$Q\bar{q}q\overline{Q}$ charmonium threshold states and QQq potentials

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- Lattice QCD
- Threshold charmonia
- Outlook I
- QQq baryonic potentials
- Outlook II

Charmonium results from GB & Christian Ehmann, arXiv:0710.0256, arXiv:0903.2947, arXiv:0911.1238, in prep.

QQq potentials from GB & Johannes Najjar, arXiv:0910.2824, in prep.

$Q\bar{q}q\overline{Q}$ potentials (not discussed): GB & Martin Hetzenegger, in prep.

Input: $\mathcal{L}_{QCD} = -\frac{1}{16\pi\alpha_L} FF + \bar{q}_f(\not{D} + m_f)q_f$

$$\begin{aligned} m_N^{\rm latt} &= m_N^{\rm phys} &\longrightarrow\quad & a \\ m_\pi^{\rm latt}/m_N^{\rm latt} &= m_\pi^{\rm phys}/m_N^{\rm phys} &\longrightarrow\quad & m_u \approx m_d \\ &\ldots \end{aligned}$$

Output: hadron masses, matrix elements, decay constants, etc...

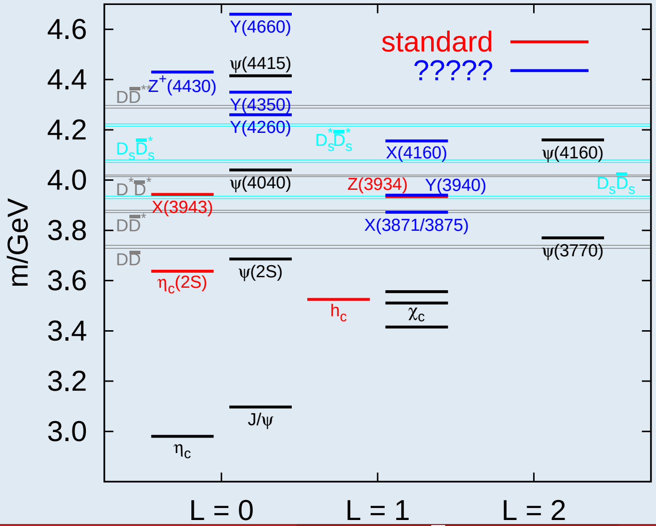
Extrapolations:

1. $a \rightarrow 0$: functional form known.
2. $L \rightarrow \infty$: harmless but often computationally expensive.
3. $m_q^{\rm latt} \rightarrow m_q^{\rm phys}$: chiral perturbation theory (χPT) but $m_q^{\rm latt}$ must be sufficiently small to start with.

   ($m_{\rm PS}^{\rm latt} = m_\pi^{\rm phys}$ has only very recently been realized.)

1974 – 1977: 10 $c\bar{c}$ resonances, 1978 – 2001: 0 $c\bar{c}$'s

2002 – 2008: $\leq$ 12 new $c\bar{c}$'s found by BaBar, Belle, CLEO-c, CDF, D0



new detectors

higher luminosity

new channels:

B decays

$\gamma\gamma$

$\psi\psi$-production

gg in $p\bar{p}$ collisions.

$c\bar{q}q\bar{c}$ in $c\bar{c}$?

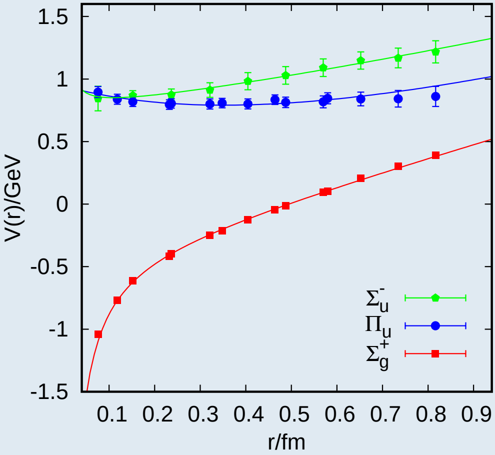
$cg\bar{c}$ hybrids ?

Hybrid mesons

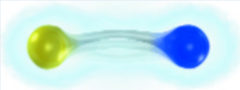
$m_c \gg \Lambda_{\rm QCD} \longrightarrow$ Adiabatic and non-relativistic approximations:

$$H\psi_{nlm} = E_{nl}\psi_{nlm} \quad , \quad H = 2m_c + \frac{p^2}{m_c} + V(r)$$

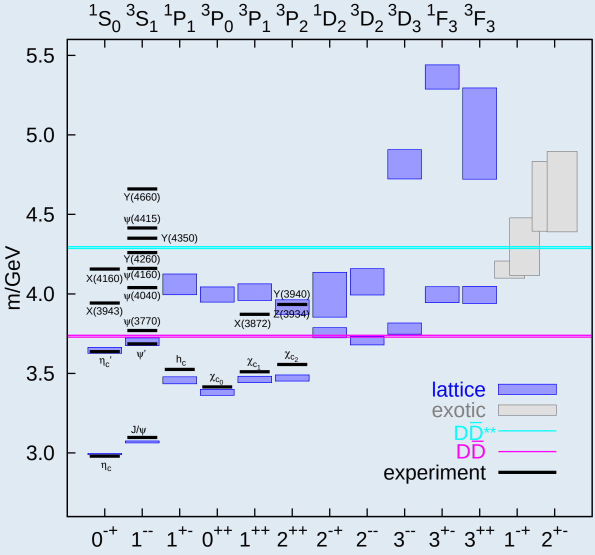
Lattice:



hybrid potential:

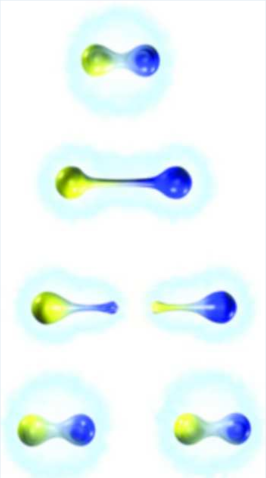


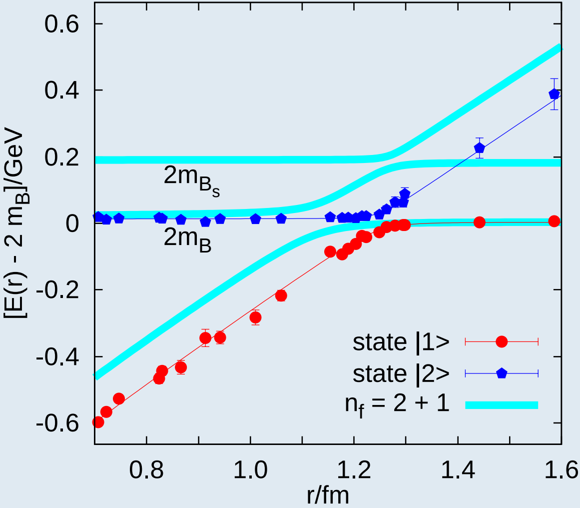
C Ehmann, GB 07 ($n_f = 2$, $a^{-1} \approx 1.73\,\text{GeV}$ from m_N)



Two state potentials

GB, H Neff, T Düssel, T Lippert, Z Prkacin, K Schilling 04/05





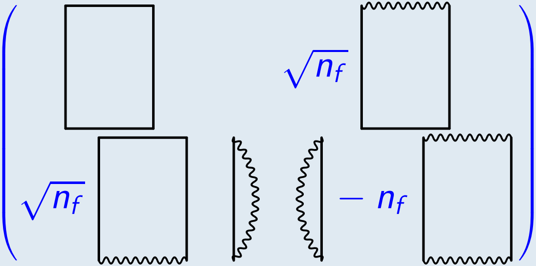
Two state system:

Eigenstates:

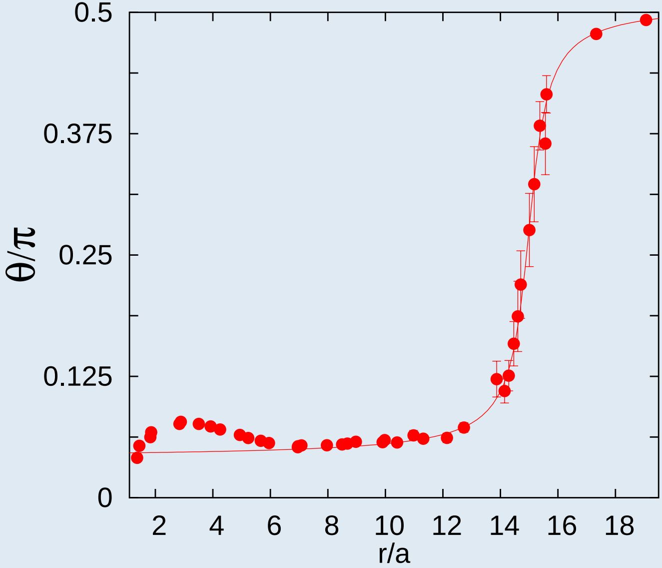
$$
\begin{aligned}
|1\rangle &= \cos\theta\,|\overline{Q}Q\rangle + \sin\theta\,|B\overline{B}\rangle \\
|2\rangle &= -\sin\theta\,|\overline{Q}Q\rangle + \cos\theta\,|B\overline{B}\rangle
\end{aligned}
$$

with $B = \overline{Q}q$.

Correlation matrix:



Mixing angle: $B\overline{B}$ content of the ground state



$a \approx 0.083\,\mathrm{fm}$

Coupled channel potential model for threshold effects ?

Many channels ($D\overline{D}, D^*\overline{D}, D_s\overline{D}_s, D^*\overline{D}^*, \cdots$) $\Rightarrow$ many parameters!

However, very good to address qualitative questions:

For what I, S and radial excitation do we get attraction/repulsion ?

Are Z^+s possible and/or likely ?

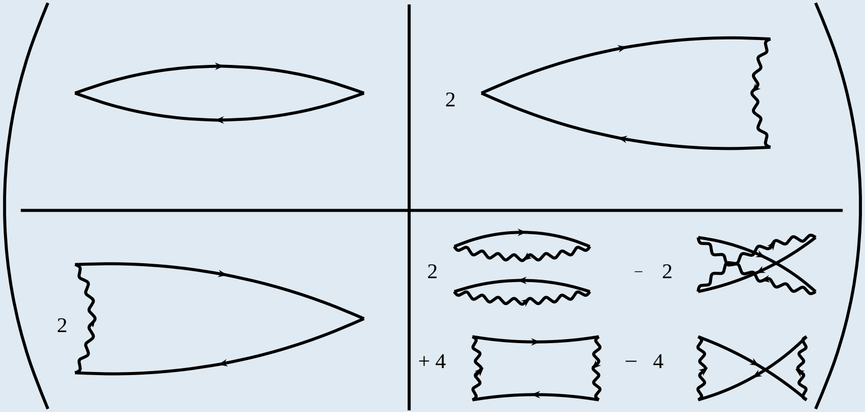
"Direct" calculation of the spectrum ?

We have to be able to resolve radial excitations!

(remember e.g. the very dense 1^{--} sector.)

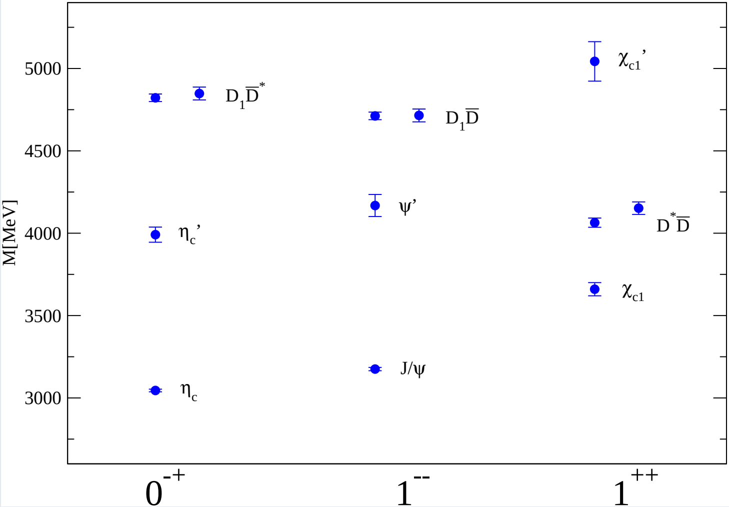
Required: large basis of test wavefunctions including $c\bar{c}$, $c\bar{q}q\bar{c}$ and $cg\bar{c}$ operators and good statistics.

$c\bar{c} \leftrightarrow \overline{D}D$ mixing (for $n_f = 2$) GB, C Ehmann 09/10:

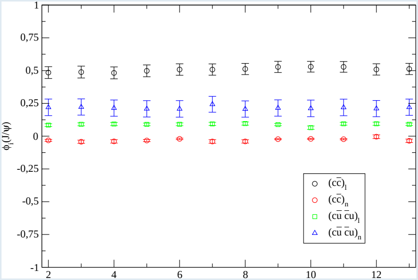


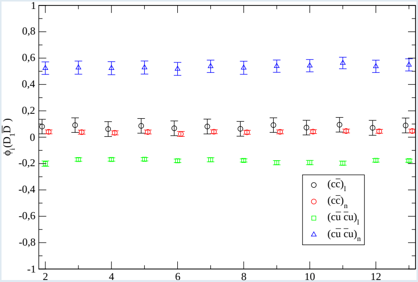
($c\bar{c}$ annihilation diagrams negelected.)

$n_f = 2,\ a^{-1} \approx 2.59\,\text{GeV},\ La \approx 1.83\,\text{fm},\ m_{\text{PS}} \approx 290\,\text{MeV}$

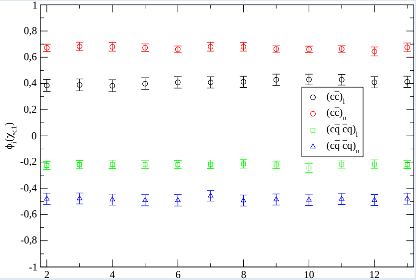


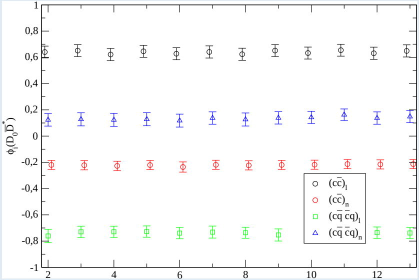
Eigenvector components of the J/ψ. Components of the $D_1\overline{D}$.





Eigenvector components of the χ_{c1}. Components of the $D^*\overline{D}$.





Outlook I

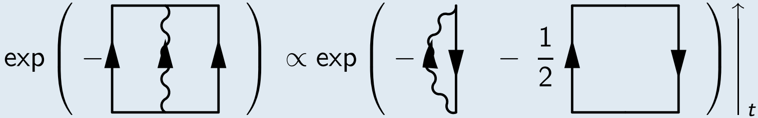
- $\exists$ first simulations near the physical m_π at $a^{-1} \approx 2$ GeV.
- $\exists$ first precision calculations of annihilation and mixing diagrams.
- Study of $c\bar{c} \leftrightarrow c\bar{q}q\bar{c}$ is well on its way.
- The continuum limit is important, in particular for the fine structure.
- There will be a lot of progress in charmonium spectroscopy below and above decay thresholds in the next years.
- Forces between pairs of static-light mesons for different S and I are being studied, to qualitatively understand 4-quark binding ($X(3872)$, $Z^+(4430)$ etc.).

QQq: factorization

Distance r between Q and Q in static-static-light baryon (QQq).

In the limit $r \to 0$ this becomes a $\overline{Q}q$ static-light meson.

For small r, the factorization



should hold:

$V_{QQq}(r) \simeq m_{\overline{Q}q} + \frac{1}{2} V_{\overline{Q}Q}(r) \quad (r \ll \Lambda^{-1})$

(NB: the $1/m$ corrections to the static limit are different, even at $r = 0$.)

Minimal string picture with QQ tension $= \frac{1}{2}\, Q\overline{Q}$ string tension:

$V_{QQq}(r) \simeq \text{const} + V_{\overline{Q}Q}(r) \quad (r \gg \Lambda^{-1})$

How does the light quark see the two static quarks?

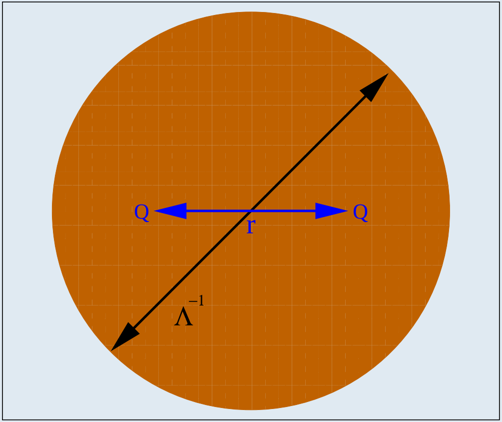


Figure: This is the HQET picture for $r \ll \Lambda^{-1}$.

How does the light quark see the two static quarks?

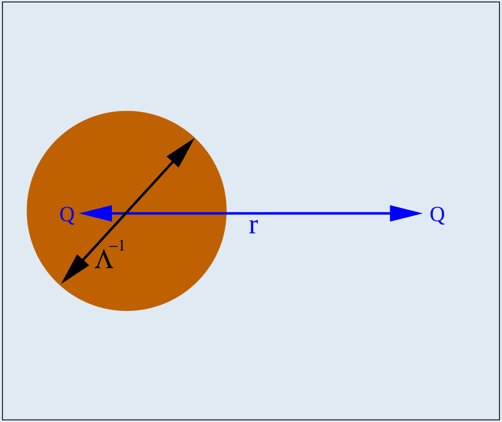


Figure: $r \gg \Lambda$: light quark is near static source.

How does the light quark see the two static quarks?

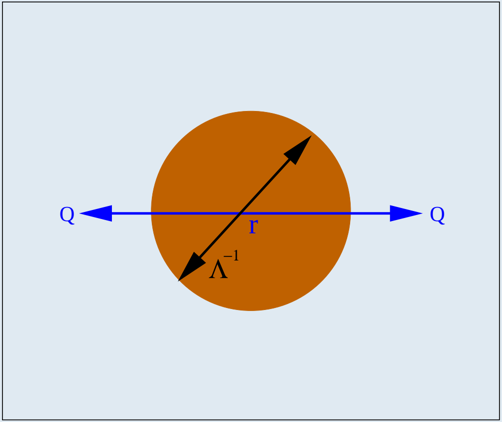


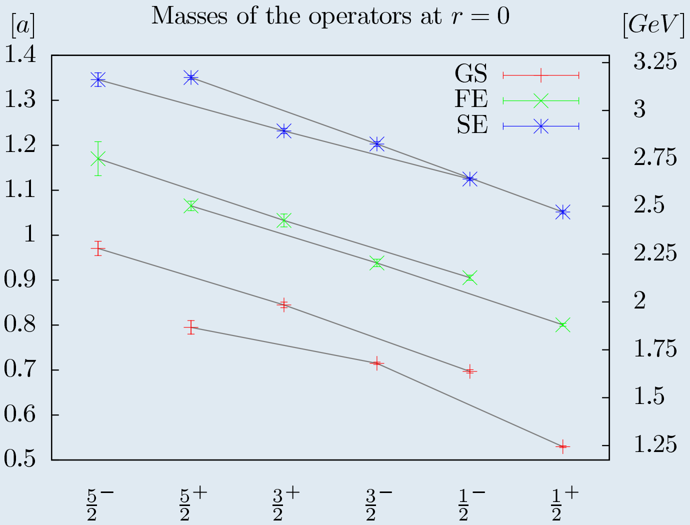
Figure: $r \gg \Lambda$: light quark is in the centre.

Construction of the states

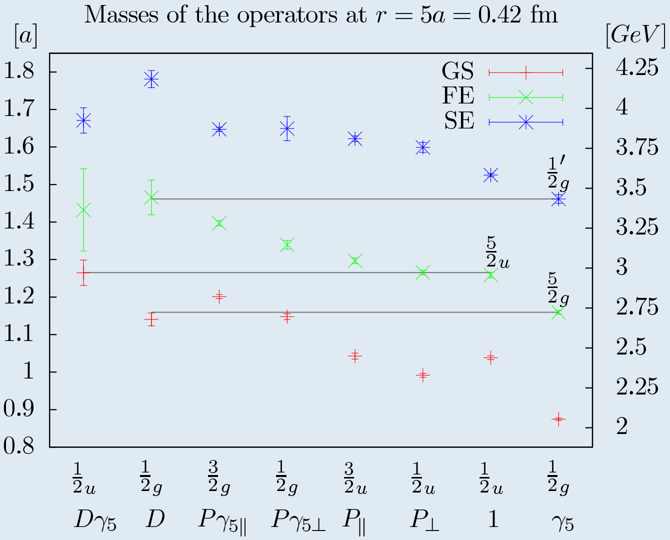
Wave	Operator	$r=0$ $O'(3), O'_h$	$r>0$ $D'_{\infty h}, D'_{4h}$
S	γ_5	$\frac{1}{2}^+, G_1^+$	$\frac{1}{2}_g, G_{1g}$
P_-	1	$\frac{1}{2}^-, G_1^-$	$\frac{1}{2}_u, G_{1u}$
P_+	$\gamma_1\Delta_1 - \gamma_2\Delta_2 \oplus$ cyclic	$\frac{3}{2}^-, H^-$	$\frac{3}{2}_u \parallel, G_{2u}$ $\frac{1}{2}_u \perp, G_{1u}$
D_-	$\gamma_5(\gamma_1\Delta_1 - \gamma_2\Delta_2) \oplus$ cyclic	$\frac{3}{2}^+, H^+$	$\frac{3}{2}_g \parallel, G_{2g}$ $\frac{1}{2}_g \perp, G_{1g}$
D_+	$\gamma_1\Delta_2\Delta_3 + \gamma_2\Delta_3\Delta_1 + \gamma_3\Delta_1\Delta_2$	$\frac{5}{2}^+, G_2^+$	$\frac{1}{2}_g / \frac{5}{2}_g, G_{1g}$
F_-	$\gamma_5(\gamma_1\Delta_2\Delta_3 + \gamma_2\Delta_3\Delta_1 + \gamma_3\Delta_1\Delta_2)$	$\frac{5}{2}^-, G_2^-$	$\frac{1}{2}_u / \frac{5}{2}_u, G_{1u}$

Table: ΓD Dirac structure.

$r = 0$: Regge trajectories

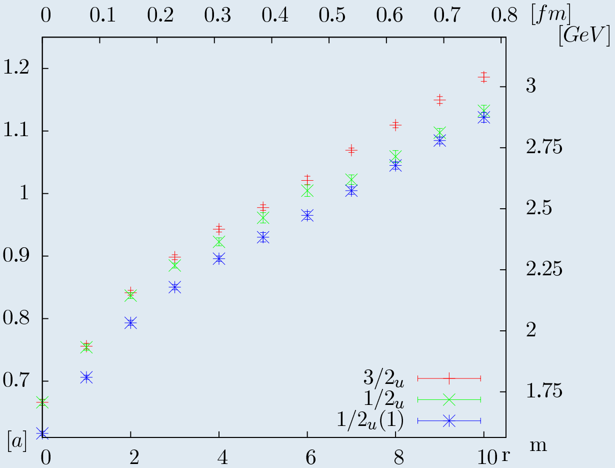


$r > 0$: overview

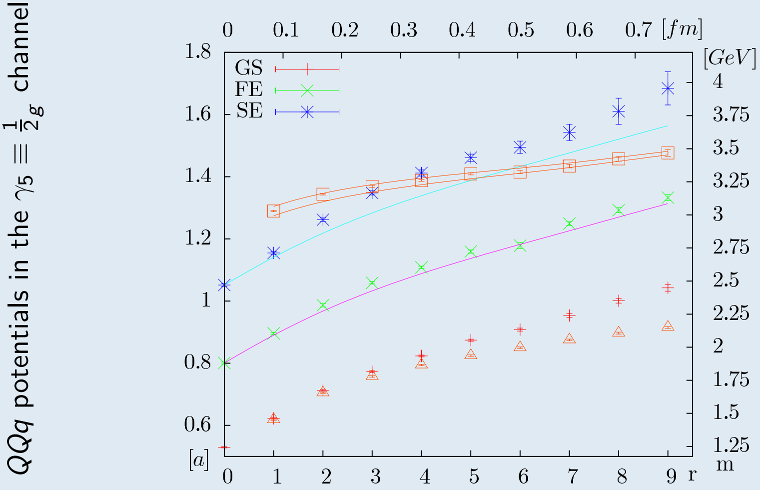

Masses of the operators at $r = 5a = 0.42$ fm
[a]
[GeV]
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
4.25
4
3.75
3.5
3.25
3
2.75
2.5
2.25
2
GS
FE
SE
$\frac{1}{2}'_g$
$\frac{5}{2}_u$
$\frac{5}{2}_g$
$\frac{1}{2}_u$ $\frac{1}{2}_g$ $\frac{3}{2}_g$ $\frac{1}{2}_g$ $\frac{3}{2}_u$ $\frac{1}{2}_u$ $\frac{1}{2}_u$ $\frac{1}{2}_g$
$D\gamma_5$ D $P\gamma_{5\|}$ $P\gamma_{5\perp}$ $P_\|$ $P_\perp$ 1 γ_5

The degeneracy problem understood

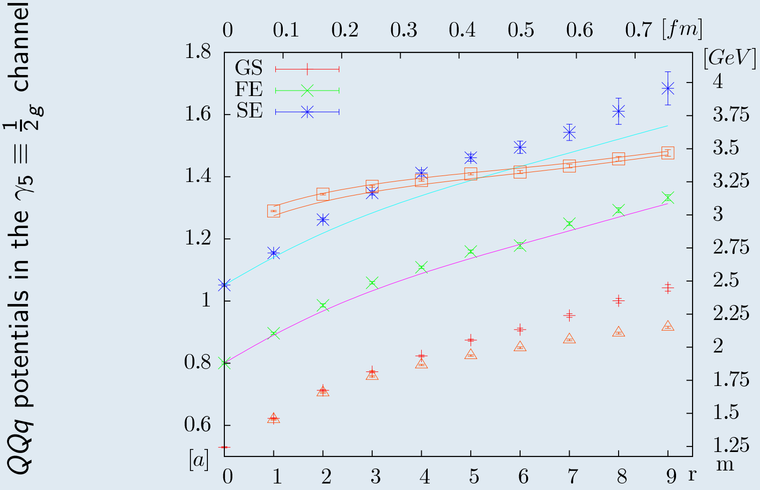
Lattice with lighter $m_\pi \approx 430\,\text{MeV}$ and $a \approx 0.08\,\text{fm}$



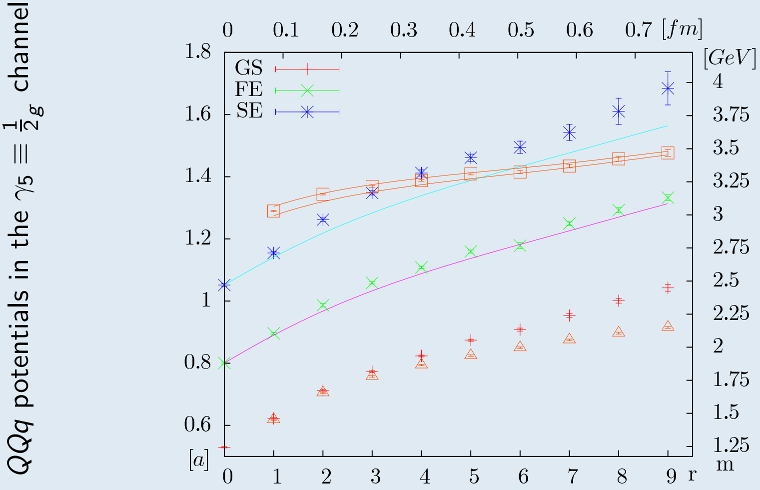
$P_-(1/2_u)$, $P_{+,\perp}(1/2_u)$ and $P_{+,\parallel}(3/2_u)$ fit results (one exponential).



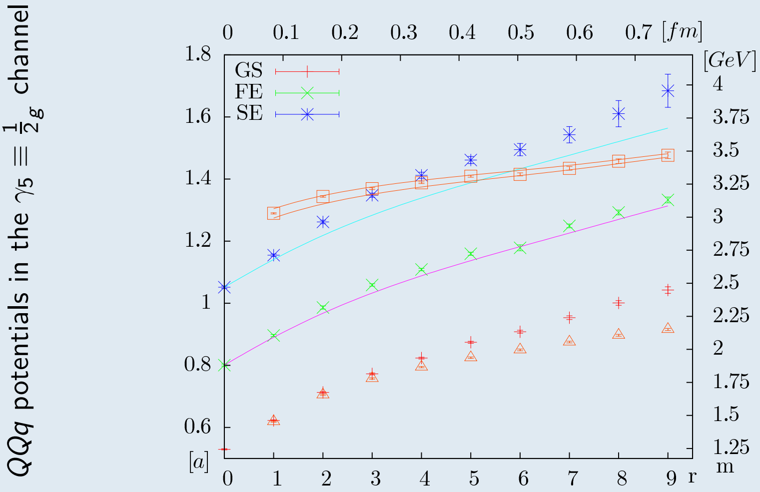
Red, green and blue crosses are QQq potentials.



Orange triangles are the factorization $m_{\overline{Q}q} + \frac{1}{2}V_{Q\overline{Q}}(r)$.

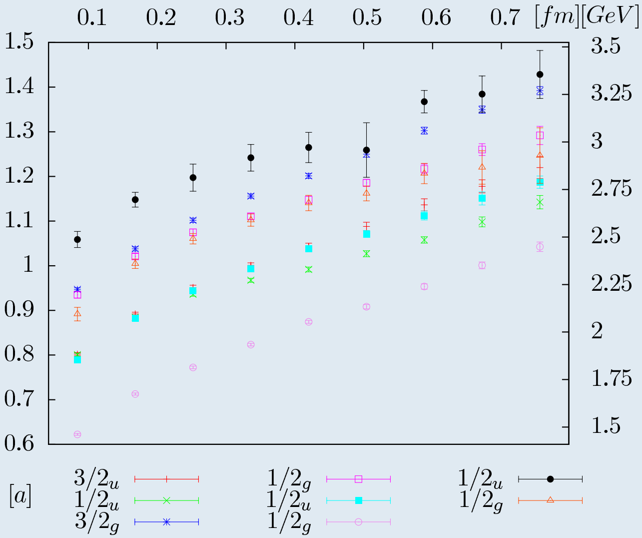


Pink and light line are ground state points, shifted by the respective static-light energy splittings.



The red band is the Nambu-Goto expectation for the first gluonic hybrid excitation: $E_2 - E_0 + GS$, where $E_n(r) = \sigma_{GS} r \sqrt{1 + \left(2n - \frac{d-2}{12}\right) \frac{\pi}{\sigma r^2}}$.

The groundstate potentials in comparison



Outlook II

- The HQET factorization applies to $r \ll \Lambda^{-1}$.
- The scale where this factorization breaks down depends on the state.
- Light quark excitations are more important than gluonic ones.
- Not shown: correlators with the light quark in the centre mostly have a better ground state overlaps $\rightarrow$ no evidence for Qq diquark formation.
- Ongoing: decreasing the light quark mass to increase Λ^{-1}.
- See also the work on the ground state QQq potential by Yamamoto and Suganuma 08.