# HYPERFINE INTERACTION IN QUARKONIA

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## Introduction

The richness of the spectra of the excited states of atoms as well as hadrons lies not only in the principal quantum number and angular momentum dependence of the states, but in the **spin–dependent** multiplicities. These arise from spin–orbit, tensor, and spin–spin interactions between the constituents. Of these three, the most interesting is the **hyperfine** structure that arises due to magnetic interactions between the spins, and leads to the splitting between **spin–singlet** $(s = s_1 + s_2 = s = 0)$ and **spin–triplet** $(s = s_1 + s_2 = s = 1)$ states. In hydrogen it gives rise to the **famous 21 cm line** which is the workhorse of microwave astronomy.

- Hyperfine interaction is equally important in hadron spectroscopy. For a purely Coulombic interaction, the hyperfine splitting for mesons made of two quarks with masses $m_1$ and $m_2$ is given by

$$\Delta M_{hf}(nS) \equiv M(n^3S_1) - M(n^1S_0) = \frac{32\pi\alpha_S}{9m_1m_2}|\psi_n(0)|^2 \qquad (1)$$

- However, we know that the $q\bar{q}$ potential is not purely Coulombic but has an **additional confinement part** which is generally parameterized as being proportional to $r$, giving rise to the Cornell potential:

$$V(q\bar{q}) = \frac{4}{3}\frac{\alpha_S}{r} + Cr \qquad (2)$$

It is assumed that the spin-dependent potentials, the spin-orbit, tensor, and spin-spin potentials all arise only from the Coulombic, $1/r$ part of the potential, and the confinement part makes no contribution to them.

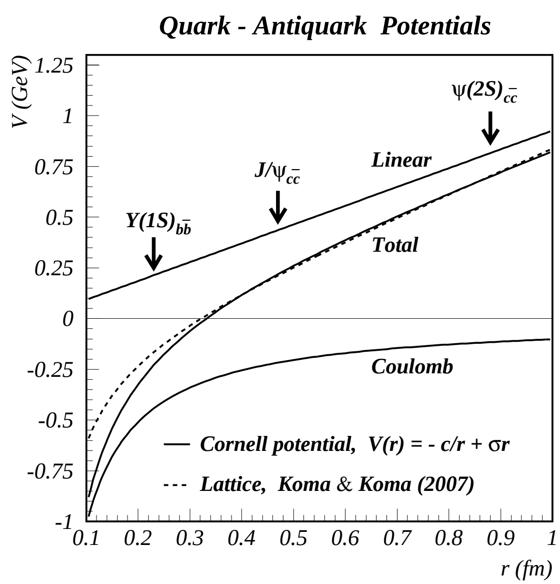
**The Spin–Spin Interaction and the Confinement Potential**

To put the question about the role of the confinement potential in the nature of the $q\bar{q}$ spin–spin potential in perspective, we note again that different $q\bar{q}$ states sample different regions of the $q\bar{q}$ potential with quite different levels of contribution from the Coulombic and confinement potentials. It ranges from being dominantly Coulombic for the bottomonium 1S states to dominantly confinement for the 2S charmonium states. This raises the following questions. **How does the hyperfine interaction change**

(a) with principal quantum number $\boldsymbol{n}$, for example between 1S and 2S states,

(b) between S–wave and P–wave states, e.g., between $\boldsymbol{\ell} = 0$ and $\boldsymbol{\ell} = 1$ states,

(c) with **quark masses**, e.g., between $c$–quark states and $b$–quark states?



# Experimental Measures of the Hyperfine Interaction

The answers to the questions posed can be provided only by experimental data. Unfortunately, there is an **experimental problem** in measuring hyperfine splittings,

$$\boldsymbol{\Delta M_{hf}(nL)} \equiv M(n^3L) - M(n^1L)$$

The problem is that while the triplet states are conveniently excited in $e^+e^-$ annihilation, the radiative excitation of singlet states is either forbidden, or possible only with **weak M1** allowed $(n \to n)$ and forbidden $(n \to n')$ transitions.

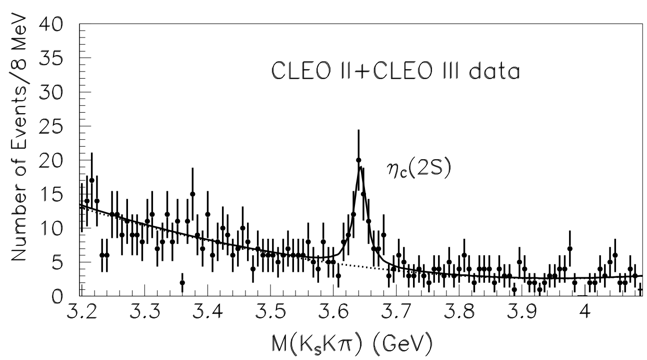
- Because of this difficulty, the identification of singlet state has lagged behind that of triplet states by many years: $\eta_c(1^1S_0)_{c\bar{c}}$, **6 years**; $\eta_c'(2^1S_0)_{c\bar{c}}$, **26 years**; $h_c(1^1P_1)_{c\bar{c}}$, **29 years**; $\eta_b(1^1S_0)_{b\bar{b}}$ **31 years**.
- But great progress **has been made in the last five years** with contributions from many laboratories. I do not have time to describe the details of these marathon efforts, but I do want to give you the important results.

## Hyperfine Splitting of Charmonium Ground State

- $\Delta M_{hf}(1S)_{c\bar{c}} = M(J/\psi, 1^3S_1) - M(\eta_c, 1^1S_0) = 116.6 \pm 1.0$ MeV.
  This remains the best measured hyperfine splitting in a heavy quark hadron.
- The recent Fermilab **unquenched** lattice calculation predicts the remarkably successful result $\Delta M_{hf}(1S)_{c\bar{c}} = 116.0 \pm 7.4^{+2.6}_{-0}$ MeV.

## Hyperfine Splitting of Charmonium Radial Excitation

- $\Delta M_{hf}(2S)_{c\bar{c}} = M(\psi', 2^3S_1) - M(\eta_c', 2^1S_0) = 49 \pm 4$ MeV.
  $\eta_c'$ was first identified in 2002 by Belle in $B$–decay, and confirmed by its formation in two-photon fusion, and decay into $K_SK\pi$, by CLEO and BaBar in 2004, and by Belle later. The figure shows the CLEO spectrum.




- This is the first measurement of hyperfine splitting in a radial excitation.

- There are numerous pQCD–based predictions for $\Delta M_{hf}(2S)_{c\bar{c}}$, and they range all over the map. However, it is fair to say that it was not expected the 2S hyperfine splitting would be $\sim 2.5$ times smaller than the 1S hyperfine splitting.

- It has been suggested that the smaller than expected 2S hyperfine splitting is a consequence of the 2S levels being very close to the **open-charm threshold**, and the consequent mixing with the continuum levels. Of course, this problem would not exist in the 2S hyperfine splitting in bottomonium, if one could ever measure it.

- The Fermilab **unquenched** lattice calculation which is successful in reproducing the 1S hyperfine splitting, is not able to reproduce the 2S hyperfine splitting with better than $\pm 100\%$ error ($65 \pm 65$ MeV). It ascribes its failure to confusion in distinguishing 2S charmonium levels with "nearby multiple open–charm levels".

  **The problem of understanding hyperfine splitting in radial excitations remains open.**

# Hyperfine Splitting in Charmonium P–wave Levels

In this case, we have a very simple, and provocative theoretical expectation, namely

$$\Delta M_{hf}(1P) \equiv M(^3P) - M(^1P) = 0 \qquad (3)$$

A non-relativistic reduction of the Bethe-Salpeter equation makes the hyperfine interaction a **contact interaction**, so that $\boldsymbol{\Delta M_{hf}(L \neq 0) = 0}$.

- We can test this prediction in charmonium by
  - identifying the **singlet**–P state $h_c(1^1P_1)$, and by determining $M(^3P)$.

  The precisely measured masses of the **triplet**–P states gives $^3P$ centroid

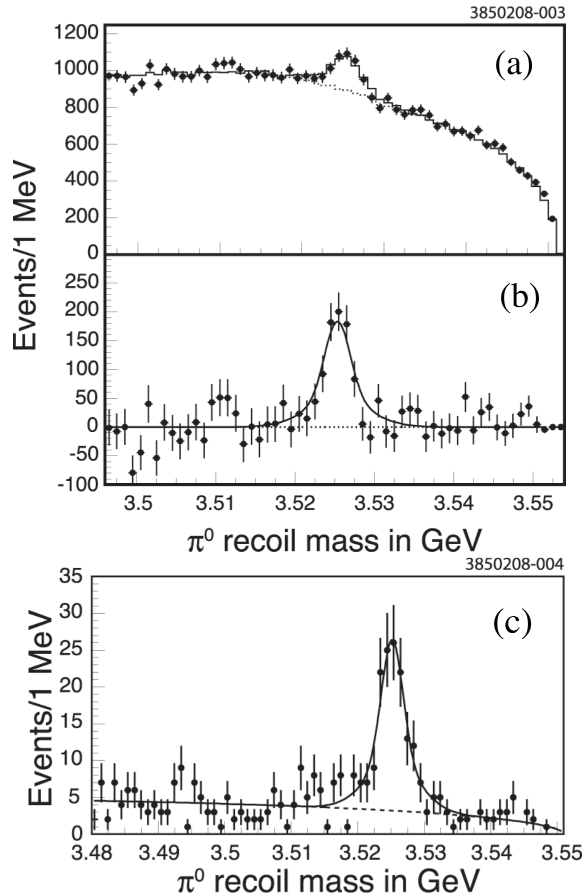
$$\langle M(^3P_J)\rangle = [5M(^3P_2) + 3M(^3P_1) + M(^3P_0)]/9 = 3525.30 \pm 0.04 \text{ MeV} \qquad (4)$$

- If Eqs. 3 and 4 are true, we expect $M(h_c) \approx 3525$ MeV, i.e., $\sim 160$ MeV below the $\psi'(2S)$ state from which it must be fed. There are two problems:
  - The radiative transition $\psi'(1^{--}) \to \gamma h_c(1^{+-})$ is forbidden by **charge conjugation** invariance.
  - A $\pi^0$ transition ($M(\pi^0) = 139$ MeV) is forbidden by **isospin conservation**, and has very little phase space.
    But this was the route taken by CLEO for the definitive identification of $h_c(^1P_1)$.

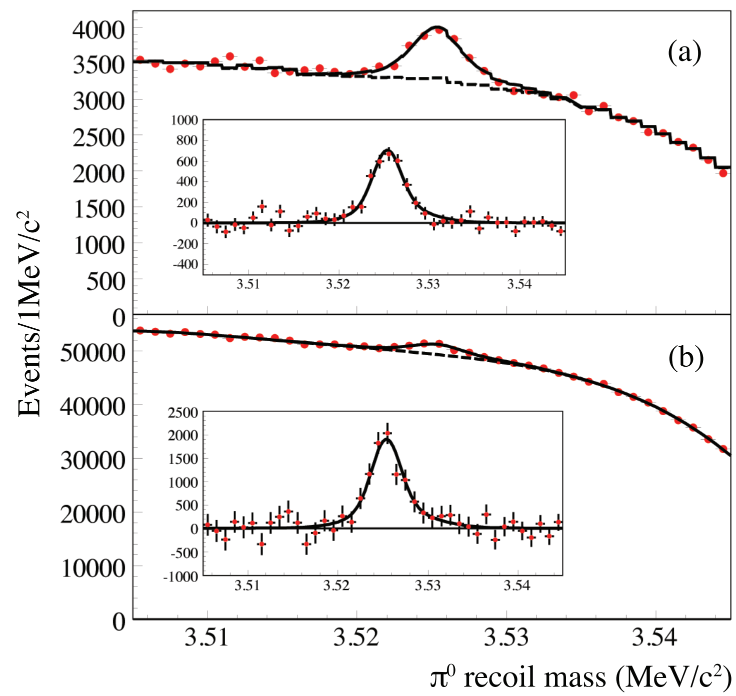
## Searches for $h_c(^1P_1)$

There is a history of $h_c(^1P_1)$ searches, extending from Crystal Ball (SLAC) search in 1982 to the Fermilab E835 search in 1992 and 2005. I will skip over this history and go on to the discovery of $h_c$ by CLEO and its recent confirmation by BES III.

- In **2005** the **CLEO Collaboration** (PRL 95, 102003 (2005)) reported a $> \boldsymbol{6\sigma}$ observation of $h_c$ in the reaction $\boldsymbol{\psi' \to \pi^0 h_c}$, $\boldsymbol{h_c \to \gamma\eta_c}$, with **3.08 million** $\boldsymbol{\psi'}$, followed in 2008 by the precision measurement of the mass of $h_c$ in the same reaction with **24.5 million** $\boldsymbol{\psi'}$ (PRL 101, 182003 (2008)).
  In inclusive and exclusive analyses, they observed $h_c$ with significance $> \boldsymbol{15\sigma}$, and obtained
  $\boldsymbol{M(h_c) = 3525.28 \pm 0.19 \pm 0.12}$ **MeV,** $\mathcal{B}_1 \times \mathcal{B}_2 = (4.19 \pm 0.32 \pm 0.45) \times 10^{-4}$**.**
- Recently, in **2010** the **BES III Collaboration** (PRL 104, 132002 (2010)) confirmed the CLEO observation with **106 million** $\boldsymbol{\psi'}$, and significance $\boldsymbol{18.6\sigma}$
  $\boldsymbol{M(h_c) = 3525.40 \pm 0.13 \pm 0.18}$ **MeV,** $\mathcal{B}_1 \times \mathcal{B}_2 = (4.58 \pm 1.3 \pm 1.0) \times 10^{-4}$**.**
  This agreement between the results of the CLEO and BES III measurements is quite remarkable.
- BES also determined $\mathcal{B}_1(\psi(2S) \to \pi^0 h_c) = (8.4 \pm 1.3 \pm 1.0) \times 10^{-4}$, $\mathcal{B}_2(h_c \to \gamma\eta_c) = (54.3 \pm 6.7 \pm 5.2)\%$, and $\Gamma < 1.44$ MeV (90% CL).




CLEO (2008)




BES III (2010)

(a) Inclusive, E1 tagged, (b) background subtracted, (c) Exclusive, $\eta_c$ decays

(a) Inclusive, E1 tagged, (b) untagged. (Inserts: background subtracted)

**If we assume $M(^3P) = \langle M(^3P_J) \rangle$,**

$$\Delta M_{hf}(1P) = +0.02 \pm 0.22 \text{ MeV} \quad \text{(CLEO)}$$
$$= -0.10 \pm 0.22 \text{ MeV} \quad \text{(BESIII)}$$

- Unfortunately, the identification $\langle M(^3P_J)\rangle = M(^3P)$ can not be correct because the centroid determination of $M(^3P)$ is only valid if the spin-orbit splitting is perturbatively small. But this is hardly true, with $M(^3P_2) - M(^3P_0) =$ **142 MeV**. Also, the pQCD prediction

$$[M(^3P_1) - M(^3P_0)] = (5/2) \times \left[M(^3P_2) - M(^3P_1)\right] = \mathbf{114\ MeV}$$

is actually 20% larger than the experimental result **96 MeV**.

- This leads to serious questions.
  - What mysterious cancellations are responsible for the wrong estimate of $M(^3P)$ giving the expected answer that $\boldsymbol{\Delta M_{hf}(1P) = 0}$.
  - Or, is it possible that the expectation is wrong? Is it possible that the hyperfine interaction is not entirely a **contact interaction**?
  - **Can Lattice help?** Not so far.

    For example, the Fermilab lattice result is that $\Delta M_{hf}(1P) \leq 10 \pm 10$ MeV.

## Hyperfine Interaction Between b–Quarks

The best system in which to study hyperfine interaction without the "contamination" due to the "confinement" part of the $q\bar{q}$ potential is the bottomonium system. The hyperfine splitting in this case is

$$\Delta M_{hf}(1S)_{b\bar{b}} \equiv M(\Upsilon(1S)) - M(\eta_b(1S))$$

and the transition photon in the radiative decay

$$\Upsilon(1S) \rightarrow \gamma\eta_b(1S)$$

is expected to have energy less than 100 MeV.

The identification of this photon in the presence of the very huge number of hadrons from the decay of $\Upsilon(1S)$, and also $\eta_b(1S)$ is a formidable task. Add to this the huge photon background, and you have a really tough job on hand.

For these reasons, although $\Upsilon(1S)$ was discovered in 1977, and numerous attempts were mae at ALEPH, DELPHI, CDF and CLEO for 31 years, $\eta_b(1S)$ was successfully identified only in 2008 by BaBar by looking not for the $<$ **100 MeV photon** from $\Upsilon(1S)$ but the $\sim$ **900 MeV photon** in the radiative decay from $\Upsilon(3S)$.

# Hyperfine Interaction Between b–Quarks

- In July 2008, BaBar announced the identification of $\eta_b$. They analyzed the inclusive photon spectrum of

$$\Upsilon(3S) \to \gamma\eta_b(1S)$$

in their data for **109 million** $\Upsilon(3S)$ (28 fb$^{-1}$ $e^+e^-$). BaBar's success owed to their very large data set and a clever way of reducing the continuum background, a cut on the so–called **thrust angle**, the angle between the transition photon and the thrust axis of the rest of the event. Their results were:

$$M(\eta_b) = 9388.9^{+3.1}_{-2.3} \pm 2.7 \text{ MeV}, \quad \Delta M_{hf}(1S)_b = 71.4^{+3.1}_{-2.3} \pm 2.7 \text{ MeV}$$

$$\mathcal{B}(\Upsilon(3S) \to \gamma\eta_b) = (4.8 \pm 0.5 \pm 1.2) \times 10^{-4}$$

- Any important discovery requires independent confirmation.
  At CLEO, we had a factor 20 smaller data set with **5.9 million $\Upsilon(3S)$**,
  compared to Babar's **109 million $\Upsilon(3S)$**.
  But we have the advantage of a symmetric collider and detector, and a better photon resolution and line shape. We therefore decided to revisit our $\Upsilon(3S, 2S)$ datasets to make a new search for $\eta_b$.

- So, how did we overcame our limitations? The important steps were

  1. We made a very detailed study of the non-peaking background,
  2. We made independent determinations of the photon peak shape parameters (Crystal Ball (CB) parameters $\sigma, \alpha, n$),
  3. And most importantly, we took full advantage of the variation of signal to background in the different regions of thrust angle distribution, and made a **joint analysis** of the data in three bins of thrust angle.

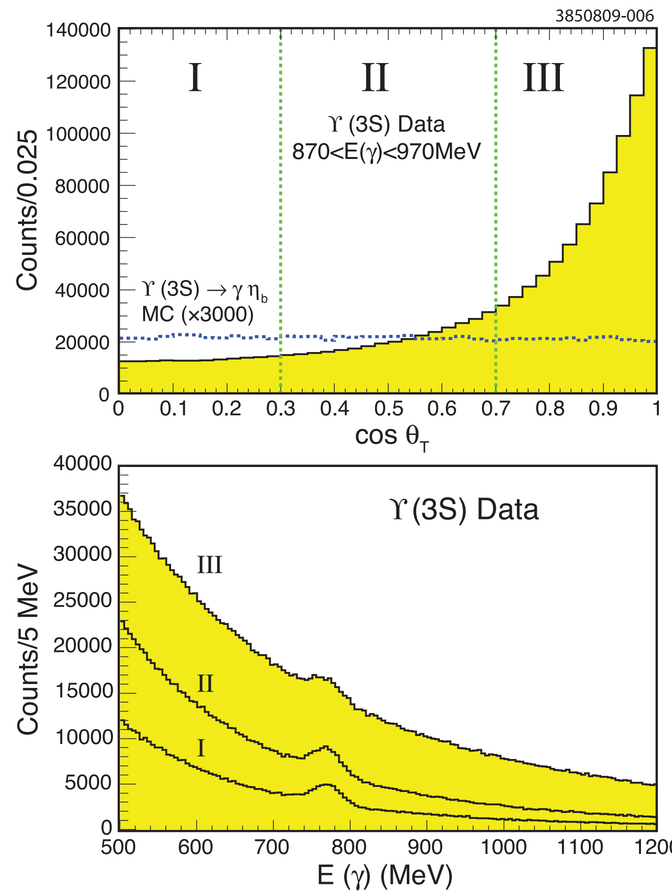
- Since our paper has been published (PRD 81, 031104(R) (2010)), I do not have to go into all the details of these improvements. Let me only describe the most important of these, namely our improved treatment of thrust angle information.

# Joint–Fit Analysis of Three $|\cos\theta_T|$ Bins

The $|\cos\theta_T|$ thrust angle distribution for the data is peaked in the forward direction, $|\cos\theta_T| \approx 1$, whereas for the $\eta_b$ it is expected to be uniform.

As a result, the data in the three different regions of $|\cos\theta_T|$ have different ratios of signal/background as shown in the bottom plot.

- Unlike BaBar, we do not cut out the $|\cos\theta_T| > 0.7$ region. Instead, we let each region contribute to the total result weighted by its individual signal-to-background. Since none of the data are rejcted, we preserve full statistics, and are free of instabilities in making thrust cuts.
- We have analyzed our data by the joint–fit method, and for comparison purposes also with $|\cos\theta_T| < 0.7$. The joint–fit method enhances the significance of the $\eta_b$ identification by $\sim 1\sigma$.



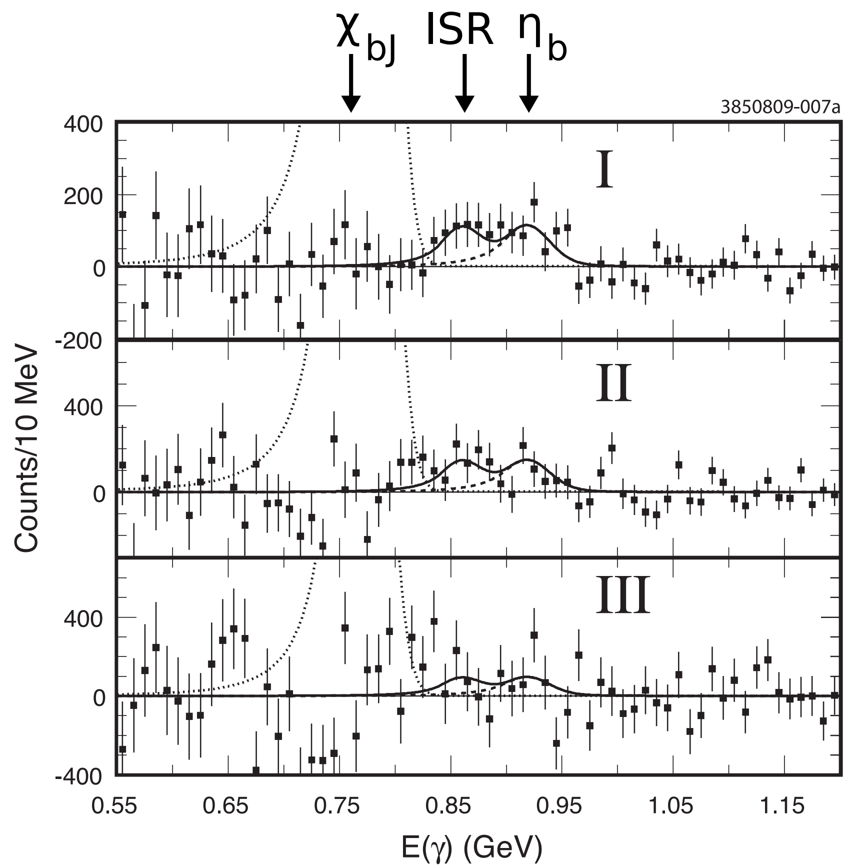
# $\Upsilon(3S) \to \gamma\eta_b(1S)$ Results

Our results are:

- $E_\gamma(\eta_b) = 918.6 \pm 6.0 \pm 1.8$ MeV, which leads to
  $M(\eta_b) = 9391.8 \pm 6.6 \pm 2.0$ MeV
  $\boldsymbol{\Delta M_{hf}(1S)_b = 68.5 \pm 6.6 \pm 2.0}$ **MeV**
  $\mathcal{B}(\Upsilon(3S) \to \gamma\eta_b) =$
  $(7.1 \pm 1.8 \pm 1.1) \times 10^{-4}$

- BaBar subsequently also searched for $\eta_b(1S)$ in $\Upsilon(2S) \to \gamma\eta_b(1S)$, and reported the observation of $\eta_b(1S)$ only at a $3\sigma$ significance level.

- We also attempted to identify $\eta_b(1S)$ in $\Upsilon(2S) \to \gamma\eta_b(1S)$, but were not successful.



# Summary of $\eta_b(1S)$ Results

Following is the summary of the $\eta_b(1S)$ results, and their comparison with theory.

| | | $\Delta M_{hf}(1S)_{b\bar{b}}$, (MeV) | $\mathcal{B}(\Upsilon(nS) \to \gamma\eta_b) \times 10^4$ | significance |
|---|---|---|---|---|
| $\Upsilon(3S) \to \gamma\eta_b$ | (CLEO) | $68.5 \pm 6.6 \pm 2.0$ | $7.1 \pm 1.8 \pm 1.1$ | $4\sigma$ |
| | (BaBar) | $71.4^{+3.1}_{-2.3} \pm 2.7$ | $4.8 \pm 0.5 \pm 0.6$ | $\geq 10\sigma$ |
| $\Upsilon(2S) \to \gamma\eta_b$ | (CLEO) | — | $< 8.4$ (90% CL) | — |
| | (BaBar) | $67.4^{+4.8}_{-4.6} \pm 2.0$ | $4.2^{+1.1}_{-1.0} \pm 0.9$ | $3.5\sigma$ |
| | Average | $69.6 \pm 2.9$ | | |
| Unquenched Lattice | | | | |
| | (UKQCD+HPQCD) | $61 \pm 14$ | (PRD 72, 094507 (2005)) | |
| | (RBC/UKQCD) | $52.5 \pm 1.5\pm$(syst) | (PRD 79, 094501 (2009)) | |
| | (FNAL/MILC) | $54.0^{+1.2}_{-0.0} \pm 12.4$ | (PRD 81, 034508 (2010)) | |
| pQCD (various) | | $35 - 100$ | $0.05 - 25$ ($\Upsilon(3S)$) | |
| | | | $0.05 - 15$ ($\Upsilon(2S)$) | |

It is very encouraging to see that unquenched lattice calculations with $b$–quarks are getting to be quite successful, even though errors are still large.

# Summary of all Hyperfine Results

To summarize, we now have well–measured experimental results for several hyperfine singlet/triplet splittings.

$|c\bar{c}\rangle$ Charmonium: $\Delta M_{hf}(1S) = 116.7 \pm 1.2$ **MeV**

$\Delta M_{hf}(2S) = 43.2 \pm 3.4$ **MeV**

$\Delta M_{hf}(1P) = 0.02 \pm 0.22$ **MeV**

$|b\bar{b}\rangle$ Bottomonium: $\Delta M_{hf}(1S) = 69.6 \pm 2.9$ **MeV**

- In charmonium, we do not have satisfactory understanding of the variation of hyperfine splitting for the S–wave radial, and P–wave hyperfine splitting.
  No unquenched lattice predictions for these are available, at present.
- For bottomonium, lattice predictions are available, and they appear to be on the right track.
- **For neither charmonium or bottomonium there are any reliable predictions of transitions strength, particular for forbidden M1 transitions.**

Our understanding of the hyperfine interaction in quarkonia has greatly improved in the last five years as a result of the experimental measurements, but many theoretical questions remain to be resolved.