Quarkonium from three-flavor lattice QCD

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Outline

- Motivation
- Lattice QCD and the Fermilab method for heavy quarks
- Splittings in the charmonium and bottomonium systems:
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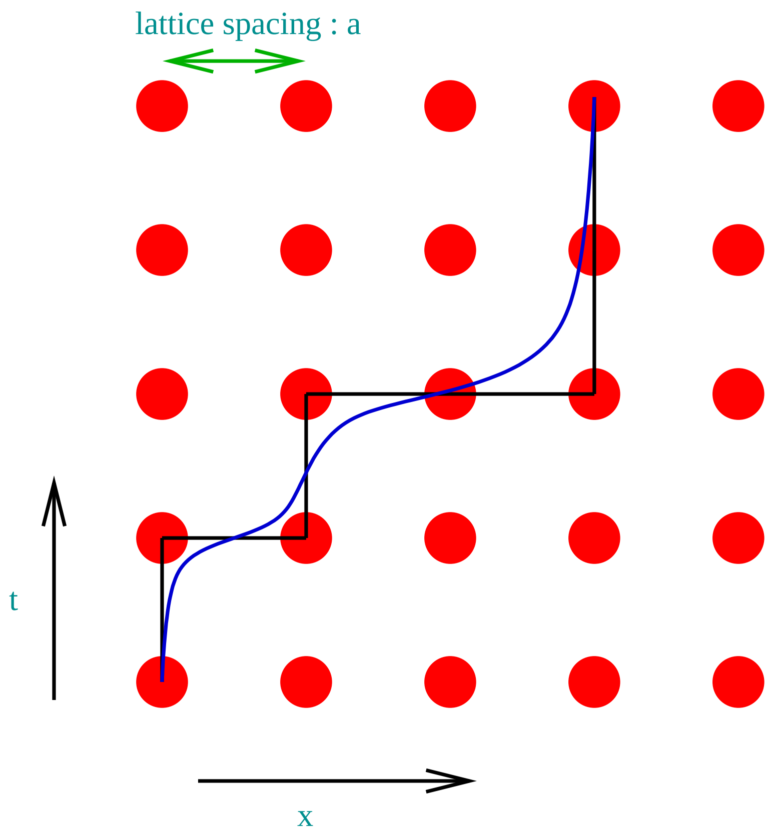
- Conclusions and future plans

Motivation

- Study the quarkonium spectrum in an *ab initio* calculation based on lattice QCD with three flavors of quarks.
- Test the Fermilab action, which treats heavy and light quarks within the same framework.
- Assess the errors that arise from our treatment of the heavy quarks.

Lattice QCD



- Discretized version of QCD on a 4d space-time lattice (Wilson, 1974).
- Nonperturbative approach to QCD.
- Solved by large scale numerical simulations using supercomputers.
- The lattice spacing is a.
  - Continuum limit taken as $a \longrightarrow 0$.
  - Momentum cut-off $O(1/a)$.

- Fermions fields live on the lattice sites. They are represented by pseudofermions (complex vectors).
- Gluons encoded in links ($SU(3)$ matrices) between sites.

Heavy-quark formulation

$$
\begin{aligned}
S = \sum_n \bar{\psi}_n \psi_n - \kappa \sum_n & \left[\bar{\psi}_n (1-\gamma_4) U_{n,4} \psi_{n+\hat{4}} + \bar{\psi}_{n+\hat{4}} (1+\gamma_4) U^{\dagger}_{n,4} \psi_n \right] \\
& - \kappa\zeta \sum_{n,i} \left[\bar{\psi}_n (r_s - \gamma_i) U_{n,i} \psi_{n+\hat{\imath}} + \bar{\psi}_{n+\hat{\imath}} (r_s + \gamma_i) U^{\dagger}_{n,i} \psi_n \right] \\
& - c_B \kappa\zeta \sum_n \bar{\psi}_n i \Sigma \cdot B_n \psi_n - c_E \kappa\zeta \sum_n \bar{\psi}_n \alpha \cdot E_n \psi_n ,
\end{aligned}
$$

- We adjust κ, ζ, r_s, c_B and c_E so that the lattice gauge theory matches the NRQCD description of continuum QCD with controllable uncertainty.
  We set $r_s = 1$, $\zeta = 1$ and $c_E = c_B = u_0^{-3}$.
- Expected errors:
  - Hyperfine splittings — $O(\alpha_s m v^4)$ and $O(v^6)$, larger for charmonium.
  - Spin-orbit part of the χ splittings — $O(a^2 m^3 v^4)$, larger for charmonium.
  - Spin-averaged splittings — $O(a^2 m^3 v^4)$, larger for bottomonium.
- All of the above errors are within a few to several percent of the splitting.

Tuning of the heavy quark mass

- The hopping parameter κ and bare quark mass m_0 are related:

  $$m_0 a = \frac{1}{2\kappa} - 1 - 3r_s\zeta$$

  To tune the quark mass we tune κ.

- Nonrelativistic interpretation of Wilson fermions:

  $$E(p) = m_1 + \frac{p^2}{2m_2} + O(p^4),$$

  The quark rest mass is m_1 and the kinetic mass is m_2. Their relation to m_0 can be calculated in perturbation theory. We have $m_1 \neq m_2$ unless $m_0 a \ll 1$

- Similarly we define for a meson:

  $$\begin{aligned} M_1 &= 2m_1 + B_1 \\ M_2 &= 2m_2 + B_2 \end{aligned}$$

- We tune κ to make $M_2(\overline{1S})$ (approximately) equal to the experimental value of a heavy-light hadron (D_s and B_s).

Constructing and fitting the quarkonium correlators

- The meson propagator at spatial momentum p:

  $$C_{ab}(p,t) = \sum_x e^{-ip\cdot x} \langle 0|O_a(x,t)O_b^\dagger(0,0)|0\rangle,$$

  with

  $$O_c(x,t) = \sum_y \bar{\psi}(x,t)\Gamma\phi_c(x-y)\psi(y,t).$$

  We use relativistic operators and non-relativistic ones for the P-wave states.

- Fitting the correlators:
  - Bayesian fits with priors from potential models.
  - Use simultaneous fits to up to three source-sink combinations.
  - Include up to 2 excited states in fitting functions.

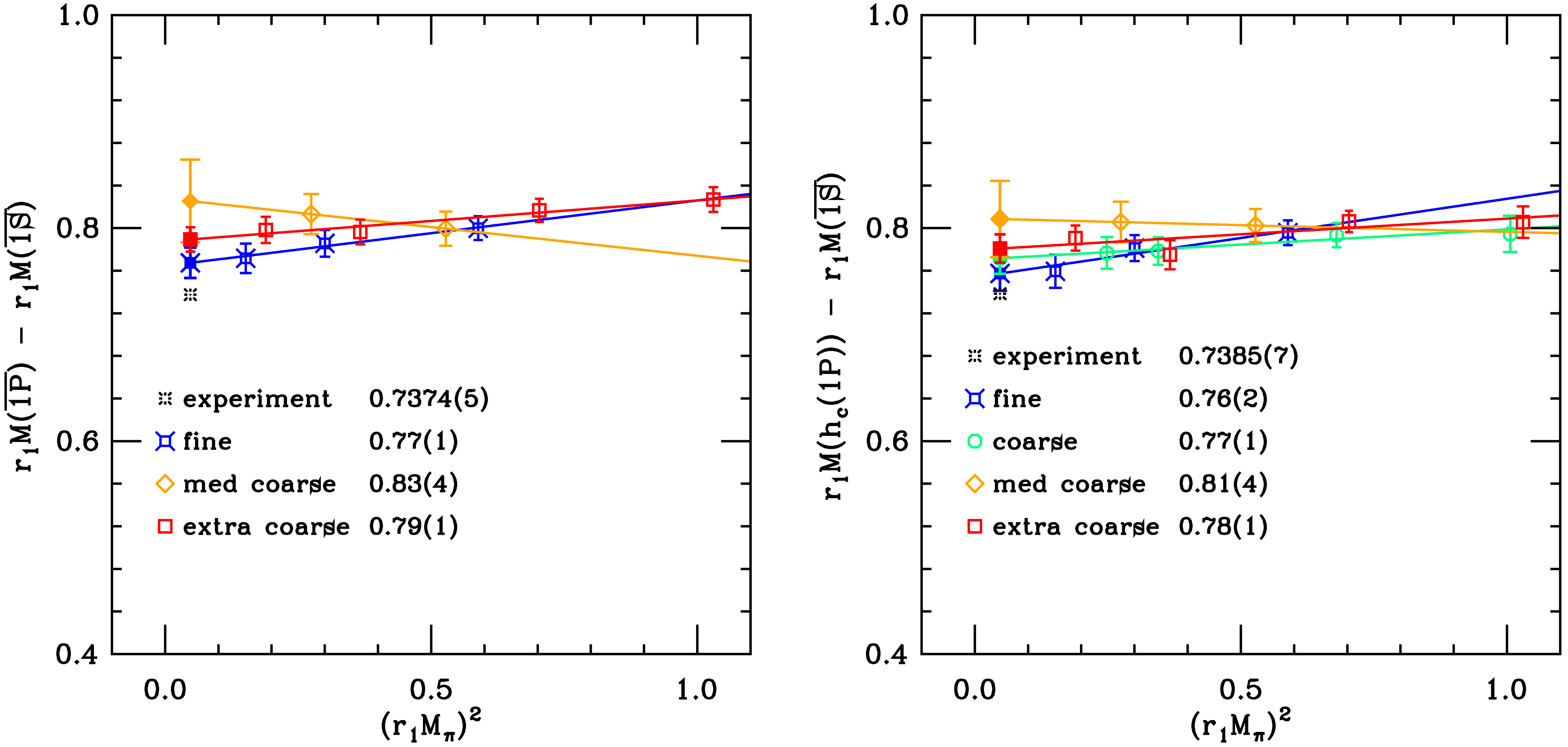
Parameters of the MILC ensembles used

- The ensembles are generated using the staggered asqtad action. There are two light quark flavors and a strange quark in the sea (2+1 flavors).
- The ensembles are also referred as: fine , coarse, medium coarse and extra coarse.

				relativistic				nonrelativistic			
a (fm)	β	am_l/am_s	$N_s^3 \times N_t$	κ_c	N^c_{conf}	κ_b	N^b_{conf}	κ_c	N^c_{conf}	κ_b	N^b_{conf}
≈ 0.18	6.503	0.0492/0.082	$16^3 \times 48$	0.120	401	—		0.120	400	—	
	6.485	0.0328/0.082	"	"	331			"	501		
	6.467	0.0164/0.082	"	"	645			"	647		
	6.458	0.0082/0.082	"	"	400			"	601		
≈ 0.15	6.600	0.0290/0.0484	$16^3 \times 48$	—		—		0.122	580	0.076	595
	6.586	0.0194/0.0484	"	0.122	631	0.076	631	"	580	"	595
	6.572	0.0097/0.0484	"	"	631	"	631	"	629	"	631
	6.566	0.00484/0.0484	$20^3 \times 48$	—		—		"	601	"	600
≈ 0.12	6.81	0.03/0.05	$20^3 \times 64$	0.122	549	0.086	549	—		—	
	6.79	0.02/0.05	"	"	460	"	460				
	6.76,a	0.01/0.05	"	"	593	"	539				
	6.76,b	0.007/0.05	"	"	403	—					
≈ 0.09	7.11	0.0124/0.031	$28^3 \times 96$	0.127	517	0.0923	517	0.127	518	0.0923	510
	7.09	0.0062/0.031	"	"	557	"	557	"	557	"	557
	7.08	0.0031/0.031	$40^3 \times 96$	"	504	"	504	"	504	"	504

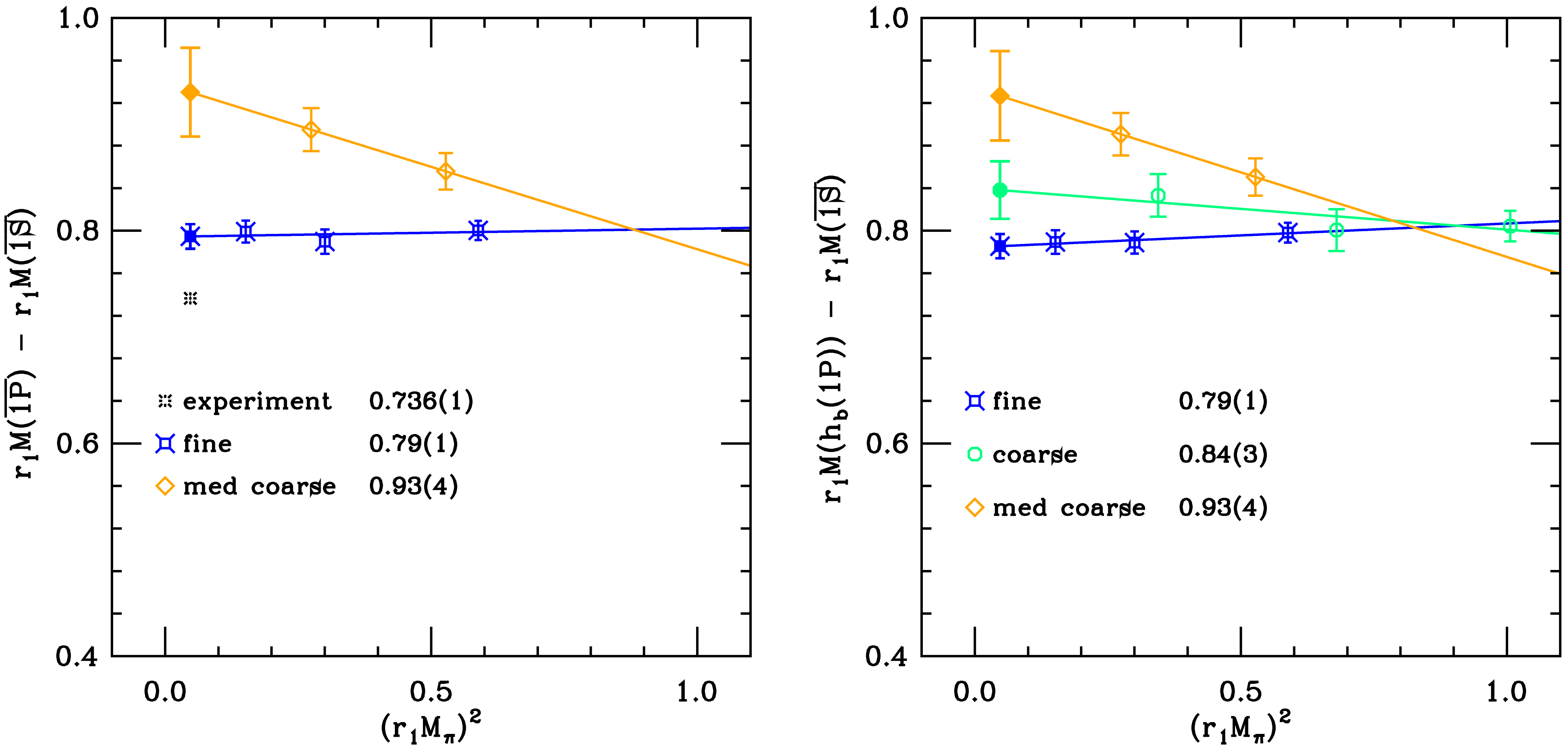
Charmonium spin-averaged splittings

$$M(\overline{1S}) = \frac{1}{4}\left(M_{\eta_c} + 3M_{J/\psi}\right), \qquad M(\overline{1^3P}) = \frac{1}{9}\left(M_{\chi_{c0}} + 3M_{\chi_{c1}} + 5M_{\chi_{c2}}\right)$$



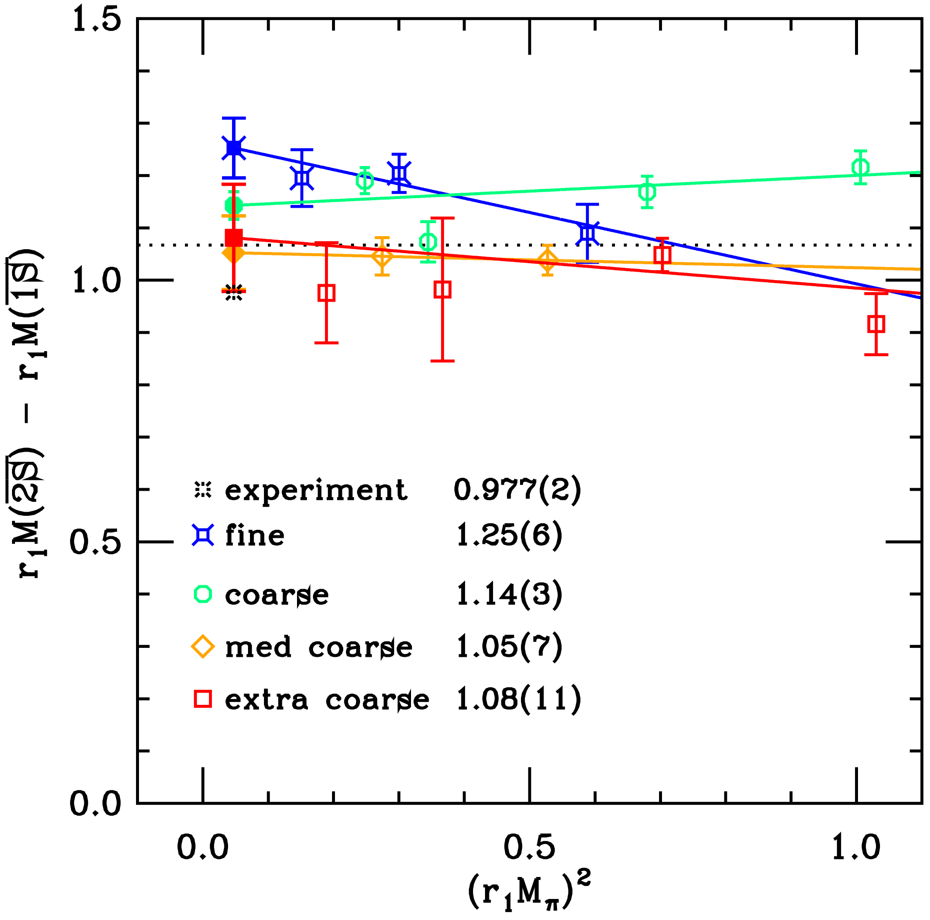
- Discretization effects of $O(v^4)$. Expected relative errors no more than 8%.
- Units: $r_1 = 0.318$ fm or 620 MeV.

Bottomonium spin-averaged splittings

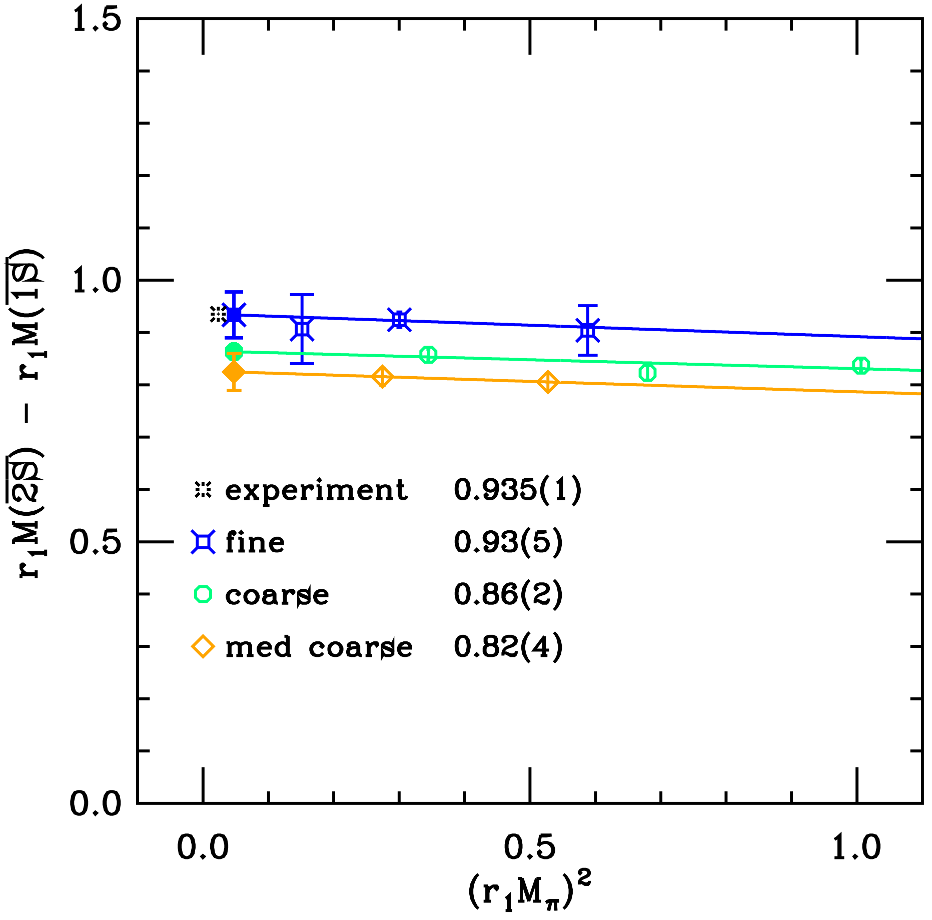


- Discretization effects of $O(v^4)$. Expected relative errors no more than 6%.

The $\overline{2S} - \overline{1S}$ splittings



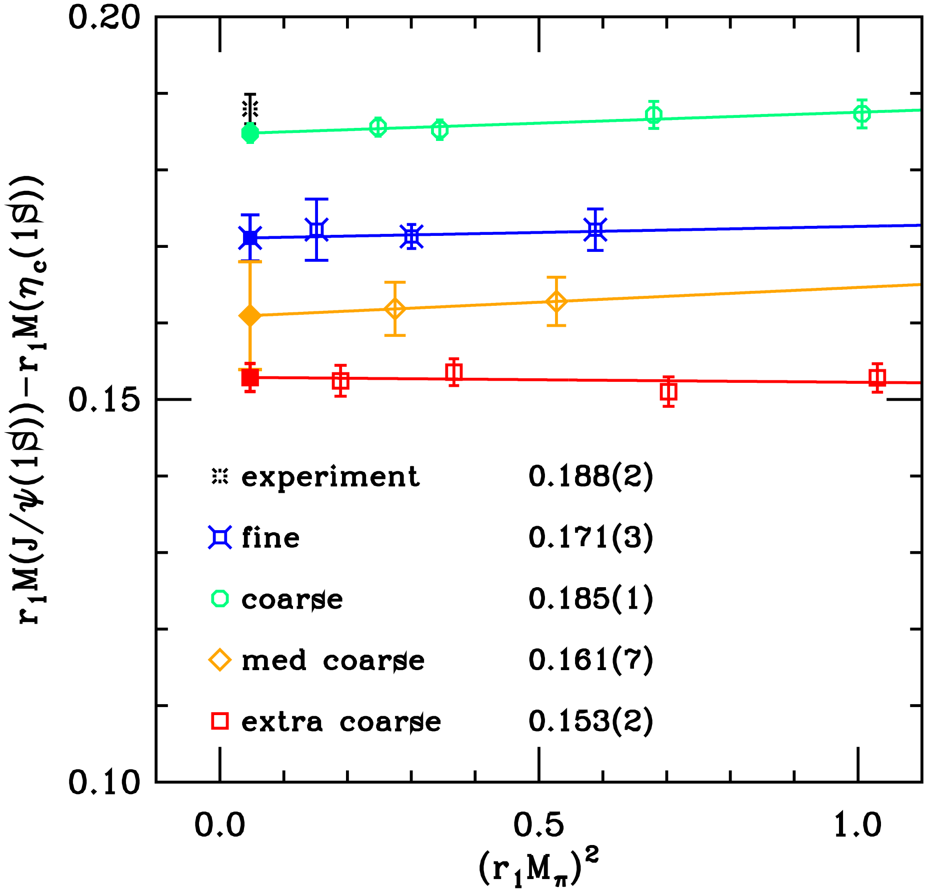
- Charmonium results. Possible contributions from open-charm levels.

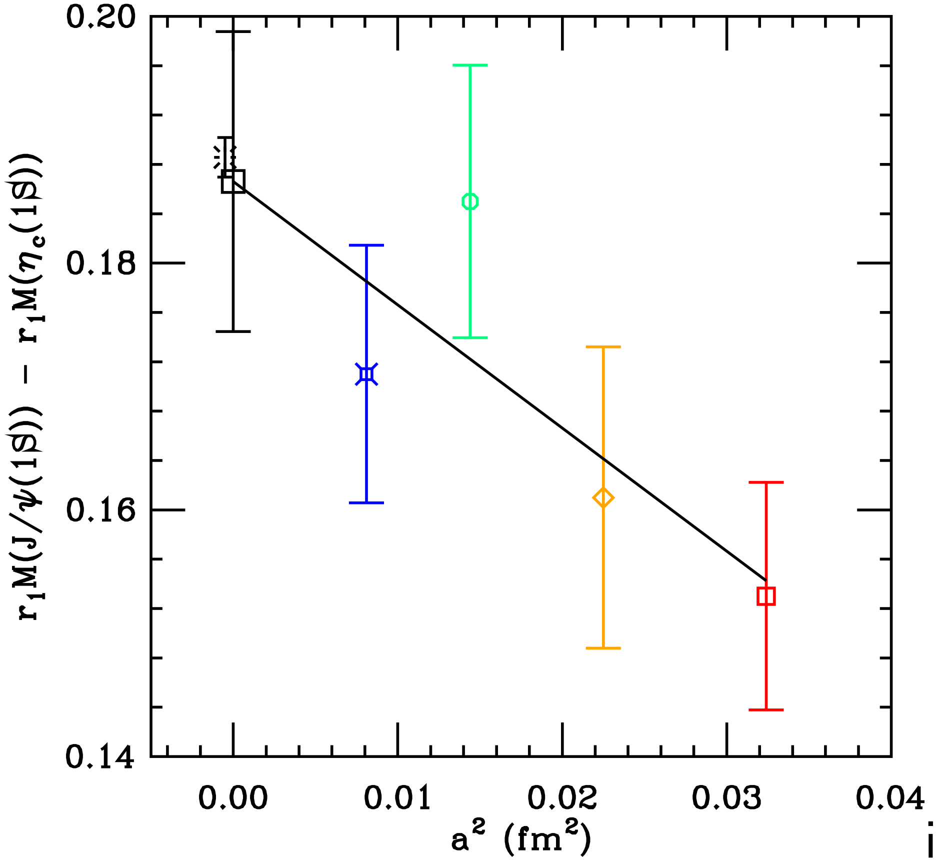


- Bottomonium results. The open-bottom levels are safely distant.

Charmonium hyperfine splitting

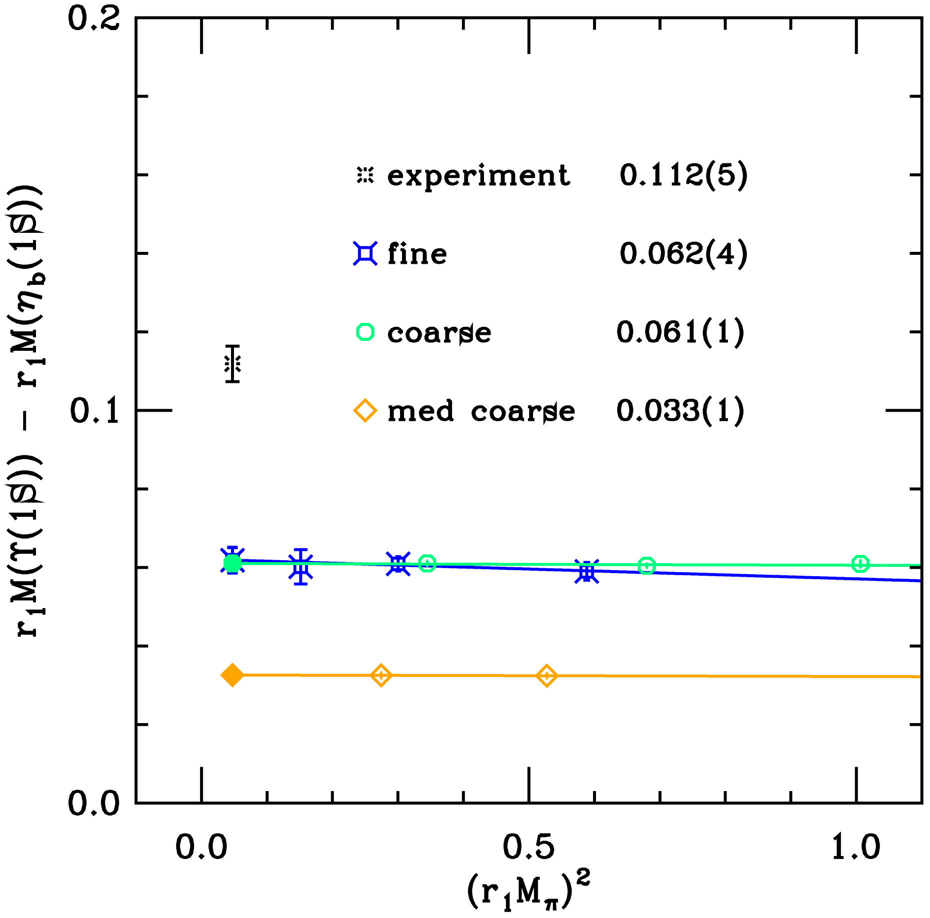
$$M(nS_{\mathsf{HFS}}) = M_{J/\psi} - M_{\eta_c}$$



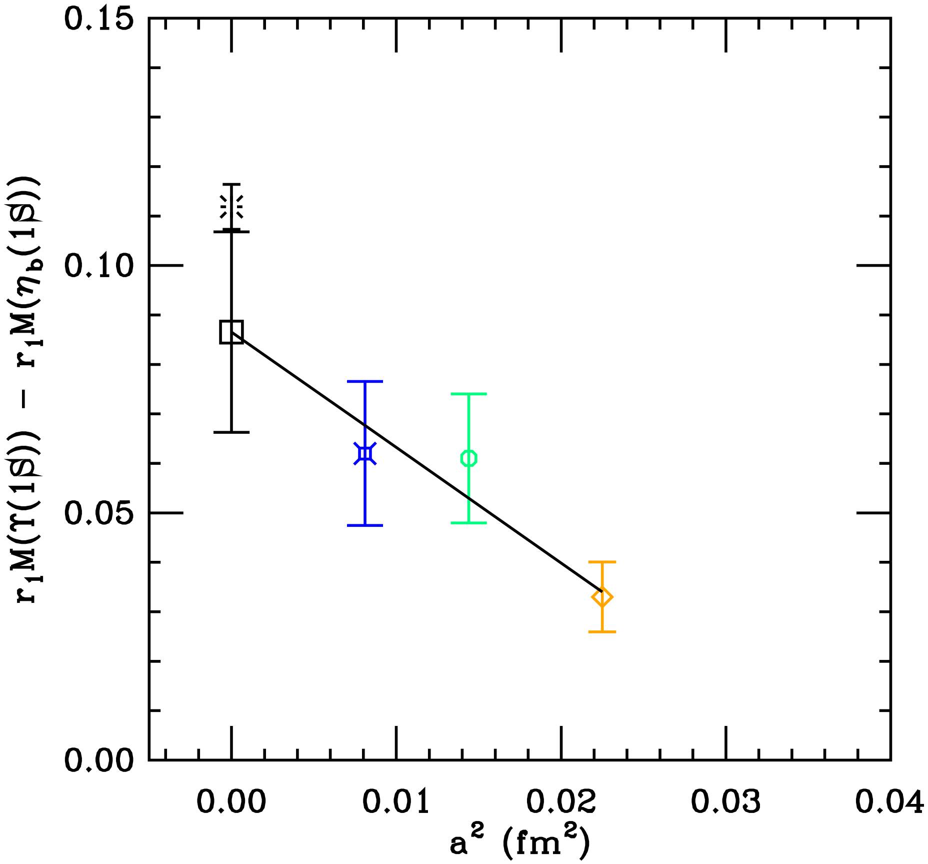


- Chirally extrapolated results with statistical errors only. Expected error of $O(\alpha_s a)$ through the tree-level tuned c_B.
- Continuum extrapolated results with κ-tuning errors added. Extrapolated value 116 ± 7.4 MeV. Experimental 116.4 ± 1.2 MeV.

Bottomonium hyperfine splitting



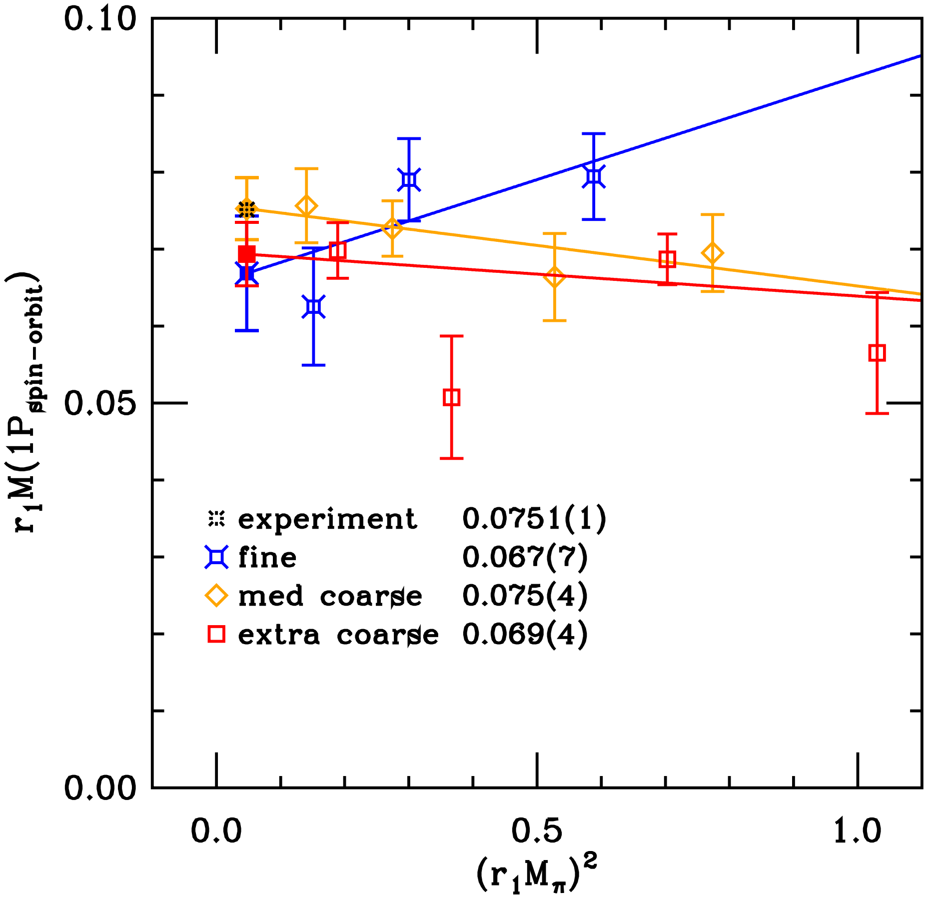
- Chirally extrapolated results with statistical errors only.



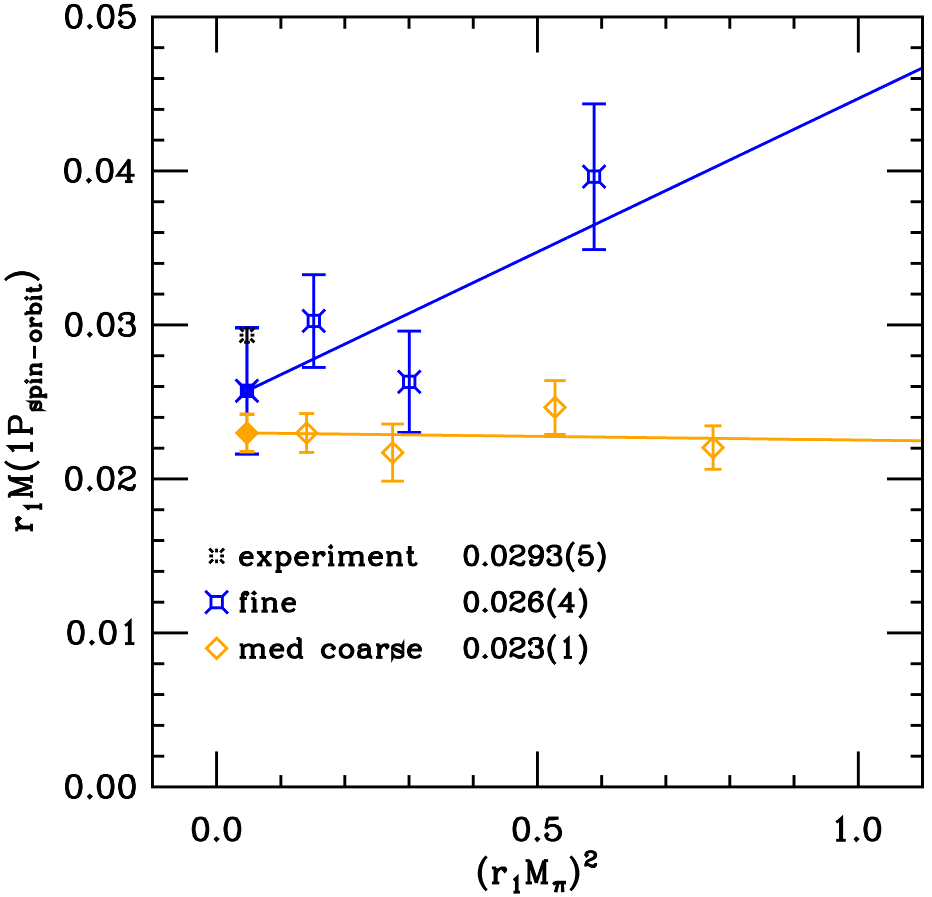
- Continuum extrapolation with κ-tuning errors added: 54.0 ± 12.4 MeV. Experimental 69.4 ± 2.8 MeV.

Spin-orbit splitting in $1P$ levels: adjusting the chromoelectric interactions

$$M(nP_{\text{spin-orbit}}) = \frac{1}{9}\left(5M_{\chi_{c2}} - 2M_{\chi_{c0}} - 3M_{\chi_{c1}}\right)$$



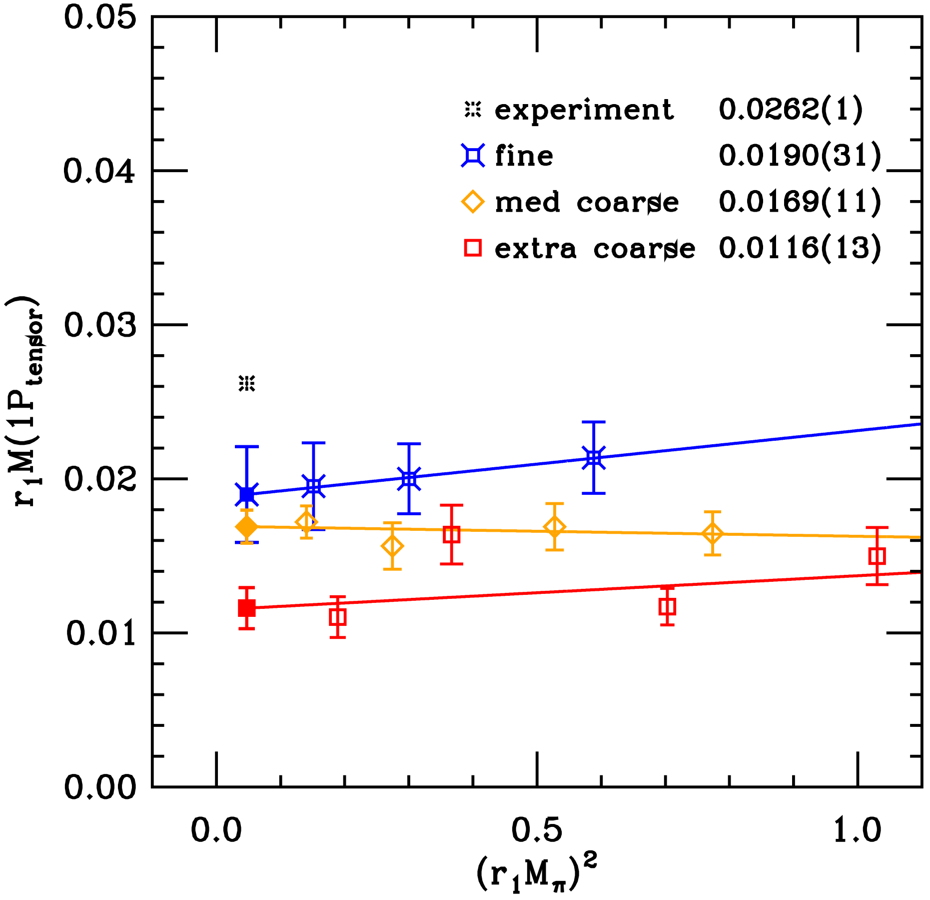
- Charmonium



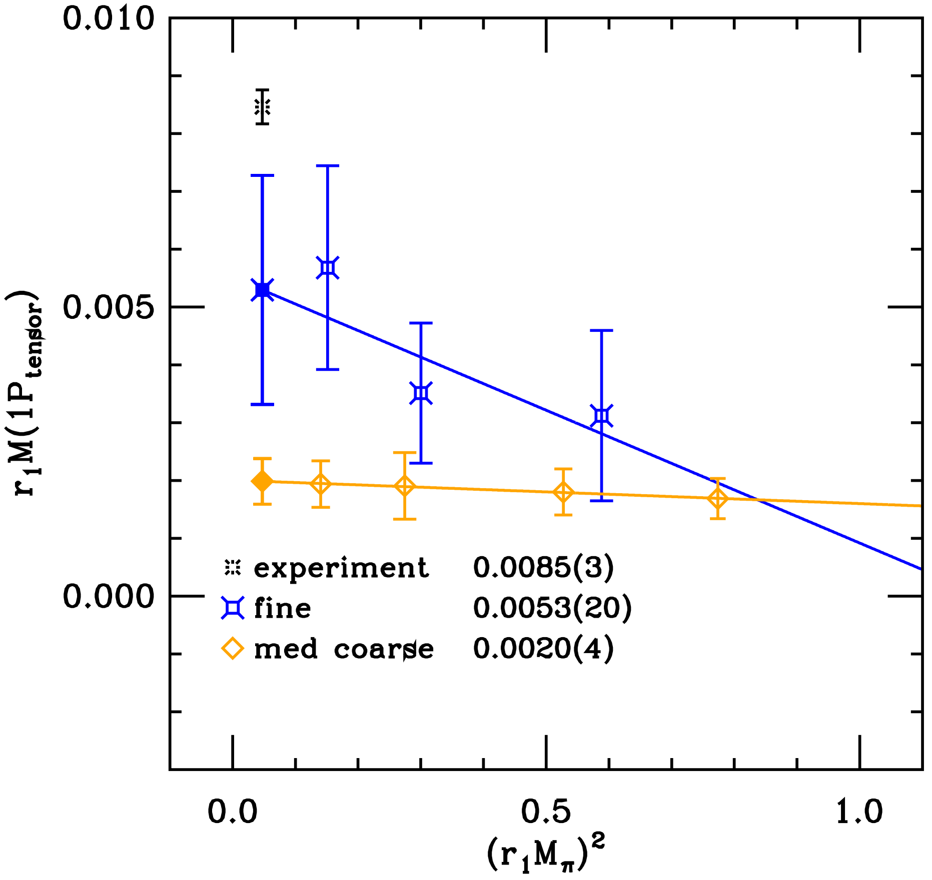
- Bottomonium

Tensor splittings in $1P$ levels: adjusting the chromomagnetic interactions

$$M(nP_{\text{tensor}}) = \frac{1}{9}\left(3M_{\chi_{c1}} - M_{\chi_{c2}} - 2M_{\chi_{c0}}\right)$$



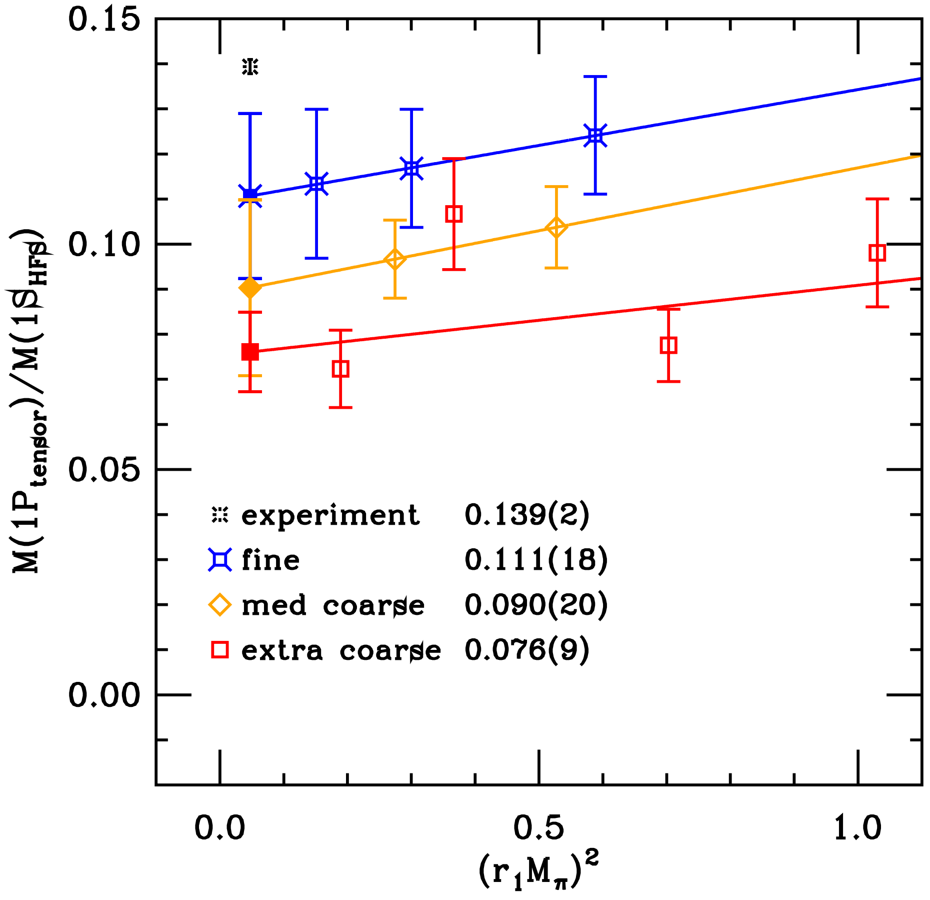
- Charmonium



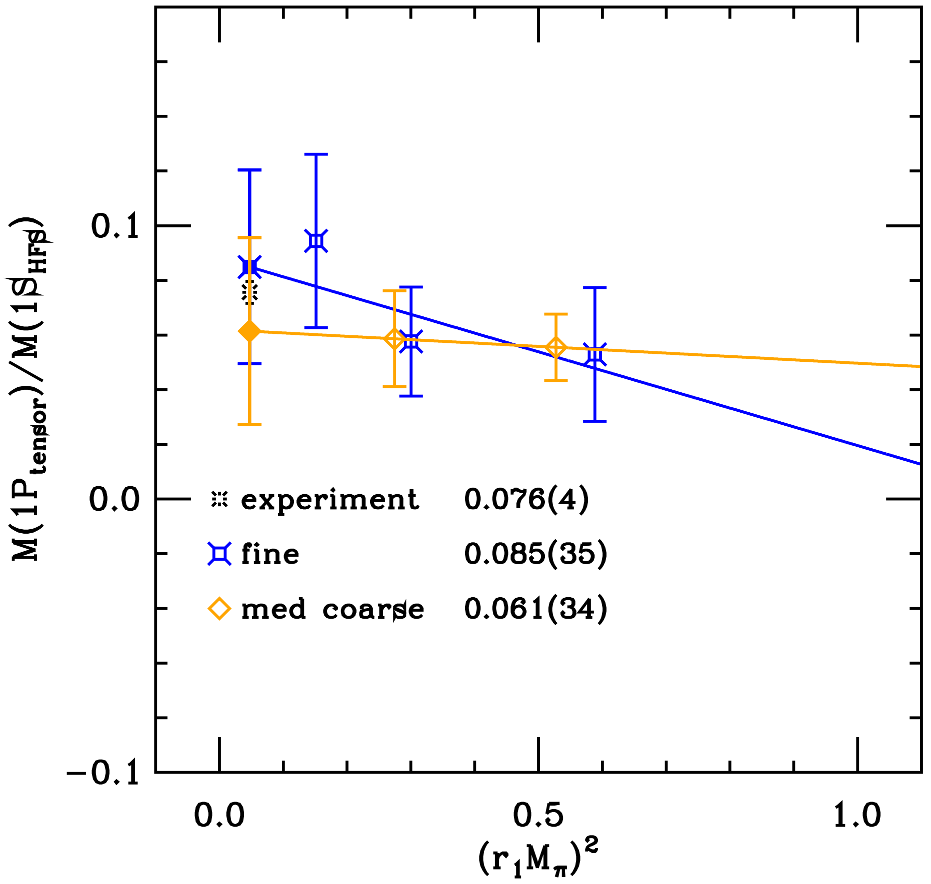
- Bottomonium

Ratio of the $1P$ tensor and $1S$ hyperfine splittings

- If there are no effects from higher order operators in the chromomagnetic interactions this ratio should be a constant agreeing with experiment.



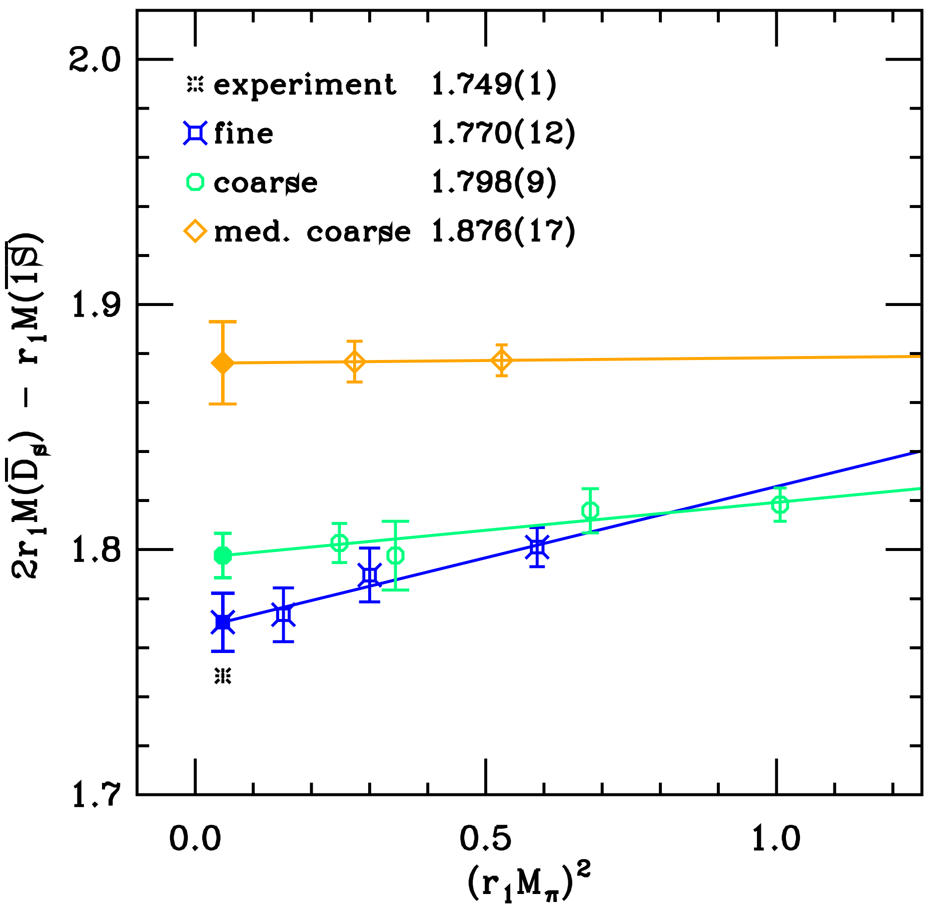
- Charmonium: possible effects ($v^2 \sim 0.3$).



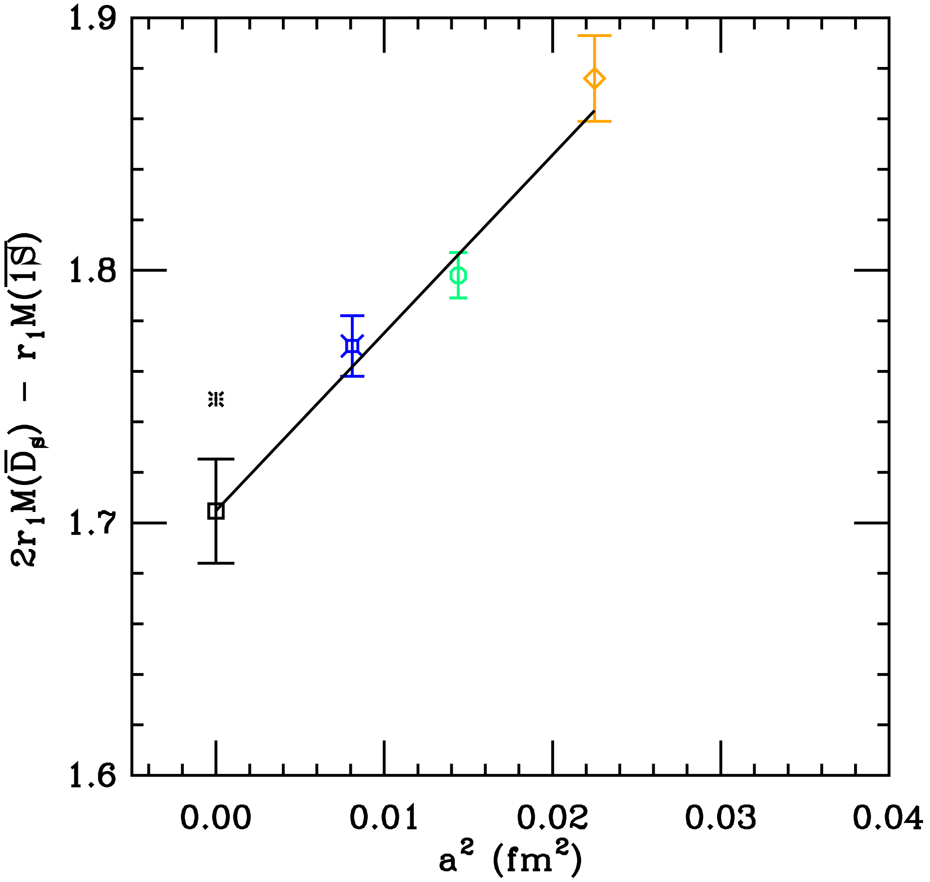
- Bottomonium: effects are supressed ($v^2 \sim 0.1$).

Charmonium-heavy-light splitting: a purely QCD quantity

$$2M(\overline{D_s}) - M(\overline{1S})$$

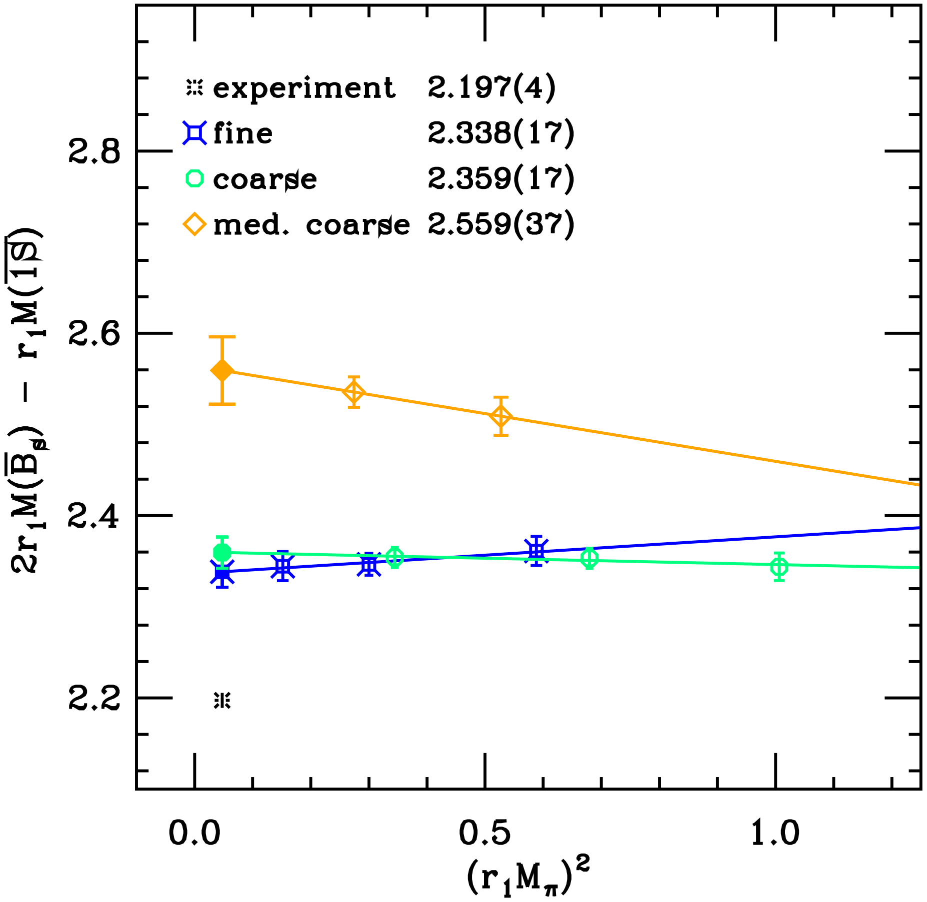


- Chiral extrapolation.

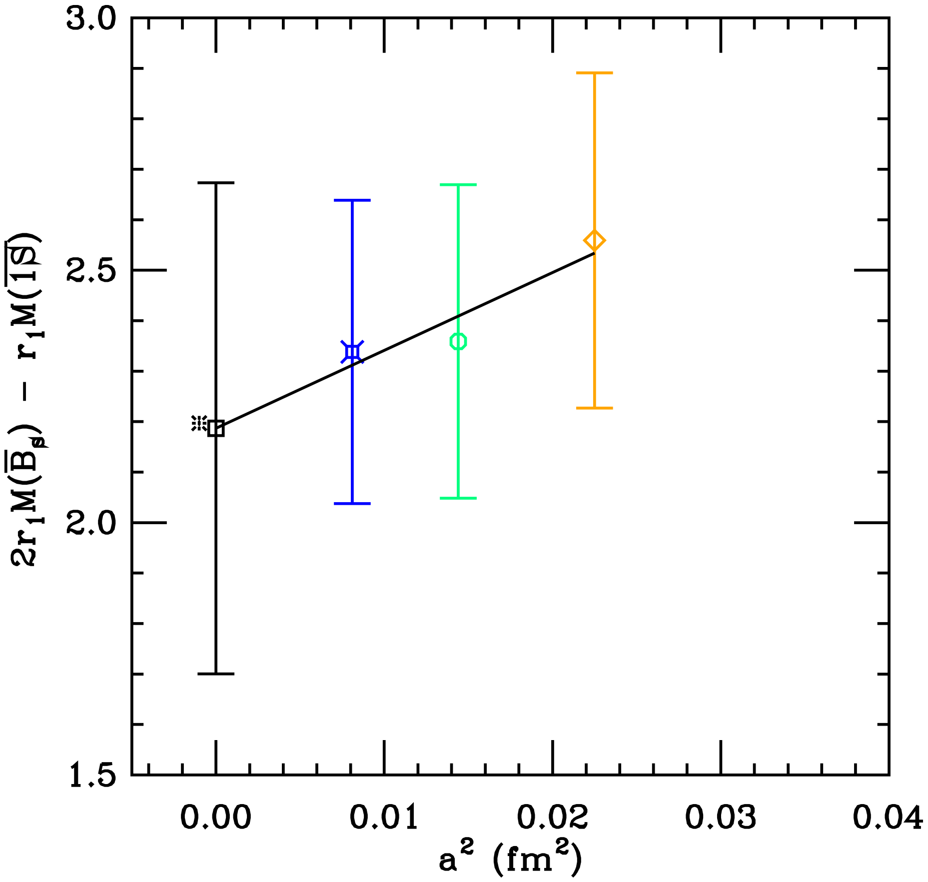


- Continuum extrapolation with κ-tuning errors added.

Botomonium-heavy-light splittings: a purely QCD quantity

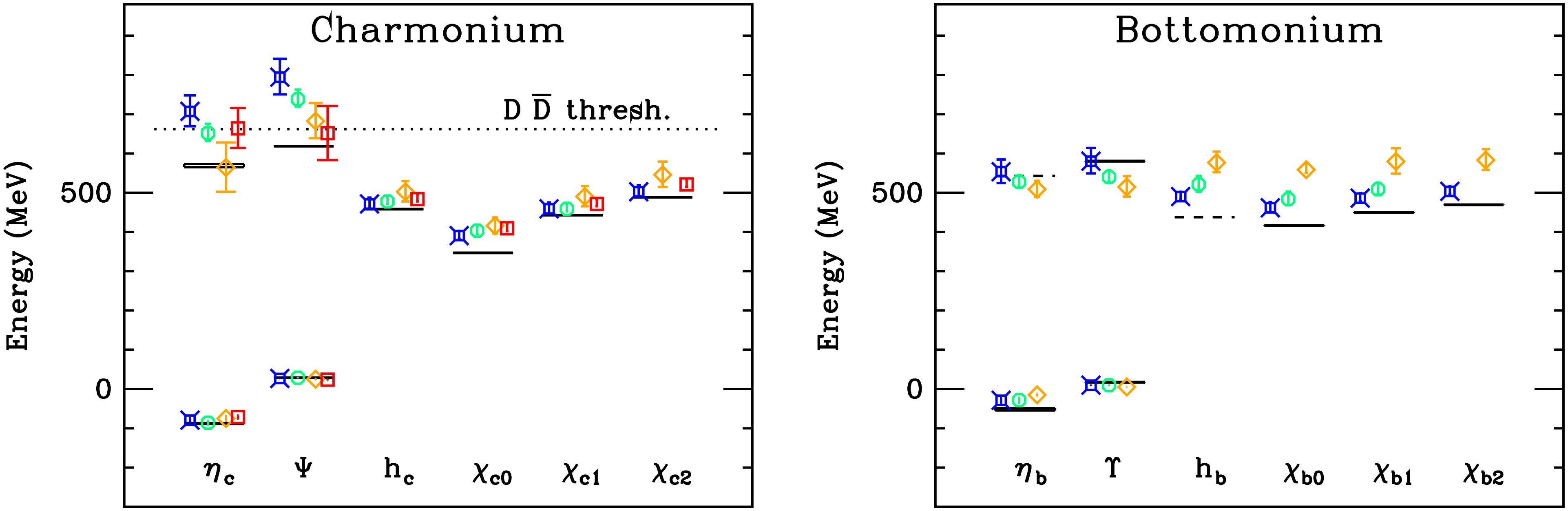


- Chiral extrapolation.



- Continuum extrapolation with κ-tuning errors added.

Quarkonium spectra



- Quarkonium spectrum as splittings from the $\overline{1S}$ level for $\bar{c}c$ (left) and $\bar{b}b$ (right). The fine-ensemble results are in blue fancy squares, the coarse in green circles, the medium-coarse in orange diamonds and the extra-coarse in red squares. Solid lines show the experimental values, and dashed lines estimates from potential models. The dotted line in the left panel indicates the physical open-charm threshold. The error on the data points combines statistical, κ-tuning, and r_1 uncertainties.

Continuum extrapolations of splittings in quarkonium

Splitting	Charmonium		Bottomonium	
	This work	Experiment	This work	Experiment
$\overline{1P}$-$\overline{1S}$	$473 \pm 12^{+10}_{-0}$	457.5 ± 0.3	$446 \pm 18^{+10}_{-0}$	456.9 ± 0.8
${}^1P_1$-$\overline{1S}$	$469 \pm 11^{+10}_{-0}$	457.9 ± 0.4	$440 \pm 17^{+10}_{-0}$	—
$\overline{2S}$-$\overline{1S}$	$792 \pm 42^{+17}_{-0}$	606 ± 1	$599 \pm 36^{+13}_{-0}$	(580.3 ± 0.8)
1^3S_1-1^1S_0	$116.0 \pm 7.4^{+2.6}_{-0}$	116.4 ± 1.2	$54.0 \pm 12.4^{+1.2}_{-0}$	69.4 ± 2.8
$1P$ tensor	$15.0 \pm 2.3^{+0.3}_{-0}$	16.25 ± 0.07	$4.5 \pm 2.2^{+0.1}_{-0}$	5.25 ± 0.13
$1P$ spin-orbit	$43.3 \pm 6.6^{+1.0}_{-0}$	46.61 ± 0.09	$16.9 \pm 7.0^{+0.4}_{-0}$	18.2 ± 0.2
$1S$ $\bar{s}Q$-$\bar{Q}Q$	$1058 \pm 13^{+24}_{-0}$	1084.8 ± 0.8	$1359 \pm 304^{+31}_{-0}$	1363.3 ± 2.2

- Continuum extrapolations of splittings in charmonium and bottomonium in MeV. The first error comes from statistics and accumulated extrapolation systematics; the second comes from the uncertainty in scale setting with $r_1 = 0.318^{+0.000}_{-0.007}$ fm.

Conclusions

- We study the bottomonium and charmonium systems with the Fermilab method and we reproduce successfully important features of the quarkonium spectrum.

- The size of the discretization effects is as expected from the theory.

- The tuning error in κ is significant for spin-dependent splittings.

- Future plans:
  - Statistics is increased up to 4 times.
  - Ensembles with lattice spacings of 0.06 and 0.045 fm are now available. Should bring bottomonium discretization effects to under 1%.
  - Further improvements of the Fermilab action (p^4 corrections).
  - Significantly improve κ-tuning errors.