Notes on Phase Errors in Linac Simulations

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Revisiting Magnetic Field Limits in Quadrupoles Arising From Losses due to H-Stripping

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PartI: Notes on Phase Error in Simulations

(With thanks to JP Carneiro for discussions)

How Did We Get Here and Why Should we Care ?

- J.P. Carneiro observed and reported some inconsistencies between simulations done with TRACK and ASTRA in presence of phase errors.
- Because losses are a major concern for a high intensity machine, we need to be confident that
  - We understand how errors are modeled
  - We understand how to use the code(s)
  - The code(s) are giving us the right answer

•Unfortunately, the codes we are using at the moment are "black-boxes".
•What follows is my attempt at summarizing our understanding of the inner working of linac codes and how we would expect the calculations to be done.
What the codes are actually doing is a separate issue; some testing is needed.

TRACK Internals (I)*

B. Brief description of the code TRACK

In the code TRACK, the transport of a charged particle is described by the equation of motion,

$$d\vec{p}/dt = q(\vec{E} + \vec{v} \times \vec{B}), \quad (1)$$

where $\vec{p}$ is the particle momentum and q is its charge, $\vec{E} = \vec{E}_{\mathrm{ext}} + \vec{E}_{\mathrm{int}}$ and $\vec{B} = \vec{B}_{\mathrm{ext}} + \vec{B}_{\mathrm{int}}$ are the sums of external and internal electric and magnetic fields, and $\vec{v}$ is the particle velocity. The structure of the TRACK code and its ideology is close to the RAYTRACE code [10] except that the TRACK code has many additional features and capabilities. Unlike RAYTRACE, TRACK integrates the equations of motion of all tracked particles for a short distance and calculates space charge fields. In the TRACK code particle motion is generally described in three coordinate systems: two rectangular Cartesian coordinate systems related to both the entrance and the exit of each ion-optics device and a third Cartesian coordinate system used for the definition of the electromagnetic field distribution of the device. The particle trajectory is calculated in the input coordinate system for all types of devices. Only the transformation from the field-distribution coordinate system to the input coordinate system of the device is needed for the description of the external fields in the equation of motion. The transformation from the input to the output coordinate system is later performed to prepare for tracking through the next device.

The fourth-order Runge-Kutta method is used for the integration of particle trajectory through an ion-optics device. TRACK uses the independent variable z for the tracking of phase-space coordinates of the particles $(x, x' = dx/dz,\ y, y' = dy/dz,\ \beta = v/c, \phi)$, where $v = |\vec{v}|$, ϕ is the particle phase with respect to the rf field at the given section of the accelerator. For static ion-optics devices ϕ represents the time difference between the particle being tracked and the reference particle (RP) which provides the beam pulse transformation through the device. In the different accelerator sections ϕ represents the particle phase with respect to the current frequency of the rf resonators. For direct current (dc) beams extracted from the ion source the ϕ coordinate is uniformly distributed within $\pm\pi$ at the frequency of the downstream rf resonator f_0.

We acknowledge that the time-dependent integration of the equation of motion including space charge fields is the most natural and physically correct method. The development of the TRACK code was mainly motivated by the dynamics of multicomponent ion beams in the linac with negligible space charge effects but noticeable nonlinear components of the external fields in the presence of strippers, field errors, and device misalignments. Therefore the use of the Cartesian z coordinate as an independent variable was justified. Most features of the code were already developed before the implementation of the space charge routine. For the applications being considered in

* P. N. OSTROUMOV,V. N. ASEEV, AND B. MUSTAPHA , PRST-AB 7, 090101 (2004)

TRACK Internals (II) * : Dynamical Equations

* P. N. OSTROUMOV,V. N. ASEEV, AND B. MUSTAPHA , PRST-AB 7, 090101 (2004)

$$\frac{dx}{dz} = x', \qquad \frac{dx'}{dz} = \chi\frac{Q}{A}\frac{h}{\beta\gamma}\left[\frac{h}{\beta c}(E_x - x'E_z) + x'y'B_x - (1 + x'^2)B_y + y'B_z\right], \qquad \frac{dy}{dz} = y',$$
$$\frac{dy'}{dz} = \chi\frac{Q}{A}\frac{h}{\beta\gamma}\left[\frac{h}{\beta c}(E_y - y'E_z) + (1 + y'^2)B_x - x'y'B_y - x'B_z\right], \qquad \frac{d\phi}{dz} = \frac{2\pi f_0 h}{\beta c},$$
$$\frac{d\beta}{dz} = \chi\frac{Q}{A}\frac{h}{\beta\gamma^3 c}(x'E_x + y'E_y + E_z), \tag{2}$$

where $\gamma = 1/\sqrt{1-\beta^2}$, $h = 1/\sqrt{1 + x'^2 + y'^2}$, $\chi = |e|/m_a c$, A is the mass number, m_a is the atomic mass unit, $Q = q/|e|$, and B_x, B_y, B_z, E_x, E_x, E_x are the components of the magnetic and electric fields. The trajectory equations (2) are directly derived from the Lagrangian of a charged particle in any time-dependent electromagnetic field (see, for example, Ref. [11]) and do not include any simplifications.

Here φ(z) is the absolute phase, a monotonically increasing quantity. Equivalent to time of flight, expressed in rf periods.

State variables: x, dx/dz, y, dy/dz, β, φ
independent (integration) variable: z

About Cavity Phasing ...

Expressed as a function of z, the field experienced by a particle in a SW cavity is

$$E_z(z) = A_i(z)\cos(\omega z/(\beta c) + (\phi - \phi_{s0,i}) + \phi_{a,i} + \phi_{a0,i})$$

$\Delta\phi_i = \phi - \phi_{s,i}$: Phase slippage w/r to a reference particle arrival phase

ϕ Phase advance of an arbitrary particle within the tracked distribution

$\phi_{s0,i}$ Arrival phase advance of the reference particle in the reference machine at cavity i.

$\phi_{a0,i}$ Phases that result in maximum energy gain (reference phases). These phases are determined by setting all the other phase terms = 0

$\phi_{a,i}$: Acceleration phases, measured w/r to max acc phases (e.g. in TRACK these are the phases specified in "fi.dat")

Dynamic Phase Errors

We now introduce dynamic phase errors, $\delta\phi_i$ (phase "jitter").

$$E_z(z) = A_i(z)\cos(\omega z/(\beta c) + (\phi - \phi_{s,i}) - \phi_{0,i} + \phi_{a,i} + \delta\phi_i)$$

To correctly describe the physics, all the phase references must remain unchanged i.e.

$\phi_{0,i}, \phi_{a,i}\ \phi_{s,i}$ **remain unchanged**

Note: When tracking a distribution, once dynamic phase errors are introduced, a particle within the distribution with the same initial conditions as those used for the reference particle in the reference machine will not arrive at the reference phase advances in the cavities, since it will experience additional errors. The code must save the static reference phase advances once they are established.

Static vs Dynamic Phase errors

- Static phase errors are additional errors that are introduced before establishing the reference acceleration phases.
- dynamic errors represent "jitter" i.e. changes in the environment seen by different bunches during machine operation
- Static errors represent in a given machine w/r to an ideal "design" machine
- The distinction between static/dynamic errors applies to transverse dynamics as well: e.g. quad misaligments (static) vs quad vibrations.

Choice of Independent Variable

- To account for space charge forces, one must evaluate the space charge distribution at a fixed instant in **time**.
- In many beam dynamics codes (e.g. TRACK), z (or "s") is used as the independent variable. This is done so that the computations can cleanly proceed sequentially through elements. If t is used (e.g. ASTRA), at given instant, one particle might lie within element i while another is within element i+1. Since the elements are indexed according to their longitudinal spatial positions, using t as an independent variable forces the code to check within what specific element a particle is located before it can evaluate the external field it experiences.
- Rigorously transforming from f(x(z), y(z), z) to g(x(t), y(t), z(t)) implies a knowledge of the transformations $t(z; X_i)$. In general, these transformations are not available.

Note: Xi in the above represents the initial conditions for particle i.

Cartoon – s vs t as Independent Variable

Before transformation

After transformation

Ballistic Approximation

- In the special case where all the particles in a distribution are known to remain "close" (time-wise) to each other, one can use a ballistic approximation to determine the spatial distribution at a fixed time t.
- With a reference particle located at z_r, the position of a particle with coordinate Δφ w/r to the latter is:

$$z \simeq z_r + \beta c \frac{\Delta\phi}{h\omega_r f}$$

$$x \simeq x_r + \frac{dx}{dz}\beta c \frac{\Delta\phi}{h\omega_r f} \qquad y \simeq y_r + \frac{dy}{dz}\beta c \frac{\Delta\phi}{h\omega_r f}$$

Second order corrections, can be neglected

Reference Particle for Ballistic Approximation

- For the ballistic approximation to be valid, the expansion must be made w/r to a point that is "close" time(phase)-wise to all the others, preferably a particle near the center of the tracked bunch.
- One way to make this choice would to use the average phase advance (same as time) of all the particles in the distribution. This requires significant additional work
- in the case where there are no dynamic errors, the phase of the nominal reference particle can be used to make this choice.
- In the presence of dynamic errors, one can simply pick an additional dedicated reference particle (distinct from the nominal one) for that purpose.
- There is no need for a transformation and therefore no need for an additional reference particle when t is the integration variable.

Part II: Magnetic Field Limits in Quadrupoles

H- Stripping: Theory and Phenomenology

•H- moving through a magnetic field experiences a force that tends to pull p and e apart. In its rest frame, the ion experiences an E field. The "outer boundary" of the $1/r^2$ potential well is lowered, resulting in a finite tuneling probability. Accordingly, the ion lifetime τ in its rest frame can be parametrized as follows:

$$\tau = \frac{A}{E} \exp \frac{C}{E}$$

$1.87 < E < 2.14$ MV/cm $A = 7.96 \times 10^{-14}$ [s MV/cm] $C = 42.56$ MV/cm

$1.87 < E < 7.02$ MV/cm $A = 2.47 \times 10^{-14}$ [s MV/cm] $C = 44.94$ MV/cm

Ref.: M.A. Furman in "Handbook of Accelerator Physics and Engineering"

Quadrupole Field Limit: How Conservative do we need to be ?

From:

P. Ostroumov, "Physics design of the 8 GeV H-minus linac", New J. Of Phys., 8 (2006), p. 281

"Tolerable magnetic field on the **pole tip** of quadrupoles"

Section	CH	SSR-1	SSR-2	TSR	S-ILC	ILC-1	ILC-2
Focusing	SR	SR	SRR	FRDR	FR²DR²ᵃ	FR⁴DR³	FR⁸DR⁸
Length of the focusing period L_P(m)	0.515–0.75	0.75	1.60	3.81	6.1	12.2	24.4
Aperture radius (cm)	1.0	1.5	1.5	2.0	3.05	3.5	3.5
B^b (T)	⩽5.8	⩽5.6	⩽6.2	⩽0.24	⩽0.15	⩽0.1	⩽0.081
Effective length $L_E{}^c$ (cm)	11.238	8.192	18.559	20.0	40.0	50.0	50.0

[a] $R^2 = RR$.
[b] Magnetic field of quadrupoles is given on the pole tip for which the radial position is equal to the aperture radius. Magnetic field of solenoids is given at the centre of solenoids.
[c] The effective length is calculated at the 6 T field level in the centre of solenoid.

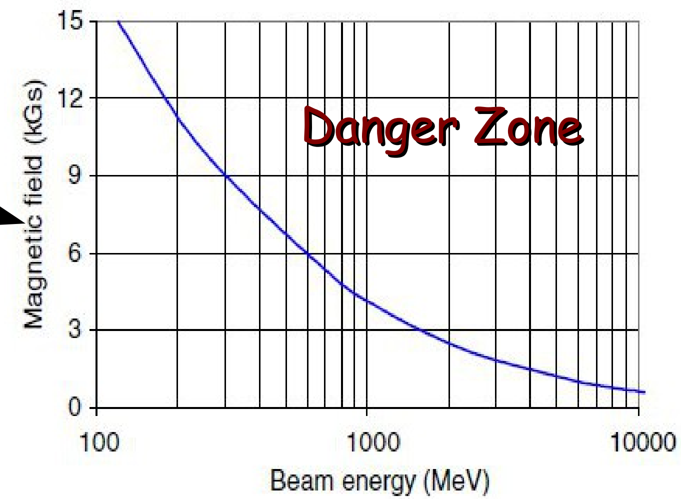

Figure 2. Tolerable magnetic field as a function of beam energy.

- Tunneling parametric model, with parameters as specified previously
- Tolerable beam losses assumed to be 0.1W/m
- Quadrupoles assumed to occupy 10% of the focusing period length.
- **Beam assumed uniformly distributed and occupying 70% of the aperture**

Comments

- Ostroumov's assumptions for PD design are very conservative.
- Beam occupies much less than 70% of aperture; as a consequence, |B| experienced by most particles gets overestimated.
- A uniform distribution is pessimistic. Most particles are likely to be near the axis, further reducing the |B| experienced by most of them.
- Average beam current is reduced for CW linac scenario, so higher fractional losses should be allowable.

Probability of Particle Loss

The electric field in the ion rest frame is related to the magnetic field in the lab frame as follows:

$$E_{\mathrm{rest}} = \kappa\beta\gamma B_{\mathrm{lab}}$$

Where κ = 0.3 GV m/T

The mean decay length in the lab frame is:

$$\lambda_d = c\beta\gamma\tau$$

where, again,

$$\tau = \frac{A}{E}\exp\frac{C}{E}$$

The lost fraction after a distance z is

$$f(z) = 1 - \exp(-z/\lambda_d) \simeq z/\lambda_d, z << \lambda_d$$

Loss Estimate

$$f = 2\pi \int_{r=0}^{r=\infty} \sigma(r)(1 - \exp \frac{-L_q}{L_d(r,\gamma)})\, r\, dr dz$$

$f(\gamma)$ Lost fraction (depends on beam energy, magnetic field)

$\sigma(r)$ Normalized (projected) bunch particle radial surface density

L_q Quadrupole Length

The power loss over a quadrupole of length Lq is estmated as follows:

$$P = f \cdot N_b \cdot f_b \cdot (\gamma - 1) m_0 c^2$$

N_b No of particles/bunch

f_b Bunch frequency

Radial Dependence of |B| in a Quad

The stripping probability depends only on the magnitude of the electric field in the ion's rest frame. Therefore, we care only about the magnitude of the B magnetic field in the lab frame.

$$B_x = -gy \qquad B_y = gx \qquad B_x^2 + B_y^2 = g^2(x^2 + y^2)$$

$$|B| = |g|r$$

Status

- A small program has been written to estimate losses based on different assumptions about aperture,

  gradient quad lengths, etc.
- Still needs a bit of time before I am ready to present results …