Matching and Optimizing the SILC / ILC sections of the CW Linac (v7)

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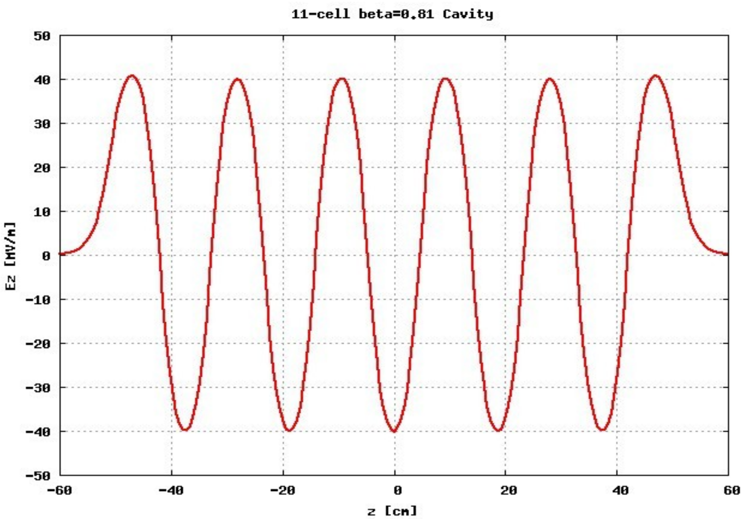
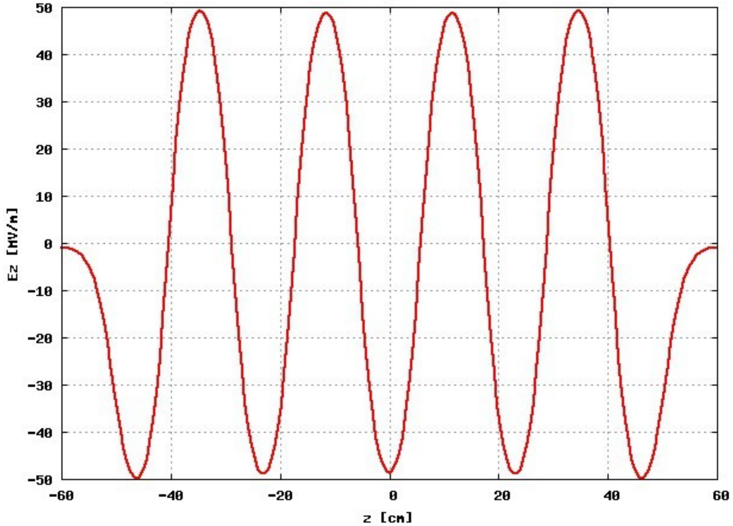
Assumptions

- Type 4 cryostat used in both sections
- SILC section: 11-cell, β=0.81,
  gradient = 16.4 MV/m,
  Leff ~ 1.0 m (1.074 m)
- ILC section: 9-cell, β=1.0,
  gradient = 18 MV/m
  Leff ~ 1.0 m (1.038 m)

Cavity Fields Extracted from TRACK binary files





11-cell β=0.81* (eh_MWS.#30)

V_0T = 21.48601 MV

E_{acc} = 20.91316 MV/m

L_c = 18.6798/2 = 9.34 cm

T = 0.729104

T(sine) = π/4 = 0.7853982

9-cell β=1.0 (eh_MWS.#26)

V_0T = 26.42866 MV

E_{acc} = 25.46680 MV/m

L_c = 23.0615/2 = 11.531 cm

T = 0.736490

T(sine) = π/4 = 0.7853982

*Assumptions: (1) βc is constant (2) Max T occurs for cavity β as specified.

“Actual” Cavity β (i.e not based on inner cells length)

Procedure

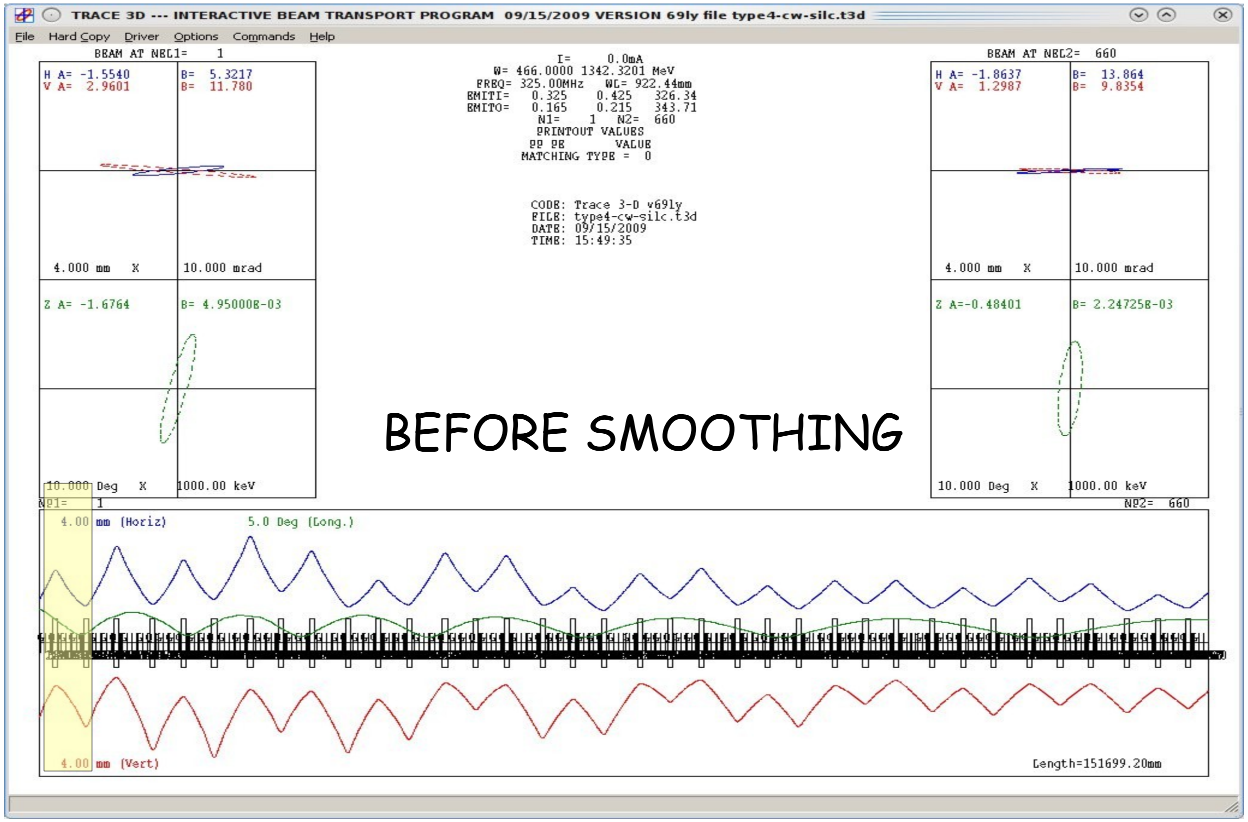
- Start by setting quad strengths in first cell to achieve <~90 deg phase advance
- Use TRACE3D to find a "periodic" solution for the first cell
- Set ALL quads in the section to the strength used in the first cell. Quad and cavity optical focusing strengths decrease as $1/\gamma$, in the smooth approximation, would one expects the beam envelope to not to vary drastically i.e. this should provide a good starting point.
- Assume I=0. RF (de)focusing is a pertubation. Smooth the envelope by tweaking quadrupoles downstream of first cell.

  When $I \neq 0$, SC introduces another perturbation (hopefully "small")
- The procedure above is repeated independently for each section (in this case: SILC and ILC). The sections are subsequently matched to each other by using a few (usually 4) quads in the vicinity of the section boundaries.

TRACK to TRACE3D Translator

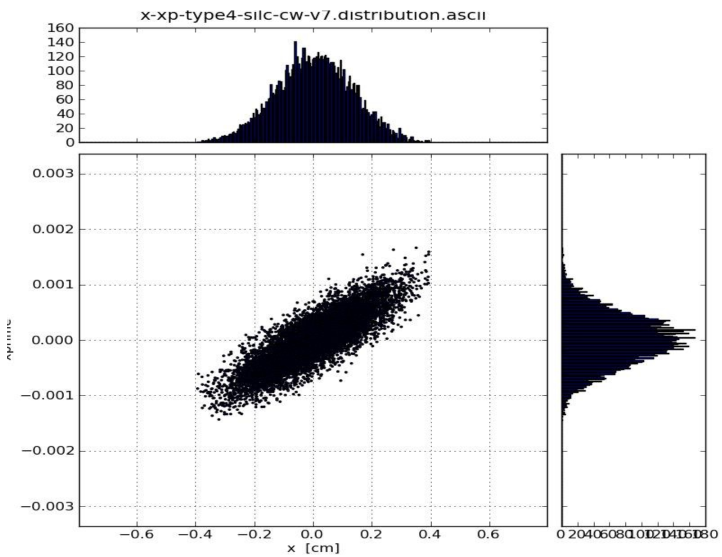
- **The TRACK input file is the authoritative one.**
- Automates translation from TRACK to TRACE3D lattice representations.
- Cavities in theTRACE3D lattice are replaced by RF gaps. The gap Voltage (E0 x L xT) is computed for each cavity and automatically substituted into the file.
- The longitudinal field profiles used to compute T (transit time factor) are extracted from the TRACK cavity binary fieldmap files.
- For the TRACK runs, (Gaussian) initial distributions (binary file) can be generated from the TRACE3D optimized lattice functions.

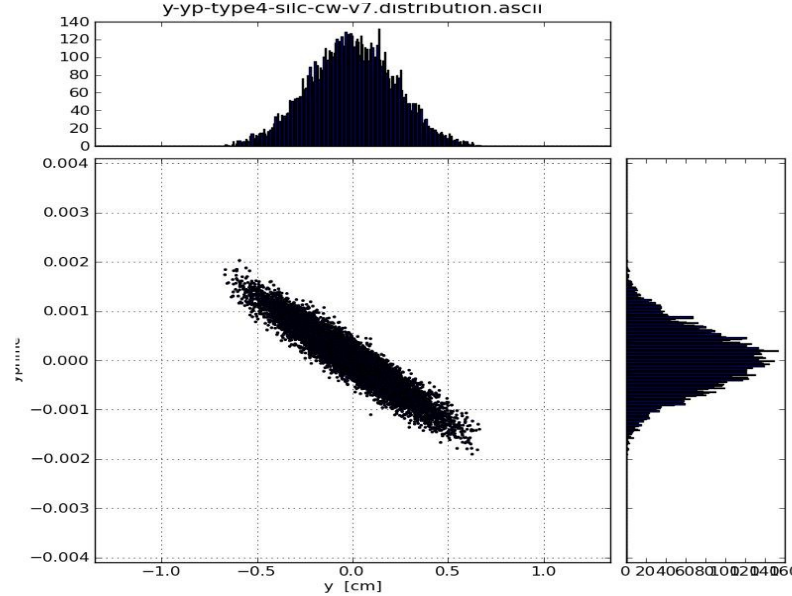
SILC Section with Matched (periodic) First Cell

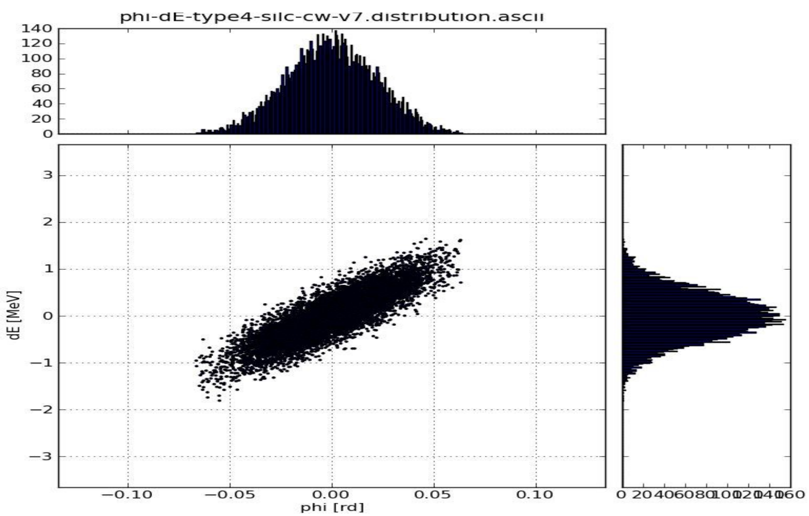


<u>RF defocusing is a significant perturbation in SILC.</u> Defocusing is proportional to the transit time factor, and varies rather rapidly with β for an 11-cell cavity. A not so adiabatic change in RF defocusing makes it necessary to optimize **all 34** quadrupole settings to obtain a smooth envelope.

Gaussian Distributions for TRACK Test

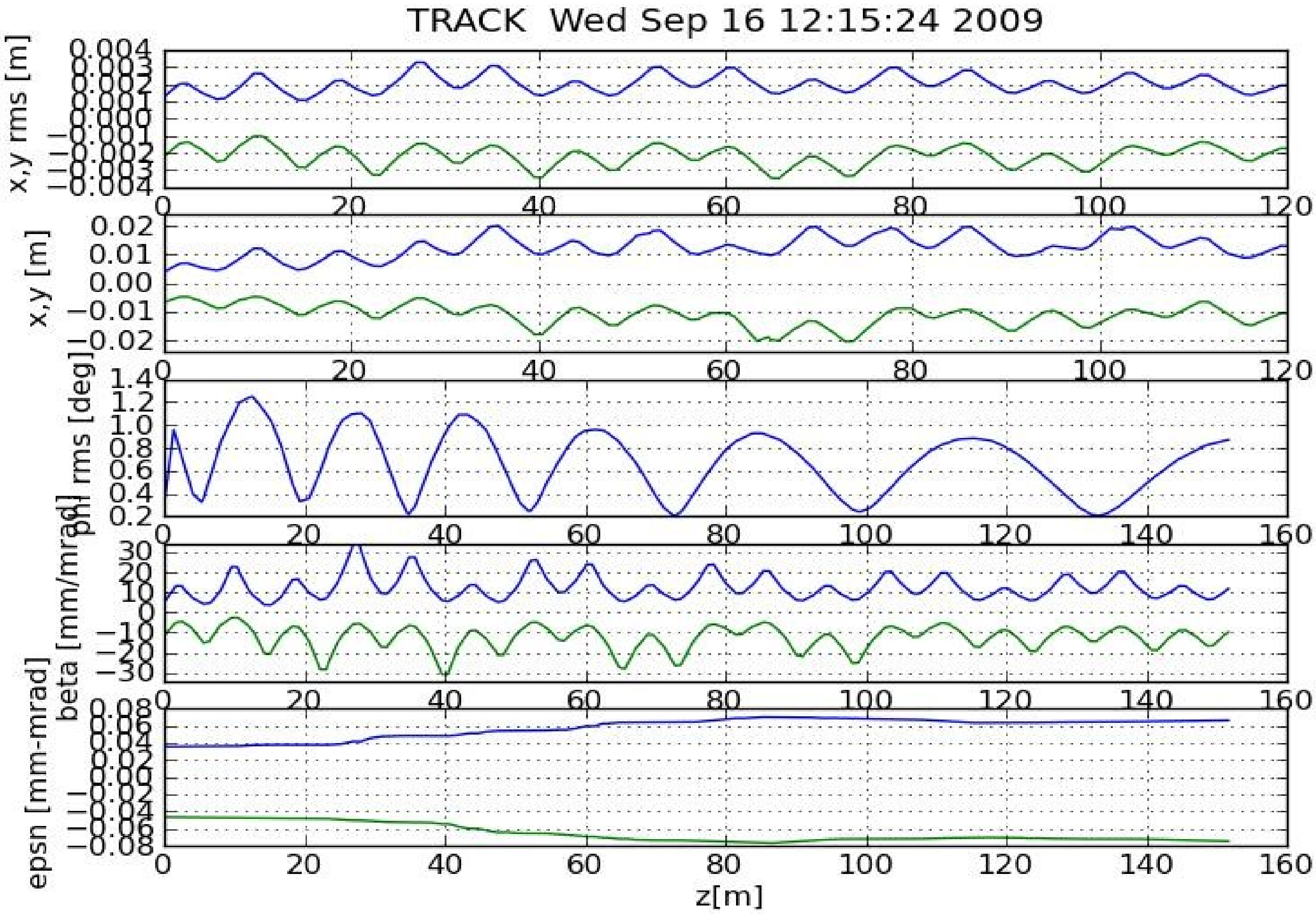






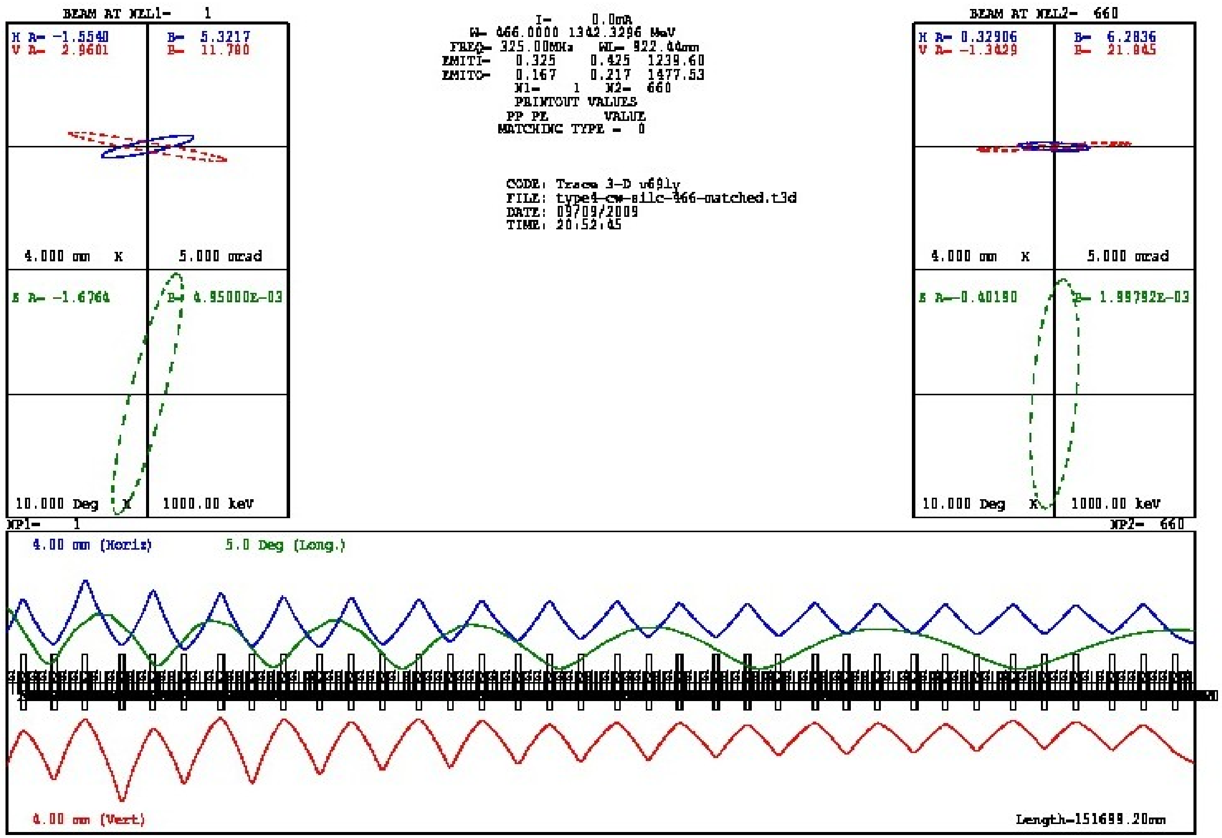
Gaussian initial distributions generated for TRACK so as to match TRACE3D beam parameters and verify correspondance between TRACE3D and TRACK models.

SILC w/Periodic 1st Cell: TRACK (I = 0mA)



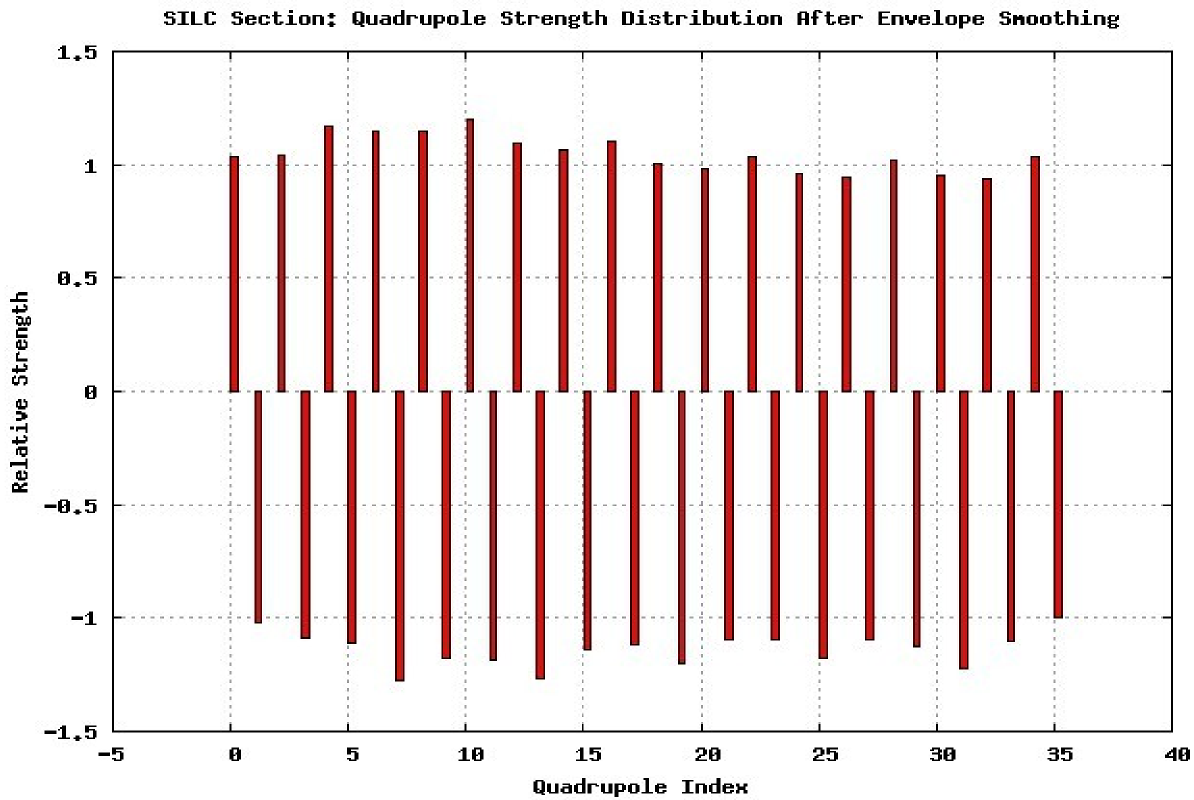
Very good agreement with TRACE3D.

SILC After Smoothing (Using a Custom Program)



A separate program was written to optimize the envelope. The objective function attempts to maintain a uniform envelope amplitude. Iterating over the 34 quads is done using the BGFS algorithm. The result after optimization is displayed here using TRACE3D.

"Optimal" Quadrupole Strengths



Relative strength quad distribution a long the SILC section after envelope smoothing. Strength = 1 corresponds to a phase advance of ~90 deg in the first cell.

Objective Function

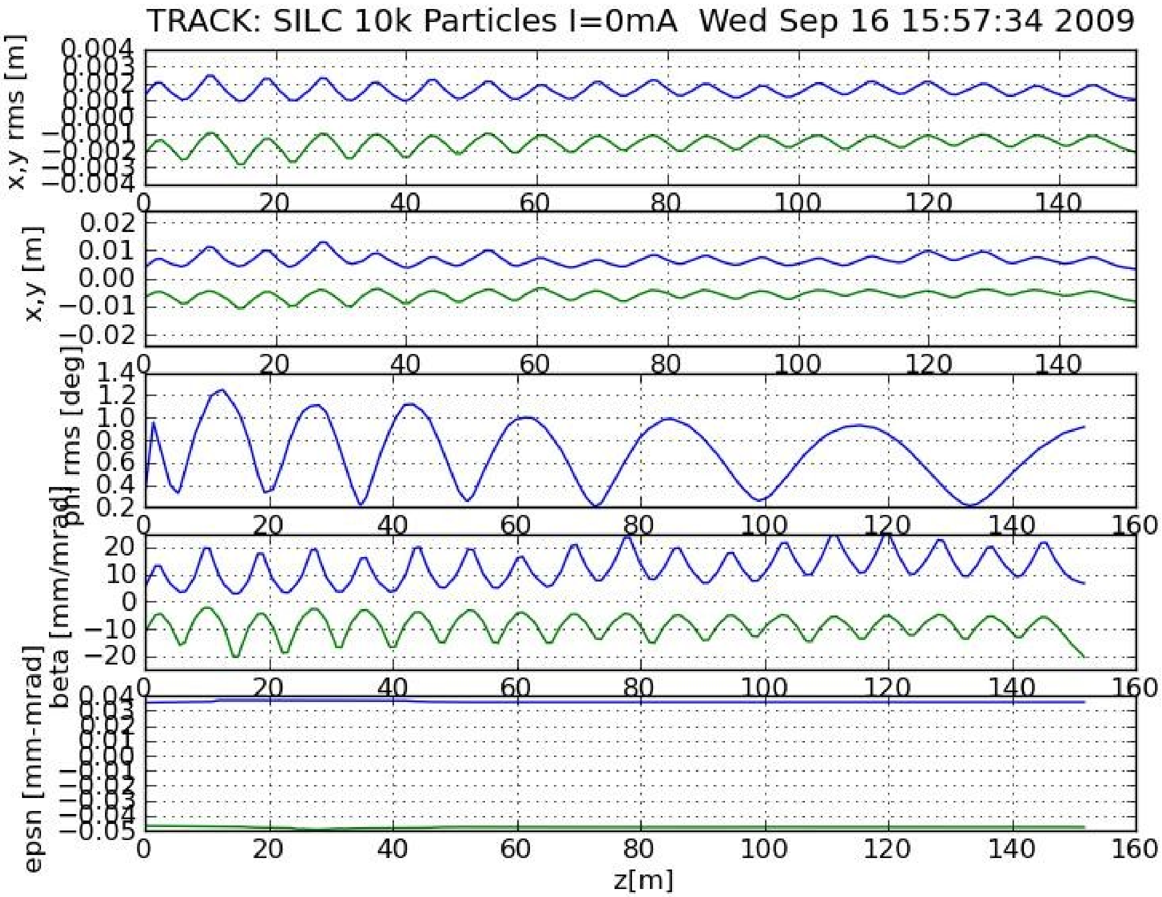
$$\sum_{i=x,y\, k=F,\ell=D} (\beta_{ik} - \beta_{iF})^2 + (\beta_{i\ell} - \beta_{iD})^2$$

Where β_{iF} are the values of the beta function in the first F quad.

The index k (l) runs over all F(D) quad positions.

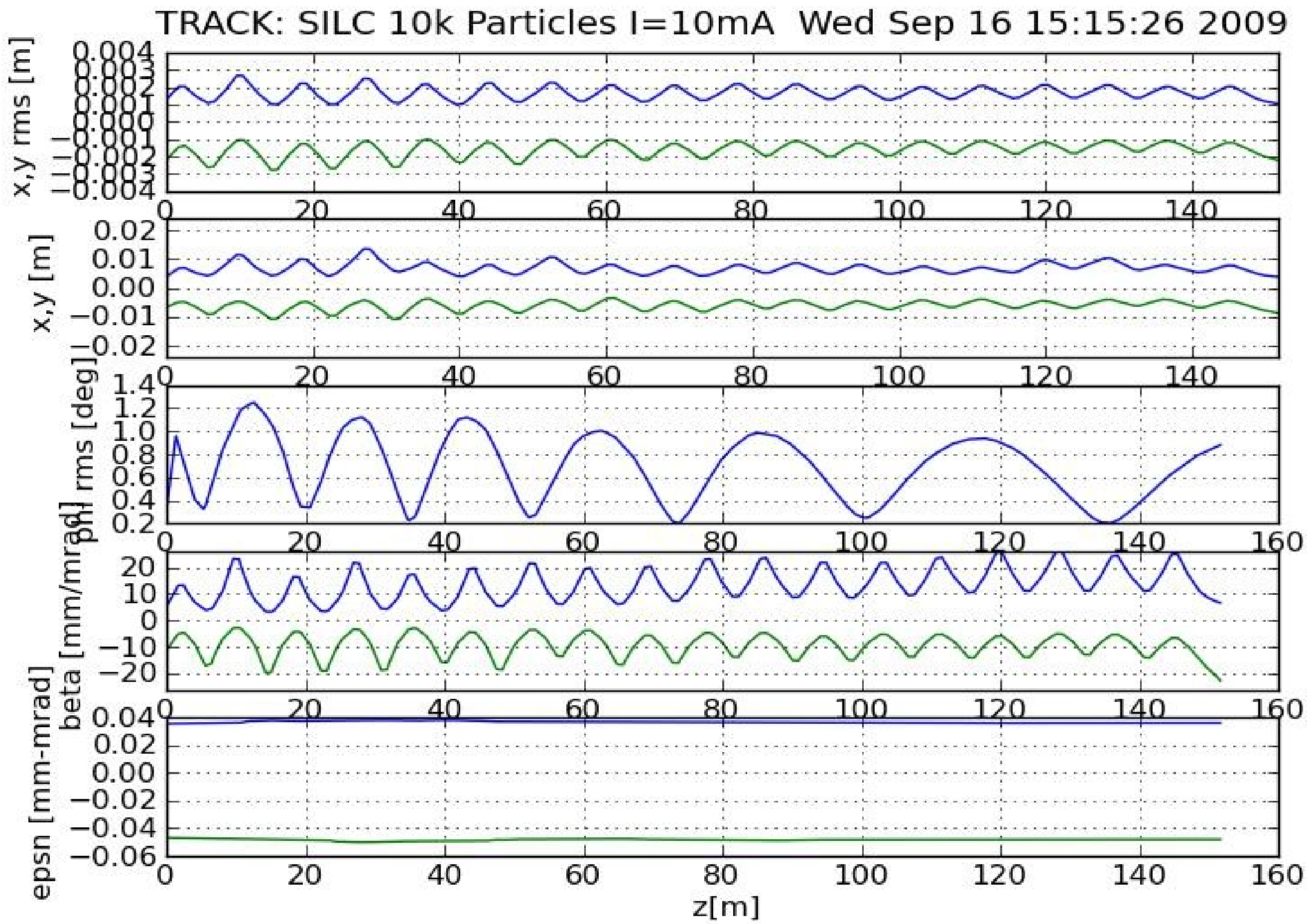
This choice produces an envelope oscillation that has a (more or less) contant amplitude.
This is only one choice; others are possible.

SILC "Optimal" Lattice: TRACK I=0mA



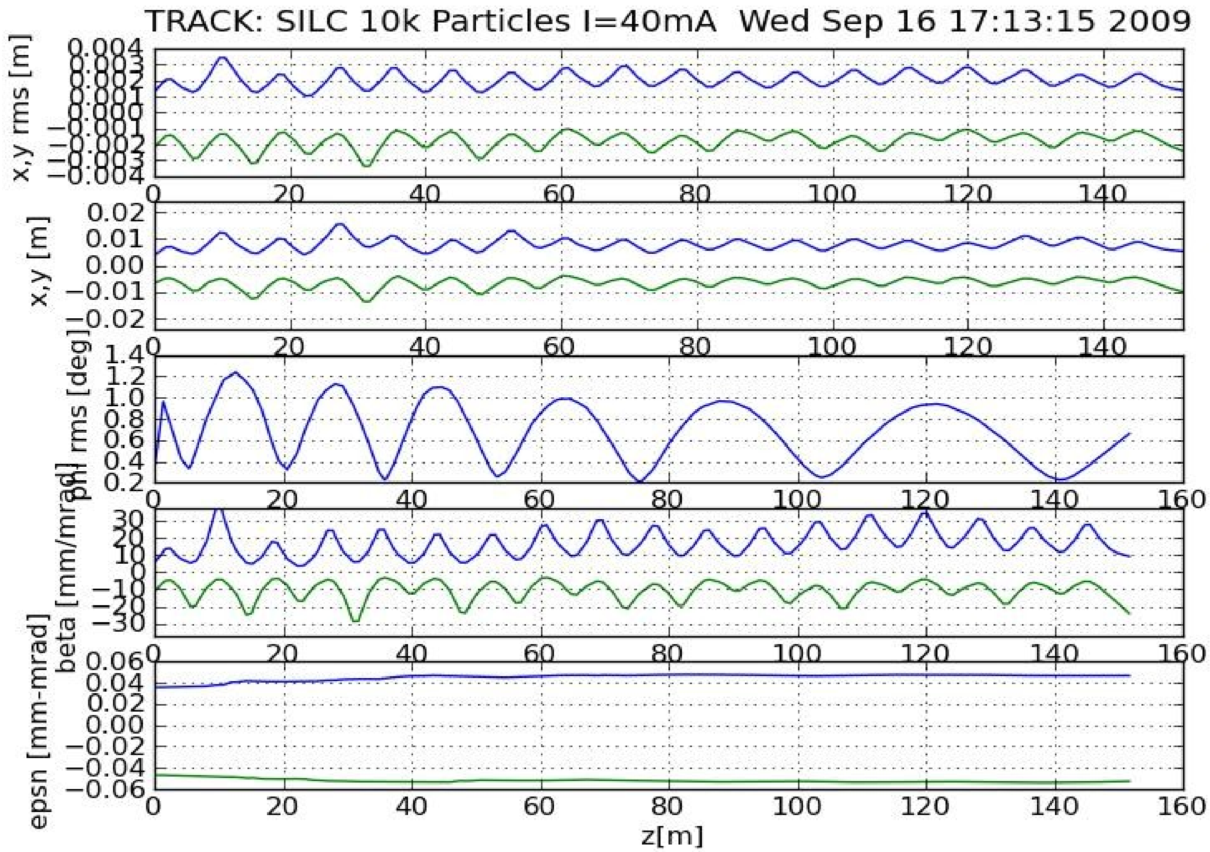
OK …, as expected.

SILC, "Optimized" TRACK I= 10 mA



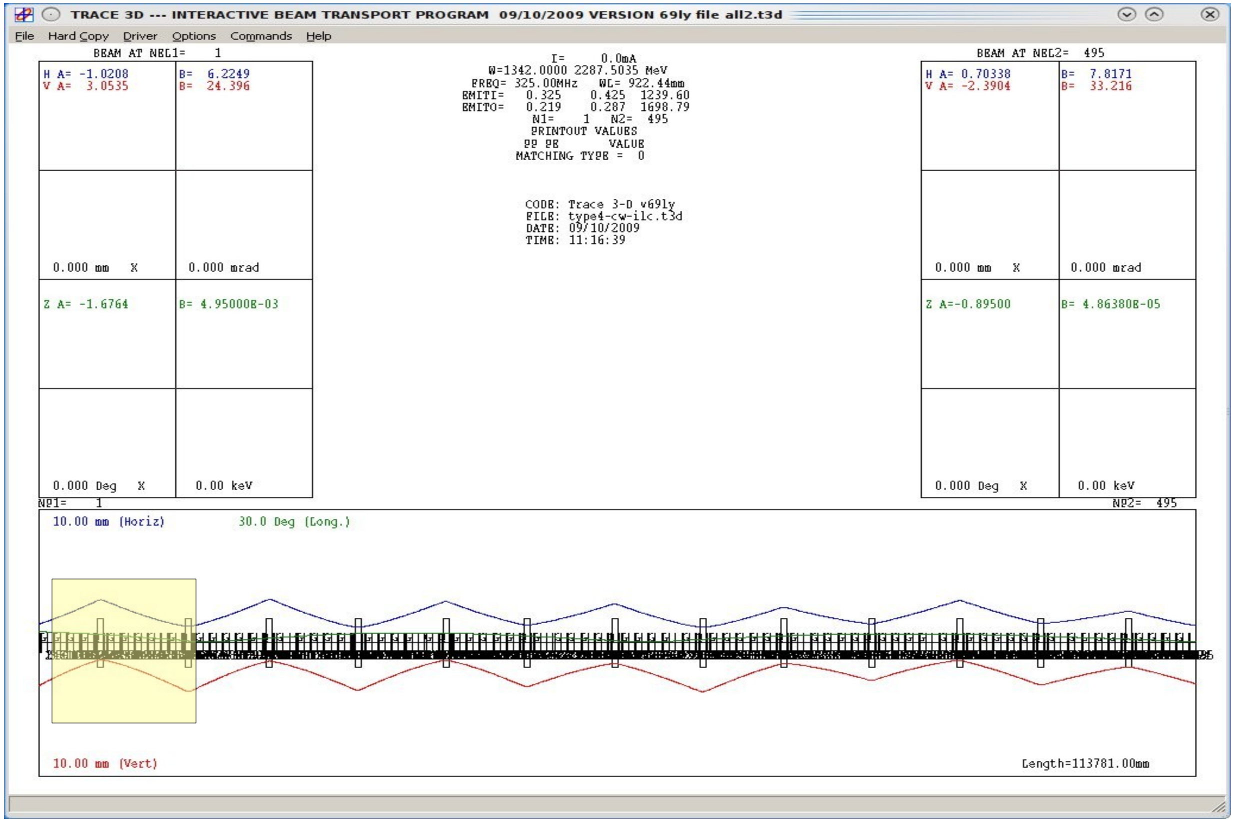
@10 mA , the 0 mA beam envelope shape is not very perturbed.

SILC 'Optimized', TRACK I = 40mA



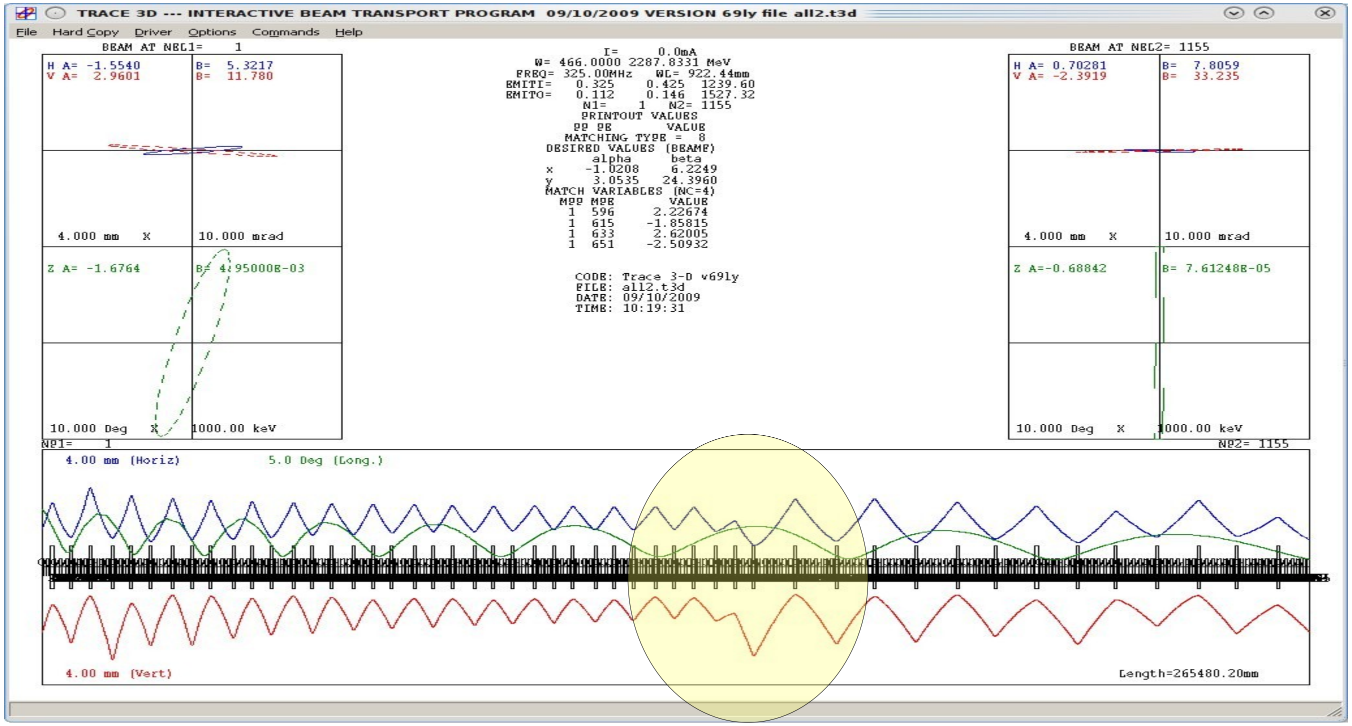
At 40mA, envelope perturbations due to SC become visible.

ILC Section



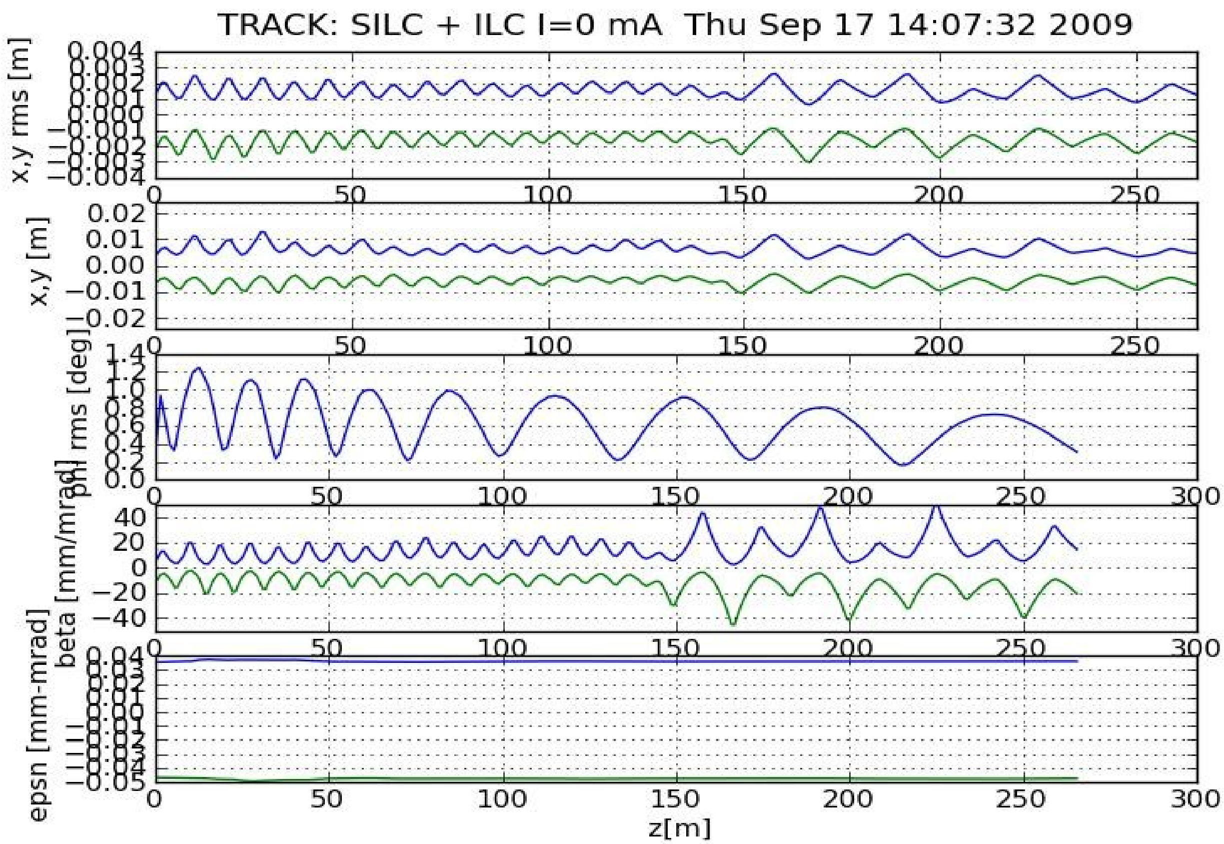
In the ILC section, rf defocusing is a smaller perturbation (and is also more adiabatic because no longer β vary rapidly). Periodic matching of the first cell yields an immediately acceptable solution.
Note that in the case shown above, all quads have identical strengths.

Attempt at Matching SILC and ILC Sections



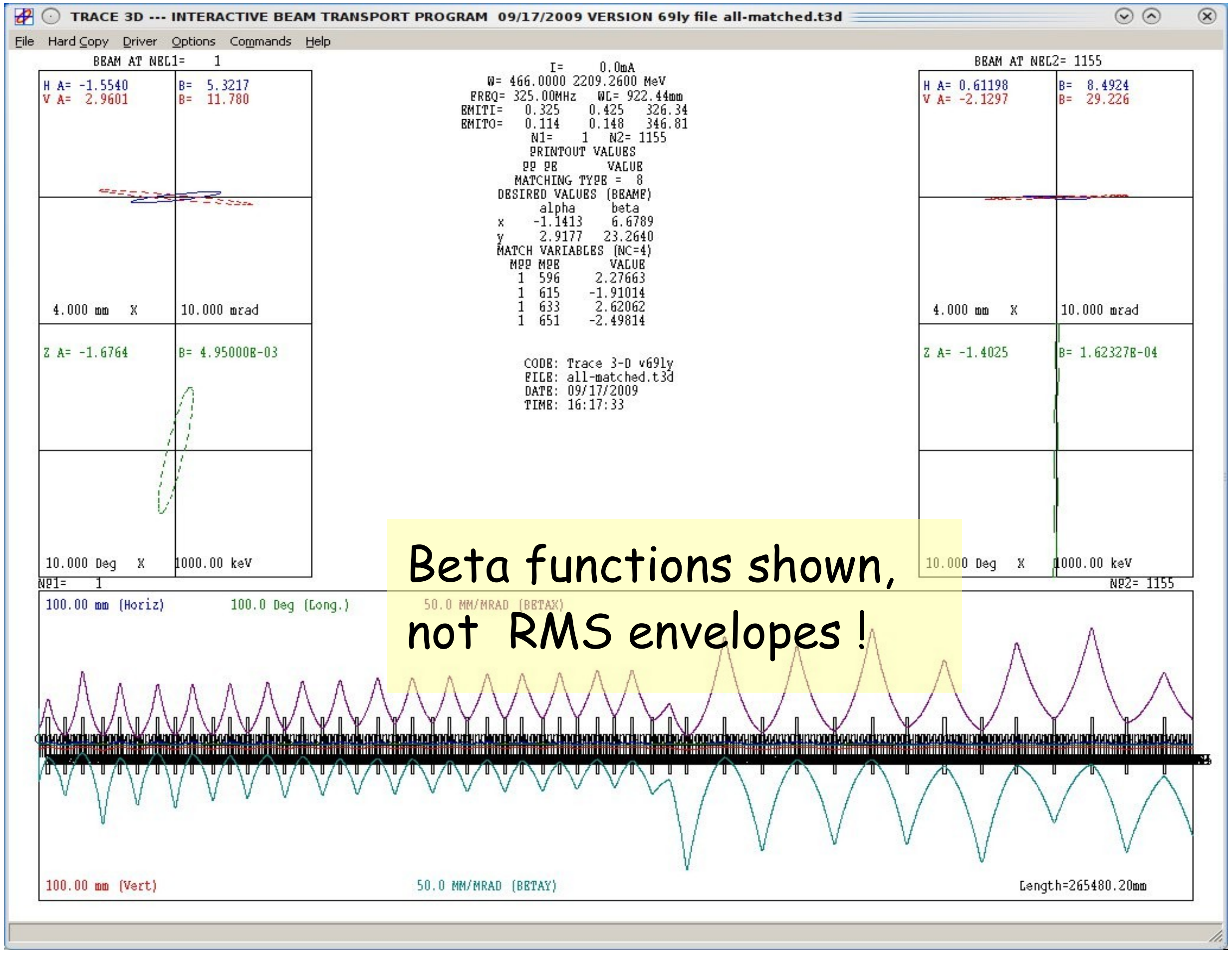
The transition match needs work. Attempts at matching with TRACE3D using 2 quads on both sides of the section interface fail to converge. TRACE3D uses a simple fixed-point solver to match. We might consider resorting to an external program with a more robust non-linear solver.

TRACK: SILC + ILC Tentative Match



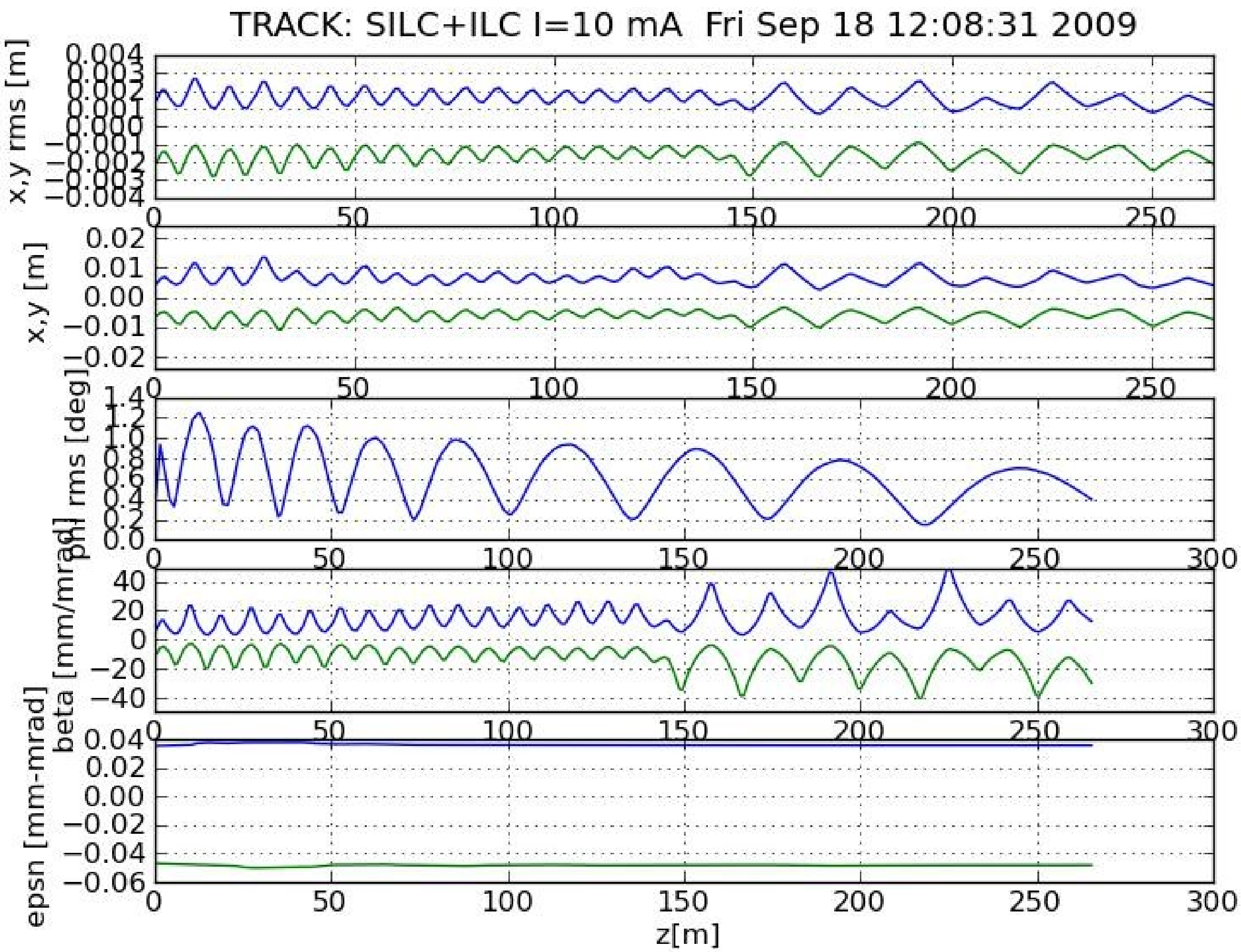
Agreement with the simplified TRACE3D model is very good …
but the β-functions in the ILC (β=1) section are not
particularly regular.

β-Functions Computed by TRACE3D



In good agreement with TRACK, but betas max/min ratios computed by TRACK are higher in the ILC section.

SILC+ ILC: Tentative Match I = 10 mA



Observations

- The match in the transition from SILC to ILC still needs to be looked at.
- At I=10 mA, the I=0 settings appear still OK.
- Acceleration efficiency issues aside, the use of 11-cell cavities in the SILC section makes the rf defocusing variation along the section not very adiabatic.
- The trade-offs of using 11-cell cavities in the SILC section need to be well understood.

Smooth Approximation

- In a smooth approximation, one averages over the modulation introduced by individual lenses.
  E.g. For a FODO cell of period 2L with a quad gradient G and a gap in the middle of the drift region one obtains

$$k_0^2 = \left(\frac{\mu_0}{2L}\right)^2 = \left[\left(\frac{qGL_q}{2mc\gamma\beta}\right)^2 - \frac{\pi q E_0 T \sin(-\phi)}{mc^2(\lambda\beta)^3}\right]$$

Where μ is the phase advance.

WKB Solution and Smooth Focusing (I)

$$\frac{d^2y}{ds^2} + f(s)y = 0$$

Let $y(s) = Ae^{i\phi(s)}$ Then: $-(\phi')^2 + i\phi'' + f = 0$

Assume f(s) is "slowly varying"

First Approximation: φ'' is small (since f(s) is slowly varing)

$$\phi' = \pm\sqrt{f} \qquad \phi(s) = \pm\int\sqrt{f(s)}ds$$

"Small" means $$|\phi''| \simeq \frac{1}{2}\left|\frac{f'}{\sqrt{f}}\right| << |f|$$

i.e. the change in focusing strength within one (betatron) wavelength should be small compared to the focusing strength |f| itself.

WKB Solution and Smooth Focusing (II)

$$\phi'' \simeq \pm \frac{1}{2} f^{-1/2} f'$$

$$\phi' \simeq f \pm \frac{i}{2} \frac{f'}{\sqrt{f}}$$

$$\phi(s) \simeq \pm \int \sqrt{f(s)}ds + \frac{i}{4} \ln f = \psi(s) + \frac{i}{4} \ln f$$

$$y(s) \simeq \frac{1}{f^{1/4}} (c_+ e^{i\psi(s)} + c_- e^{-i\psi(s)})$$

Conclusion: provided the focusing strength varies "slowly" enough, the solution **remains sinusoidal (phase modulated).** The phase variation **remains proportional to the square root of focusing strength**.
The amplitude variation (increase or decrease) is weak and affected only by a factor equal to the 4th root of the focusing f.

Phase Advance/Cell

- If the "second derivative" of the phase advance/cell (or 1/2-cell) is "small", one expects a sinusoidal oscillation of weakly varying amplitude. For example, assuming constant magnetic field strength focusing along the linac, an increase of 400% in $\beta\gamma$ implies a corresponding amplitude growth of only 40%.
- In the case where the focusing is not truly "smooth" (e.g. FODO-like), a suitable proxy for φ'' is:

  $\Phi_k = \varphi_{k+1} - 2\,\varphi_k + \varphi_{k-1}$ where k is the cell index.
- Minimizing $\Sigma\Phi^2_k$ is a possible alternative strategy to obtain a "smooth" envelope.

Computing the Phase Advance/Cell

- The notion of phase advance has a clear meaning only in the context of linear maps (elliptic beams).
- In principle, information about phase advance is contained in the change of orientation of the beam ellipse (the lattice function/ellipse parameter α : $\tan\varphi = -\alpha$).
- Unfortunately, there is some ambiguity (i.e. cannot distiguish between φ and $\varphi \pm n\pi$)