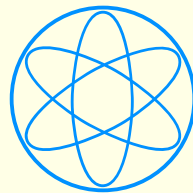


# The $\eta_c$ line shape

Antonio Vairo

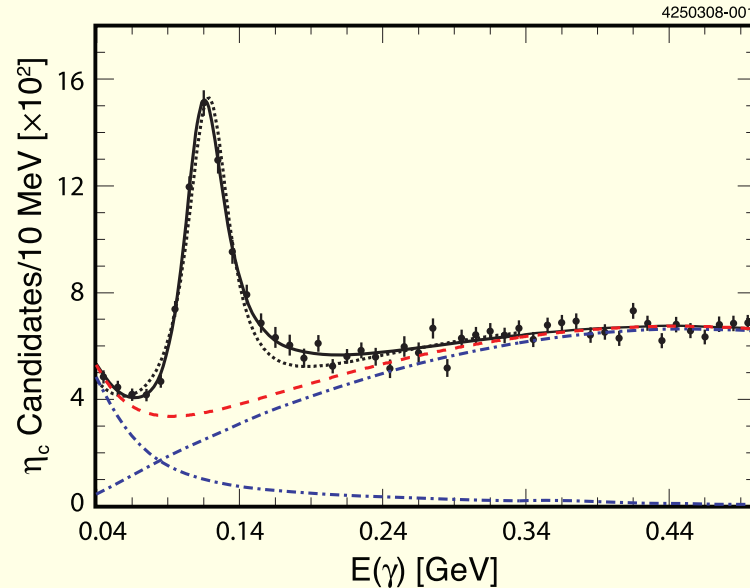
Technische Universität München



work in progress with

Nora Brambilla and Pablo Roig Garces

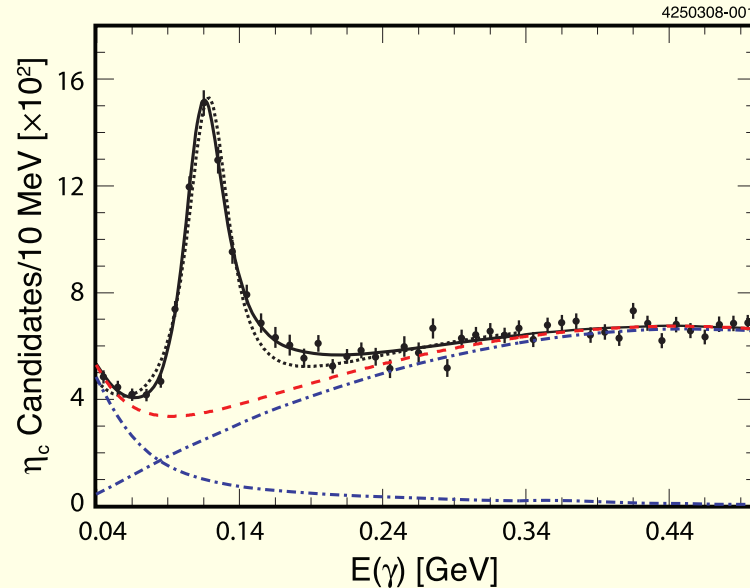
## $\eta_c$ line shape in the CLEO analysis



○ CLEO PRL 102(09)011801

- Input: two background sources
  - a MC modeled background for spurious  $J/\psi \rightarrow X$ :  
$$\text{bkg}^{(1)}(E_\gamma) = N[\exp(-5.720 E_\gamma) + 10.441 \exp(-33.567 E_\gamma)]$$
  - a freely fit background for  $J/\psi \rightarrow \pi^0 X$  and non-signal  $J/\psi \rightarrow \gamma X$ :  
$$\text{bkg}^{(2)}(E_\gamma) = A + B E_\gamma + C E_\gamma^2$$

## $\eta_c$ line shape in the CLEO analysis



○ CLEO PRL 102(09)011801

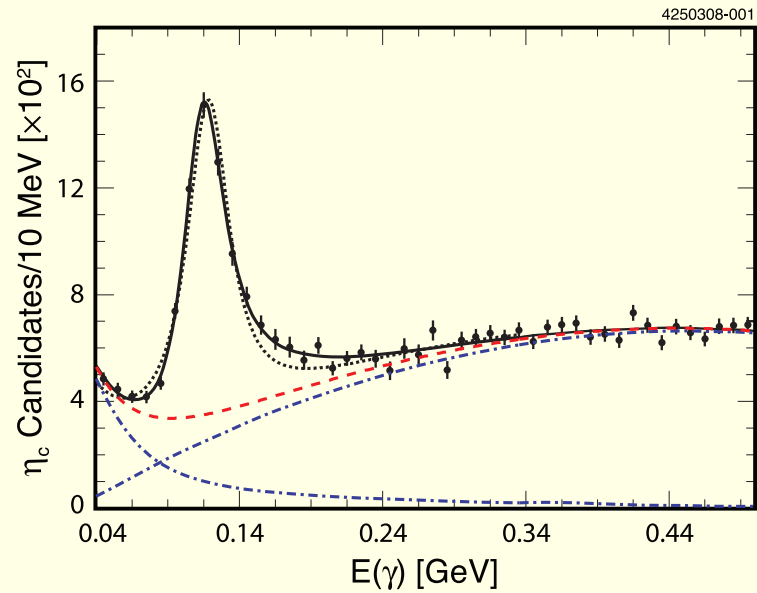
- Input: a theoretical line shape given by

$$E_\gamma^3 \times \text{BW}^{\text{rel}}(E_\gamma) \times \text{damping}(E_\gamma)$$

where

- $\text{BW}^{\text{rel}}(E_\gamma) = \frac{1}{(M_{J/\psi}^2 - 2M_{J/\psi}E_\gamma - M_{\eta_c}^2)^2 + (M_{J/\psi}^2 - 2M_{J/\psi}E_\gamma)^2\Gamma_{\eta_c}}$
- $\text{damping}(E_\gamma) = \exp[-E_\gamma^2/(8 \times (65.0 \pm 2.5 \text{ MeV})^2)]$

## $\eta_c$ line shape in the CLEO analysis

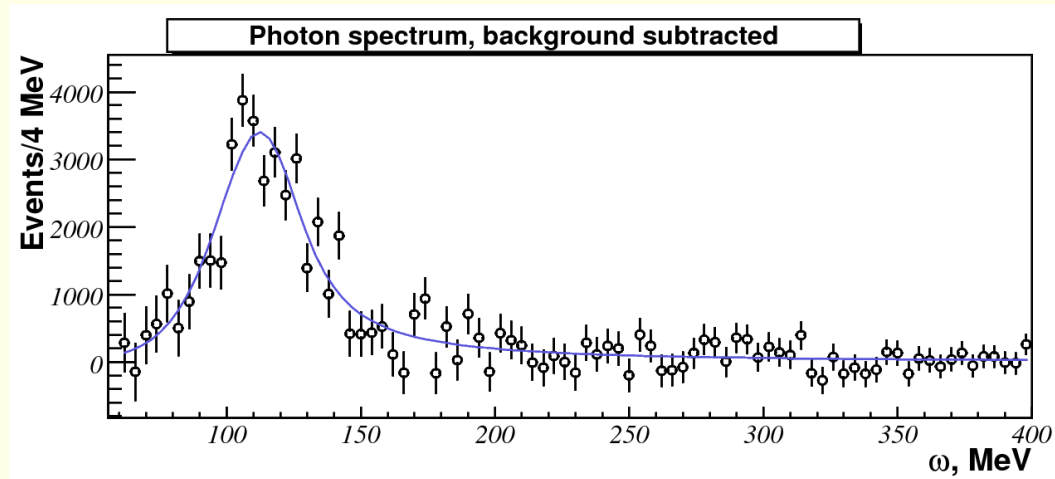


○ CLEO PRL 102(09)011801

● Output:

$$M_{\eta_c} = 2982.2 \pm 0.6 \text{ MeV}$$

## $\eta_c$ line shape in the KEDR analysis



○ KEDR arXiv:1002.2071

- Input: as before, but also

$$\text{damping}'(E_\gamma) = \frac{E_{\text{peak}}^2}{E_\gamma E_{\text{peak}} + (E_\gamma - E_{\text{peak}})^2}$$

- Output:

$$M_{\eta_c} = 2979.7 \pm 1.6 \text{ MeV}$$

$$\Gamma_{\eta_c} = 26.9 \pm 4.8 \text{ MeV}$$

$$M'_{\eta_c} = 2979.4 \pm 1.5 \text{ MeV}$$

$$\Gamma'_{\eta_c} = 27.8 \pm 5.1 \text{ MeV}$$

## An EFT approach: scales

Scales:

- $\langle p \rangle \sim 1/\langle r \rangle \sim m_c v \sim 700 \text{ MeV} - 1 \text{ GeV} \gg \Lambda_{\text{QCD}}$
- $E_{J/\psi} \equiv M_{J/\psi} - 2m_c \sim m_c v^2 \sim 400 \text{ MeV} - 600 \text{ MeV} \ll 1/\langle r \rangle$
- $M_{J/\psi} - M_{\eta_c} \sim m_c v^4 \sim 120 \text{ MeV} \ll E_{J/\psi}$
- $0 \text{ MeV} \leq E_\gamma \lesssim 400 \text{ MeV} - 500 \text{ MeV} \ll 1/\langle r \rangle$
- $\Gamma_{\eta_c} \sim 30 \text{ MeV} \ll m_c v^4$

It follows that the system is

- (i) non-relativistic,
- (ii) weakly-coupled at the scale  $1/\langle r \rangle$ :  $v \sim \alpha_s$ ,
- (iii) that we may multipole expand in the external photon energy.

# An EFT approach: degrees of freedom

Degrees of freedom at scales lower than  $m v$ :

- $c\bar{c}$  states, with energy  $\sim \Lambda_{\text{QCD}}, m_c v^2$  and momentum  $\lesssim m_c v$   
 $\Rightarrow$  (i) singlet S    (ii) octet O
- Gluons with energy and momentum  $\sim \Lambda_{\text{QCD}}, m_c v^2$
- Photons of energy and momentum lower than  $m_c v$

## An EFT approach: the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{pNRQCD}} + \mathcal{L}_\gamma$$

- $$\mathcal{L}_{\text{pNRQCD}} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} - \frac{1}{4} F_{\mu\nu}^{\text{em}} F^{\mu\nu \text{em}}$$

$$+ \text{Tr} \left\{ S^\dagger (i\partial_0 - h_s) S + O^\dagger (iD_0 - h_o) O \right\} + \dots$$

$$h_s = \text{Re } h_s + i\text{Im } h_s, \quad \text{Re } h_s = \mathbf{p}^2/m_c + \text{Re } V_s, \quad \langle H | \text{Im } h_s | H \rangle = -\Gamma_H/2$$

- Pineda Soto NP PS 64(98)428, Brambilla et al NPB 566(00)275  
Brambilla et al PRD 67(03)034018

- $$\mathcal{L}_\gamma = \text{Tr} \left\{ \overbrace{V_A^{\text{em}} S^\dagger \mathbf{r} \cdot ee_Q \mathbf{E}^{\text{em}} S}^{\text{E1}} + \overbrace{\frac{1}{2m_c} V_1 \left\{ S^\dagger, \boldsymbol{\sigma} \cdot ee_Q \mathbf{B}^{\text{em}} \right\} S}_{\text{M1}} + \dots \right\}$$

$$V_A^{\text{em}} = 1 + \dots, \quad V_1 = 1 + \dots$$

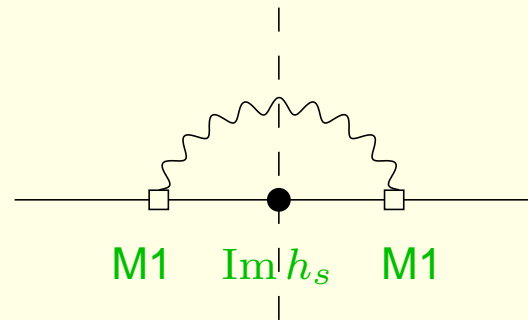
- Brambilla Jia Vairo PRD 73(06)054005



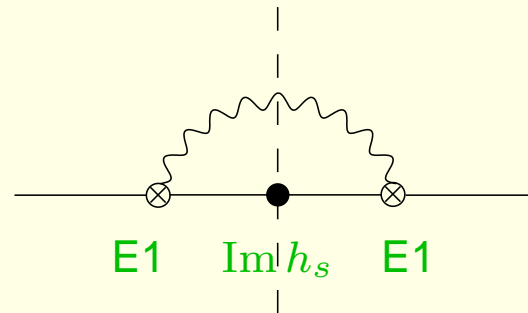
$$J/\psi \rightarrow X \gamma \text{ for } 0 \text{ MeV} \leq E_\gamma \lesssim 500 \text{ MeV}$$

Three main processes contribute to  $J/\psi \rightarrow X \gamma$  for  $0 \text{ MeV} \leq E_\gamma \lesssim 500 \text{ MeV}$ :

- $J/\psi \rightarrow \eta_c \gamma \rightarrow X \gamma$



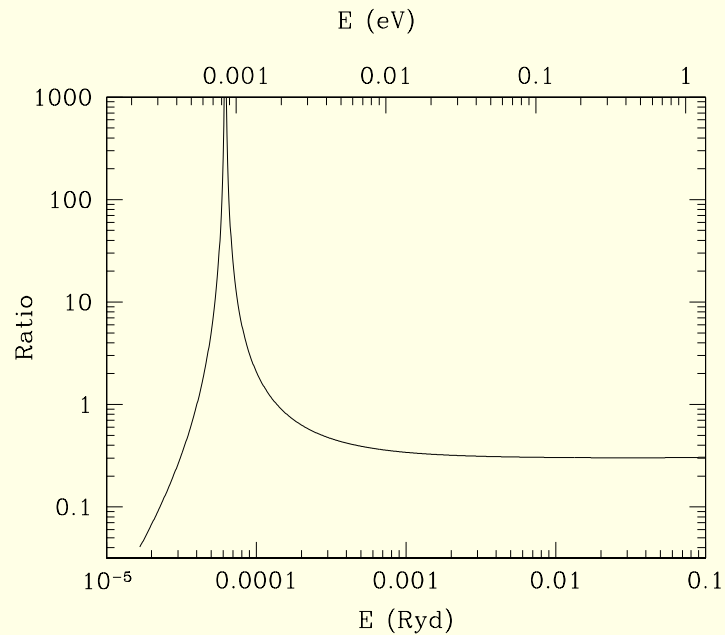
- $J/\psi \rightarrow \chi_{c0,2}(1P) \gamma \rightarrow X \gamma$



- fragmentation and other background processes, included in the bkg functions.

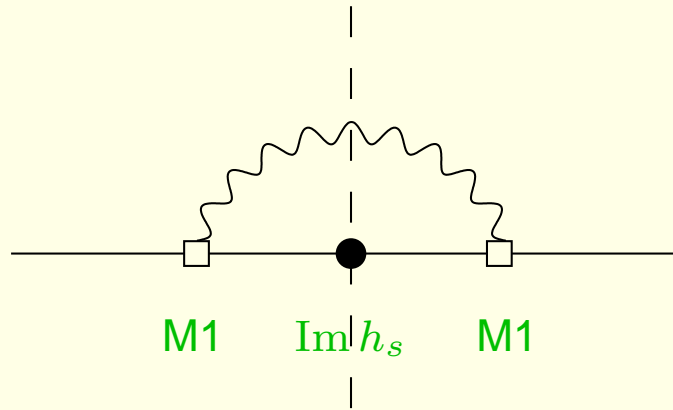
# The orthopositronium decay spectrum

The problem is analogous to the photon spectrum in orthopositronium  $\rightarrow 3\gamma$  that also exhibits a characteristically asymmetric spectrum.



- Manohar Ruiz-Femenia PRD 69(04)053003  
Ruiz-Femenia NPB 788(08)21, arXiv:0904.4875

$$J/\psi \rightarrow \eta_c \gamma \rightarrow X \gamma$$

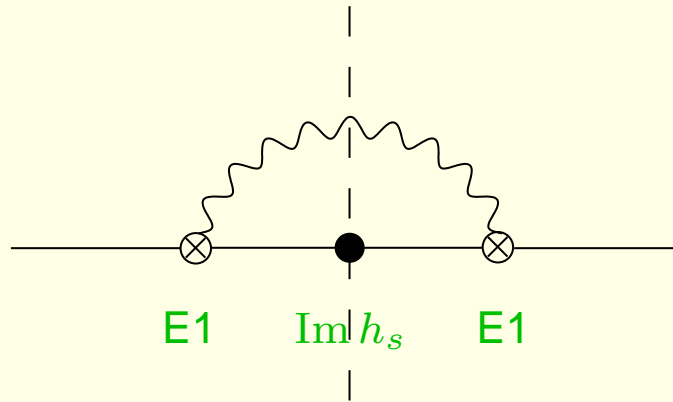


$$\frac{d\Gamma}{dE_\gamma} = \frac{64}{27} \frac{\alpha}{M_{J/\psi}^2} \frac{E_\gamma}{\pi} \frac{\Gamma_{\eta_c}}{2} \frac{E_\gamma^2}{(M_{J/\psi} - M_{\eta_c} - E_\gamma)^2 + \Gamma_{\eta_c}^2/4}$$

- For  $\Gamma_{\eta_c} \rightarrow 0$  one recovers  $\Gamma(J/\psi \rightarrow \eta_c \gamma) = \frac{64}{27} \alpha \frac{E_\gamma^3}{M_{J/\psi}^2}$
- The non-relativistic Breit–Wigner distribution goes like:

$$\frac{E_\gamma^2}{(M_{J/\psi} - M_{\eta_c} - E_\gamma)^2 + \Gamma_{\eta_c}^2/4} = \begin{cases} 1 & \text{for } E_\gamma \gg m_c \alpha_s^4 \sim M_{J/\psi} - M_{\eta_c} \\ \frac{E_\gamma^2}{(M_{J/\psi} - M_{\eta_c})^2} & \text{for } E_\gamma \ll m_c \alpha_s^4 \sim M_{J/\psi} - M_{\eta_c} \end{cases}$$

$$J/\psi \rightarrow \chi_{c0,2}(1P) \gamma \rightarrow X \gamma$$



$$\frac{d\Gamma}{dE_\gamma} = \frac{32}{81} \frac{\alpha}{M_{J/\psi}^2} \frac{E_\gamma}{\pi} \left[ \frac{21 \alpha_s^2}{2 \pi \alpha^2} \right] |a(E_\gamma)|^2$$

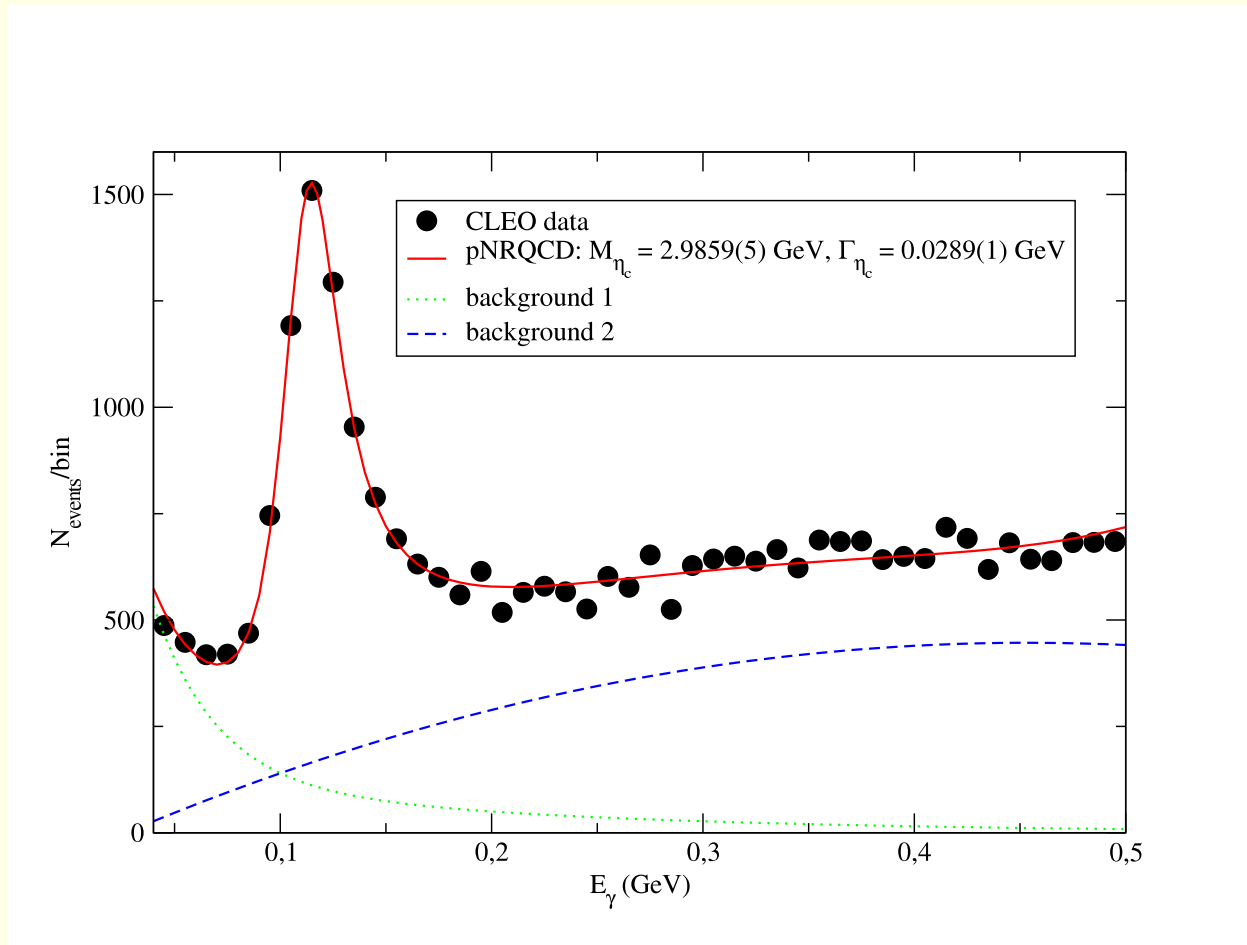
- $$a(E_\gamma) = \frac{(1-\nu)(3+5\nu)}{3(1+\nu)^2} + \frac{8\nu^2(1-\nu)}{3(2-\nu)(1+\nu)^3} {}_2F_1(2-\nu, 1; 3-\nu; -(1-\nu)/(1+\nu))$$

$$\nu = \sqrt{-E_{J/\psi}/(E_\gamma - E_{J/\psi})}$$

○ Voloshin MPLA 19(04)181

- $$|a(E_\gamma)|^2 = \begin{cases} 1 & \text{for } E_\gamma \gg m_c \alpha_s^2 \sim E_{J/\psi} \\ E_\gamma^2 / (2E_{J/\psi})^2 & \text{for } E_\gamma \ll m_c \alpha_s^2 \sim E_{J/\psi} \end{cases}$$

- The two contributions are of equal order for  
 $m_c \alpha_s \gg E_\gamma \gg m_c \alpha_s^2 \sim -E_{J/\psi}$ ;
- the magnetic contribution dominates for  
 $-E_{J/\psi} \sim m_c \alpha_s^2 \gg E_\gamma \gg m_c \alpha_s^4 \sim M_{J/\psi} - M_{\eta_c}$ ;
- it also dominates by a factor  $E_{J/\psi}^2 / (M_{J/\psi} - M_{\eta_c})^2 \sim 1/\alpha_s^4$  for  
 $E_\gamma \ll m_c \alpha_s^4 \sim M_{J/\psi} - M_{\eta_c}$ .



$$M_{\eta_c} = 2985.9 \pm 0.5 \text{ MeV}$$

$$\Gamma_{\eta_c} = 28.9 \pm 0.1 \text{ MeV}$$

- Besides  $M_{\eta_c}$  and  $\Gamma_{\eta_c}$  the fitting parameters are the overall normalization  $N$ , the signal normalization, and the background parameters  $a$ ,  $b$  and  $c$ .

## Differences with the CLEO fit

- No damping functions, which do not seem to have a theoretical justification. They account for about 50% difference in the  $\eta_c$  mass.
- Damping functions require a larger background above the  $\eta_c$  peak. The absence of the damping functions accounts for a background about 30% smaller than in the CLEO analysis.
- The use of the non-relativistic Breit–Wigner  $\times E_\gamma^3$  distribution instead of the relativistic Breit–Wigner  $\times E_\gamma^3$  distribution used in the CLEO analysis accounts for about 50% difference in the  $\eta_c$  mass.

## Outlook

- We plan to include the first relativistic correction of order  $E_\gamma/m_c$  ( $\sim \alpha_s^4$  around the  $\eta_c$  peak). Note that relativistic corrections of order  $\langle p \rangle^2/m_c^2$  are reabsorbed in the overall normalization while corrections coming from the multipole expansion are of order  $E_\gamma^2 \langle r \rangle^2$  ( $\sim \alpha_s^6$  around the  $\eta_c$  peak) and therefore suppressed in the peak region.