Proton Bunch Compression Strategies

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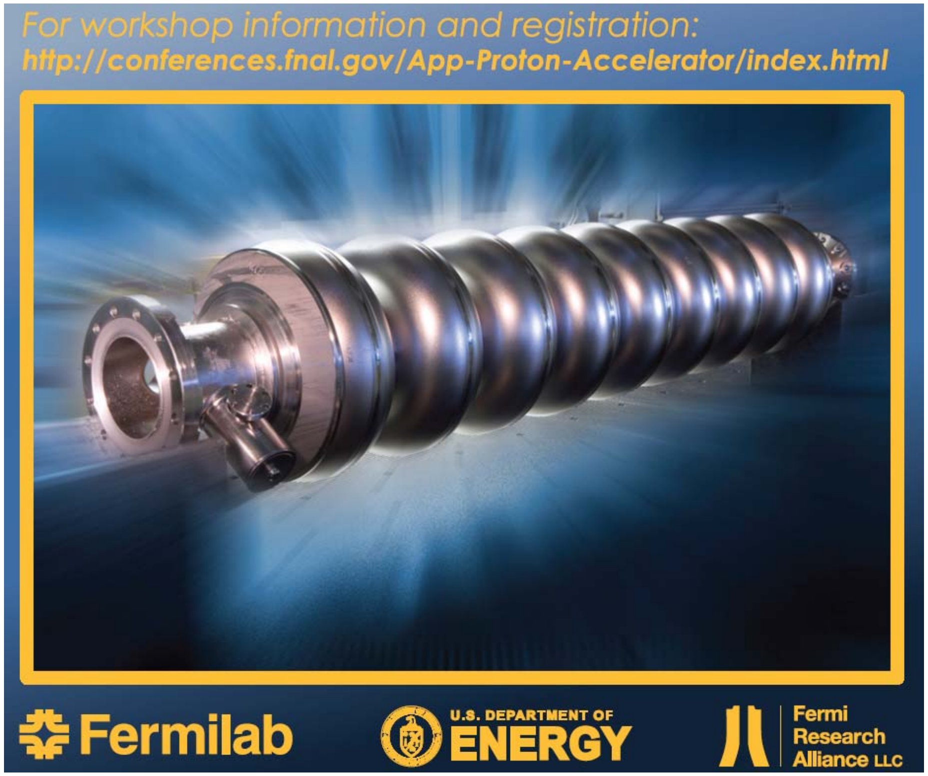

Workshop on Applications of High Intensity Proton Accelerators

Fermilab

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Main Phenomena Limiting Bunch Compression

- Longitudinal instability
- Non-linearity of compressing RF fields

Criterion of Longitudinal Beam Stability

- For continuous beam
  - Equation of motion

$$\frac{\partial \tilde f}{\partial t} + \delta\omega \frac{\partial \tilde f}{\partial \theta} + eE \frac{\partial f_0}{\partial p} = 0 \quad \Rightarrow \xrightarrow{\tilde f \propto \tilde f_n \exp(i(\omega - n\theta))} \quad i\left(\omega + n\eta\omega_0 x\right)\tilde f_n + \frac{eE_n}{p_0}\frac{\partial f_0}{\partial x} = 0$$

$$x = \frac{\Delta p}{p_0}\ , \qquad \delta\omega \equiv \omega - \omega_0 = \left(\frac{d\omega}{dx} x\right) = -\eta\omega_0 x, \qquad \eta = \alpha - \frac{1}{\gamma^2}\ .$$

$$E_n = \frac{Z_n}{2\pi R_0} I_n = \frac{Z_n}{2\pi R_0} I_0 \int \tilde f_n dx\ , \qquad \int f_0 dx = 1\ .$$

  - It results in the dispersion equation

$$\left(\varepsilon_n(y) \equiv 1 + \frac{eI_0}{2\pi i c p_0 \beta\eta\sigma_p{}^2}\left(\frac{Z_n}{n}\right)\int_{\delta\to+0} \frac{d\psi_0/dz}{y + z - i\delta\,\mathrm{sign}(n)} dz\right) = 0, \quad y = \frac{\delta\omega}{n\omega_0\eta\sigma_p}\ , \quad z = \frac{x}{\sigma_p}$$

Where we normalized the distribution function width to 1

$$f_0(x) = \frac{1}{\sigma_p}\psi_0\left(\frac{x}{\sigma_p}\right)\ , \quad \sigma_p{}^2 = \int x^2 f_0(x)dx$$ so that $\int \psi_0(x)dx = 1\ .\ \int x^2\psi_0(x)dx = 1\ .$

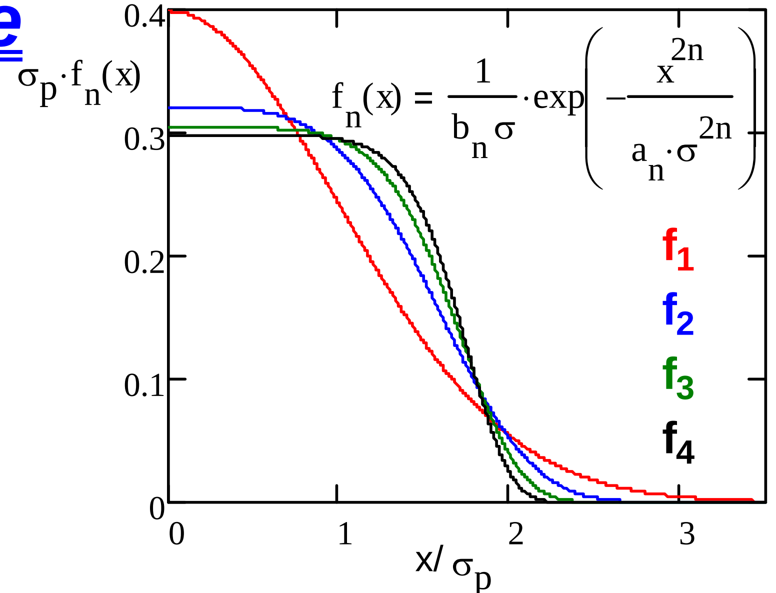
- At the stability boundary $\mathrm{Im}(\delta\omega) = 0$ or $\mathrm{Im}(y) = 0$
- Thus, the stability boundary is characterized by the distribution function shape and one parameter

Stability Criterion and Growth Rate

- Finally, stability boundary is

$$A(y) = \left(i \int_{\delta \to +0} \frac{d\psi_0 / dz}{y + z - i\delta \mathrm{sign}(n)} dz \right)^{-1} , \quad y = \frac{\delta\omega}{n\omega_0 \eta \sigma_p}$$

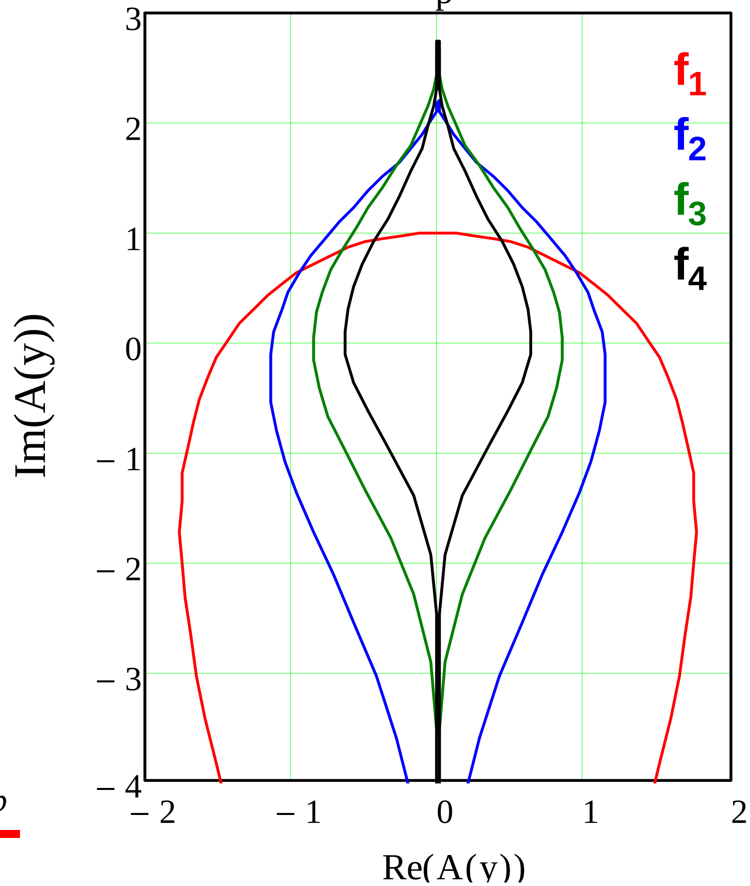
  - where $A \equiv \dfrac{eI_0}{2\pi c p_0 \beta \eta \sigma_p{}^2} \left(\dfrac{Z_n}{n} \right)$



- For "rectangular" distribution there is no significant difference in stability thresholds above and below transition
- In practice
  - $\mathrm{Im}(Z_n) \gg \mathrm{Re}(Z_n)$,
  - Longitudinal injection painting creates truncated tail

$\Rightarrow$Stability condition can be approximated as

$$\frac{eI_0}{2\pi c p_0 \beta \eta \sigma_p{}^2} \left(\frac{Z_n}{n} \right) \le 2$$



- Well above stability threshold: $\lambda_n \approx n\,\eta\omega_0\sigma_p$

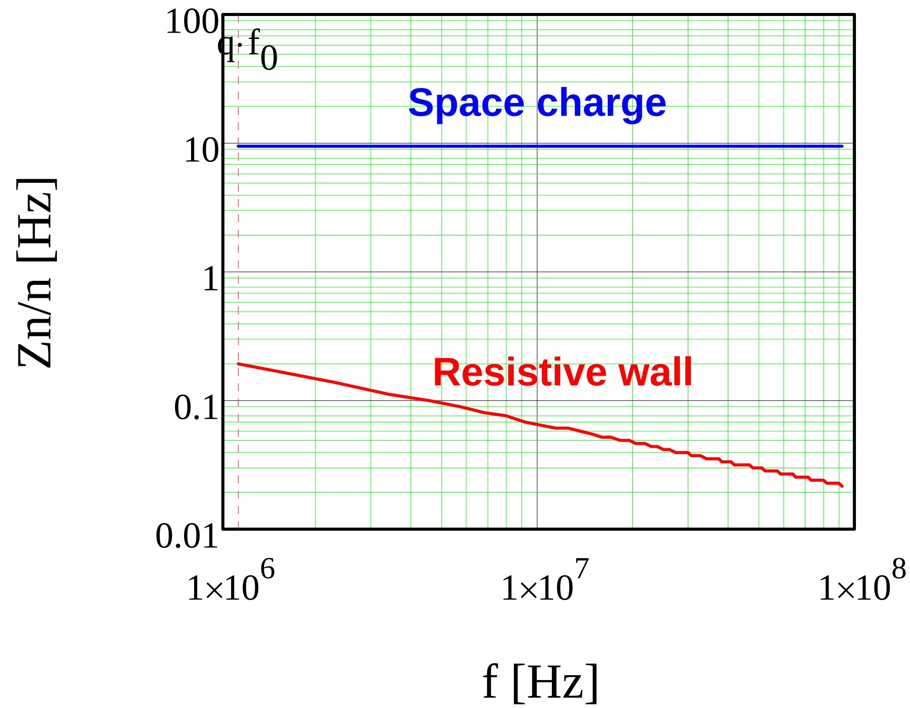
Longitudinal Impedance

- Longitudinal impedance has three major contributions
  - Space charge
    - For round beam & vacuum chamber with radius *a*

$$\frac{Z(\omega_n)}{n} = i\frac{Z_0}{\beta\gamma^2}\ln\left(\frac{a}{1.06\sigma}\right)$$

  - Resistive wall
    - For round beam & vacuum chamber

$$\frac{Z(\omega_n)}{n} = \left(1 - i\,\mathrm{sign}(\omega_n)\right)\frac{Z_0\beta c}{2a\sqrt{2\pi\sigma\omega_n}}$$



Copper chamber, f_0 = 1.13 MHz, a = 4.8 cm, E=8 GeV

  - Effect of RF cavities, vacuum chamber discontinues, etc. can be controlled by machine design and dampers (f < 100 MHz)
- Space charge contribution does not depend on frequency and dominates at all frequencies if an appropriate attention was paid to the vacuum chamber electrodynamics and E ≤ 20 GeV

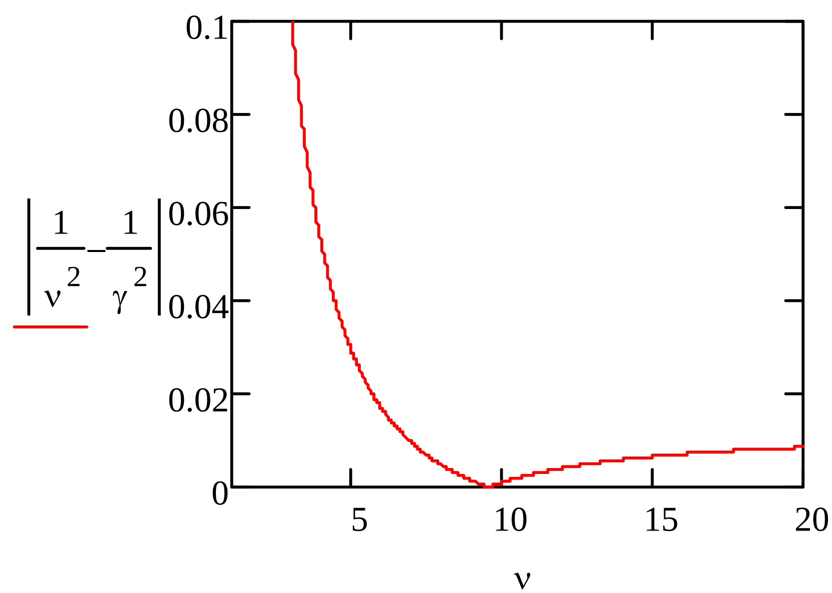
Simple Stability Criterion

- At high frequencies, $\lambda_n \approx n\omega_0\eta(\Delta p/p) >> \omega_s$, the continuous beam theory can be used for bunched beam
- We assume
  - The space charge impedance dominates (γ< 20)
  - Before compression the bunch has uniform density and length L_b
  - Conservation of longitudinal impedance:

    $\sigma_p L_b = const$ - before and after compression
- Then $$\frac{r_p N L_b}{\beta^2\gamma^3\eta\left(\sigma_p L_b\right)_{fin}^2}\ln\left(\frac{a}{1.06\sigma}\right) \le 1$$
- Let's rewrite it for beam power

$$P_{\max} \le m_p c^2(\gamma-1) f_{rep}\frac{\beta^2\gamma^3\eta\left(\sigma_p L_b\right)_{fin}^2}{r_p L_{init}\ln\left(\dfrac{a}{1.06\sigma}\right)}$$

  - Very steep dependence on beam energy
  - For 8 GeV and above an operation well above transition maximizes η



Dependence of sllip-factor on tune in the smooth focosing approximation for 8 GeV beam

Dependence of Maximum Power on Parameters

- Assume
  - Operation well above transition
    - Small tune
    - Large dispersion reduces the space charge tune shift
  - Slip-factor (momentum compaction) increase is limited by the horizontal beam size in dipoles $\eta \approx \alpha \approx \frac{D}{R_0} \approx \frac{\Delta x}{\Delta p\, R_0}$,
  - Momentum spread is limited by machine chromaticity
- For rough estimate it yields:

$$P_{\max} \approx 0.72\,\mathrm{MW}\left(\frac{E}{8\,\mathrm{GeV}}\right)^4\left(\frac{f_{rep}}{15\,\mathrm{Hz}}\right)\left(\frac{L_b \sigma_p}{60\,\mathrm{cm}\times 1\%}\right)^2\left(\frac{10\,\mathrm{m}}{L_{init}}\right)$$

What RCS can do?

- 340 kW with 3 turn injection from RCS to the compressor ring at 8 GeV
  - Compressor ring length = 1/3 of RCS
  - 2 ns single bunch as required for muon collider
- 1 MW will require
  - 3 mA current from CW linac
    - 6 MV installed CW RF
    - Or Transition to pulsed linac
    - Or Combination of CW and pulsed RF
  - 20 Hz repetition rate
    - tripling RCS RF power
    - doubling RCS RF voltage
    - Foil is OK
- Pulsed linac of ICD-I at 15 Hz will result approximately the same power
- Pulsed linac of ICD-2 can achieve 1 MW with laser injection only
  - Thin flat stream of liquid Li requires more insight

Numerical example for “ultimate” 4 MW ring

- Maximum possible slip factor
  - Its value is limited by beam size at high dispersion regions
- 4 T superferic dipoles
- 2T at beam boundary for quadrupoles
- Racetrack with minimum reasonable length of straight lines

E_k=12 GeV, C=208 m

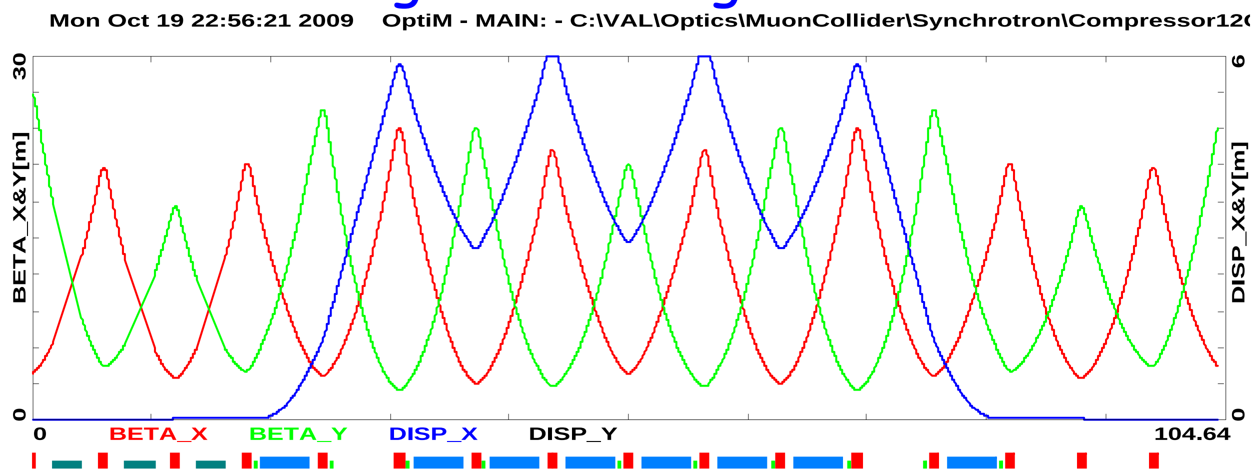
α=0.0957, η=0.090

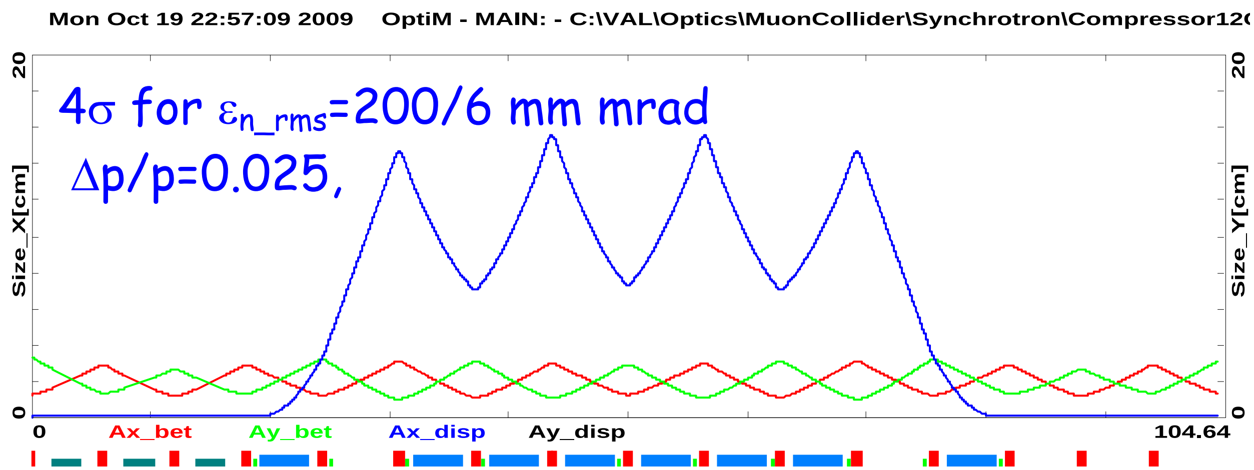
P=4 MW, $N_p=1.4\cdot10^{14}$

ν_x,ν_y=4.42/4.42

$\Delta\nu_x,\Delta\nu_y$=0.17/0.22

($\Delta\nu$=0.366 for D=0)





Major parameters of longitudinal dynamics

Compressed beam

Desired: σ_p=0.01, σ_s=60 cm

RF voltage: V=3 MV (q=1)

Bunch field: $E_{max}C$ = 350 kV

Rotation time: 136 turns ($1/(4\nu_s)$)

Bucket height: $\Delta p/p$=0.041

Peak beam current: 4.4 kA

Number of Cavities = 5

Per cavity: R/Q=25 Ω, Q=350, P_{gen}=20 MV

Injected beam

$\eta_{inj}=L_b/C=0.22$ $\Rightarrow$ Peak beam current=150 A

Injection time: 1800 turns

for 80 mA peak linac current (18 mA average)

Initial momentum distribution assumed to be close to Gaussian one

Stability margin ~ 3 ($\sigma_{pinj}=5.3\cdot10^{-4}$ requires linac beam debunching)

$Z_n/n_{allowed}$=14 Ω (for Gaussian distribution)

$Z_n/n_{SpaceCharge}$=5 Ω

Simulations of injection and bunch rotation

Injected:

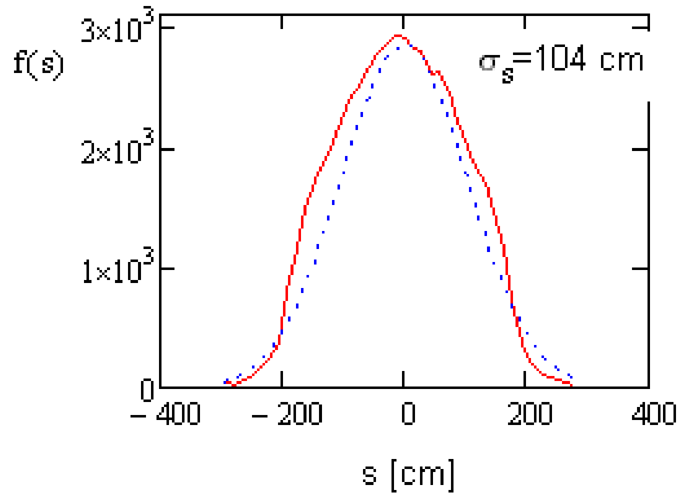
$\sigma_p \approx 5.3 \cdot 10^{-4}$,

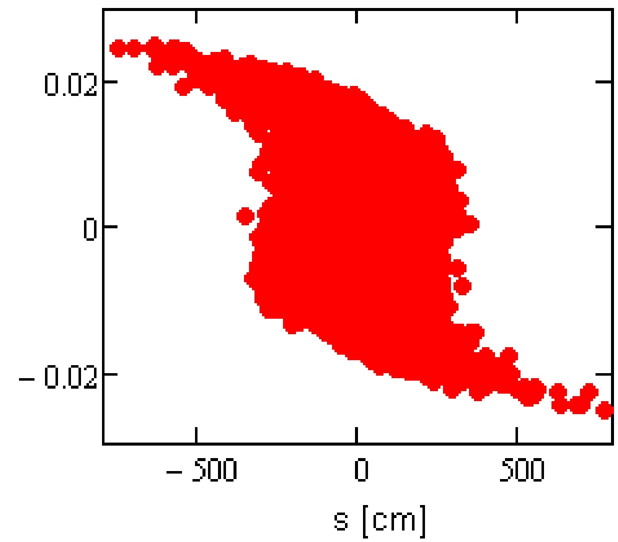
$\eta_{inj} = L_b / C = 0.22$

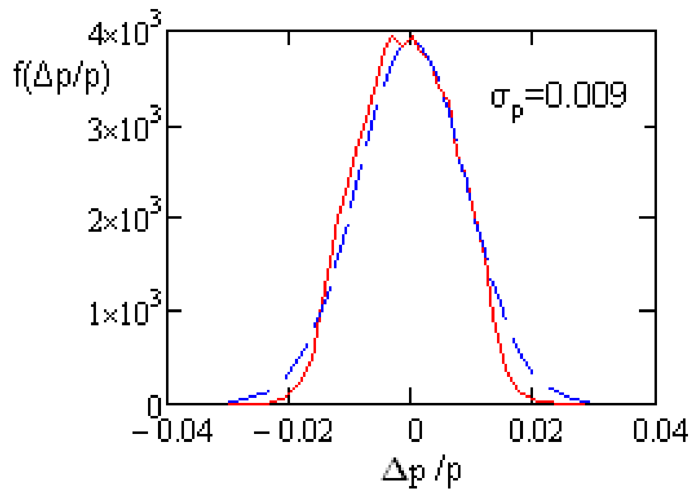
After rotation:

$\sigma_p \approx 0.01$,

$\sigma_s \approx 100$ cm







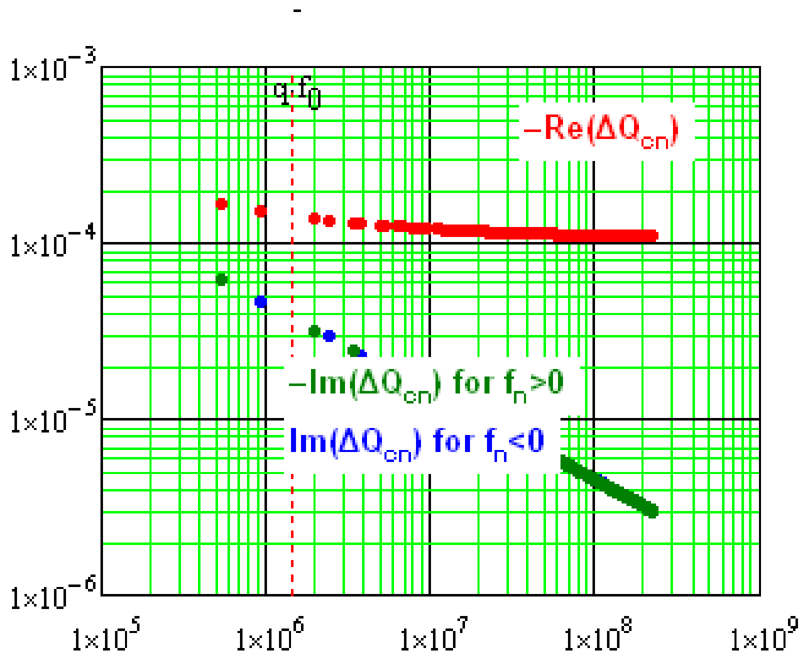
Nonlinearity of RF wave form does not make a major contribution

Further reduction of bunch length will reduce the stability margin to ~1

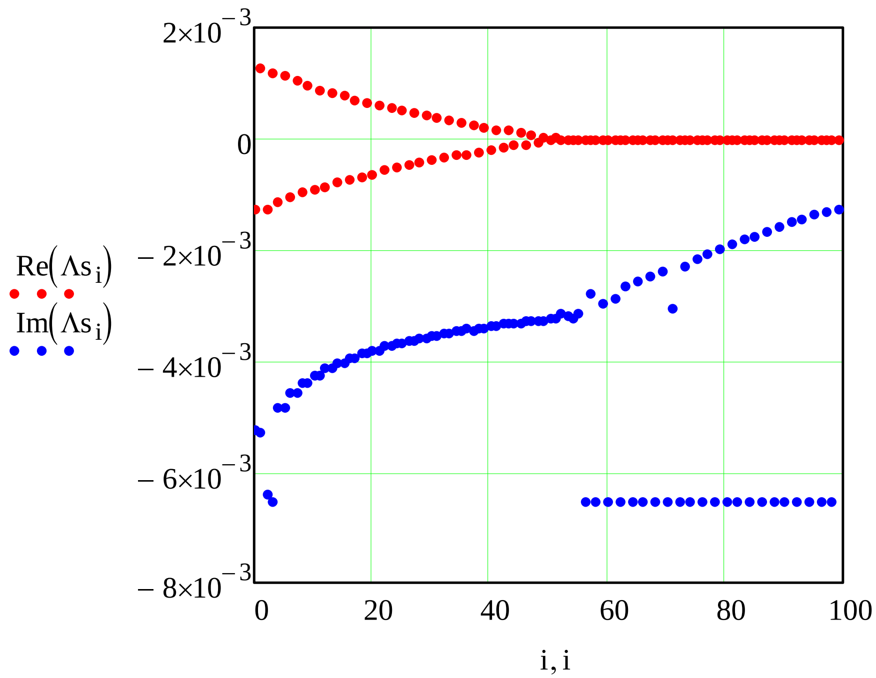
Transverse stability

- Assume
  - resistive wall makes major contribution (Cu, 2h=6 cm)

Eigen-values



For continuous beam



For compressed beam

  - Transverse damper is required during injection and bunch compression

Conclusions

- ICD-2 can deliver ~340 kW at 10 Hz with required bunch length
- It looks feasible to achieve 4 MW at 15 Hz with 12 GeV beam
- It requires
  - 12 GeV linac with large beam current from linac (~30-100 mA) which cannot be supported CW linac of ICD-2
  - Or synchrotron with ~20 GeV energy