voltage 2GV/turn)

Particle lost on collimator even for small transverse emittance...
**Summary on Harmonic Number Jump**

**Works well...**

- Large transverse acceptance.
- Large longitudinal acceptance, and no emittance degradation during acceleration.
- Excursion reduction of a factor 3 is already possible.
- Possible with RF frequency in the 200 MHz to 400 MHz range.

...But not yet for both charge in the same time.
Part II

Scaling FFAG lattices for

Stationary bucket acceleration
Stationary bucket acceleration principle:

**Figure 16 - Longitudinal phase space showing a 6 turns acceleration cycle (in red) as well as the equi-hamiltonian lines (in black).**
Lattice example with super-periodicity of 6:

\[ B_{max} \quad 3 \text{ T} \]
Horizontal tune \quad 36.8
Vertical tune \quad 11.02

Figure 17 - Schematic view of a 3 to 10 GeV muon FFAG ring with 6 “almost-straight” insertions.
Lattice for stationary bucket acceleration

Mean radius 90 m
Number of cells 6 × 21
Cell opening angle 2.6 deg.
Field index $k$ 500
$B_{max}$ 3 T
Horiz. phase adv. per cell 85.9 deg.
Vert. phase adv. per cell 19.3 deg.

Figure 17 - Schematic view of a 3 to 10 GeV muon FFAG ring with 6 “almost-straight” insertions.
Figure 17 - Schematic view of a 3 to 10 GeV muon FFAG ring with 6 “almost-straight” insertions.

- Mean radius: 360 m
- Number of cells: 6 × 6
- Cell opening angle: 0.9 deg.
- Field index $k$: 2003
- $B_{\text{max}}$: 3 T
- Horiz. phase adv. per cell: 67.7 deg.
- Vert. phase adv. per cell: 42.2 deg.
Scaling FFAG Lattices For Muon Acceleration

Figure 18 - **Horizontal beta function at 6 GeV (1/6 of a turn).**

Figure 19 - **Vertical beta function at 6 GeV (1/6 of a turn).**
Tracking results

4D tracking results: RF frequency = 200 MHz, peak voltage 2GV/TURN. Acceleration within only 4 turns.

Figure 20 - Longitudinal phase space showing a 4 turns acceleration cycle from 3 to 10 GeV with an initial beam 4D emittance of 0.1 eV.sec x 24 000 π.mm.mrad.

Figure 21 - Horizontal phase space showing the injected beam profile (red) and the same beam after a 4 turns acceleration cycle (green) with (4D emittance of 0.1 eV.sec x 24 000 π.mm.mrad).
Tracking results

2D tracking results: RF frequency = **100 MHz**, peak voltage **2 GV/TURN**. Acceleration within only 4 turns.

**Open a way for 1 to 12 GeV acceleration at 100 MHz in a single FFAG ring!**

Figure 22 - **Longitudinal phase space showing a 7 turns acceleration cycle (2D longitudinal tracking) from 3 to 10 GeV with an initial beam 2D emittance of 0.2 eV.sec.**

Scaling FFAG Lattices For Muon Acceleration

T. Planche - FFAG09 - Sept. 2009
**Summary on Stationary Bucket Acceleration**

A single 1 to 12 GeV scaling FFAG ring!
Summary

Advanced scaling FFAG lattices can be used for both:

Harmonic number jump acceleration of muons

Stationary bucket acceleration of muons
Thank you!
Additional material...