FFAG for High Intensity Proton Accelerator

Yoshiharu Mori
Kyoto University, Research Reactor Institute (KURRI)

Contents

- Introduction
  - Features
- Transeverse focusing
  - Zero chromaticity (scaling: fixed tunes)
    - *"zero chromaticity"*
  - Non-zero chromaticity(non-scaling: variable tunes)
    - *"fast resonance crossing"*
  - History of development
- Acceleration(RF) cf. induction: inefficinet
  - Variable frequency
  - Constant frequency
- Advancement of FFAG
- Proton driver
- Summary: beam power efficiency

FFAG: Fixed Field Alternating Gradient

- **Static magnetic field**

  It is like cyclotron, but not much orbit excursion

  - Fast acceleration
    - *Fixed magnetic field allows the beam acceleration only by RF pattern. No needs of synchronization between RF and magnets.*
  - Large repetition rate
    - *High intensity with large repetition rate and modest number of particles in the ring*
    - *Space charge and collective effects are below threshold.*

- **6D-Strong focusing (AG focusing, phase focusing)**

  It is like synchrotron.

  - Large acceptance with small gap magnet
  - Various longitudinal RF gymnastics become possible.
    - *Bunching, Stacking, Coalescing, etc.*

Type of FFAG optics

- Zero chromaticity
  - Fixed betatron tunes
    - *Fields are non-linear.*
  - Free from betatron resonance crossing
- Non-zero chromaticity
  - Varied betatron tunes
    - *Linear optics*
  - Fast resonance crossing

Zero chromaticity FFAG

- Betatron eqs. in cylindrical coordinate

$$\frac{d^2x}{d\theta^2}+\frac{r^2}{\rho^2}\left(1-K\rho^2\right)x=0$$

$$\frac{d^2z}{d\theta^2}+\frac{r^2}{\rho^2}\left(K\rho^2\right)z=0 \qquad K=-\frac{1}{B\rho}\frac{\partial B}{\partial r}$$

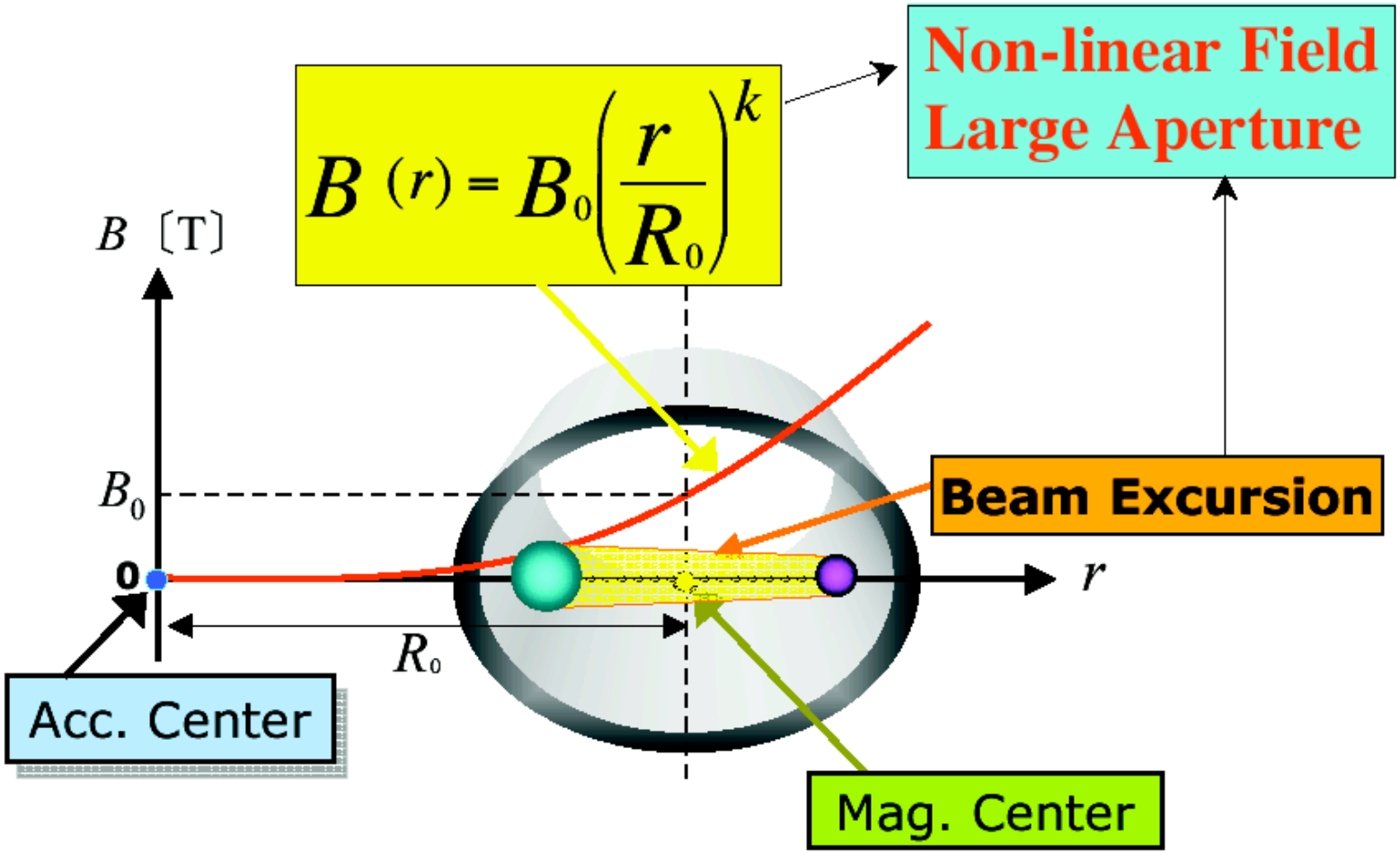
- Zero chromacitiy: Constant betatron tunes
  - Sufficient condition --> Scaling

$$\begin{cases}\dfrac{d\left(r^2/\rho^2\right)}{dp}=0\\ \dfrac{d\left(K\rho^2\right)}{dp}=0\end{cases}\longrightarrow\begin{cases}r\propto\rho\\ \dfrac{r}{B}\left[\dfrac{\partial B_z}{\partial x}\right]_{z=0}=k\end{cases}$$

$$B_z=B_0\left(\frac{r}{r_0}\right)^k f(\theta)$$

  - Note: Above is not necessary & sufficient condition!

Magnetic field of scaling FFAG



Momentum compaction: 1/k+1

Scaling FFAG lattice

Original idea ---> Ohkawa (1953)

"Zero chromaticity"

betatron eq.

$$x'' + g_x x = 0 \; ; \quad g_x = \frac{K^2}{K_0^2}(1-n)$$

$$z'' + g_z z = 0 \; ; \quad g_z = \frac{K^2}{K_0^2} n$$

geometrical field index

$$\left.\frac{\partial}{\partial p}\left(\frac{K}{K_0}\right)\right|_{\theta=const.} = 0 \qquad \left.\frac{\partial n}{\partial p}\right|_{\theta=const.} = 0$$

orbit similarity *no p-dependence*

$$B(r,\theta) = B_i\left(\frac{r_i}{r}\right)^{n_0} F\left(\theta - \varsigma \ln\frac{r}{r_i}\right)$$

radial sector

negative bend
FODO(DFDO)

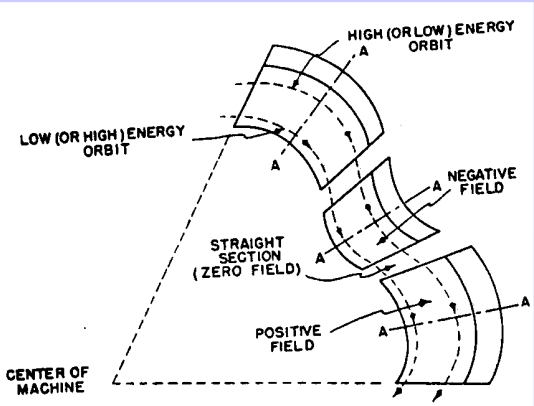


FIG. 2. Plan view of radial-sector magnets.

spiral sector

edge focus
FFDO

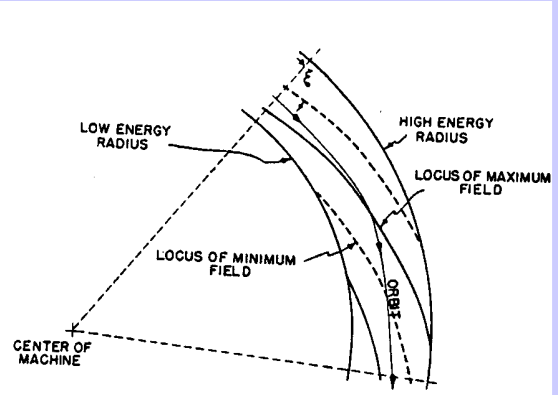
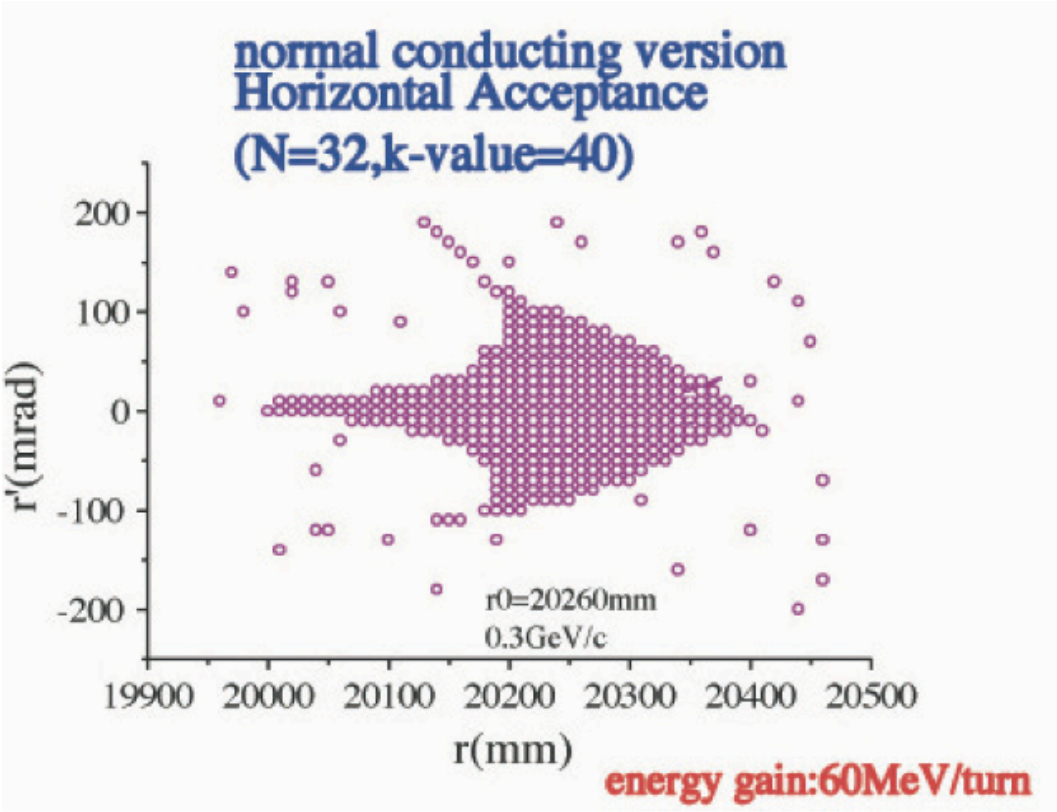
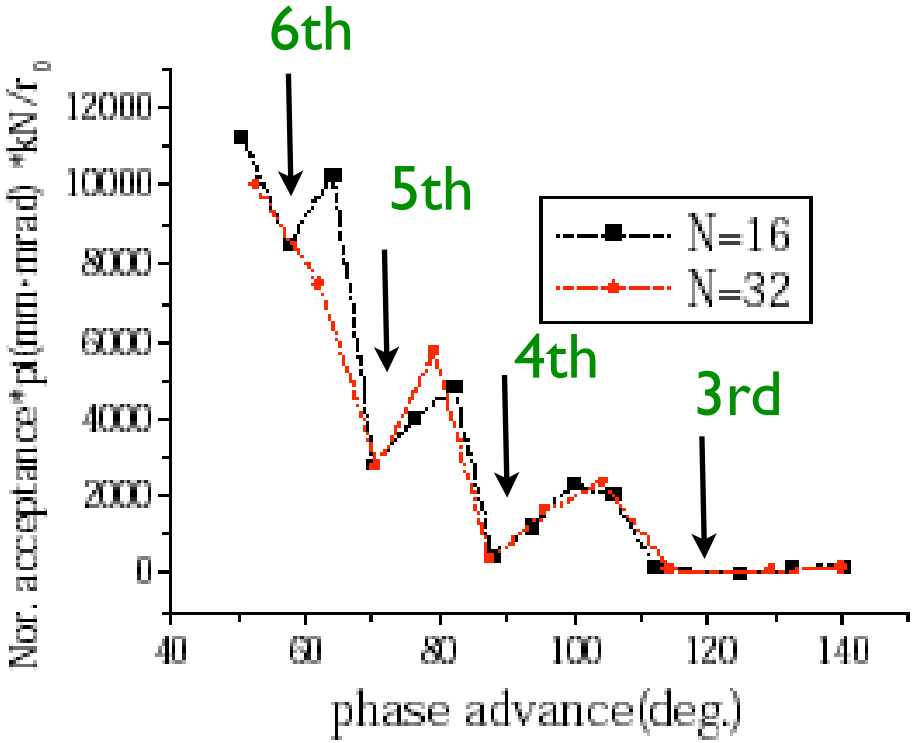


FIG. 3. Spiral-sector configuration.

AG focusing

Dynamic Aperture of Scaling FFAG





Quite large!

cf. A>10,000mm.mrad for phase advance of ~90degree/cell

$\alpha = \frac{1}{k+1}$: momentum compaction factor

Momentum Comapction: no higher orders

-------- momentum acceptance : large

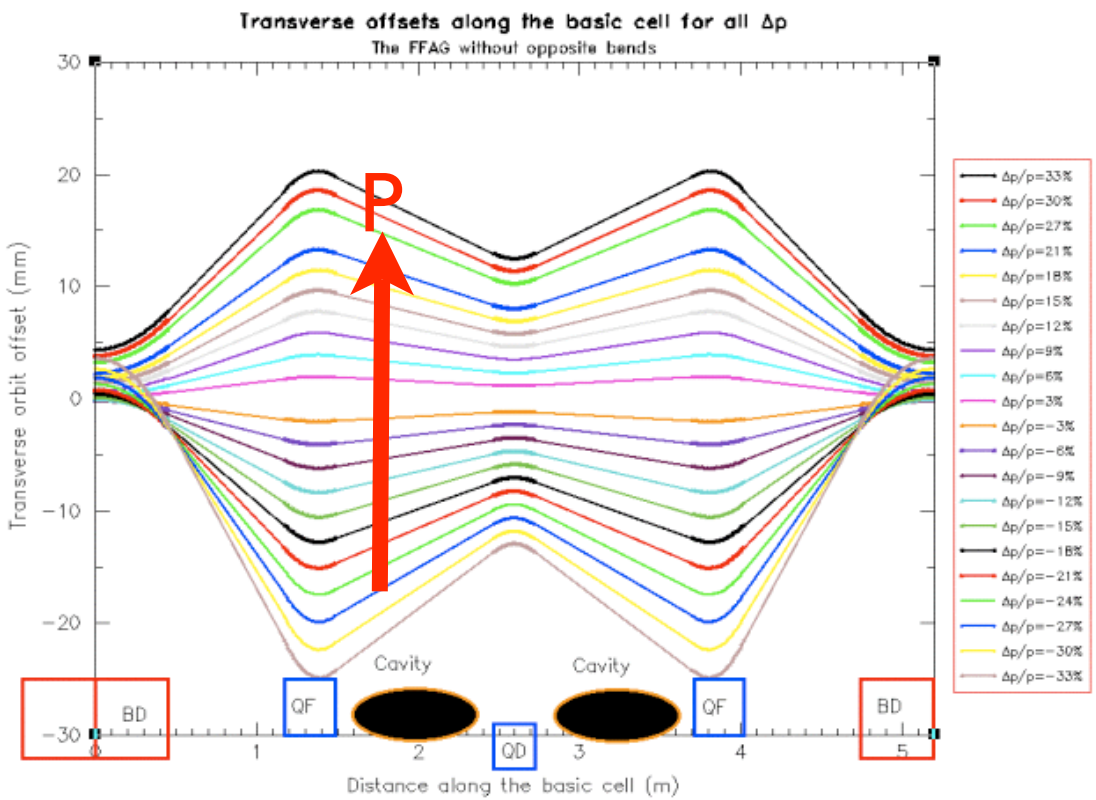
-------- kinematic effects : small enough

Non-zero chromaticity FFAG

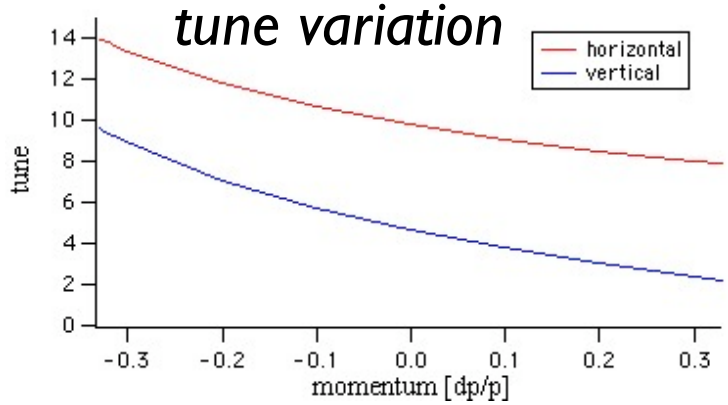
non-scaling

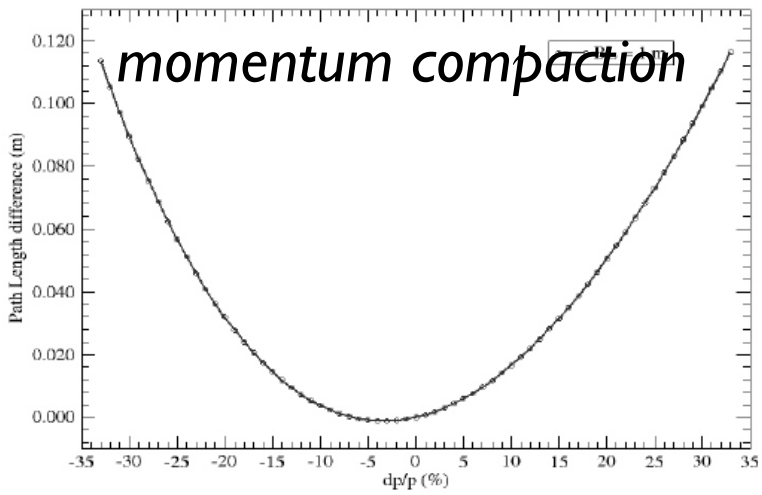
- Fields are linear: B,Q fields.
  - momentum compaction: small enough ~parabolic
- Tunes are varied: Fast resonance crossing

$$\alpha \cong C_1 \xi^2 , \xi = \frac{\Delta p}{p}$$



transverse offset for all momentum



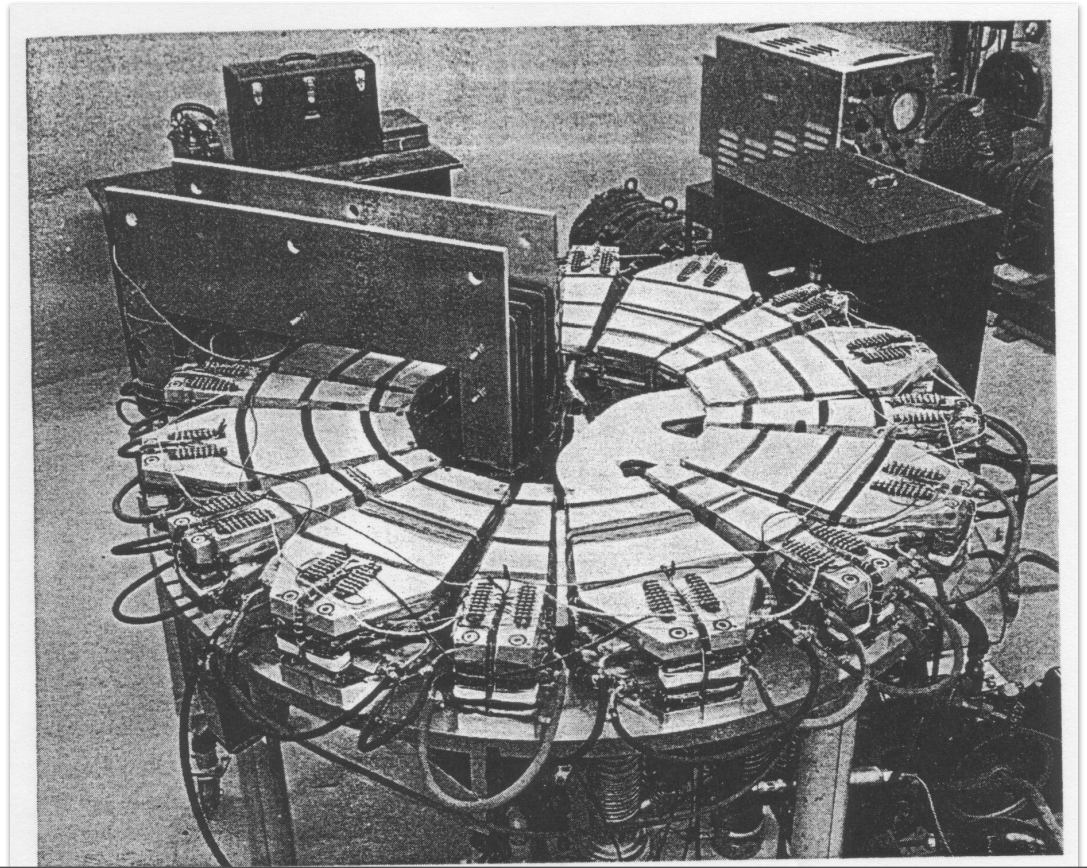


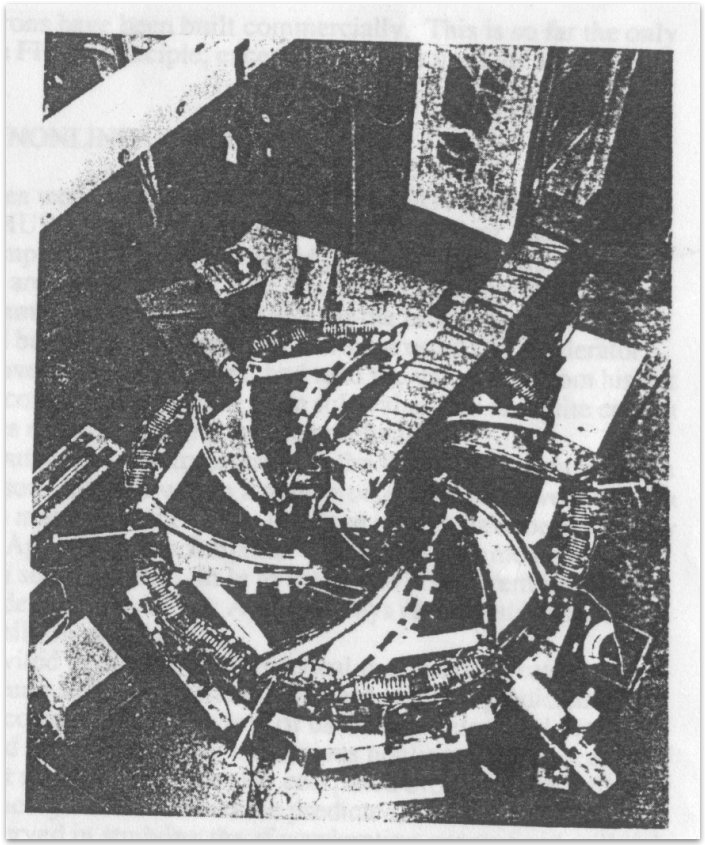
FFAG Accelerators :history

- *Ohkawa (1953), Kerst & Symon, Kolomenski*
  - *MURA project e-model, induction acceleration ~'60s*
  - *No proton FFAG for 50years!*
- *Proton FFAG (POP:World first p-FFAG, Mori et al.,2000)*
  - *Complicated field configuration : 3D design*
  - *MA(Magnetic Alloy) RF cavity :Variable Frequency & High Gradient.*
  - *150MeV p-FFAG (Mori et al.,2004)*
  - *PRISM FFAG(Kuno et al.,2008,Osaka)*
  - *p-FFAG for ADSR study, ERIT neutron source (KURRI,2008)*
  - *EMMA(e-FFAG for nuFact:World first non-scaling FFAG, England,under development)*

MURA FFAG ('60) Electron Model

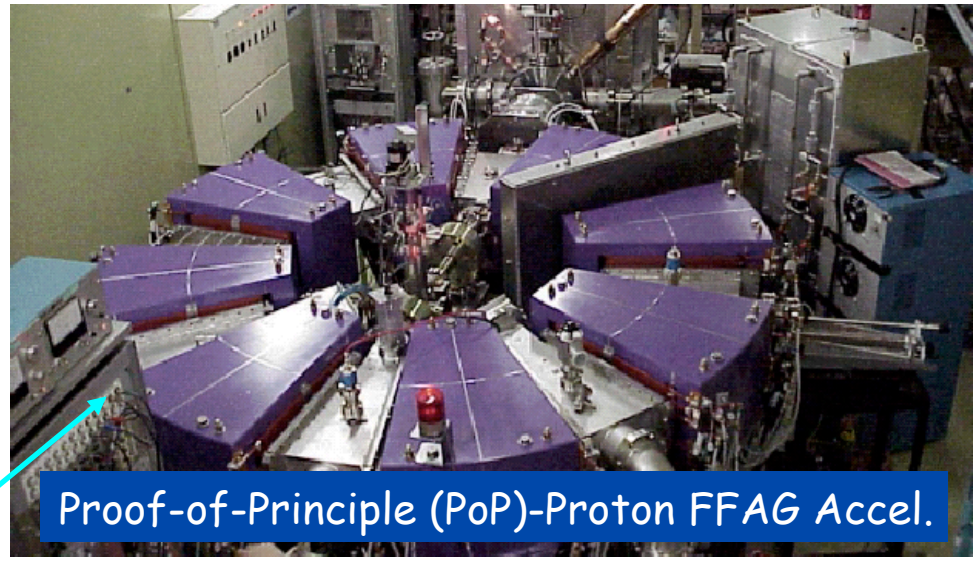
- Radial & Spiral Induction & RF (const. f)
- No proton acceleration

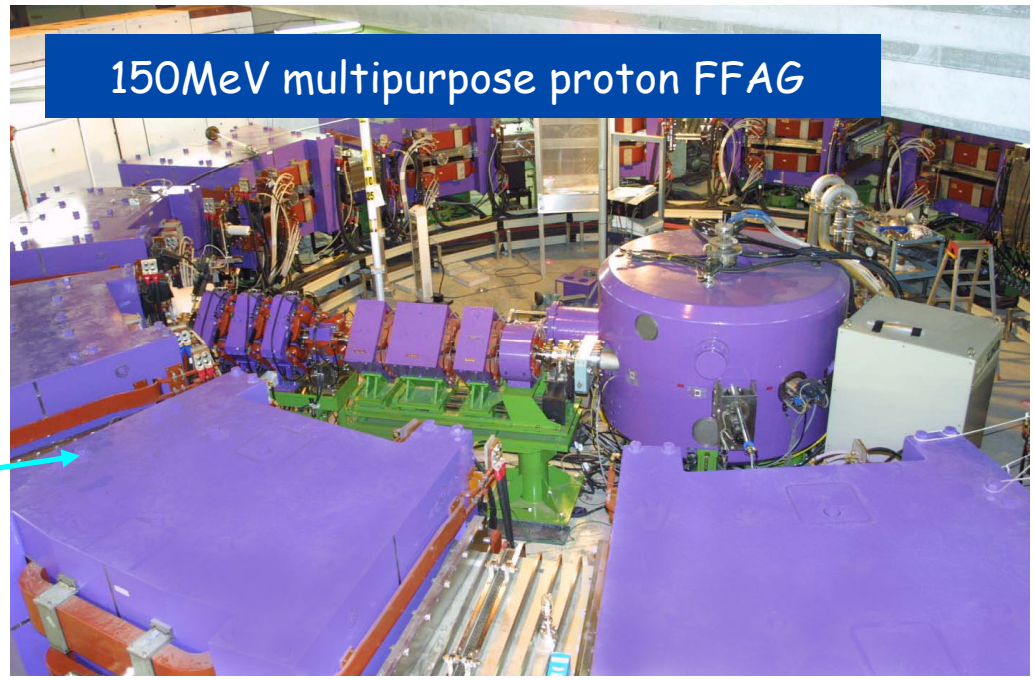




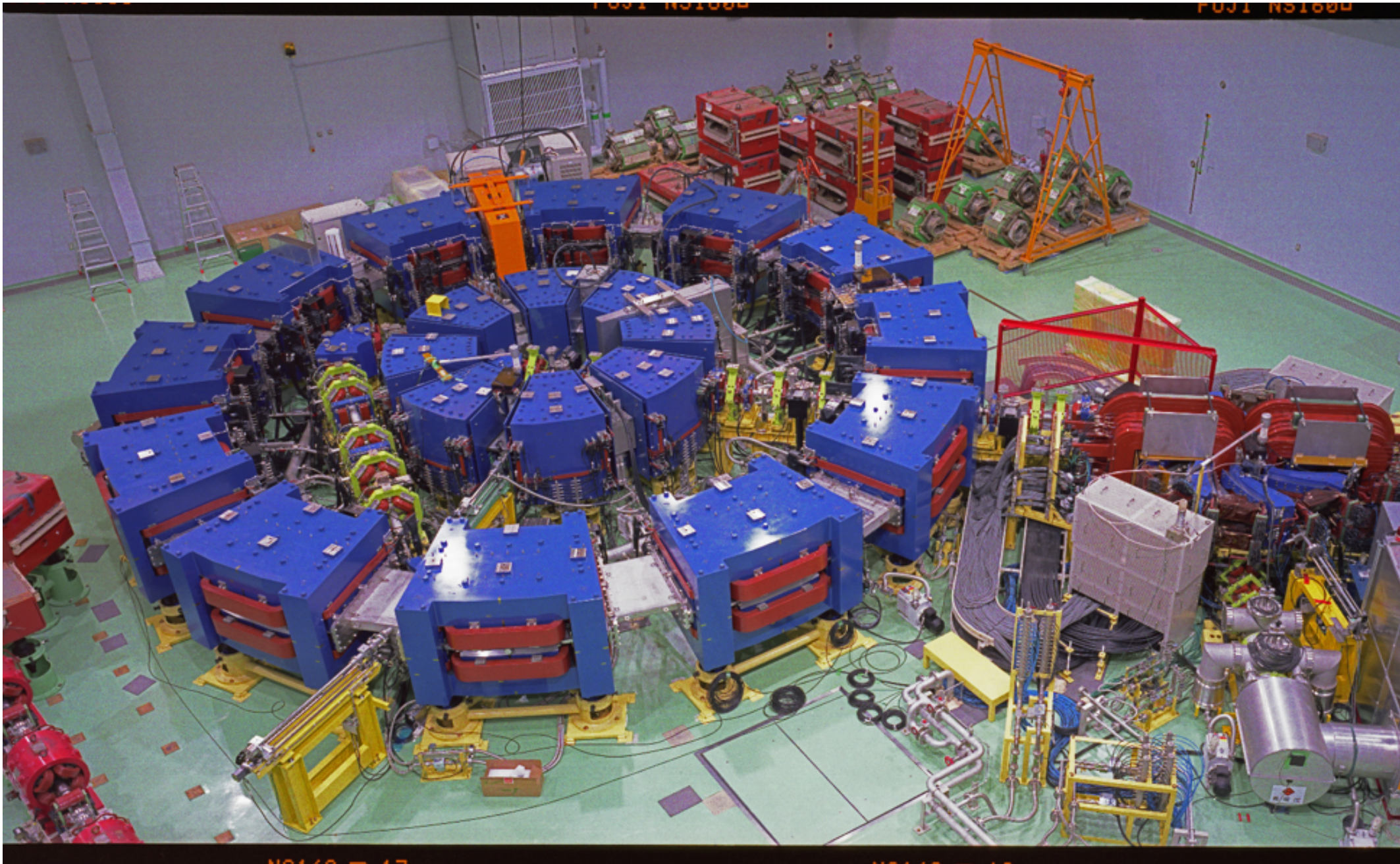
History of FFAG Proton Accelerator

- 1953: Basic concept by Ohkawa
  Proton FFAG accelerator was not successful until recent
  →**difficulty in fabricating RF cavity with variable frequency & high gradient field**
- 1998: Development of RF cavity using Magnetic Alloy
  Grant-in-Aid for Scientific Res. by MEXT: Y. Mori, KEK
- 2000: Demonstration of Proton FFAG Accelerator -POP FFAG-
  Grant-in-Aid for Scientific Res. by MEXT: Y. Mori, KEK
  WORLD's FIRST PROTON FFAG!
- 2004: Development of 150MeV multipurpose FFAG accelerator
  100Hz Operation!
  Grant-in-Aid for Creative Basic Res.: Y.Mori

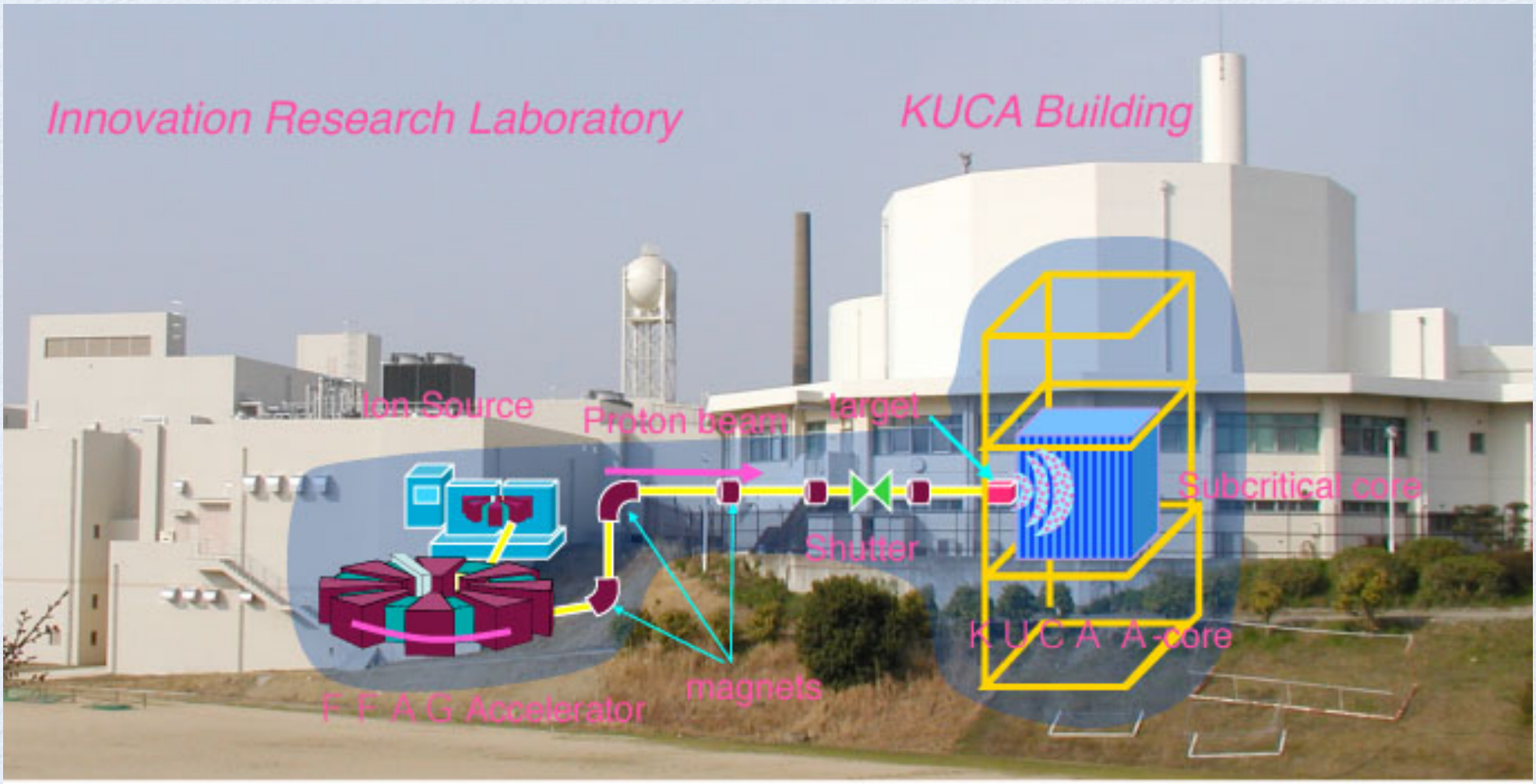

Proof-of-Principle (PoP)-Proton FFAG Accel.


150MeV multipurpose proton FFAG

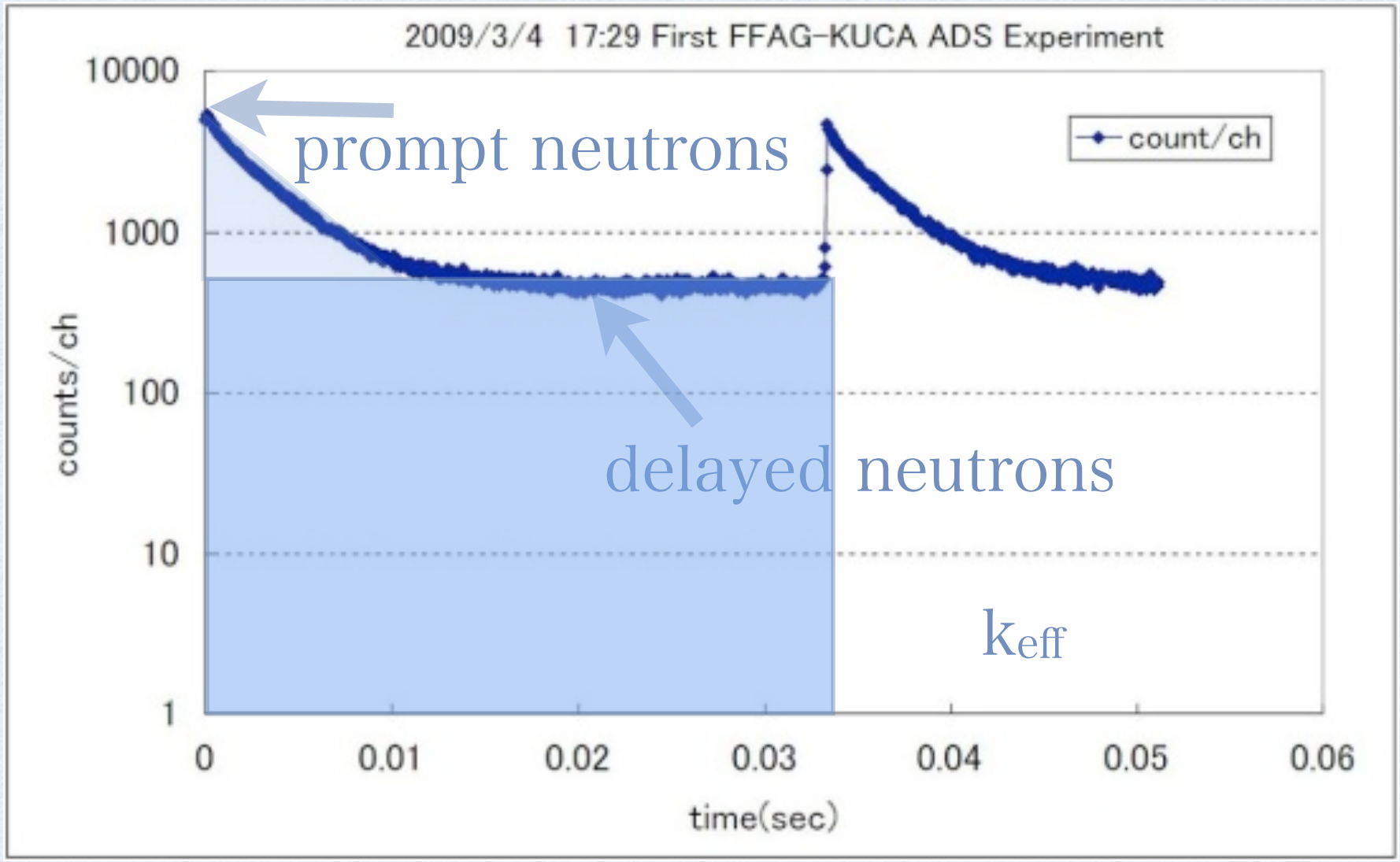
FFAG complex for ADSR study at KURRI



FFAG-ADSR project at KURRI



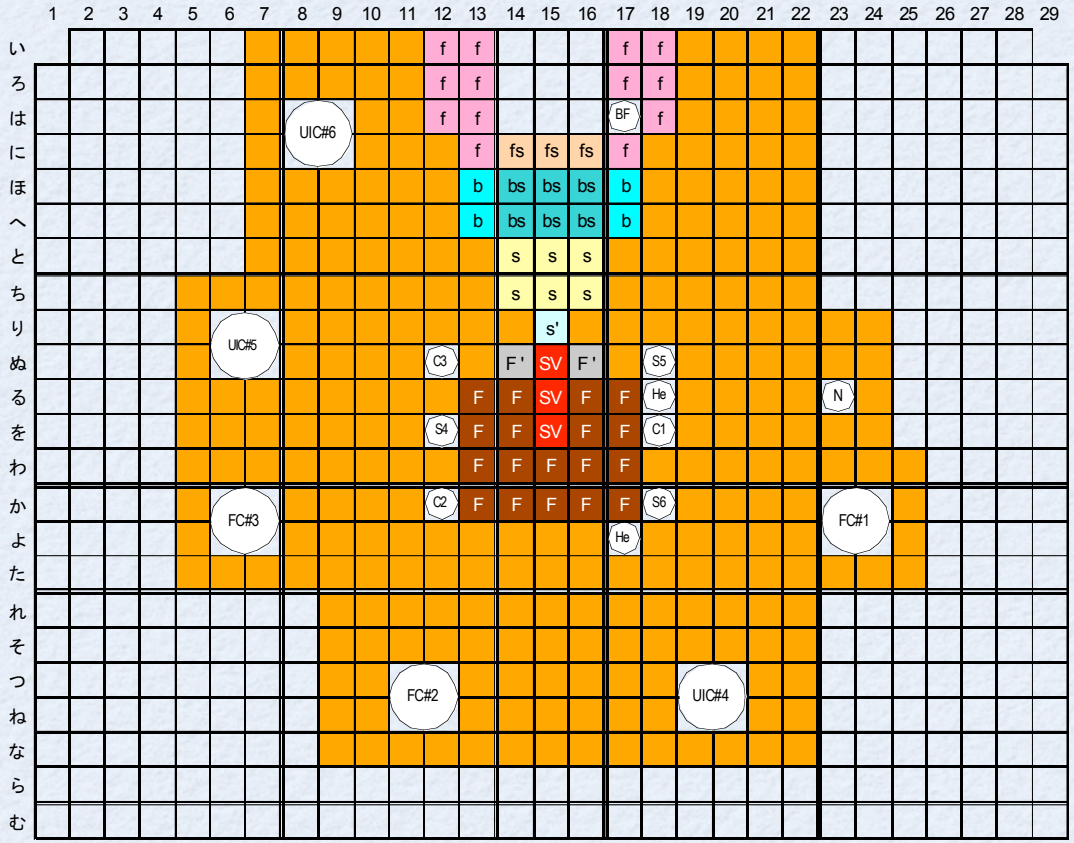
First experimental data
FFAG-KUCA for ADSR study

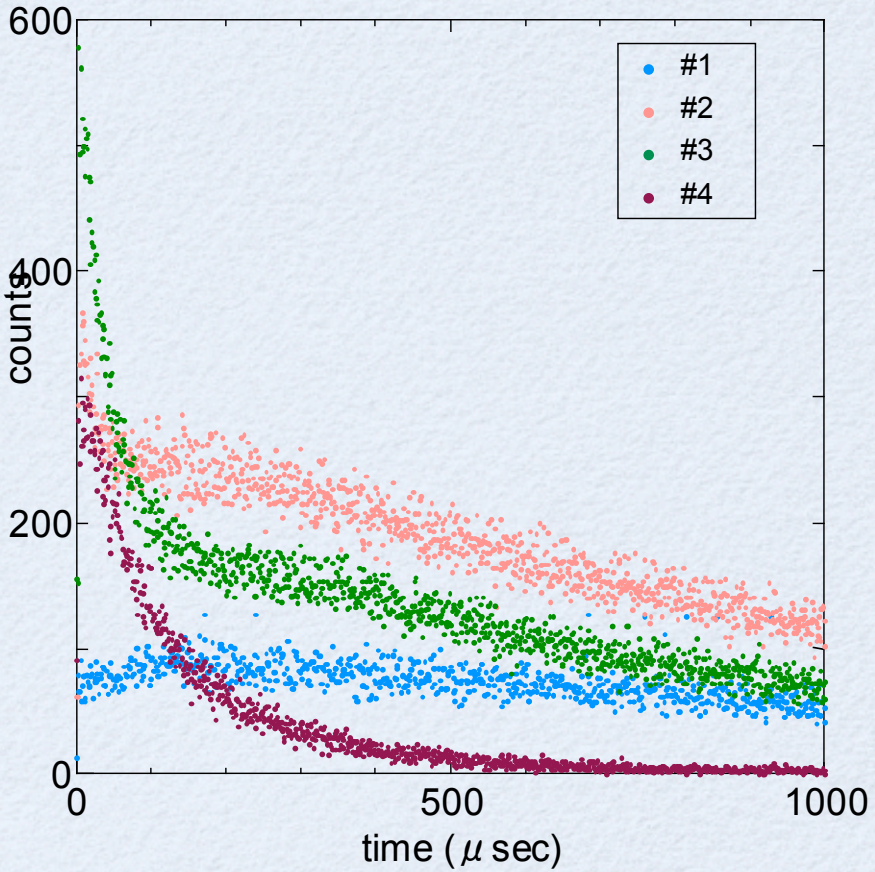


C.H.Pyeon et al., Journ. of Jap.Atom.Ene.Soc.

Neutron time sturucture

At various positions in the reactor



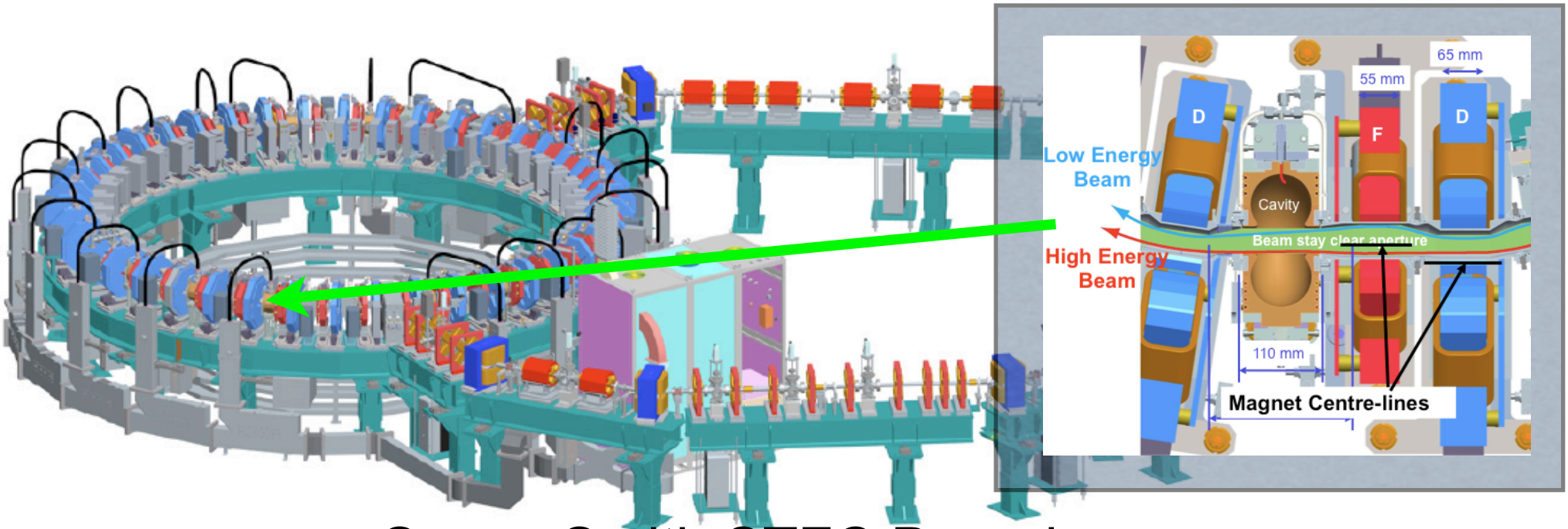


KUCA reactor core

Non-zero chromatic FFAG

EMMA

the World's First Non-Scaling FFAG Accelerator



Susan Smith STFC Daresbury Laboratory

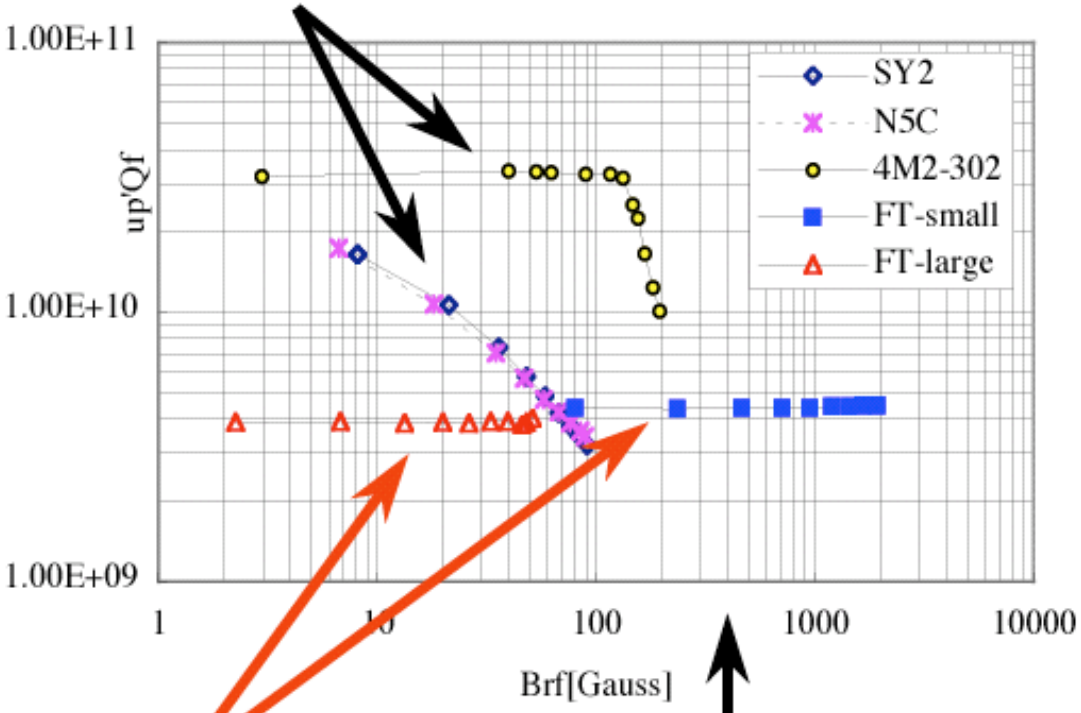
Acceleration(RF)

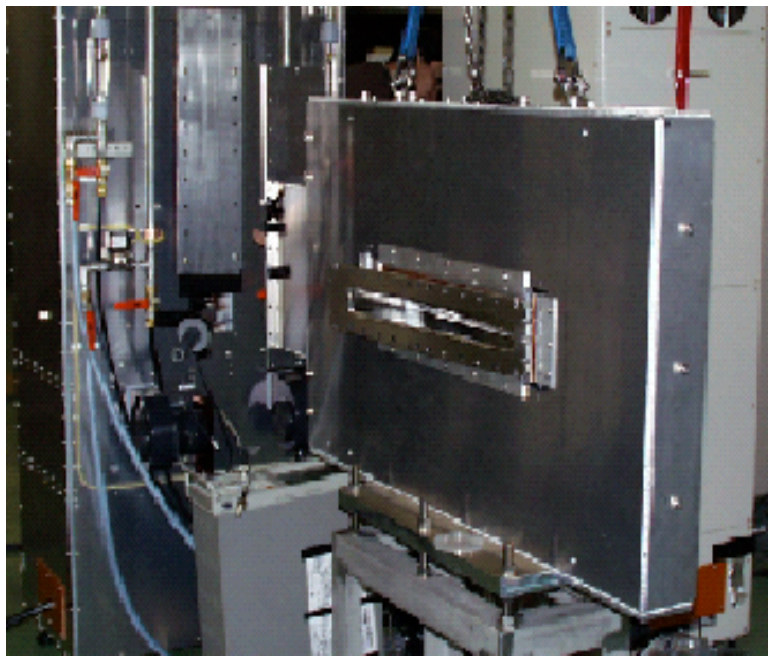
- Beam acceleration in FFAG: various and flexible
  - Momentum compaction can be tuned along orbit swing.
    - *Keeping phase stability like synchrotron*
    - *Realizing isochronism like cyclotron*
- Variable frequency RF
  - Broad-band RF cavity : Scaling & Non-scaling
    - *MA(magnetic alloy) cavity Q~1*
- Fixed frequency RF
  - Stationary RF bucket acceleration : Scaling
    - *constant momentum compaction(MC)*
  - Gutter RF acceleration : Non-scaling
    - *relativisitc beam & small MC(parabolic) :semi-isochronous*
  - Harmonic number jump acceleration : Scaling (non-scaling)
    - *non-zero slippage factor*

Variable RF frequency

- Broad-band RF cavity : MA(magnetic alloy) cavity
  - Fast acceleration requires fast frequency(phase) change.
    - *Low Q (Q~1) is essentianl !*
  - Adequate both for scaling and non-scaling FFAGs.

Ferrites

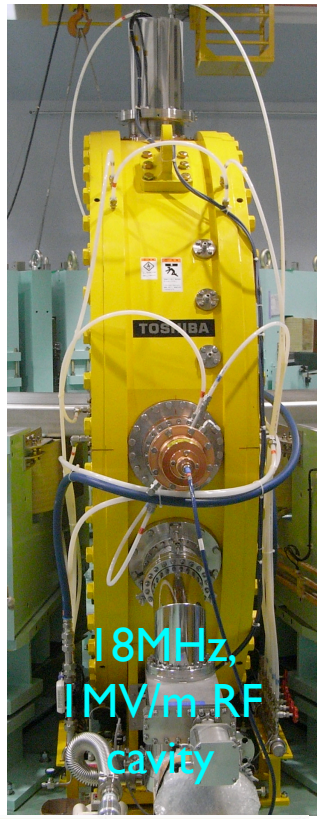




Magnetic Alloys

$B = V/\omega S = {}_{25kV/2pX5MHzX5cmX40cm} = 400Gauss$

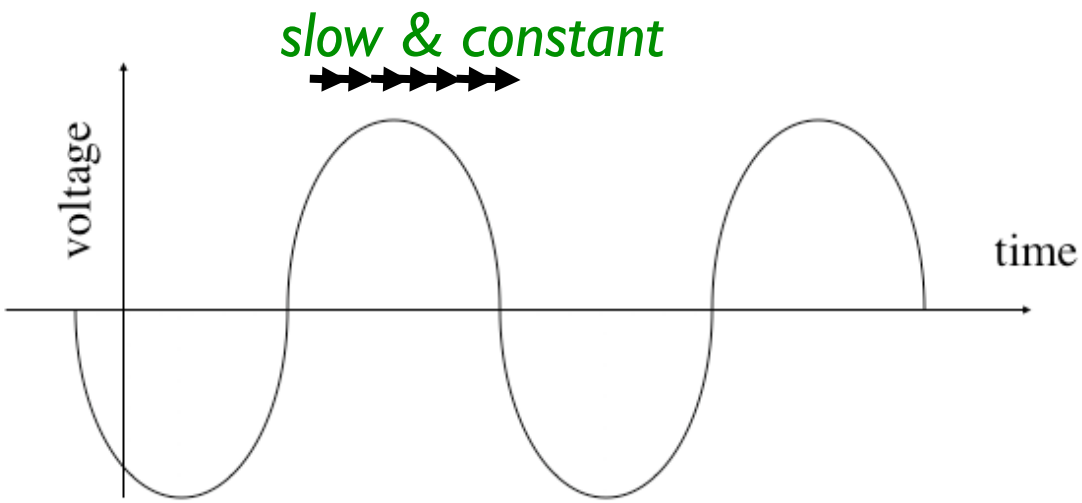
Fixed RF frequency(1)

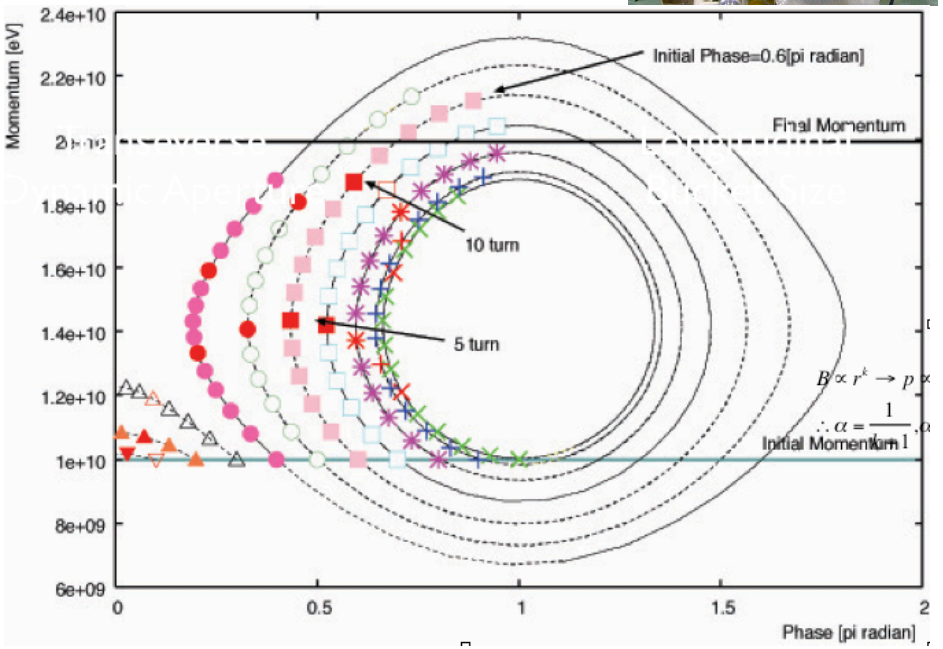


Stationary bucket acceleration

- Constant & small enough phase slip --- Large energy gain
  - *relativistic beam*
  - *constant Momentum Compaction*
- Adequate for scaling FFAG

$$\eta = \frac{1}{\gamma^2} - \alpha \cong -\alpha = -\frac{1}{k+1}$$





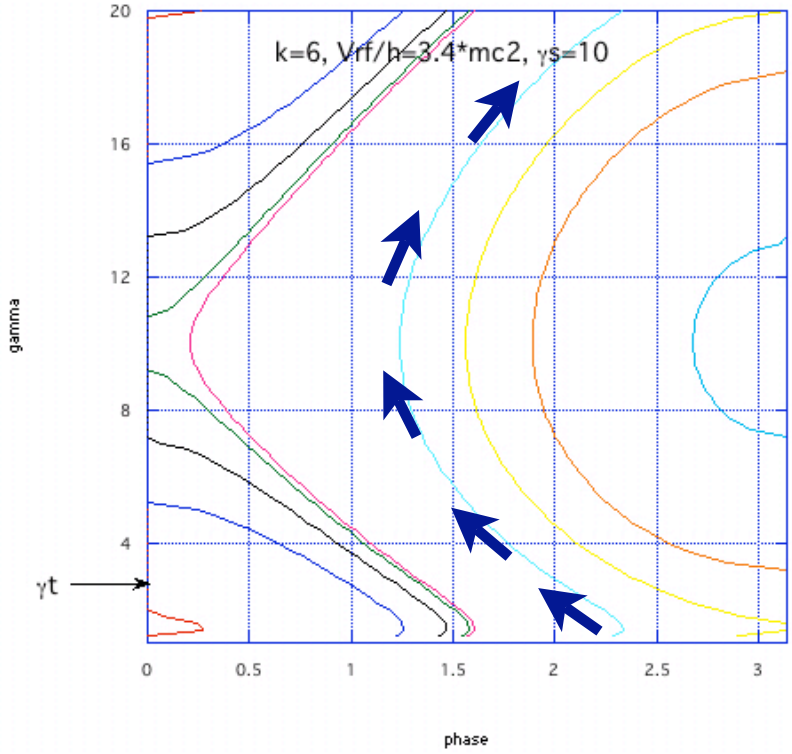
Fixed frequency (2)

- Stationary bucket acceleration
  - Non-relativistic to relativistic
  - Longitudinal Hamiltonian in scaling FFAG

$$H = 2\pi m_0 c^2 \left[\frac{\left(\gamma_s^2 - 1\right)^{\lambda}}{2\gamma_s} \frac{\left(\gamma^2 - 1\right)^{-\lambda+1}}{\left(1-\lambda\right)} + \gamma \right] + e\frac{V_{rf}}{h} f_0 \cos\phi$$

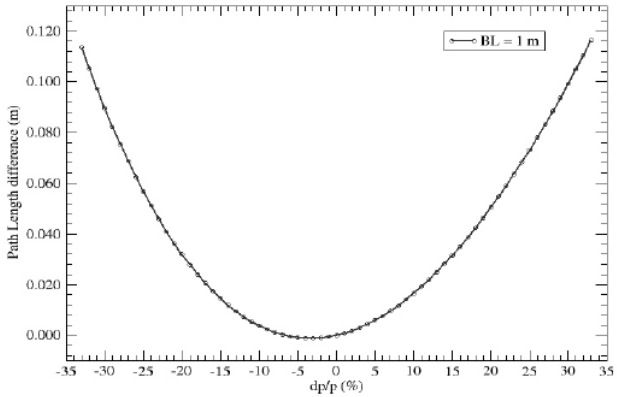
$$\lambda = \frac{k}{2\left(k+1\right)}$$

$$\frac{dp}{dT} = 0: \quad p = \gamma_1 \text{ and } \gamma_2$$

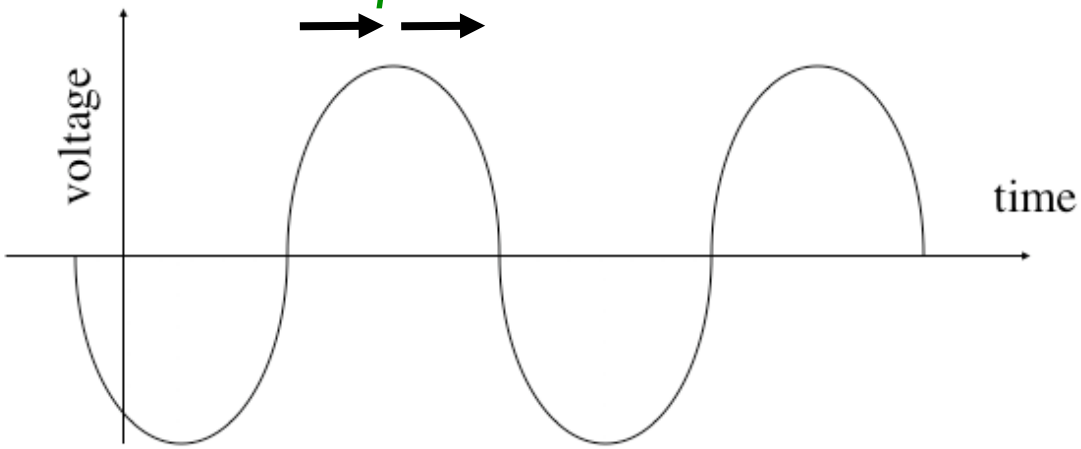


Fixed RF frequency(3)

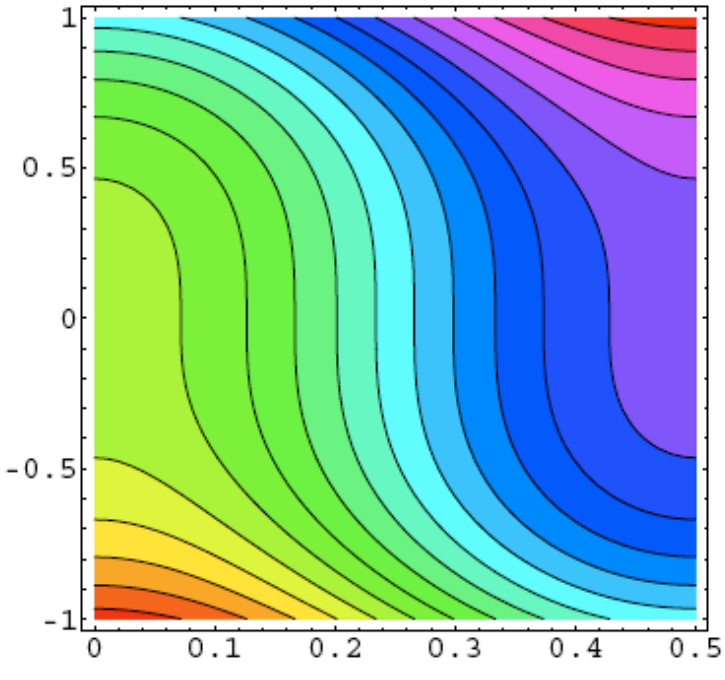
- **Gutter RF acceleration**
  - Parabolic & small enough phase slip
    - *relativistic beam*
    - *small parabolic Momentum Compaction*
  - Adequate for non-scaling FFAG



slow & parabolic



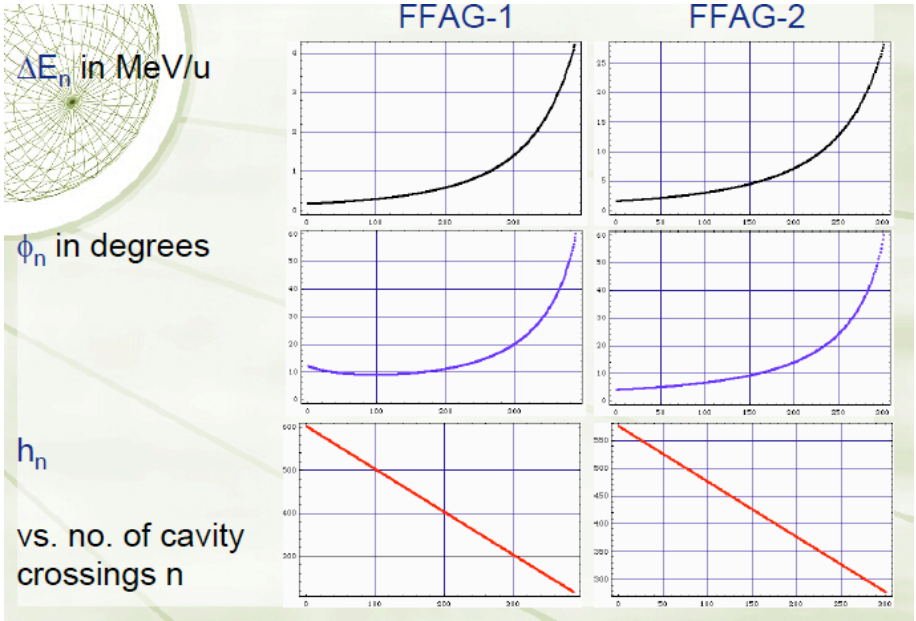
cf. S.Machida



Fixed RF frequency(4)

- Harmonic number jump acceleration
  - *m:integer, m<0: before transition, m>0: after transition*
  - Energy gain/turn can be automatically tuned if the RF voltage is high enough. ---> Phase stability
- Time slip/turn: m x Trf

$$T_{i+1} - T_i = \frac{m}{f_{RF}}$$



cf. A.Ruggiero(BNL)

Advancement of FFAG

- Zero chromaticity (scaling) FFAGs

- Pro/
  - Fixed field & Strong focusing
  - Zero chromaticity
    - *constant betatron tunes → no-resonance crossing*
  - Large acceptance (longitudinal & transverse)
- Con/
  - Relative large dispersion:Orbit excursion is large.
    - *Large horizontal aperture magnet*
    - *Large horizontal aperture rf cavity → Low frequency*
  - Short straight section
    - *Injection/Extraction difficulties → Kicker/Septum needs large apertures.*
    - *Available space for rf cavity is limited.*

- Need long straight section with small dispersion keeping "Zero Chromaticity".

Scaling FFAG linear line

- Is it possible to make a linear FFAG straight line?
  - keeping a scaling law: zero chromaticity
  - reducing dispersion: dispersion suppressor
  - making a good match with ring: insertion
- Magnetic field configuration for FFAG linear line?
  - Obviously not:

$$B = B_0 \left(\frac{r}{r_0}\right)^k f(\theta)$$

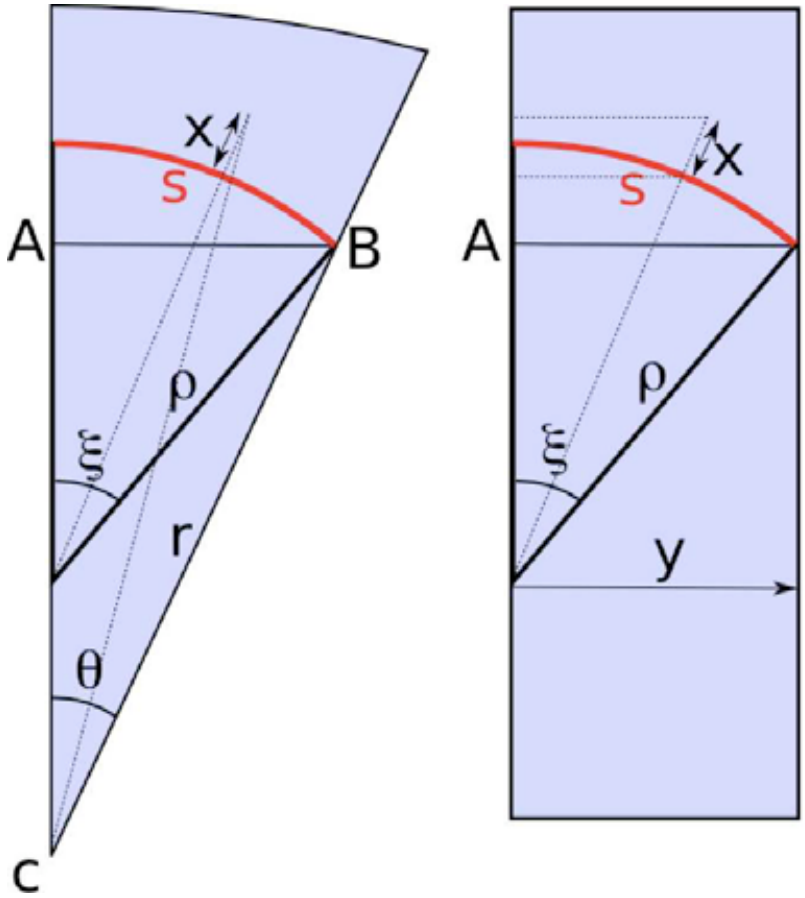
Scaling field

linear (straight) transport line

- **Betatron eqs.**

$$\frac{d^2x}{dy^2}+\frac{1}{\rho^2}\left(1-K\rho^2\right)x=0$$

$$\frac{d^2z}{dy^2}+\frac{1}{\rho^2}\left(K\rho^2\right)z=0$$



- **Scaling conditions:** zero-chromaticity

$$\begin{cases}\dfrac{d\left(1/\rho^2\right)}{dp}=0\\[2ex]\dfrac{d\left(K\rho^2\right)}{dp}=0\end{cases}\longrightarrow\begin{cases}\rho=const.\\[2ex]\dfrac{1}{B}\left[\dfrac{\partial B_z}{\partial x}\right]_{z=0}=\dfrac{n}{\rho}\end{cases}$$

  - sufficient cond.
  - Magnetic field

$$B_z=B_0\exp\left[\frac{n}{\rho}x\right]$$

$$\left[\lim_{r_0\to\infty}\left(\frac{r}{r_0}\right)^k=\lim_{r_0\to\infty}\left[\left(1+\frac{x}{r_0}\right)^{\frac{r_0}{x}}\right]^{\frac{x}{r_0}k}=\lim_{r_0\to\infty}\left[\left(1+\frac{x}{r_0}\right)^{\frac{r_0}{x}}\right]^{\frac{n}{\rho}x}=\exp\left(\frac{n}{\rho}x\right)\right]$$

Scaling linear line

Example (JB. Lagrange)

- Perfect scaling(zero-chromatic) FFAG linear transport line
- proton 80-200MeV

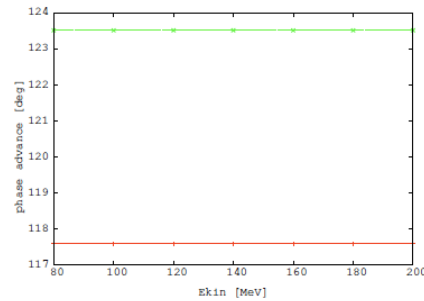
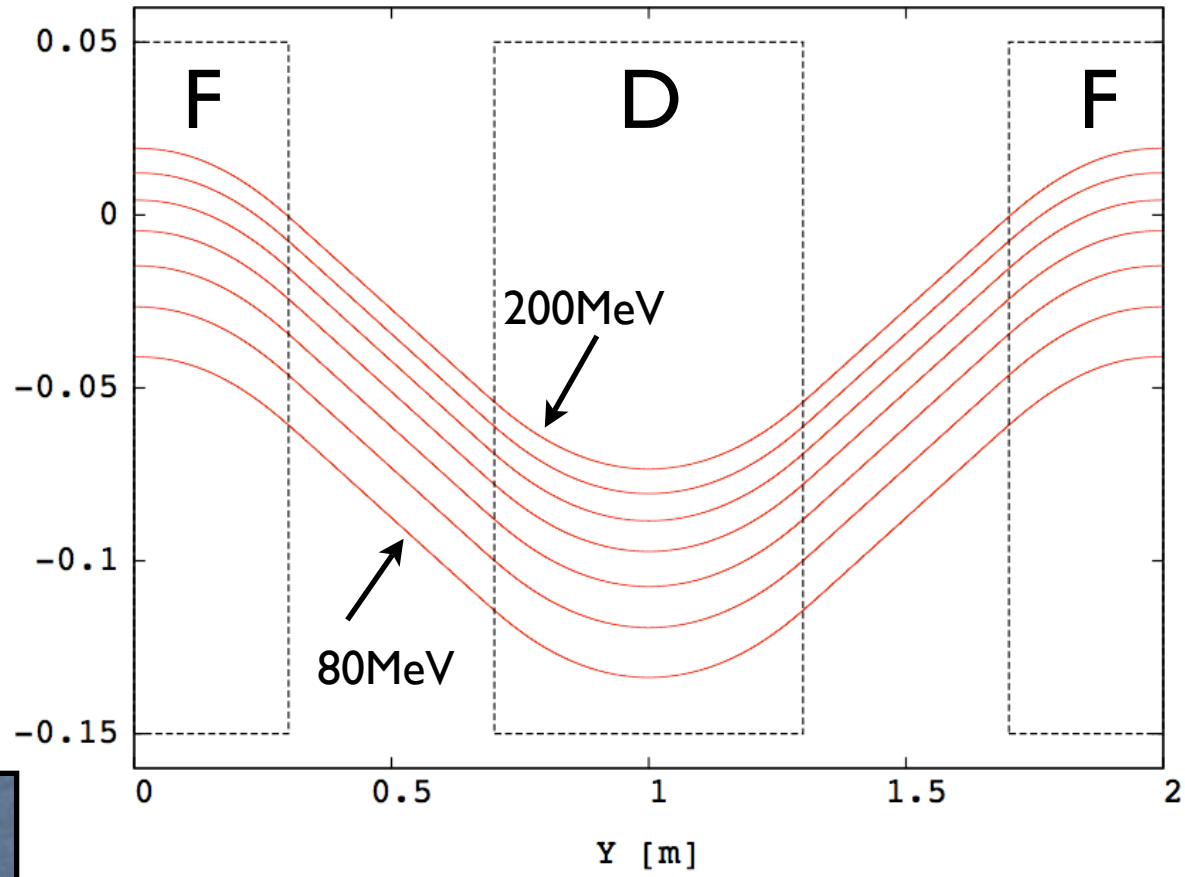


Table 1: Tracking parameters

Length of the magnets	60 cm
Drift	40 cm
Kinetic energy range	80 to 200 MeV (proton)
Field index	17
Local curvature radius	2.1 m
Step size	1 mm
Phase advances:	
horizontal μ_x	104.8 deg.
vertical μ_z	112.5 deg.



B-field

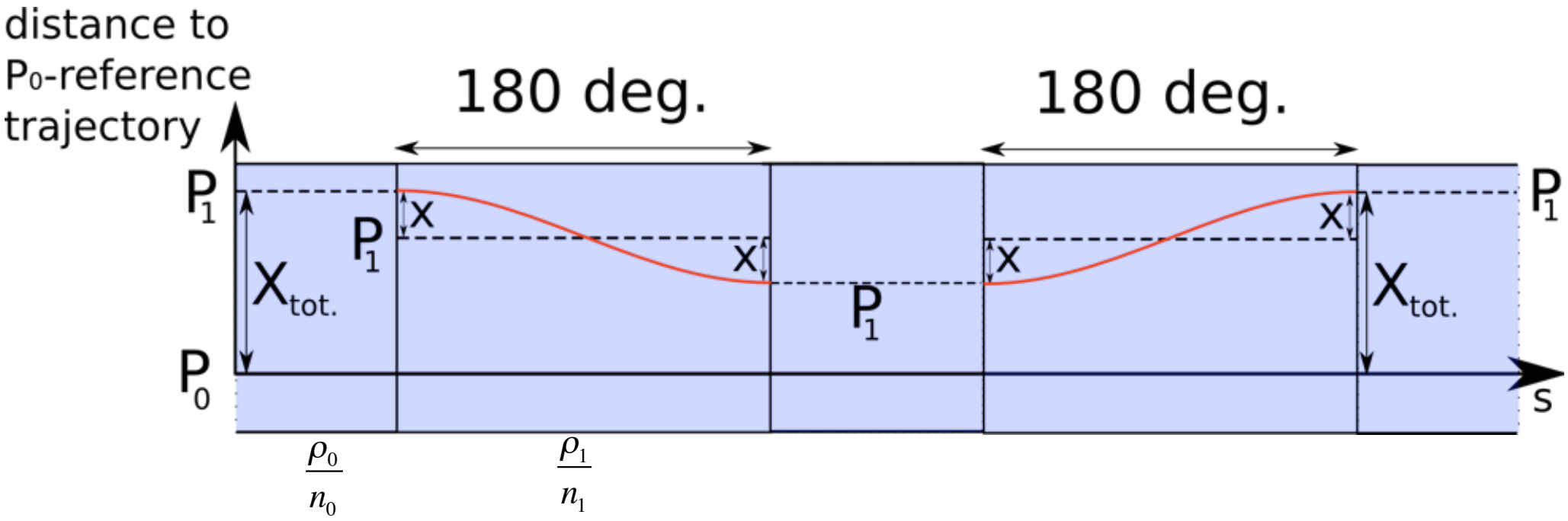
$$B_z = B_0 \exp\left[\frac{n}{\rho} x\right]$$

Dispersion suppressor

Dispersion suppressor (Planche,Lagrange,Mori)

- successive π-cells in the horizontal plane can suppress the dispersion.

$$X_{tot} = X_1 - X_0 = \frac{1}{n/\rho}\ln\left(\frac{P_1}{P_0}\right) \qquad x = \ln\left(\frac{P_1}{P_0}\right)\left(\frac{\rho_0}{n_0} - \frac{\rho_1}{n_1}\right)$$



Insertion Matching

btw. ring & straight line

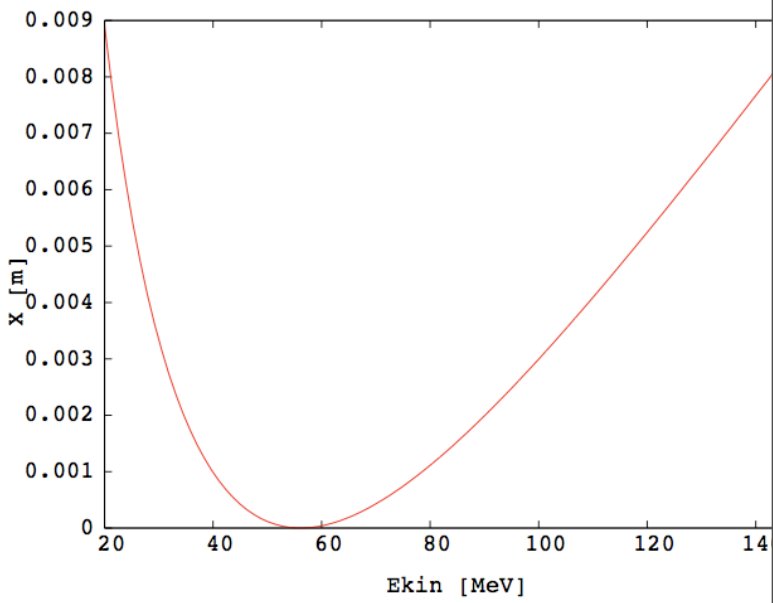
- B(closed orbit) matching condition

$$\left(1+\frac{x}{r_m}\right)^{k+1} = \exp\left(\frac{n}{\rho}x\right)$$

ring linear line

$$\frac{k+1}{r_m} = \frac{n}{\rho}$$

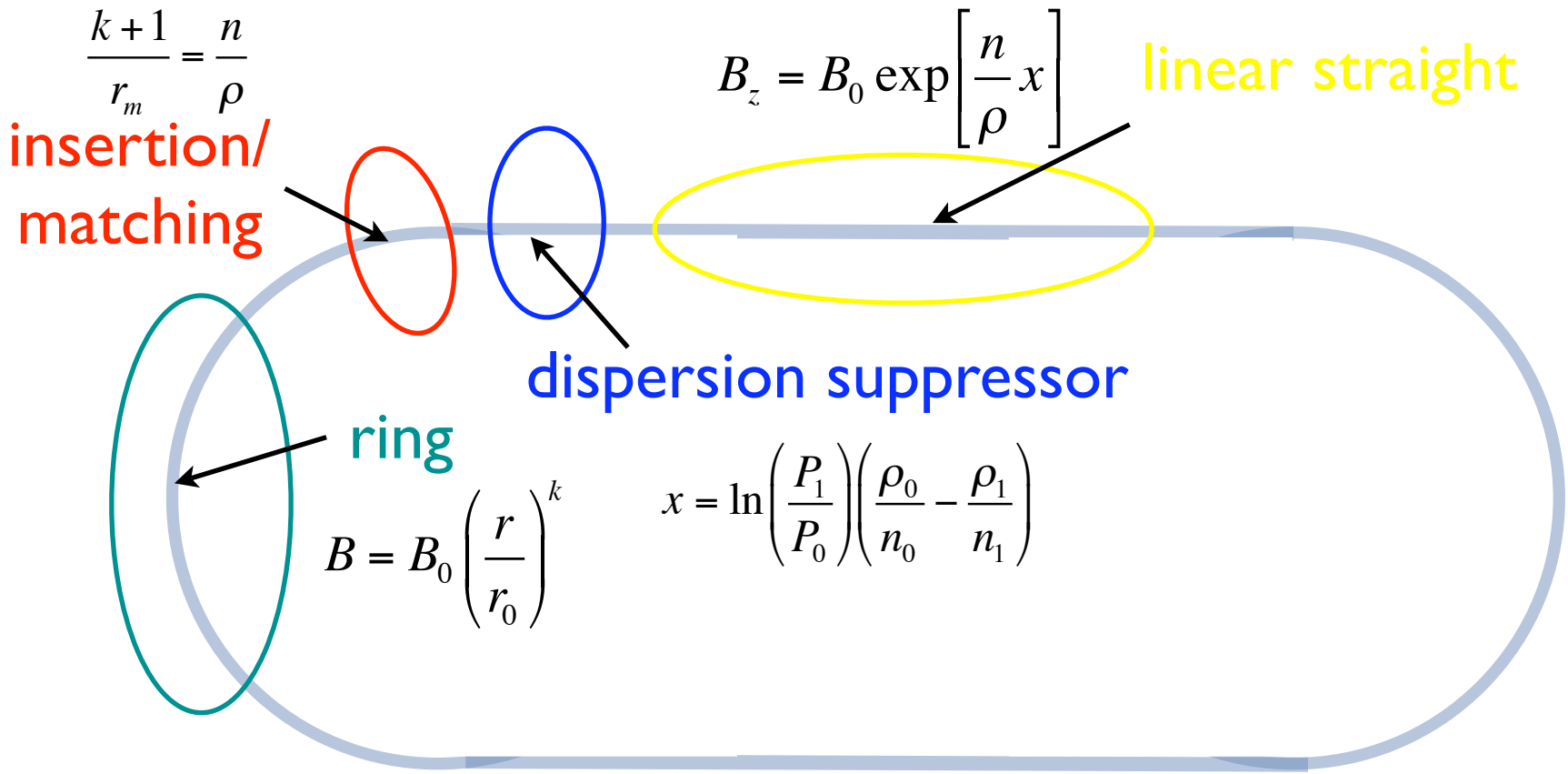
← 1st order



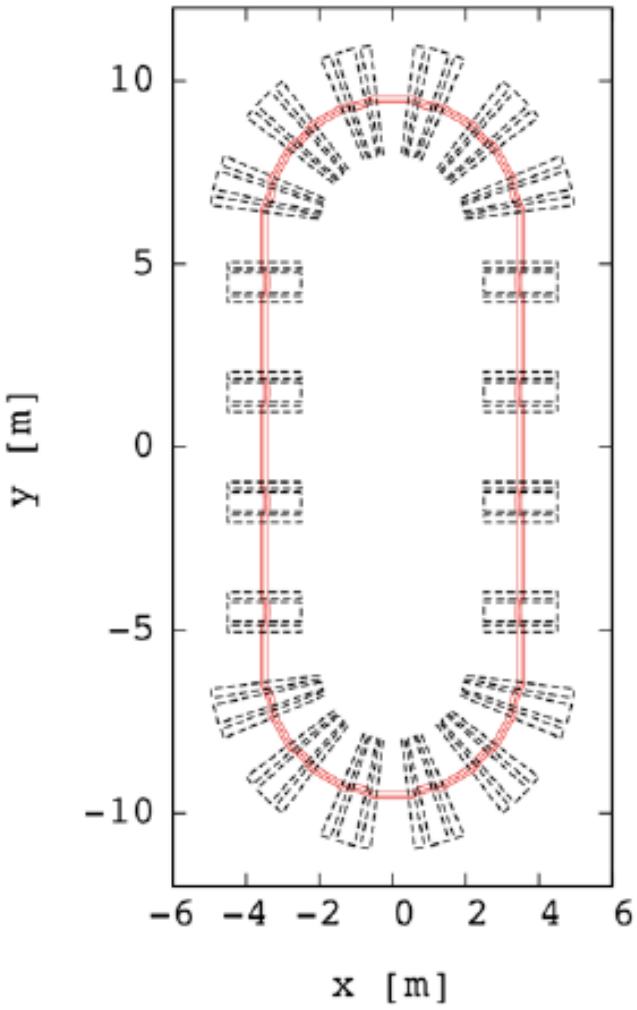
Example: 150MeV p-FFAG ring(KURRI) with insertion

CO mismatch
higher order error:
→ smaller for larger ring

Advanced scaling FFAG

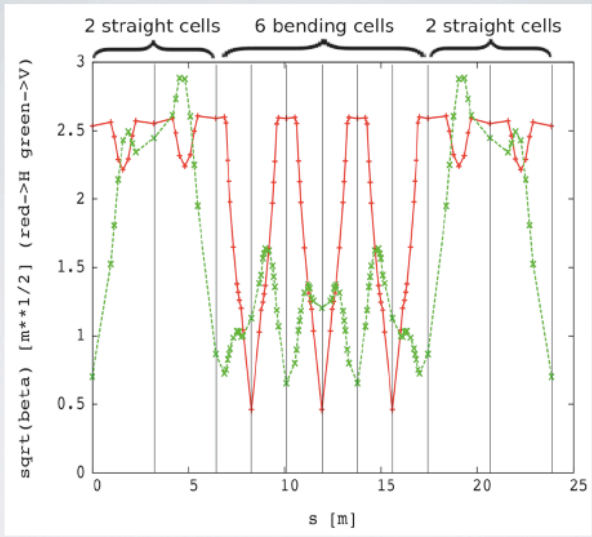


Muon phase rotation PRISM ring

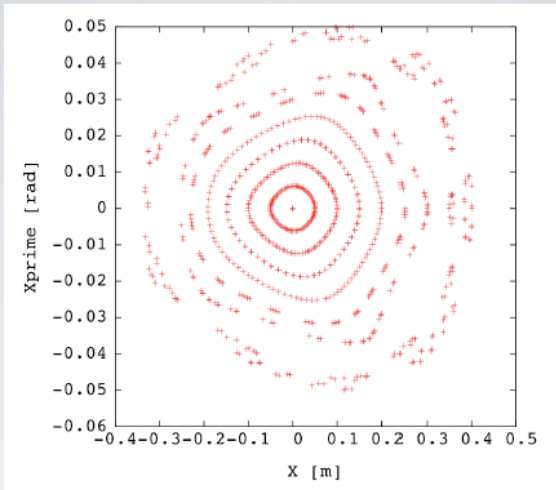


PRISM LATTICE

Bending cell	
k	6.5
Average radius	3.5 m
Phase advances:	
horizontal μ_x	90 deg.
vertical μ_z	90 deg.
Dispersion	0.47 m
Straight cell	
n/ρ	2.14 m^{-1}
Length	3 m
Phase advances:	
horizontal μ_x	24 deg.
vertical μ_z	87 deg.



Betafunctions of bending and straight cells (half ring)
(red: horizontal, green: vertical)



by Lagrange, Mori

Muon accelerator neutrino factory

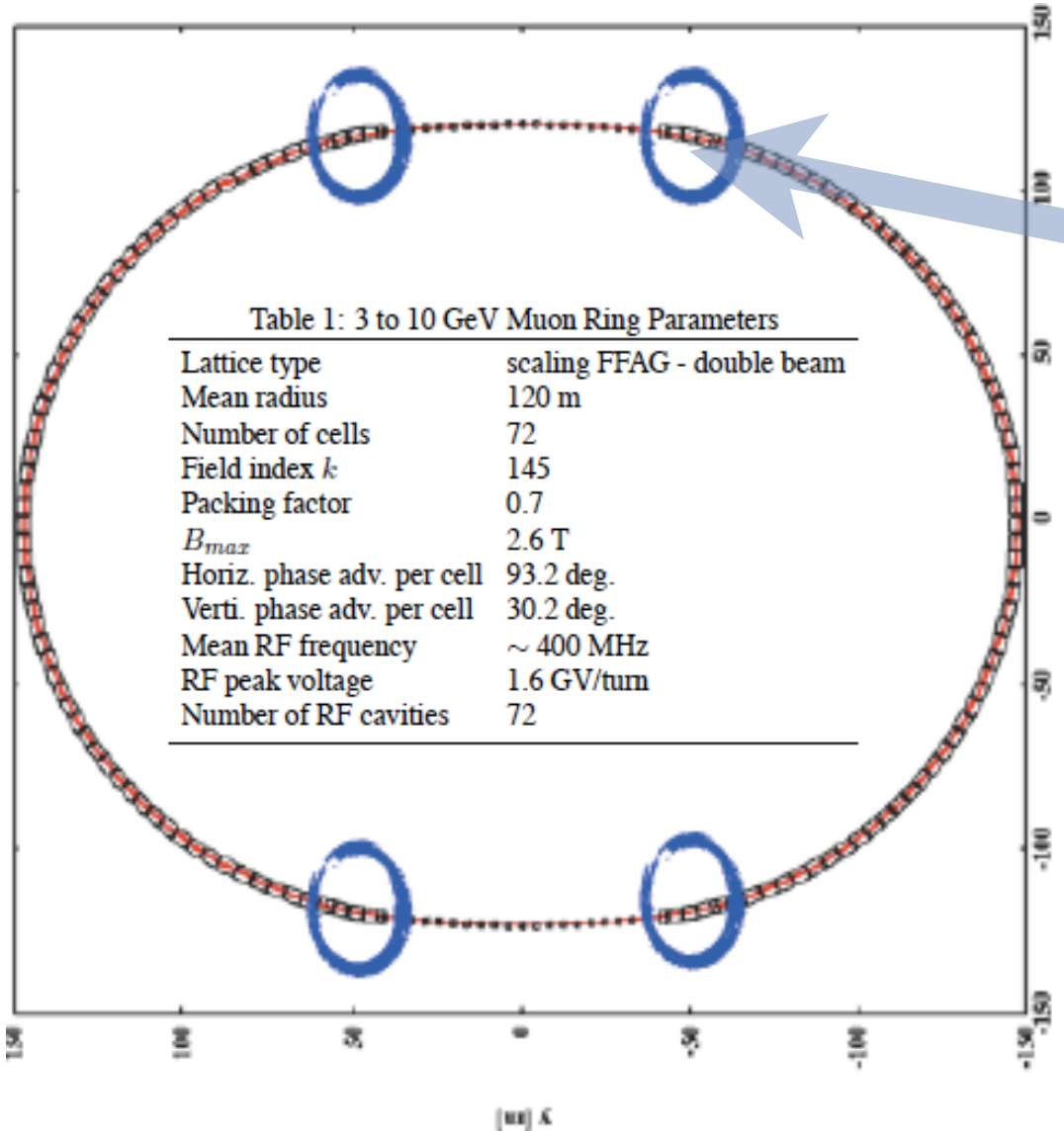
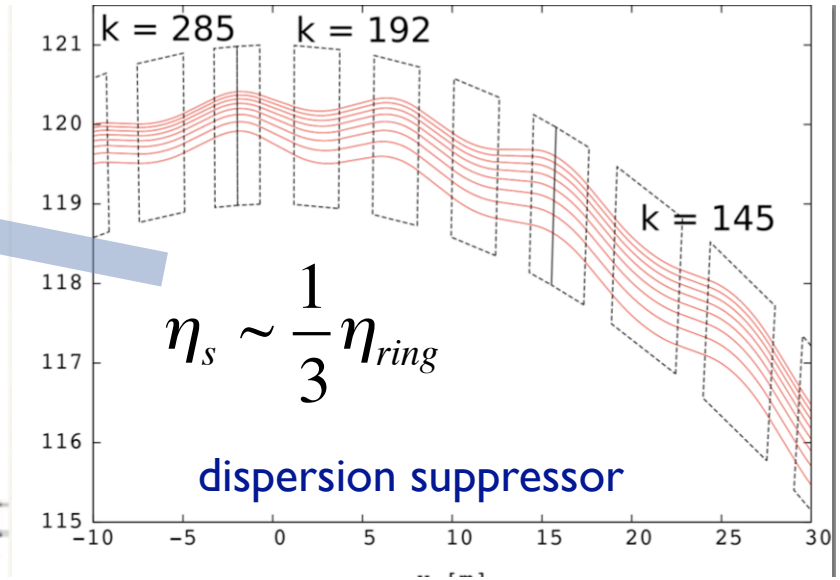


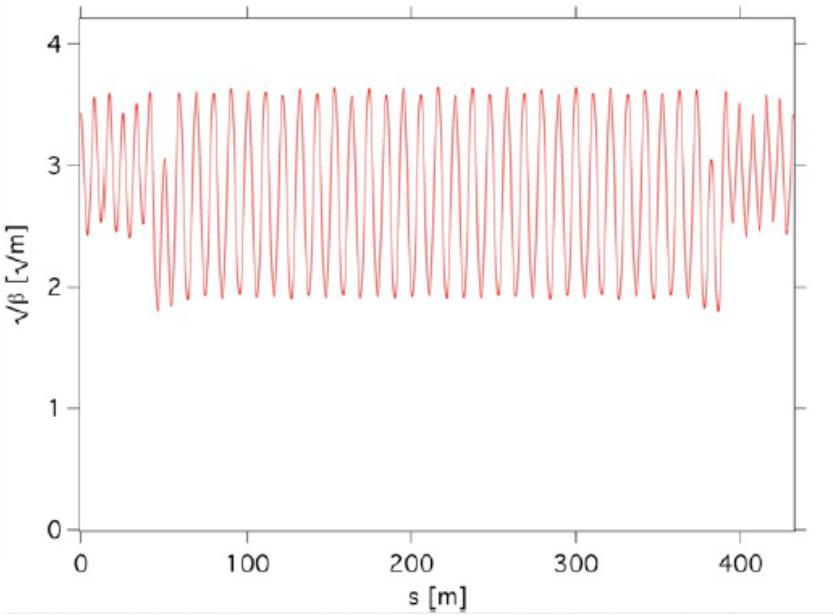
Table 1: 3 to 10 GeV Muon Ring Parameters

Lattice type	scaling FFAG - double beam
Mean radius	120 m
Number of cells	72
Field index k	145
Packing factor	0.7
B_{max}	2.6 T
Horiz. phase adv. per cell	93.2 deg.
Verti. phase adv. per cell	30.2 deg.
Mean RF frequency	~ 400 MHz
RF peak voltage	1.6 GV/turn
Number of RF cavities	72



$$\eta_s \sim \frac{1}{3}\eta_{ring}$$

dispersion suppressor



Planch, Mori

Proton driver

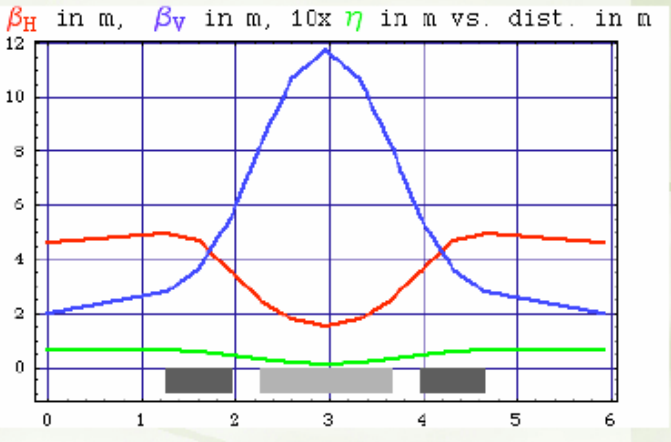
- Muon Source (neutrino factory, muon collider etc.)
  - Pulsed muon source
    - Beam energy :5-20GeV
    - Beam power: >4MW
    - Bunched beam: 1nsec , ~10Hz
- Accelerator Driven Sub-critical Reactor(ADSR)
  - Neutron source
    - *Beam energy:1-2GeV*
    - *Beam power:>10MW*
    - *CW/High Rep. Rate >kHz*

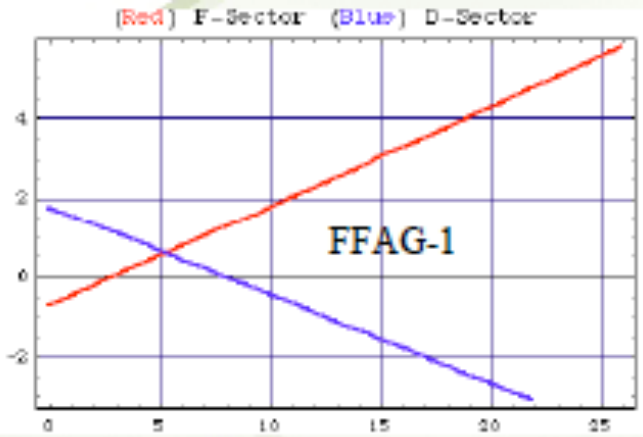
Proton driver for neutrino factory

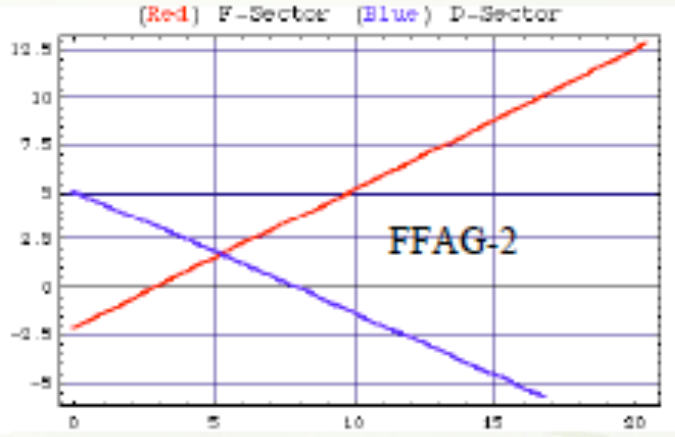
- Design works
  - Non-zero chromatic(linear) FFAG :A.G.Ruggiero (BNL)
    - *E=11.6GeV (two rings)*
    - *Lattice: O-BF-BD-BF-O, MC=linear for momentum*
    - *Harmonic number jump acceleration*
  - Zero chromatic (non-linear) FFAG :G.Rees(Rutherford Lab.)
    - *E=10GeV, 50Hz*
    - *Lattice:O-bd-BF-BD-BF-bd-Q including non-linear bd*
    - *variable frequency RF acceleration*

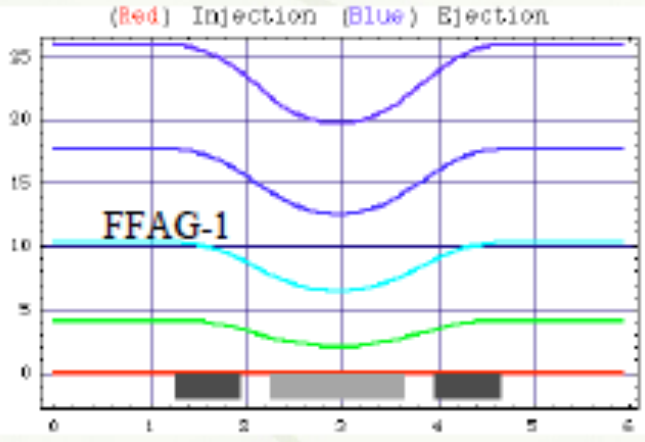
Non-scaling FFAG by A.G.Ruggiero (BNL)

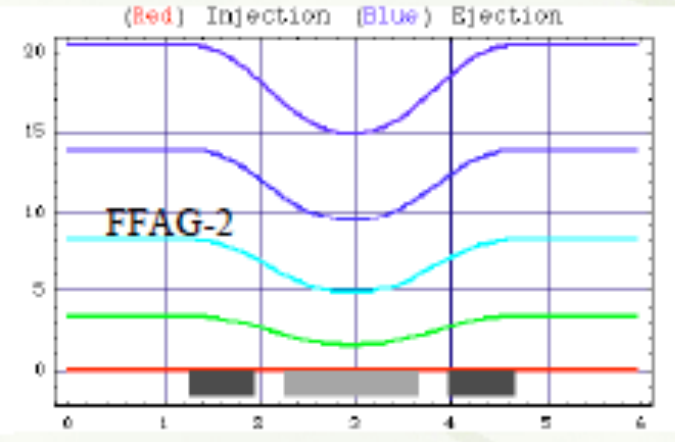
Beam Parameters







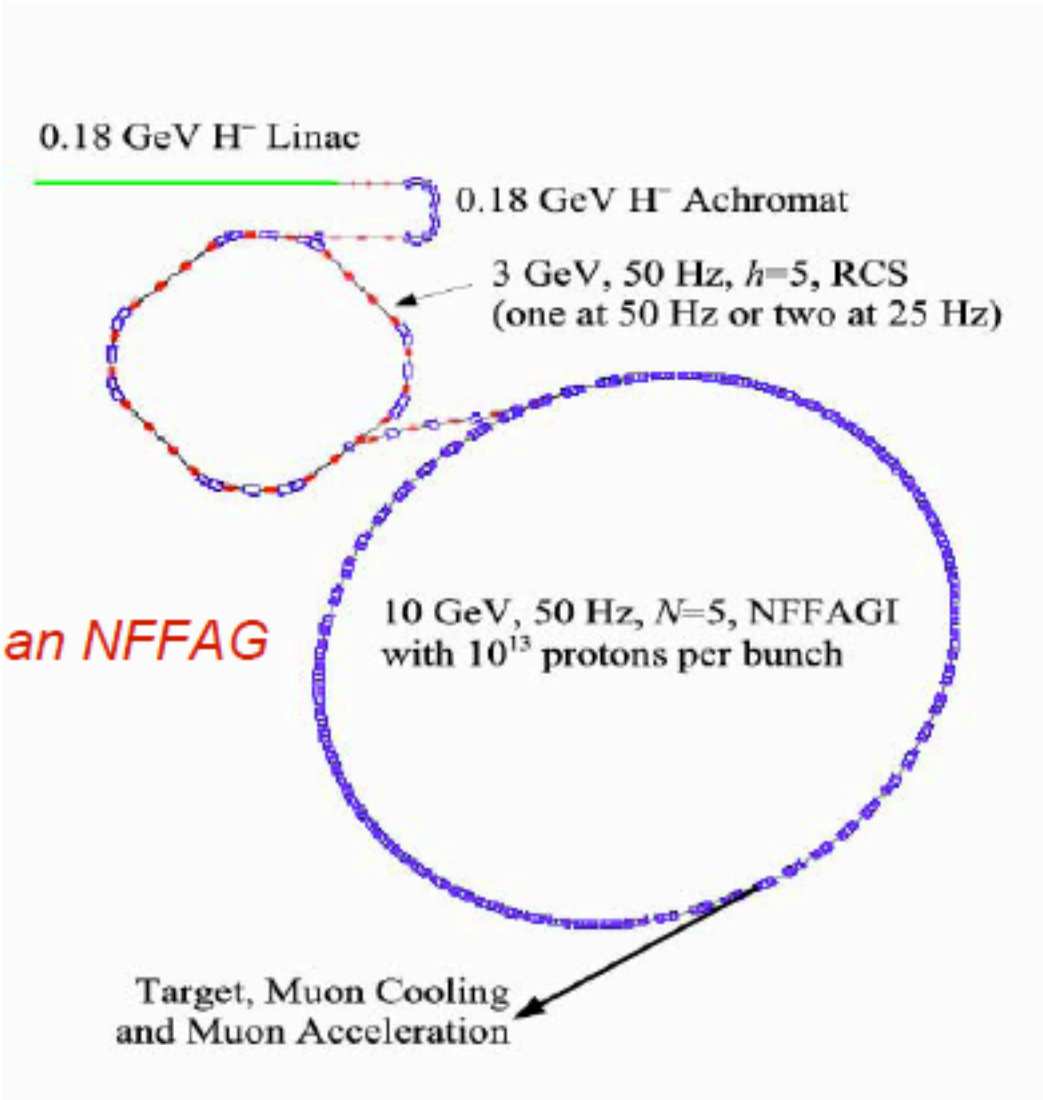




E=11.6GeV

Tune variations & orbit excursion

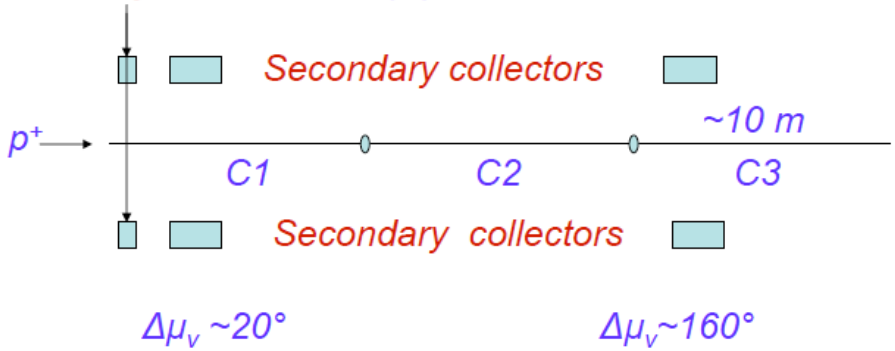
Semi-scaling(achromatic) FFAG by G.Rees(Rutherford Lab.)



NFFAG and IFFAG Lattice Cells

- *Cells have the arrangement: O-bd-BF-BD-BF-bd-O.*
- *NFFAG: non-isochronous ;* $\xi_v = 0, \xi_h = 0.$
- *IFFAG: isochronous* ($\gamma_t = \gamma$) ; $\xi_v = 0, \xi_h = +ve.$
- *NFFAGI and IFFAGI have normal & insertion cells.*
- *Different length straights in normal & insertion cells.*
- *There is closed orbit matching between single cells.*

Primary collimators (upstream end of 4.4 m straigh

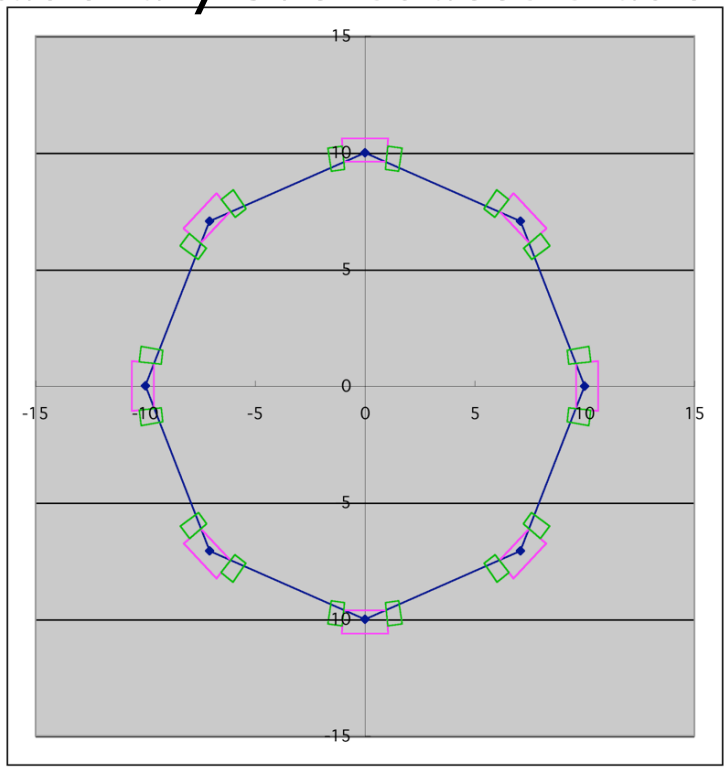


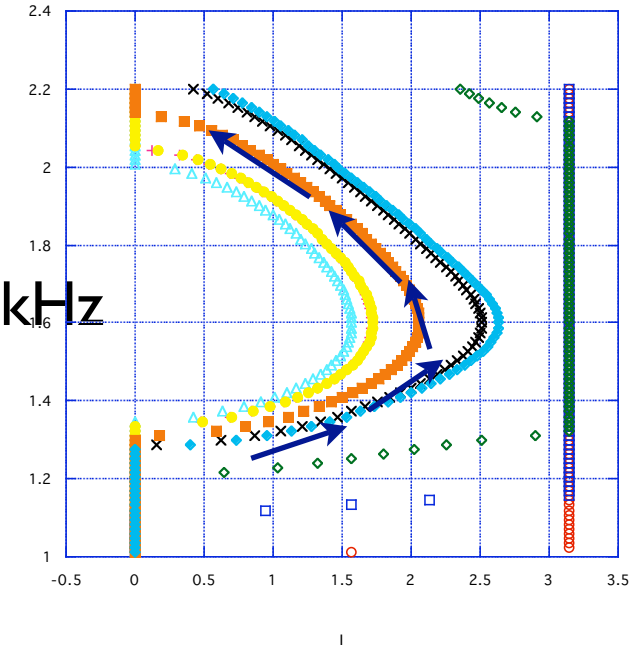
Proton driver for ADS

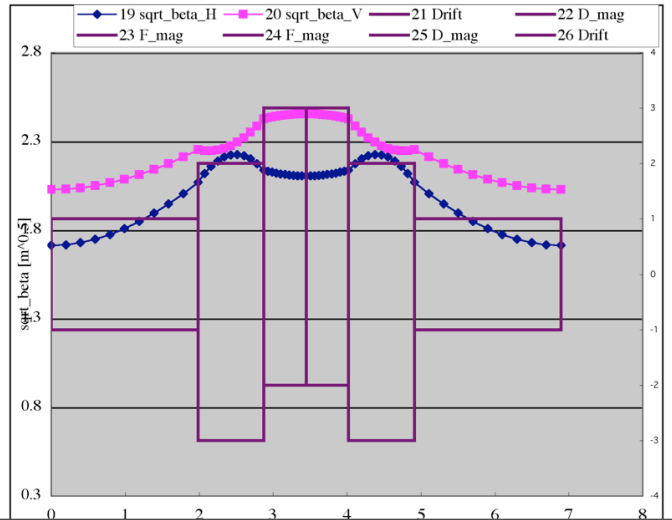
- Design works
  - Non-zero chromatic (linear) FFAG: A.G.Ruggiero (BNL)
  - Zero chromatic (isochronous) FFAG: C. Johnstone(FNAL)
  - Zero chromatic (non-linear) FFAG: Y. Mori(KURRI)
    - 1-2GeV, 10MW (single ring)
- Development for basic ADS study
  - Scaling FFAGs at Kyoto University(KURRI)
  - 150MeV
  - Combined experiment with KUCA(sub-critical reactor)

Zero chromatic(scaling) FFAG for ADSR (1)

- Energy ~1GeV
- k=3.7 (FDF lattice)
- Radius: 10m
- B ~3T : Super ferric (High temperature
- Variable frequency acceleration: f=2.5~5MHz, 1MV, 1kHz
- Stationary bucket acceleration: f=25MHz, 100MV, cw

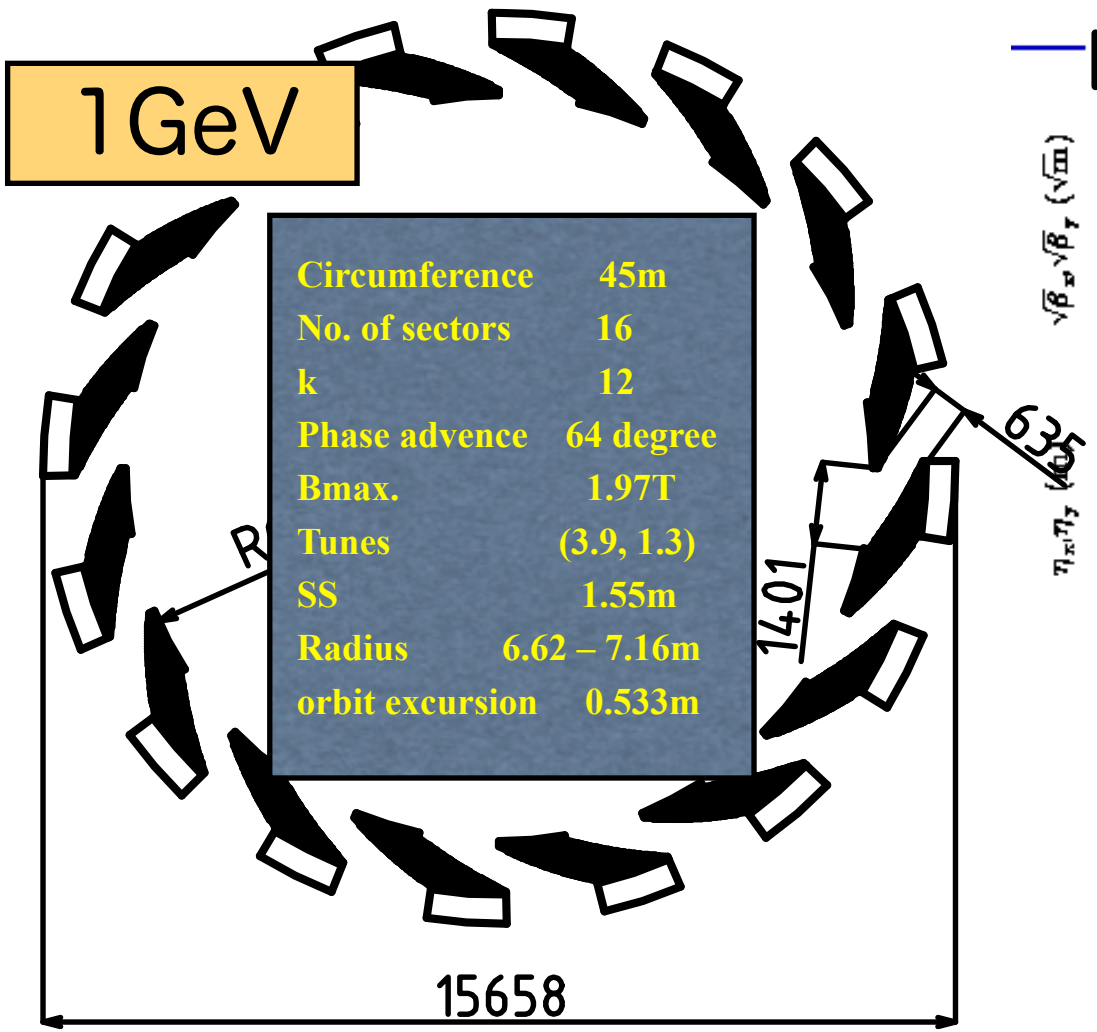


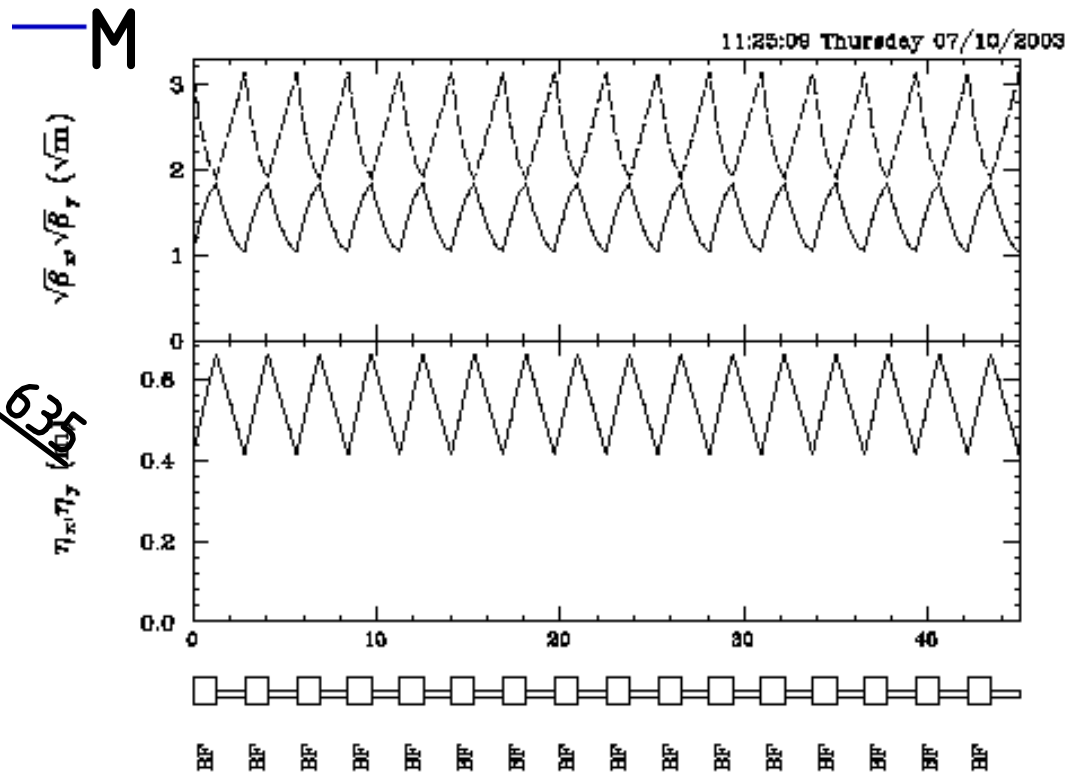




Zero chromatic(scaling) FFAG for ADSR (2)

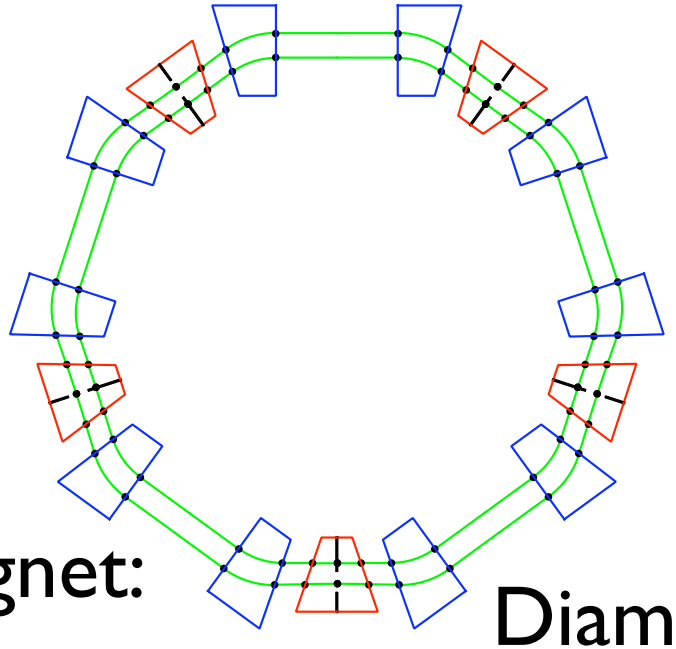
- Spiral lattice





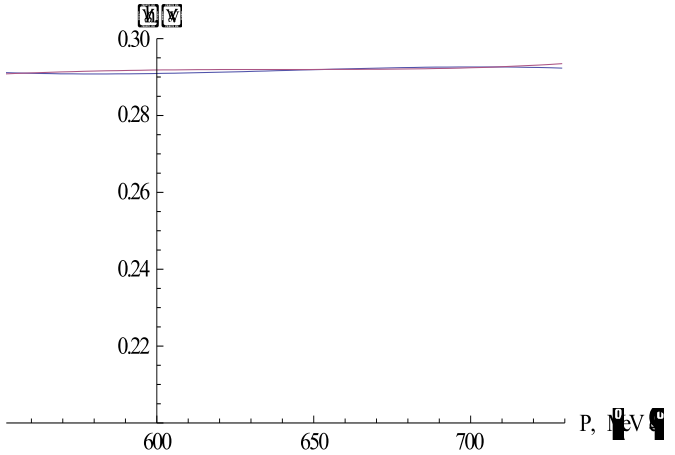
Zero-chromatic(isochronous)FFAG for ADSR

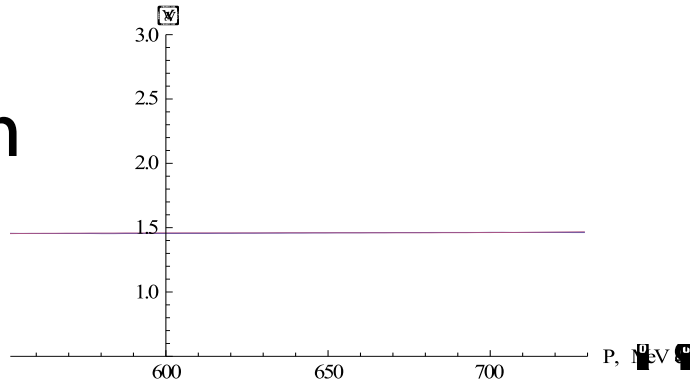
C.Johnstone(FNAL)



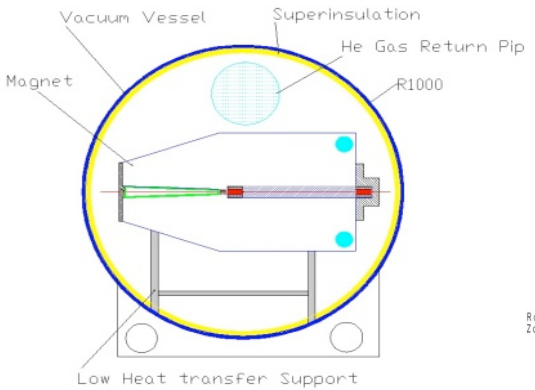
SC magnet:
4T

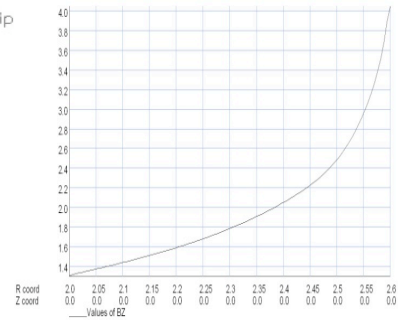
Diameter ~10m





Tune per cell with up to duo-decapole (top) and ring tune (bottom)





Summary

Beam power efficiency

- Beam power efficiency is an issue for high intensity accelerator.

$$BPE = \frac{\text{beam power } \left(\mathrm{E} \times \mathrm{I}_{\text{beam}}\right)}{\text{total operatonal power}}$$

- BPE>30% for P_b>10MW, otherwise
  - Environment problem:CO2
  - ADSR becomes nonsense ; Creating nuclear wastes more than treating!
- Superconducting magnet
  - High temperature SC is very attractive.