B-mesons and charmonium masses from lattice QCD

Elvira Gámiz for **HPQCD** collaboration

Fermi National Accelerator Laboratory

Batavia, Illinois

International Workshop on Heavy Quarkonium

· Fermilab, May 18 (2010) ·

1. Introduction: Lattice QCD

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`Precise lattice calculations`: **for stable (or almost stable) hadron masses and amplitudes with no more then one initial (final) state hadron.**

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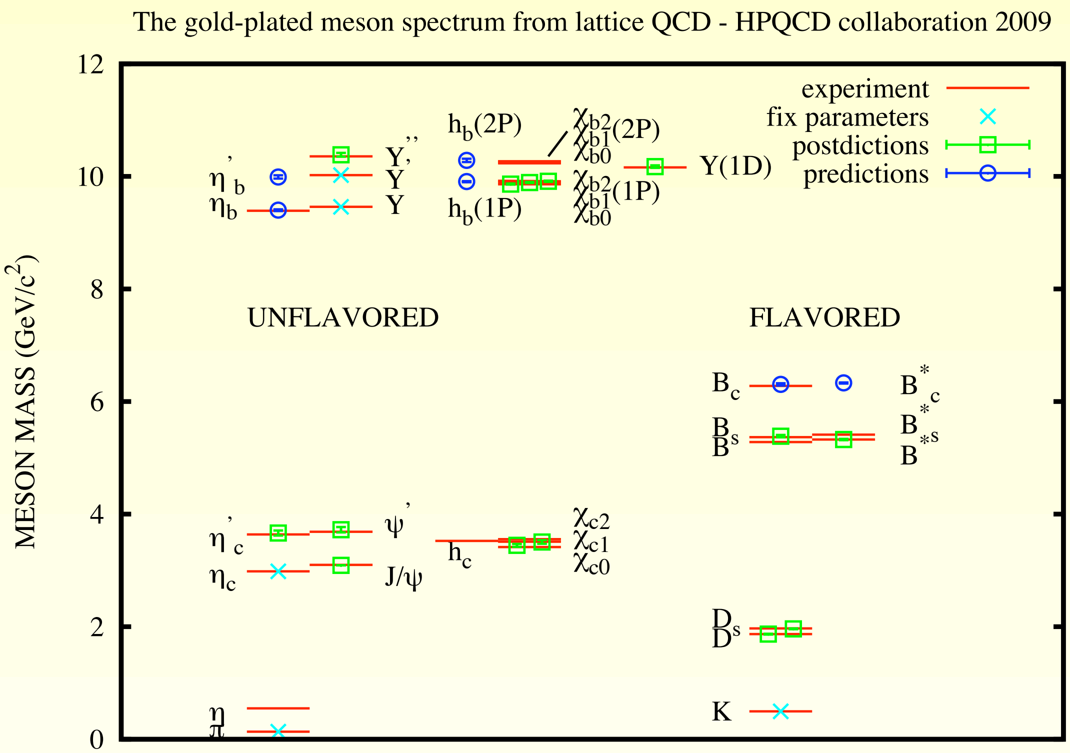
Precise lattice studies of $c\bar{c}$ and $b\bar{c}(s)$ systems:

* Provide stringent tests of lattice techniques and formulations, and of our understanding of strong interactions.

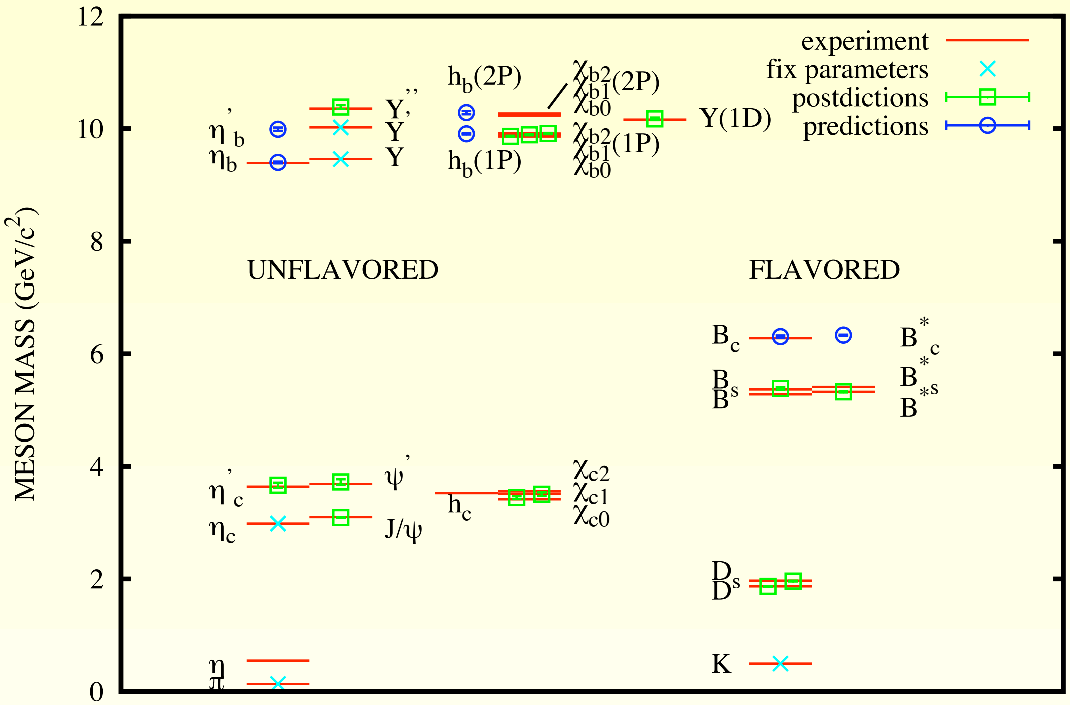
Spectrum of gold-platted mesons from HPQCD



E. B. Gregory et al. [HPQCD] PRL.104:022001(2010)

Spectrum of gold-platted mesons from HPQCD

The gold-plated meson spectrum from lattice QCD - HPQCD collaboration 2009



\# Using the same values for $m_{u,d}$, m_s, m_c and m_b

E. B. Gregory et al. [HPQCD] PRL.104:022001(2010)

\# Lattice **FNAL/HPQCD** predictions:

* $m_{B_c}^{lat} = (6304 \pm 12^{+18}_{-0}) MeV$ $\quad$ $m_{B_c}^{exp.} = (6277 \pm 6) MeV$

* $\left(m_\Upsilon - m_{\eta_b}\right)^{lat} = (61 \pm 14) MeV$ $\quad$ $\left(m_\Upsilon - m_{\eta_b}\right)^{exp.} = (68.5 \pm 6.9) MeV$

2. Lattice description of heavy quarks

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2.1. Effective theories: NRQCD

Heavy quark is non-relativistic in bound states

$\rightarrow$ $m_b a$ is not an important dynamical scale

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Non-relativistic expansion of the Dirac lagrangian.

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On lattice, hamiltonian is (improved through $\mathcal{O}(1/M^2)$, $\mathcal{O}(a^2)$):

$$aH_0 = -\frac{\Delta^{(2)}}{2(aM_0)} \quad \textbf{non-relat. kinetic energy oper.}$$

$$a\delta H = -c_1 \frac{(\Delta^{(2)})^2}{8(aM_0)^3} + c_2 \frac{i}{8(aM_0)^2}\left(\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla\right)$$

$$-c_3 \frac{1}{8(aM_0)^2} \boldsymbol{\sigma} \cdot (\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla})$$

relativistic and discretization corrections

$$-c_4 \frac{1}{2(aM_0)} \boldsymbol{\sigma} \cdot \tilde{\mathbf{B}} + c_5 \frac{\Delta^{(4)}}{24(aM_0)} - c_6 \frac{(\Delta^{(2)})^2}{16n(aM_0)^2} + \cdots$$

* Spin-independent terms to order v_b^4 and leading spin-dependent terms with discretization errors through a^2

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* Spin-independent terms to order v_b^4 and leading spin-dependent terms with discretization errors through a^2

* c_i fixed pert. or non-pert. matching to QCD

2.2. Charm quarks

Charm quark is in between the heavy and light mass regimes

* Heavy quark effective theories do not give accurate results.

* Relativistic descriptions: Maintain cut-off effects under control requires

  ** Improved actions and currents.

  ** Fine enough lattices

2.3. Staggered fermions: HISQ action

E. Follana et al, HPQCD coll., Phys.Rev.D75:054502 (2007)

- Highly improved staggered action (relativistic).
- Much improved control of discretization errors.
  - * No tree level a^2 errors (Asqtad). Highly reduce $\mathcal{O}(a^2\alpha_s)$ errors (an order of magnitude)
  - * No tree-level $\mathcal{O}((am)^4)$ at first order in the quark velocity v/c

    $\longrightarrow$ accurate results for charm quarks (can use Hisq for $a \leq 0.15\ fm$)

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    * **Relativistic bottom ($am_b < 1$) possible if $a < 0.04\ fm$ lattices are generated** (current values $a \geq 0.045\ fm$)

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  * **Relativistic bottom ($am_b < 1$) possible if $a < 0.04\ fm$ lattices are generated** (current values $a \geq 0.045\ fm$)

  * Current status: Simulations at masses $m_c \leq m_h < m_b$ and several lattice spacings $\rightarrow$ fit heavy quark mass dependence (HQET) including a corrections

    ** Comparison of extrapolated results with those using NRQCD and experiment

3. Charmonium states

Unquenched simulations with $N_f = 2 + 1$ MILC configurations.

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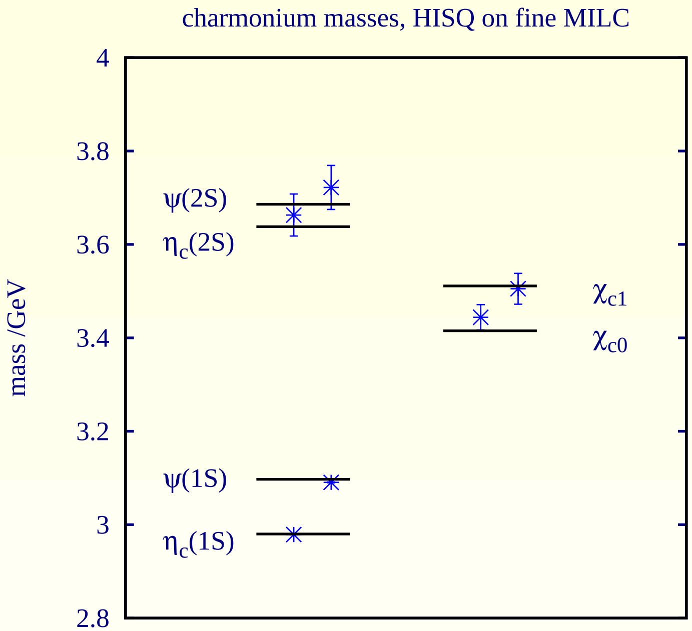

* One lattice spacing $a = 0.09\ fm$.
* No optimization of higher order states yet $\longrightarrow$ needs more statist.

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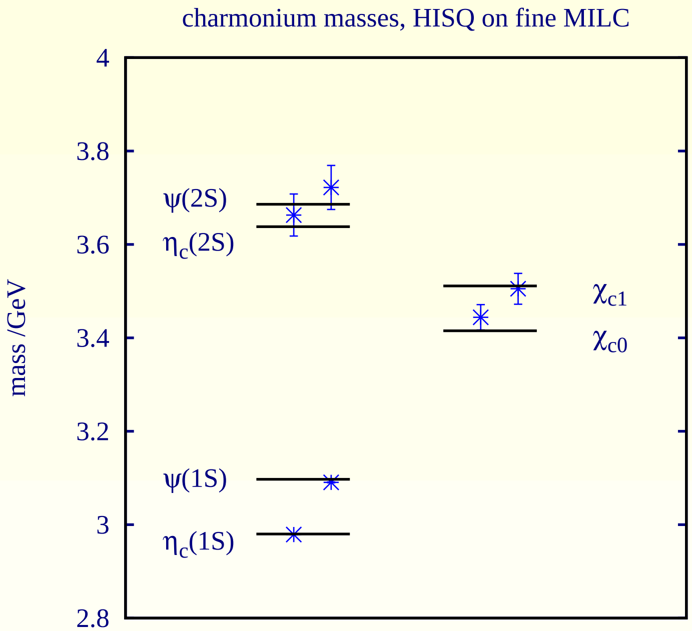

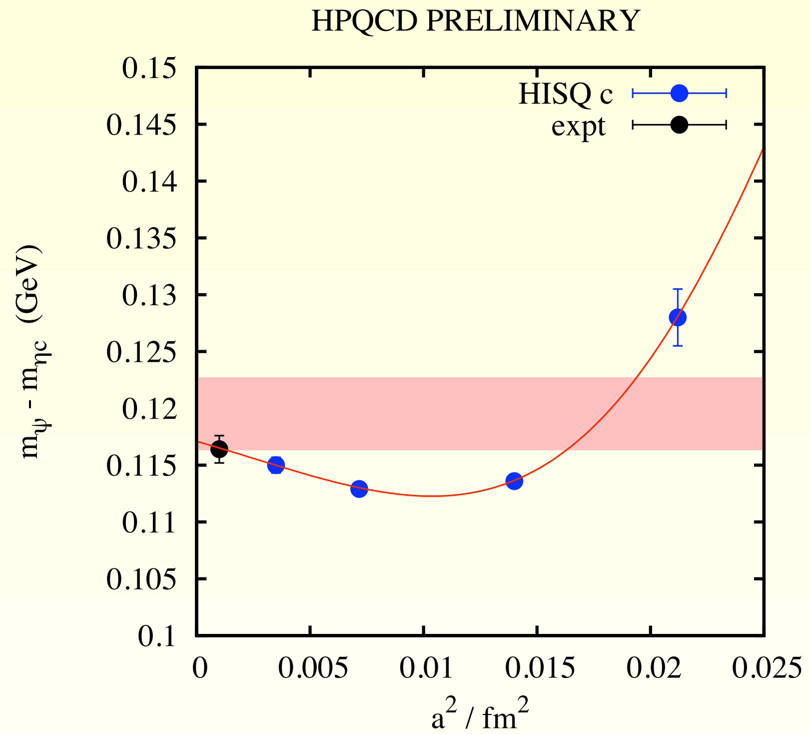

* One lattice spacing $a = 0.09\ fm$.
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* Band: final result after including $c - \bar{c}$ annihilation and em effects (error $\simeq 3.5\mathrm{MeV}$)

4. $B - meson$ states

No free parameters

The same parameters can be used for `heavy-heavy`, `light-light` and `heavy-light` states $\rightarrow$ important cross-checks

$$\begin{array}{lrcl} \Upsilon & 2S-1S\,\texttt{splitting},\, m_{D_s}-m_{\eta_c},\, f_{\eta_s}, & \rightarrow & a^{-1} \\ & V^{hh}(r),\, m_{\eta_s} & & \\ & m_{\Upsilon}, m_{\eta_b} & \rightarrow & m_b \\ & m_{\eta_c} & \rightarrow & m_c \\ & m_{\pi} & \rightarrow & m_{u/d} \\ & m_K & \rightarrow & m_s \end{array}$$

4.1 Pseudoscalar $B - mesons$: B_s and B_c

(**bottom described with NRQCD, light and charm with Hisq**)

Energies of the ground state (E_1) and excited states can be extracted from 2-point correlation functions.

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* Two methods used (differences enable high accuracy)

$$\text{hh method} \quad M_{B_c} = \left(E_{B_c} - \tfrac{1}{2}(E_{b\bar{b}} + E_{c\bar{c}})\right)_{\text{latt}} + \tfrac{1}{2}\left(M_{b\bar{b}} + M_{c\bar{c}}\right)$$

$$\text{hl method} \quad M_{B_c} = \left(E_{B_c} - (E_{B_s} + E_{D_s})\right)_{\text{latt}} + \left(M_{B_s} + M_{D_s}\right)$$

with $E_{b\bar{b}(c\bar{c})}$ and $M_{b\bar{b}(c\bar{c})}$ the spin-averaged lattice energies and experimental masses of $b\bar{b}(c\bar{c})$ states respectively.

$$M_{b\bar{b}} = (3M_{\Upsilon} + M_{\eta_b})/4, \qquad M_{c\bar{c}} = (3M_{\Psi} + M_{\eta_c})/4,$$

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* Differences reduce the sensitivity to a (needed to convert lattice results to physical units)

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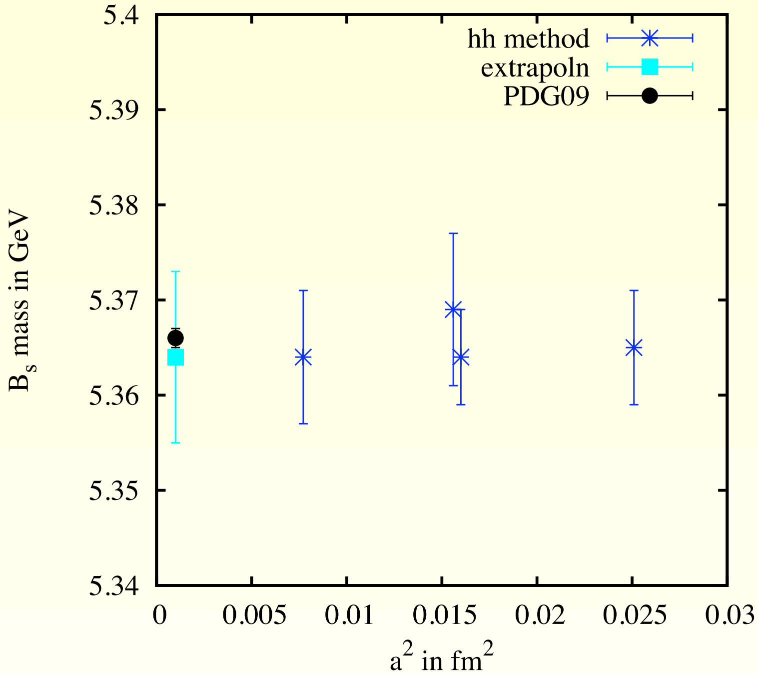
Eric Gregory et al. HPQCD collaboration

preliminary results for m_{B_s}



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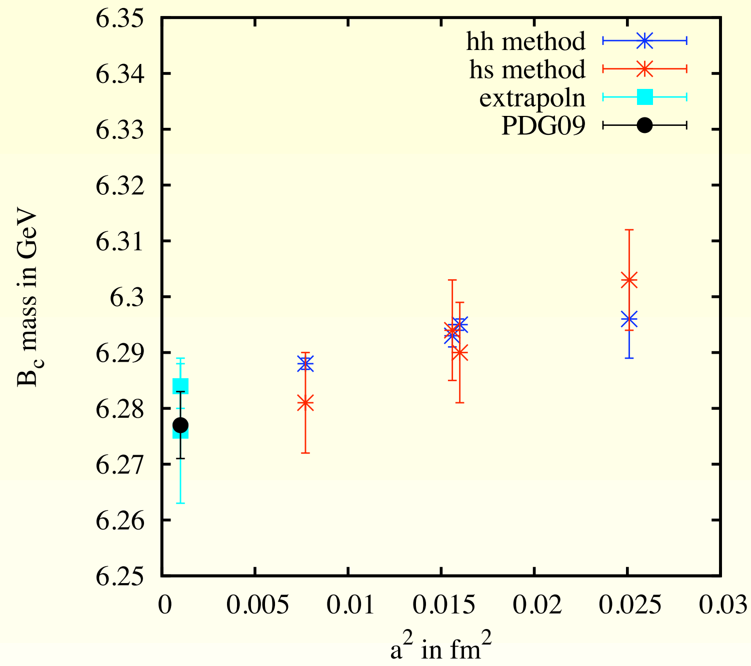

$$m_{B_s}^{preliminary} = 5.364(9)(5)\ GeV$$
$$m_{B_s}^{experiment} = 5.3663(6)\ GeV$$

$$m_{B_c}^{preliminary}(hh) = 6.284(4)(5)(2)\ GeV$$
$$m_{B_c}^{preliminary}(hl) = 6.276(13)\ GeV$$
$$m_{B_c}^{experiment} = 6.277(6)\ GeV$$

* Results for m_{B_c} from hh and hl methods agree
$\rightarrow$ consistent description of `hh` and `hl` systems.

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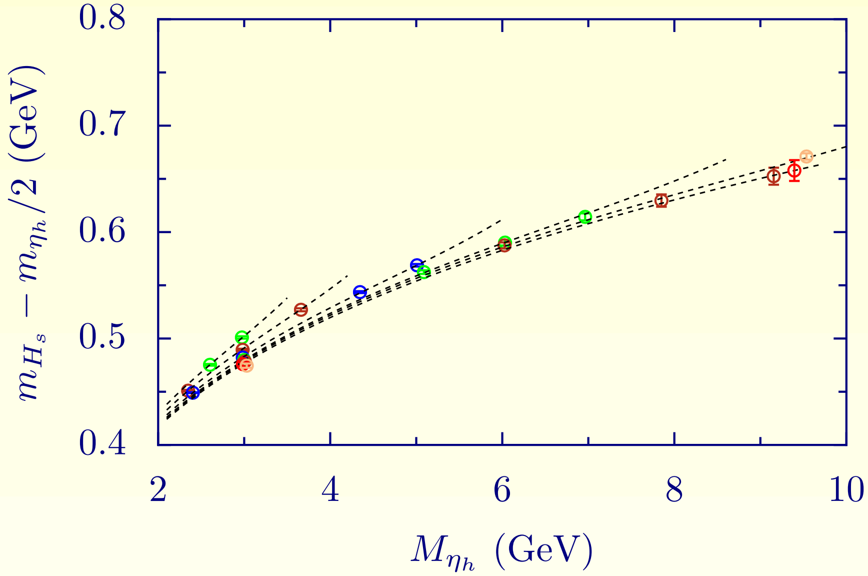
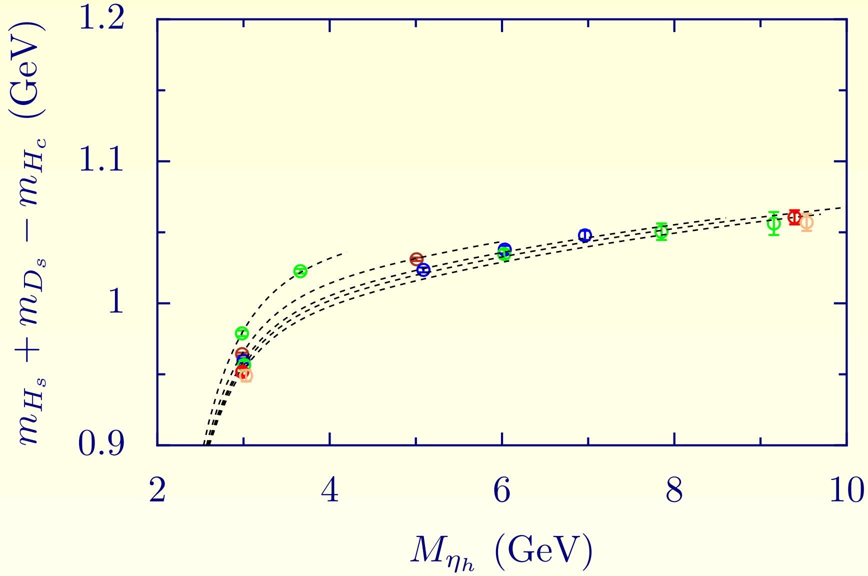
(light, charm, and bottom described with Hisq)

hh method for m_{B_s}



hl method for m_{B_c}



* Use five(four) values of a: $(0.15\ fm)$, $0.12\ fm$, $0.09\ fm$, $0.06\ fm$ and $0.045\ fm$ from left to right.
* Red circles: Interp./extrap. values at the physical M_{η_b} and M_{η_c}.
* Apricot points: Experimental values (without `annihilation` and `em` effects)
* Dashed lines: Fits to functions of M_{η_h} from HQET.

4.2 B_c^* mass: Hyperfine pseudoscalar-vector splittings **E.B. Gregory et at**, Phys.Rev.Lett.104:022001(2010)

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B_c^* not yet seen experimentally.

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Correct the NRQCD energy shift by taking the difference $M_{B_c} - M_{B_c^*}$.

Problem: Hyperfine pseudoscalar-vector splitting generated by NRQCD term

$$-c_4 \frac{\vec{\sigma} \cdot \vec{B}}{2m_Q}$$

* We use c_4 tree-level value

→ radiative corrections uncertainty $\mathcal{O}(\alpha_s) \sim 20\%$

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* We use c_4 tree-level value
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Solution: Take the ratio of B_c and B_s splittings → uncertainty cancel.

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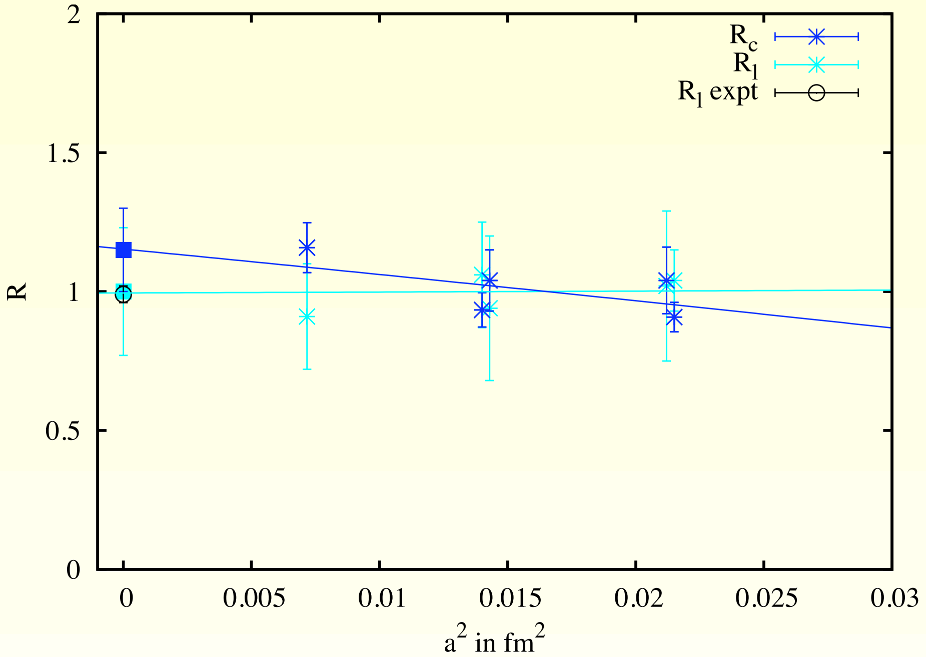
Results



* Simulations at three different values of the lattice spacing.

$$R_c = \frac{m_{B_c^*} - m_{B_c}}{m_{B_s^*} - m_{B_s}}$$

$$R_l = \frac{m_{B^*} - m_B}{m_{B_s^*} - m_{B_s}}$$

After extrapolation to the continuum and physical masses and using experimental (PDG) value of $B_s^* - B_s$ and B_c: $m_{B_c^*} = 6.330(7)(2)(6)$ GeV

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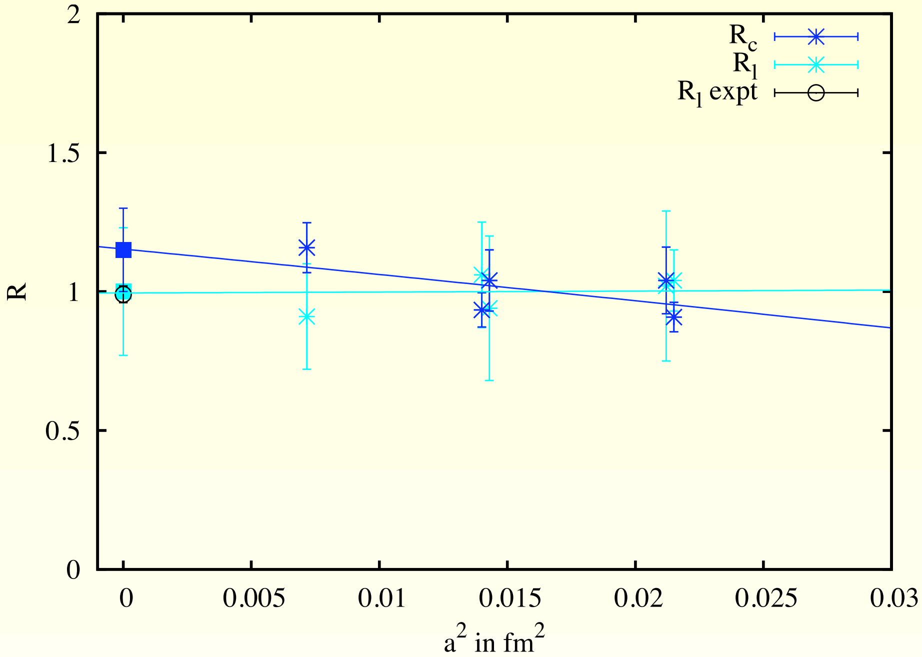

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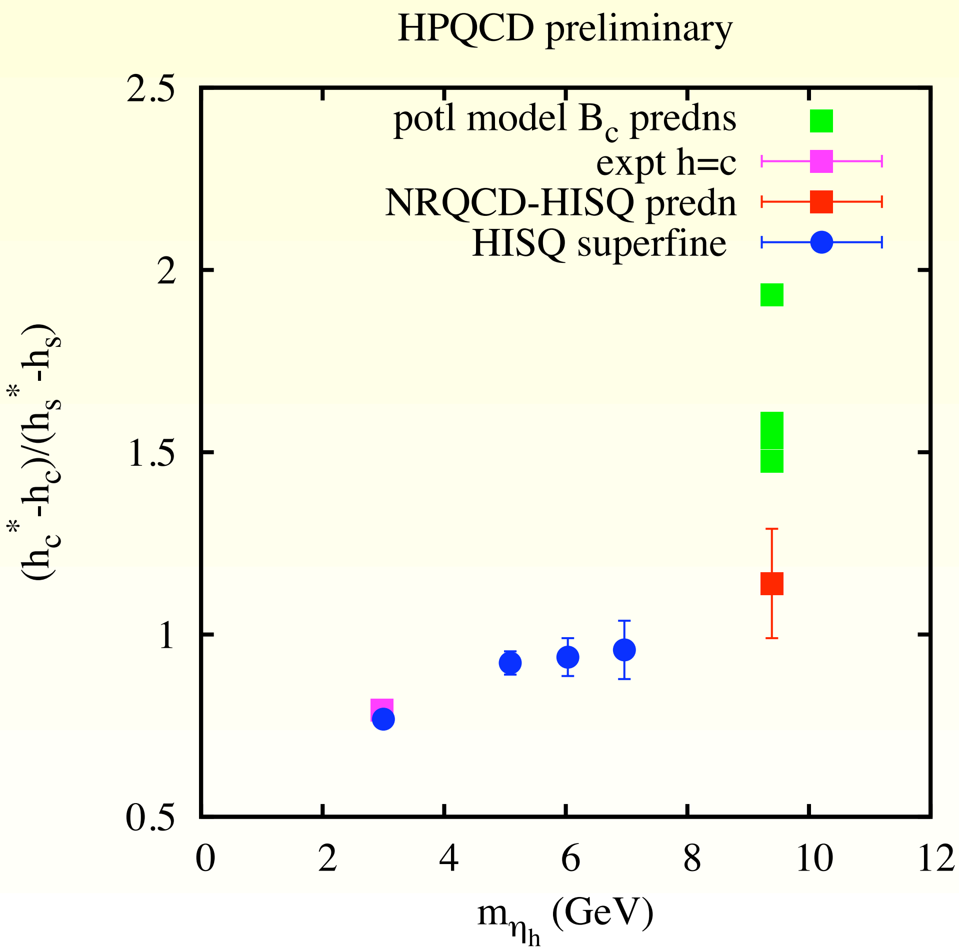
$M_{B_c^*} - M_{B_c}$ not very different from $M_{B_s^*} - M_{B_s}$. Potential models generally find much larger differences

4.2 B_c^* mass: Hyperfine pseudoscalar-vector splittings E.B. Gregory et at, Phys.Rev.Lett.104:022001(2010)

(light, charm, and bottom described with Hisq)

Comparison NRQCD-Hisq, Hisq-Hisq and potential models



* $h_{c(s)}$ is a meson with quark content $h\bar{c}(\bar{s})$, where $m_c \leq m_h < m_b$
* Hisq-Hisq results at one a
* Need extrapolation to m_{η_b}
* Potential models results from Eichten and Quigg, PRD49:5845,1994

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Highly improved actions (Hisq) will allow us to treat bottom relat.

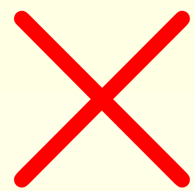
* Preliminary results for B-meson masses and decay constants with $m_h < m_b$ and extrapolation (HQET) to the physical m_b agree well with experiment and $NRQCD - Hisq$ results

* Eliminate the errors associated to higher terms in NRQCD/HQET descriptions and (in some cases) renormalization
  $\rightarrow$ very promising for achieving high accuracy results

5. Conclusions and outlook

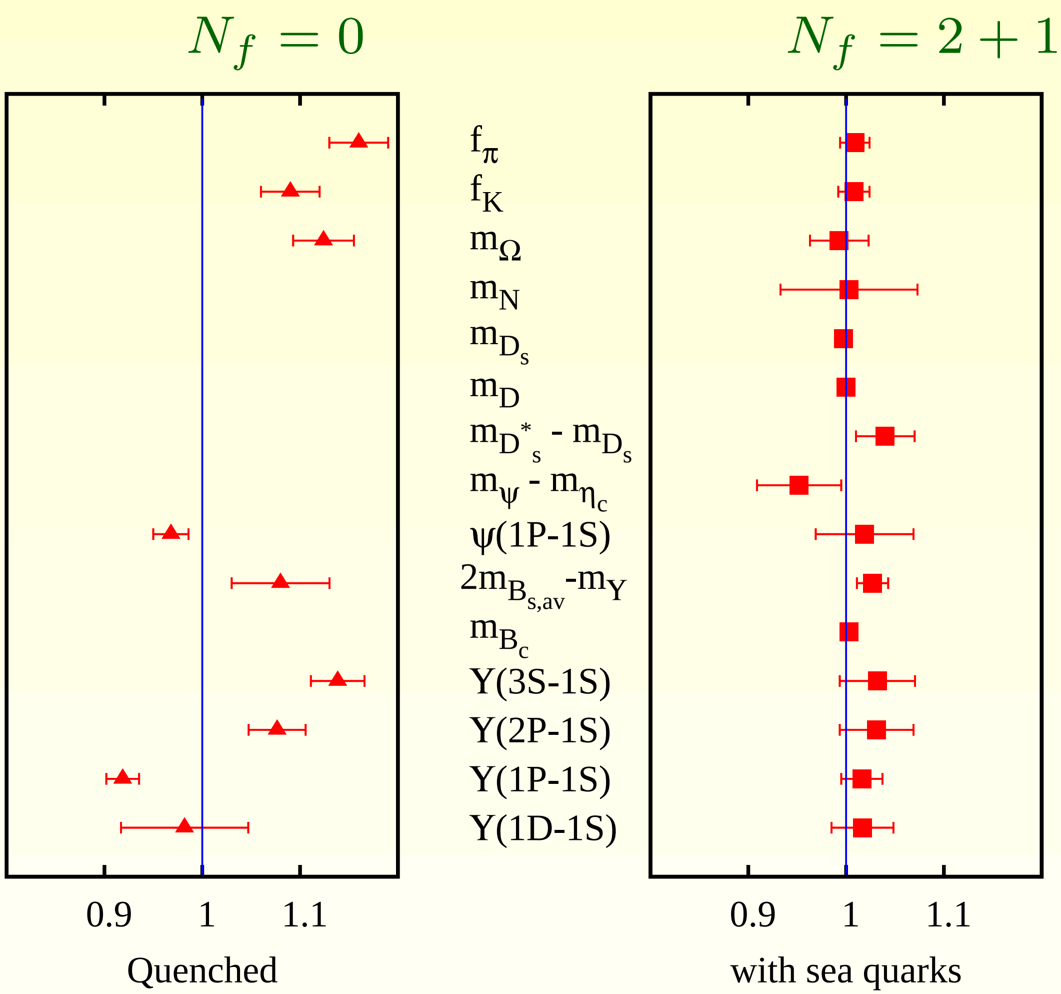
Studies of spectrum provide tests of lattice formulations, techniques, and error analyses, and accurate methods to fix lattice parameters

$\longrightarrow$ increase confidence in calculations of other phenomenologically important quantities (decay constants, form factors, ...)



Quenched approximation: neglect vacuum polarization effects

→ uncontrolled and irreducible errors



Experimental quantities are quite well reproduced by lattice

when including realistic sea quark effects

Heavy quark formalisms for D mesons

Fermilab action: `Relativistic` clover action with Fermilab (HQET) interpretation

  * Smooth interpolation between static limit and light quarks

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→ Very precise results for charm physics: charmonium and D

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Twisted mass QCD at maximal twist (tuning a single parameter)

  * Meson masses and decay constants $\mathcal{O}(a)$ improved.
  * No need for renormalization for decay constants (PCAC)
  * Mass renormalization multiplicative and calculated NP

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$\mathcal{O}(a)$ **improved Wilson**: improvement in action and currents.

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Relativistic simulations at masses $\sim m_c$ $\longrightarrow$ **fit functions determined by HQET** bottom

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Step Scaling Method (HQET):

* Simulate b in a small volume: calculate an observable $O(L_0, m_b)$.
* Eliminate finite size effects through SS functions:
  ** $\sigma(L, s, m_h) = \frac{O(sL, m_h)}{O(L, m_h)}$ for $s > 1$ and $m_h < m_b$
  ** Assume mild dependence of finite size effects on high energy scale
* Extrapolate SS functions in $1/m_h$ to m_b

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* Simulate b in a small volume: calculate an observable $O(L_0, m_b)$.

* Eliminate finite size effects through SS functions:

** $\sigma(L, s, m_h) = \frac{O(sL, m_h)}{O(L, m_h)}$ for $s > 1$ and $m_h < m_b$

** Assume mild dependence of finite size effects on high energy scale

* Extrapolate SS functions in $1/m_h$ to m_b

HQET: static + $1/M$