Exclusive decays of χ_{bJ} and η_b into two charmed mesons

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in collaboration with R. Azevedo and B. Long; Phys. Rev. D **80** 074026 (2009)

Outline

Motivation

Production of charmed hadrons in bottomonium decays

- multi-scale problem
  $m_b \gg m_b w \sim m_c \gg m_b w^2 \sim \Lambda_{QCD}$

fun playground for EFT!

Theoretical and experimental efforts on **inclusive** $c\bar{c}$ production

- sizeable production of $c\bar{c}$ in χ_{bJ} and Υ decays

State	$\mathcal{B}(\chi_{bJ}(nP) \to D^0 X)$
$\chi_{b0}(1P)$	$5.6 \pm 3.6 \pm 0.5\%$
$\chi_{b1}(1P)$	$12.6 \pm 1.9 \pm 1.1\%$
$\chi_{b2}(1P)$	$5.4 \pm 1.9 \pm 0.5\%$
	$\mathcal{B}(\Upsilon(1S) \to D^{*\pm} X)$
$\Upsilon(1S)$	$2.52 \pm 0.13 \pm 0.15\%$

CLEO, Phys. Rev. D **78**:092007 (2008);
BaBar, Phys. Rev. D **81**:011102 (2010).

- good agreement with the NRQCD prediction

$$\chi_{bJ}(nP) \to c\bar{c} + X$$

G. T. Bodwin *et al.*, Phys. Rev. D **76**:054001 (2007);

$$\Upsilon(nS) \to c\bar{c} + X$$

D. Kang *et al.*, Phys. Rev. D **76**:114018 (2007);

Motivation

For **exclusive** processes $\chi_{bJ} \to DD$, $\eta_b \to DD^*$

- highly energetic & massive final states
- need for collinear degrees of freedom

Use non relativistic (NRQCD, pNRQCD) and collinear (SCET, bHQET) effective theories

1. provides factorization formula at leading order
   - extract information on bottomonium and D structure
2. allows to resum logs of the ratio of scales involved in process
3. can be extended beyond LO, to account for perturbative and power corrections
4. gives the tools to (at least) investigate the factorization of power corrections

Recent applications

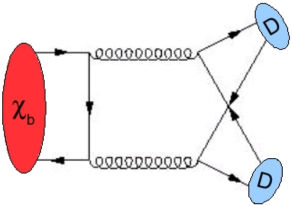
- $\Upsilon \to J/\psi + X$ at $x \to 1$

  X. Liu, Phys. Lett. B **685**:151 (2010).
- double charmonium production in bottomonium decays

  V. V. Braguta *et al.*, Phys. Rev. D **80**:094008 (2009); Phys. Rev. D **81**:014012 (2010).

see V. Braguta talk in Production

Scales and Effective Theories



bottomonium

- decay $2m_b$,
- structure $m_b w$, $m_b w^2$, Λ_{QCD}

D **meson**

- mass m_c

$$p_c^\mu = m_c v^\mu + k^\mu$$
$$p_l^\mu \sim k^\mu$$

- *D* meson rest frame
  $v^\mu = (1, \mathbf{0})$

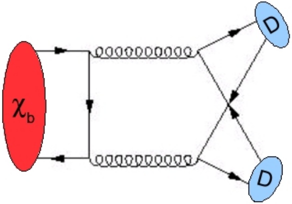
  $k^\mu = \Lambda_{QCD}$

- bottomonium rest frame
  $v^\mu = 2m_b/m_c(\lambda^2, \lambda^0, \lambda)$

  $k^\mu = 2m_b\Lambda_{QCD}/m_c(\lambda^2, \lambda^0, \lambda)$

$$\lambda = m_c/2m_b$$

Scales and Effective Theories



bottomonium

- decay $2m_b$,
- structure $m_b w$, $m_b w^2$, Λ_{QCD}

boosted D meson

- mass m_c

$$p_c^\mu = m_c v^\mu + k^\mu$$
$$p_l^\mu \sim k^\mu$$

- D meson rest frame
  $v^\mu = (1, \mathbf{0})$

  $k^\mu = \Lambda_{QCD}$

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  $v^\mu = 2m_b/m_c(\lambda^2, \lambda^0, \lambda)$

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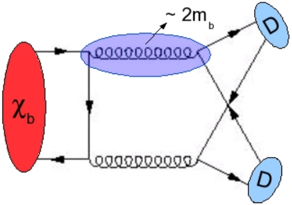
$$\lambda = m_c/2m_b$$

Scales and Effective Field Theories



- integrate out $2m_b$
- $m_b w$, m_c still dynamical

EFT I: NRQCD & SCET

NRQCD

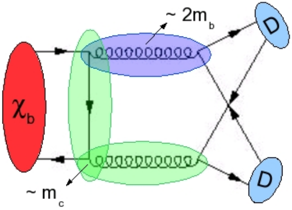
	$(E,\ \mathbf{p})$
• non relativistic b and $\bar{b}$	$(m_b w^2, m_b w)$
• potential gluons	$(m_b w^2, m_b w)$
• soft gluons	$(m_b w, m_b w)$
• ultrasoft gluons	$(m_b w^2, m_b w^2)$

SCET

	$(p^+, p^-, p_\perp)$
• collinear c and $\bar{c}$	$2m_b(\lambda^2, 1, \lambda)$
• collinear gluons	$2m_b(\lambda^2, 1, \lambda)$
• soft gluons	$2m_b(\lambda, \lambda, \lambda)$
• ultrasoft gluons	$2m_b(\lambda^2, \lambda^2, \lambda^2)$

$$m_b w \sim 2m_b\lambda \sim m_c \gg \Lambda_{QCD}$$

Scales and Effective Field Theories



- integrate out $2m_b$
- $m_b w$, m_c still dynamical

EFT I: NRQCD & SCET

- integrate out m_c and $m_b w$ (the soft modes)

EFT II: pNRQCD & bHQET

pNRQCD

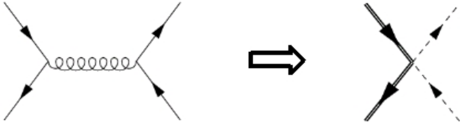
	$(E,\ \mathbf{p})$
• non relativistic b and $\bar{b}$	$(m_b w^2, m_b w)$
• ultrasoft gluons	$(m_b w^2, m_b w^2)$
• non local potentials	

$$m_b w \sim 2m_b\lambda \sim m_c \gg \Lambda_{QCD}$$

bHQET

	$(p^+, p^-, p_\perp)$
• ucollinear c and $\bar{c}$	$Q(\lambda^2, 1, \lambda)$
• ucollinear u and d	$Q(\lambda^2, 1, \lambda)$
• ucollinear gluons	$Q(\lambda^2, 1, \lambda)$
• ultrasoft gluons	$Q(\lambda^2, \lambda^2, \lambda^2)$

$$Q \sim 2m_b\Lambda_{QCD}/m_c$$



$$iJ_{\rm QCD} = iC(\mu)J_{\rm EFT_I}(\mu)\ .$$

Matching

- At leading order in the EFT and tree level in α_s :

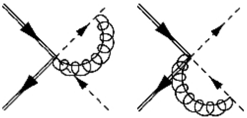
$$J_{\rm EFT_I} = \chi_{\bar b}^{\dagger}\sigma_{\perp}^{\mu}\, t^a \psi_b\ \bar\chi_{\bar n}^{c}\, S_{\bar n}^{\dagger}\ \gamma_{\mu\,\perp} t^a\ S_n\ \chi_n^{\bar c} \quad \text{and} \quad C(\mu = 2m_b) = \frac{\alpha_s(2m_b)\pi}{m_b^2}\ .$$

and Running

- μ-dependence driven by RGE in NRQCD + SCET

$$J_{\rm QCD} = C(\mu)J_{\rm EFT_I}(\mu) = C(\mu_b = 2m_b)\left(\frac{2m_b}{\sqrt{n\cdot p_c \bar n\cdot p_{\bar c}}}\right)^{g} \exp U(2m_b, m_c) J_{\rm EFT_I}(\mu = m_c)\ ,$$

Running in EFT I



collinear loops

usoft loops

$$\gamma_{EFT_I} = -2\left\{ \gamma(\alpha_s) + \Gamma_{\rm cusp}(\alpha_s)\ln\left(\frac{\mu}{\sqrt{n\cdot p_c\,\bar{n}\cdot p_{\bar{c}}}}\right)\right\}$$

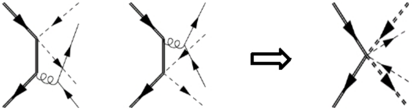
process dependent — universal, known to three loops

- resum Sudakov logarithms

Leading Log (LL)	$\alpha_s^n \ln^{n+1} m_c/2m_b$	one loop	$\Gamma_{\rm cusp}(\alpha_s)$	✓
Next to Leading Log (NLL)	$\alpha_s^n \ln^{n} m_c/2m_b$	two loop	$\Gamma_{\rm cusp}(\alpha_s)$	✓
		one loop	$\gamma(\alpha_s)$	✓

- *numerically* LL and NLL approximately equal

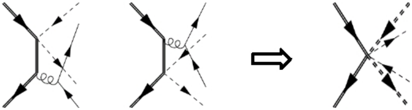
Effective Field Theory II.



$$i\mathcal{M}_{EFT_I} = C(\mu)\; T_{AB}(\omega,\bar{\omega},\mu,\mu',\,{}^{2S+1}L_J) \otimes F^2(\mu')\,\langle D_A D_B|\mathcal{O}_{AB}^{{}^{2S+1}L_J}\,(\omega,\bar{\omega};\mu')\,|\,\bar{b}b({}^{2S+1}L_J)\rangle$$

$$A, B \in \{P, V_L, V_T\}$$

Effective Field Theory II.



$$i\mathcal{M}_{EFT_I} = C(\mu)\, T_{AB}(\omega, \bar{\omega}, \mu, \mu', {}^{2S+1}L_J) \otimes F^2(\mu')\, \langle D_A D_B | \mathcal{O}_{AB}^{2S+1 L_J}(\omega, \bar{\omega}; \mu') \,|\, \bar{b}b({}^{2S+1}L_J)\rangle$$

$$A, B \in \{P, V_L, V_T\}$$

- P wave

$$T_{AA}(\omega, \bar{\omega}, \mu, \mu' = m_c, {}^3P_J) = \frac{C_F}{N_c^2} \frac{4\pi\alpha_s(\mu')}{m_b} \frac{1}{\omega + \bar{\omega}} \qquad A \in \{P, V_L\},\ J = 0, 2$$

$$A = V_T,\ J = 2$$

- S wave

$$T_{PV_L}(\omega, \bar{\omega}, \mu, \mu' = m_c, {}^1S_0) = \frac{C_F}{N_c^2} \frac{4\pi\alpha_s(\mu')}{m_b} \frac{1}{2} \frac{\omega - \bar{\omega}}{\omega + \bar{\omega}}$$

- non trivial dependence on ω and $\bar{\omega}$ at tree level

Non-perturbative matrix elements

$$F^2(\mu')\,\mathcal{O}_{PP}^{^3P_J}(\omega,\bar{\omega},\mu') = \chi_b^\dagger\,\mathbf{p}_b\cdot\sigma_\perp\psi_b\,\bar{\mathcal{H}}_{\bar{n}}^{c}\,\frac{\not{n}}{2}\gamma^5\,\delta\left(-\bar{\omega}-n\cdot\mathcal{P}\right)\chi_{\bar{n}}^{\bar{l}}\,\bar{\chi}_n^{l}\delta\left(\omega-\bar{n}\cdot\mathcal{P}^\dagger\right)\,\frac{\not{\bar{n}}}{2}\gamma^5\,\mathcal{H}_n^{\bar{c}}\,,$$

- usoft decoupling: $\mathcal{H}_n^{\bar{c}} \to Y_n\mathcal{H}_n^{\bar{c}}$, $\bar{\chi}_n^l \to \bar{\chi}_n^l Y_n^\dagger$

Non-perturbative matrix elements

$$\langle PP|F^2(\mu')\,\mathcal{O}^{{}^3P_J}_{PP}(\omega,\bar\omega,\mu')|{}^3P_0\rangle$$

$$= \langle 0|\chi^\dagger_{\bar b}\,\mathbf{p}_b\cdot\sigma_\perp\psi_b|{}^3P_0\rangle\;\langle P|\bar{\mathcal{H}}^c_{\bar n}\,\frac{\not{n}}{2}\gamma^5\,\delta\left(-\bar\omega-n\cdot\mathcal{P}\right)\chi^{\bar l}_{\bar n}|0\rangle\;\langle P|\bar\chi^l_n\delta\left(\omega-\bar n\cdot\mathcal{P}^\dagger\right)\,\frac{\not{\bar n}}{2}\gamma^5\,\mathcal{H}^{\bar c}_n|0\rangle\;,$$

- usoft decoupling: $\mathcal{H}^{\bar c}_n \to Y_n\mathcal{H}^{\bar c}_n$, $\bar\chi^l_n \to \bar\chi^l_n Y^\dagger_n$
- factorization of initial and final state:

$$|{}^3P_0\rangle = |0\rangle_n\,|0\rangle_{\bar n}|{}^3P_0\rangle_{\rm us}\,,\qquad \langle PP| =_n\langle P|\,_{\bar n}\langle P|\,_{\rm us}\langle 0|$$

Non-perturbative matrix elements

$$\langle PP|F^2(\mu')\,\mathcal{O}^{^3P_J}_{PP}(\omega,\bar\omega,\mu')|^3P_0\rangle$$
$$= \langle 0|\chi_{\bar b}^\dagger\,\mathbf{p}_b\cdot\sigma_\perp\psi_b|^3P_0\rangle\;\langle P|\bar{\mathcal{H}}^c_{\bar n}\frac{\not n}{2}\gamma^5\,\delta\left(-\bar\omega-n\cdot\mathcal{P}\right)\chi^{\bar l}_{\bar n}|0\rangle\;\langle P|\bar\chi^l_n\delta\left(\omega-\bar n\cdot\mathcal{P}^\dagger\right)\frac{\not{\bar n}}{2}\gamma^5\,\mathcal{H}^{\bar c}_n|0\rangle\;,$$

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$$|^3P_0\rangle=|0\rangle_n\,|0\rangle_{\bar n}|^3P_0\rangle_{\rm us}\,,\qquad \langle PP|={}_n\langle P|\,{}_{\bar n}\langle P|\,{}_{\rm us}\langle 0|$$

- quarkonium wavefunctions

$$\langle 0|\chi_{\bar b}^\dagger\mathbf{p}_b\cdot\sigma_\perp\psi_b|\chi_{b0}\rangle=\frac{2}{\sqrt3}\sqrt{\frac{3N_c}{2\pi}}R'_{\chi_{b0}}(0,\mu'),$$

and for S-wave

$$\langle 0|\chi_{\bar b}^\dagger\psi_b|\eta_b\rangle=\sqrt{\frac{N_c}{2\pi}}R_{\eta_b}(0,\mu').$$

D-meson distribution amplitudes

$$\langle PP|F^2(\mu')\,\mathcal{O}^{^3P_J}_{PP}(\omega,\bar{\omega},\mu')|^3P_0\rangle$$
$$= \langle 0|\chi_{\bar{b}}^{\dagger}\,\mathbf{p}_b\cdot\sigma_{\perp}\psi_b|^3P_0\rangle\,\langle P|\bar{\mathcal{H}}^{c}_{\bar{n}}\,\frac{\not{n}}{2}\gamma^5\,\delta\left(-\bar{\omega}-n\cdot\mathcal{P}\right)\chi^{\bar{l}}_{\bar{n}}|0\rangle\,\langle P|\bar{\chi}^{l}_{n}\delta\left(\omega-\bar{n}\cdot\mathcal{P}^{\dagger}\right)\,\frac{\not{\bar{n}}}{2}\gamma^5\,\mathcal{H}^{\bar{c}}_{n}|0\rangle\,,$$

- *D*-meson light-cone distribution amplitudes (DA)

$$\langle P|\bar{\chi}^{l}_{n}\,\frac{\not{\bar{n}}}{2}\gamma^5\,\delta\left(\omega-\bar{n}\cdot\mathcal{P}^{\dagger}\right)\mathcal{H}^{\bar{c}}_{n}|0\rangle = iF_P(\mu')\frac{\bar{n}\cdot v}{2}\phi_P(\omega,\mu')$$

and for vector mesons

$$\langle V_L|\bar{\chi}^{l}_{n}\,\frac{\not{\bar{n}}}{2}\,\delta\left(\omega-\bar{n}\cdot\mathcal{P}^{\dagger}\right)\mathcal{H}^{\bar{c}}_{n}|0\rangle = F_{V_L}(\mu')\frac{\bar{n}\cdot v}{2}\phi_{V_L}(\omega,\mu')$$

$$\langle V_T|\bar{\chi}^{l}_{n}\,\frac{\not{\bar{n}}}{2}\gamma^{\mu}_{\perp}\,\delta\left(\omega-\bar{n}\cdot\mathcal{P}^{\dagger}\right)\mathcal{H}^{\bar{c}}_{n}|0\rangle = F_{V_T}(\mu')\frac{\bar{n}\cdot v}{2}\varepsilon^{\mu}_{\perp}\,\phi_{V_T}(\omega,\mu')$$

heavy quark limit $F_A(\mu'=m_c)=f_D\sqrt{m_D}$,
$f_D = 205.8\pm 8.5\pm 2.5$ MeV

Factorization of the decay rate

- P wave

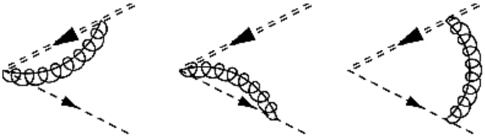
$$\Gamma\left(\chi_{b0} \to AA\right) = \frac{4}{3} \frac{m_D^2 \sqrt{m_{\chi_{b0}}^2 - 4m_D^2}}{8\pi m_{\chi_{b0}}} \frac{3N_c}{2\pi} \left|C\left(\mu\right)\right|^2 \left|R'_{\chi_{b0}}(0,\mu')\right|^2$$
$$\left[F^2(\mu') \frac{n\cdot v'}{2} \frac{\bar{n}\cdot v}{2} \int \frac{d\omega}{\omega} \frac{d\bar{\omega}}{\bar{\omega}} T\left(\omega,\bar{\omega},\mu,\mu';\, {}^3P_J\right) \phi_A(\bar{\omega},\mu')\phi_A(\omega,\mu')\right]^2$$

$A \in \{P, V_L\}$

- analogous expressions for $\chi_{b2} \to PP, V_L V_L$ or $V_T V_T$.
- S wave

$$\Gamma\left(\eta_b \to PV_L + \text{c.c.}\right) = \frac{m_D^2 \sqrt{m_{\eta_b}^2 - 4m_D^2}}{8\pi m_{\eta_b}} \frac{N_c}{2\pi} \left|C\left(\mu\right)\right|^2 \left|R_{\eta_b}(0,\mu')\right|^2 \frac{1}{2} \left[F^2(\mu') \frac{n\cdot v'}{2} \frac{\bar{n}\cdot v}{2}\right.$$
$$\left.\int \frac{d\omega}{\omega} \frac{d\bar{\omega}}{\bar{\omega}} T\left(\omega,\bar{\omega},\mu,\mu';\, {}^1S_0\right) \left(\phi_{V_L}(\bar{\omega},\mu')\phi_P(\omega,\mu') - \phi_{V_L}(\omega,\mu')\phi_P(\bar{\omega},\mu')\right)\right]^2.$$

Running in EFT II



- pNRQCD graphs do not contribute to NLL running
- from bHQET graphs, convolution running

$$\gamma_{EFT_{II}} = 2\gamma_F \delta(\omega - \omega')\delta(\bar{\omega} - \bar{\omega}') + \gamma_{\mathcal{O}}(\omega, \omega'; \bar{\omega}, \bar{\omega}'; \mu')$$

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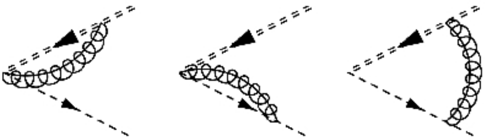

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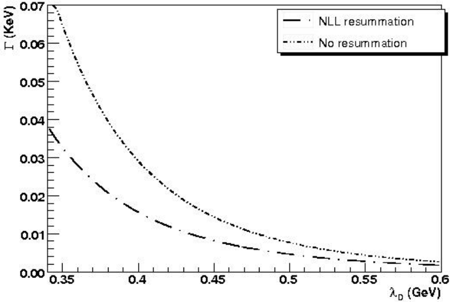
$$\gamma_{EFT_{II}} = 2\gamma_F \delta(\omega - \omega')\delta(\bar{\omega} - \bar{\omega}') + \gamma_{\mathcal{O}}(\omega, \omega'; \bar{\omega}, \bar{\omega}'; \mu')$$

- γ_F governs the running of D-meson decay constant
- $\gamma_{\mathcal{O}}$ running of the D-meson DA

B. Lange and M. Neubert, Phys. Rev. Lett. **91**:102001 (2003).

- analytical (though gory) solution for the evolved matching coefficients.

Results. $\Gamma(\chi_{b0} \to DD)$

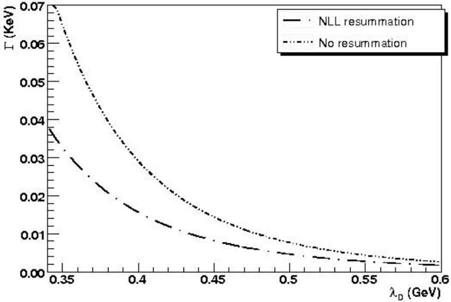


$\lambda_B = 0.460 \pm 0.110$ GeV

Non perturbative parameters

- $|R'_{\chi_{bJ}}(0)|^2 = 2.3$ GeV^5 lattice
- D-meson
  $\phi_D(\omega, \mu' \sim 1\,\text{GeV}) = \frac{\omega}{(\bar{n}\cdot v)^2 \lambda_D^2} e^{-\frac{\omega}{\bar{n}\cdot v \lambda_D}}$
- choice of λ_D inspired by B-physics!

Results. $\Gamma(\chi_{b0} \to DD)$



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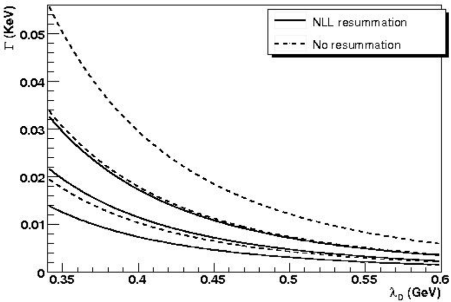
- choice of λ_D inspired by B-physics!

- relevant impact of the NLL resummation
- strong dependence on DA parameters $\sim \lambda_D^{-6-4g}$

Results. $\Gamma(\chi_{b0} \to DD)$



$\lambda_B = 0.460 \pm 0.110$ GeV
$\sigma_B = 1.4 \pm 0.4$

Non perturbative parameters

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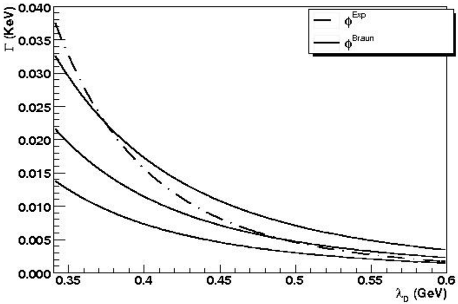
$$\phi_D(\omega, \mu' \sim 1\,\mathrm{GeV}) = \frac{4}{\pi\bar{n}\cdot v\lambda_D}\frac{\tilde{\omega}}{1+\tilde{\omega}^2}\left[\frac{1}{1+\tilde{\omega}^2} - \frac{2(\sigma_D - 1)}{\pi^2}\ln\tilde{\omega}\right]$$

- strong dependence on DA parameters $\sim \lambda_D^{-4}$
- relevant impact of the NLL resummation

Results. $\Gamma(\chi_{b0} \to DD)$



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$$\phi_D(\omega, \mu' \sim 1\,\mathrm{GeV}) =$$
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- strong dependence on DA parameters $\sim \lambda_D^{-4}$
- relevant impact of the NLL resummation
- two DAs in rough agreement

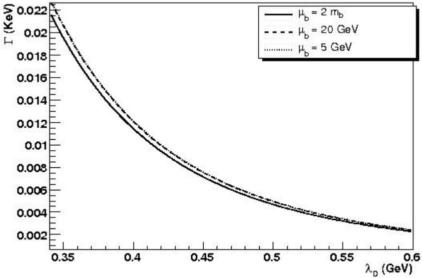
Results. $\Gamma(\chi_{b0} \to DD)$

Uncertainties

- non-perturbative corrections: $\Lambda_{QCD}/m_c \sim 30\%$

perturbative corrections

- QCD onto EFT I matching: $\alpha_s(2m_b) \sim 10\%$
- EFT I onto EFT II matching: $\alpha_s(m_c) \sim 30\%$



- mild dependence on variation of μ ($\sim 5\%$)

$$\phi(\omega) = \phi^{\text{Braun}}(\omega)$$

One loop EFT I to EFT II matching for stable result!

Results. $\Gamma(\chi_{b0} \rightarrow DD)$

Uncertainties

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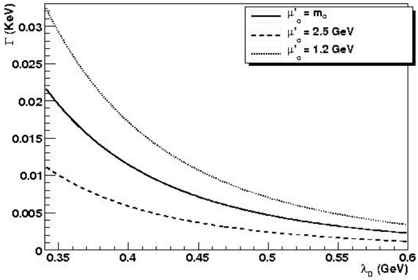

- mild dependence on variation of μ ($\sim 5\%$)
- wild dependence on variation of μ' ($\sim 50\%$)

$$\phi(\omega) = \phi^{\text{Braun}}(\omega)$$

One loop EFT I to EFT II matching for stable result!

Results. $\Gamma(\chi_{b0} \to DD)$

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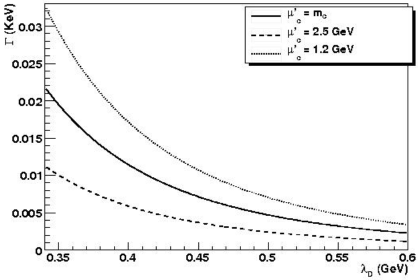

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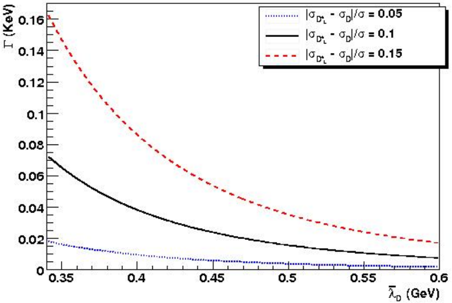
One loop EFT I to EFT II matching for stable result!

Results. $\Gamma(\eta_b \to DD^* + \text{c.c.})$



$$\bar{\lambda}_D = \frac{1}{2}(\lambda_D + \lambda_{D^*_L})$$
$$\delta = \frac{1}{2}\frac{\lambda_D - \lambda_{D^*_L}}{\bar{\lambda}_D}$$
$$\sigma = 2\sigma_D$$

Non perturbative parameters

- bottomonium
  $|R_{\eta_b}(0)|^2 = 6.92 \pm 0.38\,\text{GeV}^3$
  Υ decay
- D-meson $\phi_D = \phi_D^{\text{Braun}}$, $\phi_{D^*} = \phi_{D^*}^{\text{Braun}}$

- strong dependence on DA parameters $\sim \bar{\lambda}_D^{-4}$

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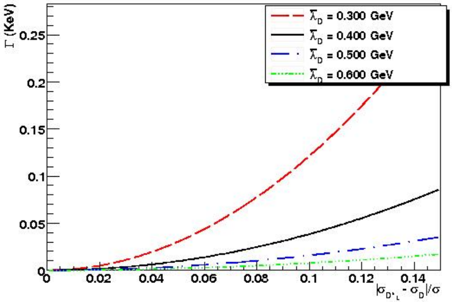

Non perturbative parameters

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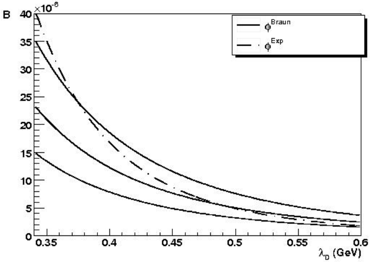
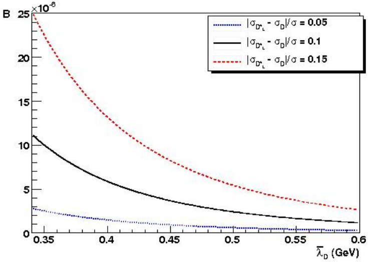
- D-meson $\phi_D = \phi_D^{\text{Braun}}$, $\phi_{D^*} = \phi_{D^*}^{\text{Braun}}$

- strong dependence on DA parameters $\sim \bar{\lambda}_D^{-4}$
- vanishes in spin-symmetry limit $\phi_D = \phi_{D^*}$
- strong dependence on the functional form

Branching Ratios





- $\mathcal{B}(\chi_{b0} \to PP)$ in the range $4 \cdot 10^{-6} - 4 \cdot 10^{-5}$
- $\mathcal{B}(\eta_b \to PV_L + \text{c.c.})$ in the same range
- too small in this range for λ_D ...

... but hope if λ_D in the $0.25 - 0.35$ GeV range

- $\eta_b \to DD^*$, $\chi_{bJ} \to DD$ *do not* dominate $\eta_b \to c\bar{c} + X$, $\chi_{bJ} \to \bar{c}c + X$

Conclusions

EFT approach:

- allows factorization of dynamics of different scales; $2m_b$, m_c, Λ_{QCD}.
- can be extended to C-odd bottomonium decays, $\Upsilon \to DD$, $\Upsilon \to D^* D^*$.
- and to power-suppressed processes, $\eta_b \to D^* D^*$, $\chi_{b2} \to DD^* +$ c.c.

Exclusive, charmed decays of bottomonium

- good candidates for extraction of bottomonium and D-meson parameters

strong dependence of DA paramaters

but

- quite large theoretical error

include perturbative and non-perturbative corrections!

- rather small branching ratio

Backup Slides

Anomalous dimension in EFT I

$$\gamma_{EFT_I} = -2\left\{\gamma(\alpha_s) + \Gamma_{\text{cusp}}(\alpha_s)\ln\left(\frac{\mu}{\sqrt{n\cdot p_c\,\bar{n}\cdot p_{\bar{c}}}}\right)\right\}$$

with

$$\gamma(\alpha_s) = 3C_F + 2N_c + i\frac{\pi}{N_c}$$

and

$$\Gamma_{\text{cusp}}(\alpha_s) = \frac{\alpha_s}{4\pi}\Gamma^{(0)}_{\text{cusp}} + \left(\frac{\alpha_s}{4\pi}\right)^2\Gamma^{(1)}_{\text{cusp}}$$

$$\Gamma^{(0)}_{\text{cusp}} = 4C_F\ , \qquad \Gamma^{(1)}_{\text{cusp}} = 4C_F\left[\left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c - \frac{10}{9}n_f\right]\ ,$$

Running Factors in EFT I

$$U(\mu_0,\mu) = -2\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)} \left\{ \gamma(\alpha) + \Gamma_{\rm cusp}(\alpha) \int_{\alpha(\mu_0)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')} \right\} \,,$$

$$g(\mu_0,\mu) = -2\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)} \Gamma_{\rm cusp}(\alpha) \,.$$

Explicitly, at NLL

$$U(\mu_b,\mu) = \frac{2\pi\Gamma_{\rm cusp}^{(0)}}{\beta_0^2} \left[\frac{r-1-r\ln r}{\alpha_s(\mu)} + \frac{\beta_0\gamma_{\rm Re}^{(0)}}{2\pi\Gamma_{\rm cusp}^{(0)}} \ln r + \left(\frac{\Gamma_{\rm cusp}^{(1)}}{\Gamma_{\rm cusp}^{(0)}} - \frac{\beta_1}{\beta_0} \right) \frac{1-r+\ln r}{4\pi} + \frac{\beta_1}{8\pi\beta_0} \ln^2 r \right] + \frac{\gamma_{\rm Im}^{(0)}}{\beta_0} \ln r \,,$$

and

$$g(\mu_b,\mu) = \frac{\Gamma_{\rm cusp}^{(0)}}{\beta_0} \left[\ln r + \left(\frac{\Gamma_{\rm cusp}^{(1)}}{\Gamma_{\rm cusp}^{(0)}} - \frac{\beta_1}{\beta_0} \right) \frac{\alpha_s(\mu_b)}{4\pi} (r-1) \right] \,,$$

where $r = \alpha_s(\mu)/\alpha_s(\mu_b)$.

The anomalous dimension in EFT is

$$\gamma_{\mathrm{EFT_{II}}}(\omega,\omega';\bar{\omega},\bar{\omega}';\mu') = 2\gamma_F\,\delta\left(\omega-\omega'\right)\delta\left(\bar{\omega}-\bar{\omega}'\right) + \gamma_{\mathcal{O}}(\omega,\omega';\bar{\omega},\bar{\omega}';\mu')\ ,$$

with

$$\begin{aligned}
&\gamma_{\mathcal{O}}(\omega,\omega';\bar{\omega},\bar{\omega}';\mu') \\
&= \frac{\alpha_s}{4\pi}\,4C_F\delta\left(\omega-\omega'\right)\delta\left(\bar{\omega}-\bar{\omega}'\right)\left[-1+\ln\left(\frac{\mu' n\cdot v'}{\bar{\omega}'}\right)+\ln\left(\frac{\mu'\bar{n}\cdot v}{\omega'}\right)\right] \\
&-\frac{\alpha_s}{4\pi}4C_F\delta\left(\omega-\omega'\right)\left[\theta\left(\bar{\omega}-\bar{\omega}'\right)\left(\frac{1}{\bar{\omega}-\bar{\omega}'}\right)_+ +\theta\left(\bar{\omega}'-\bar{\omega}\right)\theta\left(\bar{\omega}\right)\frac{\bar{\omega}}{\bar{\omega}'}\left(\frac{1}{\bar{\omega}'-\bar{\omega}}\right)_+\right] \\
&-\frac{\alpha_s}{4\pi}4C_F\delta\left(\bar{\omega}-\bar{\omega}'\right)\left[\theta\left(\omega-\omega'\right)\left(\frac{1}{\omega-\omega'}\right)_+ +\theta\left(\omega'-\omega\right)\theta\left(\omega\right)\frac{\omega}{\omega'}\left(\frac{1}{\omega'-\omega}\right)_+\right]\ .
\end{aligned}$$

Running factors in EFT II

$$F^2(\mu')\,T(\omega,\bar\omega,\mu,\mu';{}^3P_J) = F^2(\mu'_c)\frac{C_F}{N_c^2}\frac{4\pi\alpha_s(\mu'_c)}{m_b}\exp\left[V(\mu'_c,\mu')\right]\left(\frac{\mu_c'^2\bar n\cdot v n\cdot v'}{\omega\,\bar\omega}\right)^g$$

$$\frac{\theta(\bar\omega-\omega)}{\bar\omega}\left\{\frac{\Gamma(1+g)\Gamma(2+g)}{\Gamma(1-g)\Gamma(-g)}\left[1-\ln\frac{\omega}{\bar\omega}+\psi(1-g)-\psi(-g)+\psi(1+g)-\psi(2+g)\right]\right.$$

$$+\frac{1}{2}\frac{\omega}{\bar\omega}\frac{\Gamma(g+2)\Gamma(g+3)}{\Gamma(1-g)\Gamma(2-g)}\,{}_4F_3\left(1,1,g+2,g+3;3,1-g,2-g;-\frac{\omega}{\bar\omega}\right)$$

$$\left.-\left(\frac{\omega}{\bar\omega}\right)^{1+g}4\cos(g\pi)\frac{\Gamma(2+2g)^2}{g+2}\,{}_3F_2\left(g+1,2g+2,2g+3;2,g+3;-\frac{\omega}{\bar\omega}\right)\right\}+(\omega\to\bar\omega)\ ,$$

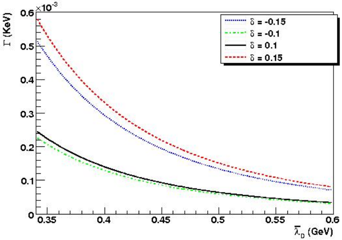
- at NLL

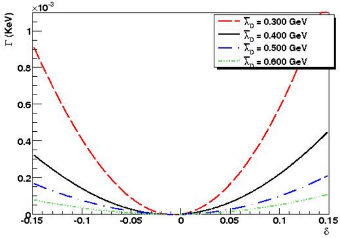
$$g(\mu'_0,\mu') = -\frac{\Gamma^{(0)}_{\rm cusp}}{2\beta_0}\left\{\ln r+\left(\frac{\Gamma^{(1)}_{\rm cusp}}{\Gamma^{(0)}_{\rm cusp}}-\frac{\beta_1}{\beta_0}\right)\frac{\alpha_s(\mu'_0)}{4\pi}(r-1)\right\}\ ,$$

$$V(\mu'_0,\mu') = -\Gamma^{(0)}_{\rm cusp}\frac{2\pi}{\beta_0^2}\left\{\frac{r-1-r\ln r}{\alpha_s(\mu')}+\left(\frac{\Gamma^{(1)}_{\rm cusp}}{\Gamma^{(0)}_{\rm cusp}}-\frac{\beta_1}{\beta_0}\right)\frac{1-r+\ln r}{4\pi}+\frac{\beta_1}{8\pi\beta_0}\ln^2 r\right\}$$

$$+\frac{C_F}{\beta_0}(2-8\gamma_E)\ln r\ ,$$

$\Gamma(\eta_b \to DD^*)$ with exponential DA





- one or two orders smaller than the result with ϕ_D^{Braun}