Static $Q\bar{Q}$ correlators in hot QCD

Yannis Burnier

Bielefeld University

Talk based on work done with
M.Laine, M. Vepsäläinen

[YB, M. Laine and M. Vepsäläinen, *Dimensionally regularized Polyakov loop correlators in hot QCD*, JHEP **1001** (2010) 054]

May 19, 2010

Outline

1. Introduction
   - Quarkonium in QGP
   - Description of bound states in media
2. Polyakov loop correlators
   - Interesting properties
   - Perturbation theory calculation
   - Comparison with lattice
3. Conclusion

1. Introduction
Quarkonium in QGP

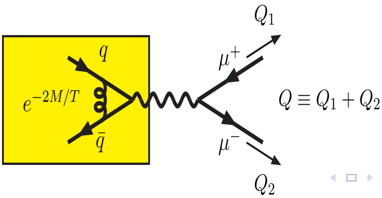
Heavy quarkonium is an important probe of the properties of a quark-gluon plasma. [T. Matsui, H. Satz (1986)]

- In heavy ion collisions $\longrightarrow$ short lived quark-gluon plasma.
- In the primary collisions heavy quarkonium is created.
- Depending if it survives the high T it eventually decays (to muons for instance).

$\Leftarrow$ Description of bound state at $T > 0$.

- Muon escape $\leftrightarrow$ carry information out of the plasma.

$\Leftarrow$ Spectral function.



The description of quarkonium bound states at $T > 0$

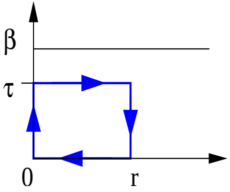
Many different theoretical approaches:

1. Potential models
   - Heavy quark $\longrightarrow$ non-relativistic
     $\longrightarrow$ interaction described by a potential.
   - Successful at $T = 0$, but how to define a potential at $T > 0$?
2. Perturbation theory
   - Heavy quark effective theory
     $\longrightarrow$ define the potential as matching coefficient.
   - Convergence of perturbation theory questionable.
3. Lattice QCD
   - Contains perturbative and non-perturbative physics.
   - Need to analytically continue results from Euclidean to real time.
4. AdS/QCD

Perturbative potential for heavy quarks

In the heavy quark limit, the potential is given by a Wilson loop of Euclidean time extend $\tau \to it$:

$$
\begin{aligned}
C_E(\tau, r) &= \langle Tr[W_E(\tau, r)] \rangle \\
V_{pert}(t, r) &= \frac{i\partial_t C_E(it, r)}{C_E(it, r)}
\end{aligned}
$$



Static potential

$$\lim_{t\to\infty} V_{pert}(t, r) = -\alpha C_F \left[m_{\mathrm{D}} + \frac{\exp(-m_{\mathrm{D}} r)}{r} + iT\,\phi(m_{\mathrm{D}} r) \right] + \mathcal{O}(g^4)$$

- 2×thermal mass correction for heavy quarks.
- Second term → standard Debye-screened potential.
- Third *imaginary* term → heavy quark damping.

[Laine, Philipsen, Romatschke, Tassler (2007); Brambilla, Ghiglieri, Vairo and Petreczky (2008); Beraudo, Blaizot, Ratti (2008)]

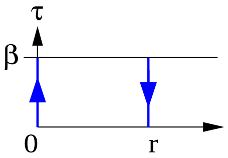
Quarkonium on the lattice

- Standard: Compute directly the spectral function $\Rightarrow$ Needs to analytically continue the Euclidean correlator (MEM).
- Potential from its perturbative definition also needs an analytical continuation.
- From the position of the first peak of the spectral function $\rightarrow$ real part of the potential.
- Find some Euclidean observable that matches the potential $\rightarrow$ Classical observable: "singlet quark-antiquark free energy".

2. Polyakov loop correlators

The singlet free energy is defined form the Coulomb gauge Polyakov loop correlator

$$\Psi_C = \frac{1}{N_c}\langle Tr[P_0 P_r^\dagger]\rangle_{Coulomb}$$



as

$$F_C = -T\log\left(\frac{\Psi_C}{\Psi_P^2}\right)$$

with the normalization $\Psi_P = \frac{1}{N_c}\langle Tr[P_0]\rangle$.

Interesting properties (lattice):

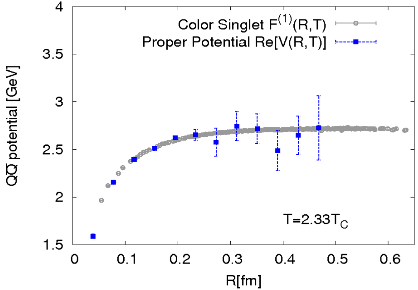
- Displays good scaling properties (lattice size and spacing).
- Matches the $T = 0$ potential in the limit $r \to 0$.

Relation between F_C and V_{pert}?

Singlet free energy as real part of the potential?

Potential out of the lattice spectral function

- The position of the first peak → real part of the potential.
- Width of the first peak → imaginary part.



[Rothkopf, Hatsuda, Sasaki, 2009]

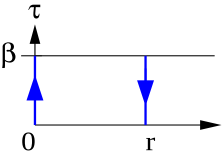
The real part of the potential matches the free energy.

Can we learn something from perturbation theory?

We consider a more general set of correlators:

1. Singlet free energy in Coulomb gauge:

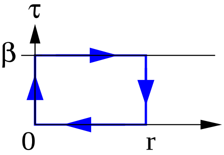
$$\Psi_C = \frac{1}{N_c}\langle \mathrm{Tr}[P_r P_0^\dagger]\rangle_{Coulomb}$$



$\Leftarrow$ Standard quantity measured on the lattice.

2. The singlet free energy in covariant gauge Ψ_ξ.
3. Cyclic Wilson loop:

$$\Psi_W = \frac{1}{N_c}\langle \mathrm{Tr}[P_r W_\beta P_0^\dagger W_0^\dagger]\rangle$$



$\Leftarrow$ Gauge invariant.

Leading order perturbation theory

- The free energy is gauge invariant,

$$F_C = F_\xi = -\alpha C_F \frac{e^{-m_D r}}{r}.$$

- It equates the Wilson loop

$$\Psi_C = \Psi_W.$$

- It is equal to the real part of the potential up to some constant.

Do all these nice properties extend to NLO?

Perturbation theory calculation at NLO

We calculated these different observables using finite T perturbation theory at NLO: [YB, M. Laine and M. Vepsäläinen, 2009]

Difficulties:

- UV divergences:
  - $\frac{1}{N_c}\langle \mathrm{Tr}[P_r P_0^\dagger]\rangle$ depends only on g.

  $\Rightarrow$ Charge renormalization alone should cancel UV divergences.
- IR divergences:
  - Color electric modes at the scale gT.

  $\Rightarrow$ Needs resummation: systematically done from EQCD.

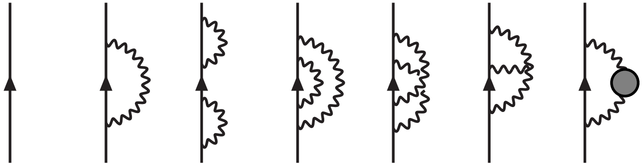
$$\Psi = [\Psi_{QCD} - \Psi_{EQCD}]_{unresummed} + [\Psi_{EQCD}]_{resummed}$$

  - Color magnetic modes at the scale $g^2 T$.

  $\Rightarrow$ No prominent role here.

Calculation: One Polyakov loop as example

$$[\Psi_P]_{QCD} = \left[\frac{1}{N_c}\langle Tr[P_r]\rangle\right]_{QCD} =$$



$$+\,\mathcal{O}(g^6)$$

$$= 1 - \frac{g^2 C_F}{2T}\int_k \frac{1}{\mathbf{k}^2} - \frac{g^4 C_F}{2}\int_{\mathbf{k}} \frac{2}{\mathbf{k}^4}\int_{\mathbf{q}} \cdots + \ldots$$

The IR divergent $1/k^4$ and further logarithmic divergences in the ... are treated with EQCD.

EQCD resummation

The Lagrangian of EQCD reads

$$\mathcal{L}_{\mathrm{E}} = \frac{1}{2}\,\mathrm{Tr}\,[\tilde{F}_{ij}^2] + \mathrm{Tr}\,[\tilde{D}_i, \tilde{A}_0]^2 + m_{\mathrm{D}}^2\,\mathrm{Tr}\,[\tilde{A}_0^2] + \dots\ .$$

The Polyakov loop operator is represented as

$$P_{\mathbf{r}} = [\mathbb{1}\,\mathcal{Z}_0] + ig\tilde{A}_0\beta\,\mathcal{Z}_1 + \frac{1}{2}\,(ig\tilde{A}_0\beta)^2\,\mathcal{Z}_2 + \dots + (g^2\tilde{F}_{ij}\beta^2)^2\mathcal{X}_4 + \dots$$

g^3, g^4 corrections to the Polyakov loop in EQCD:

$$[\Psi_P]_{EQCD} = -\frac{g^2 C_{\mathrm{F}}}{2T}\int_k \frac{1}{\mathbf{k}^2 + m_D^2} - \frac{g^4 C_{\mathrm{F}}}{2}\int_{\mathbf{k}} \frac{2}{(\mathbf{k}^2 + m_D^2)^2}\int_{\mathbf{q}} \dots$$

The divergences are regularized and reappear in the $m_D \to 0$ limit. The expression $\Psi_P = [[\Psi_P]_{QCD} - [\Psi_P]_{EQCD}]_{m_D\to 0} + [[\Psi_p]_{EQCD}]$ is finite and contains the correct color electric physics.

NLO results

All quantities Ψ_C, Ψ_ξ, Ψ_W have "problems":

- Polyakov loop correlator is not gauge invariant at $\mathcal{O}(g^4)$.
- Ψ_C is finite after charge renormalization but not Ψ_ξ nor Ψ_W.
- Ψ_C, Ψ_ξ have a power law tail $\propto \frac{\alpha^2}{T^2 r^2}$.

$\longrightarrow$ Gauge artefact since there is a finite screening length in QGP.

- The gauge invariant Ψ_W decreases like $e^{-m_\mathrm{D} r}$.
- Perturbation theory breaks down at large r: $[\Psi_C]^{NLO}_{EQCD} > [\Psi_C]^{LO}_{EQCD}$ at $r \gg \frac{\pi}{g^2 T}$.

However: For $rT \ll 1$, Ψ_C reproduces the $T = 0$ potential

$$V(r) = -\frac{g^2 C_F}{4\pi r} + \frac{g^4 C_F}{(4\pi)^2} \int_\mathbf{k} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2} \left[\frac{2N_\mathrm{f}}{3} \left(\ln \frac{\bar{\mu}^2}{k^2} + \frac{5}{3} \right) - \frac{11N_\mathrm{c}}{3} \left(\ln \frac{\bar{\mu}^2}{k^2} + \frac{31}{33} \right) \right].$$

NLO results for the free energy in the Coulomb gauge

$$F_C(r) = -\frac{\alpha(\bar{\mu}) C_F \exp(-m_D r)}{r} \left\{ 1 + \alpha(\bar{\mu}) \left[\frac{11 N_c}{3} \left(2 \ln \frac{\bar{\mu} e^{\gamma_E}}{4\pi T} + 1 \right) \right. \right.$$

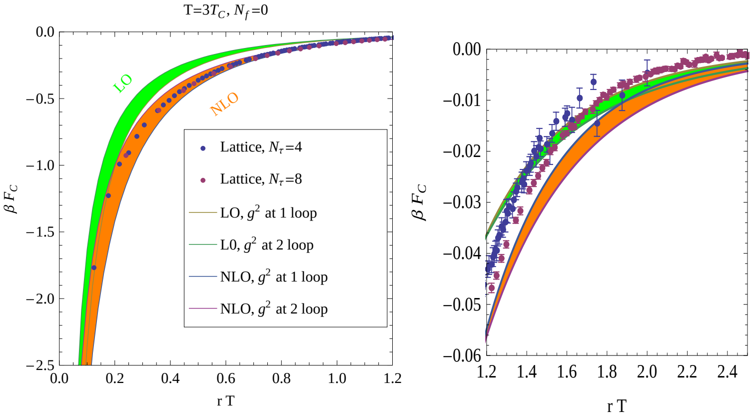
$$\left. \left. - \frac{2 N_f}{3} \left(2 \ln \frac{\bar{\mu} e^{\gamma_E}}{\pi T} - 1 \right) \right] \right\} - \alpha(\bar{\mu})^2 C_F N_c \left\{ -\frac{\exp(-2 m_D r)}{8 T r^2} \right.$$

$$\frac{1}{12 T r^2} + \frac{\mathrm{Li}_2(e^{-4\pi T r})}{(2\pi r)^2 T} + T \exp(-m_D r) \Big[2 - \ln(2 m_D r) - \gamma_E$$

$$\left. + e^{2 m_D r} E_1(2 m_D r) \Big] + \frac{1}{\pi r} \int_1^\infty \mathrm{d}x \left(\frac{1}{x^2} - \frac{1}{2x^4} \right) \ln\left(1 - e^{-4\pi T r x} \right) \right\}$$

$$-\alpha(\bar{\mu})^2 C_F N_f \left[\frac{1}{2\pi r} \int_1^\infty \mathrm{d}x \left(\frac{1}{x^2} - \frac{1}{x^4} \right) \ln \frac{1 + e^{-2\pi T r x}}{1 - e^{-2\pi T r x}} \right] + \mathcal{O}(g^5) \, .$$

Comparison with lattice



Perturbative calculation shows a good convergence and fits lattice data very well. [Lattice data from Kaczmarek, Karsch, Petreczky, Zantow, 2002]

4. Conclusion and Outlook

- The singlet free energy in Coulomb gauge reproduce the correct $Tr \rightarrow 0$ behavior.
- This observable might be quite close to the real part of the potential.
- However shows a non physical $1/r^2$ behavior at large distance both in perturbation theory and in the lattice data.

$\Rightarrow$ Using the free energy probably overestimates the binding energy.

- Perturbation theory seems to converge well.

$\Rightarrow$ Computations for the quarkonium decay from perturbative potential should be reliable. [YB, Laine, Vepsäläinen, 2007, 2008]

- Motivation to calculate the perturbative potential to $\mathcal{O}(g^4)$.