Bottomonium and Ω_{bbb} from lattice QCD

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Introduction

Lattice actions used in this work:

- light quarks: domain wall action
- gluons: Iwasaki action
- b quarks: NRQCD

Bottomonium spectrum:

- important test of lattice methods / test of universality (HPQCD collaboration: same NRQCD action, but Lüscher-Weisz gluons and rooted staggered light quarks)
- provide independent determination of lattice spacing
- used as input for further calculations with NRQCD b quarks
- calculation at $L = 24$, $a \approx 0.113$ fm: PRD **79**, 094501 (2009) [arXiv:0903.3224]

Introduction

Ω_{bbb}:

- first lattice calculation of the Ω_{bbb} mass
- provides test of continuum models

Heavy-light hadrons (will not be covered in this talk):

- this is where the good chiral symmetry of DW fermions really matters
- preliminary spectrum results: PoS(**LAT2009**), 105 (2009) [arXiv:0909.3837]

RBC/UKQCD gauge field ensembles

- 2+1 flavors of domain wall fermions, exact chiral symmetry for $L_5 \to \infty$ even at finite a, no doubling problem
- better control over operator renormalization and chiral extrapolation, automatic $O(a)$ improvement
- Iwasaki gluon action - suppresses residual chiral symmetry breaking at finite L_5

RBC/UKQCD gauge field ensembles

Ensembles with $a \approx 0.113$ fm, $am_{\text{res.}} \approx 0.003$

- $16^3 \times 32 \times 16$, $L \approx 1.8$ fm:

$a\, m_{\text{strange}}$	$a\, m_{\text{light}}$	m_π (MeV)
0.04	0.01	420
0.04	0.02	560
0.04	0.03	670

C. Allton et al., PRD **76**, 014504 (2007)
[arXiv:hep-lat/0701013]

RBC/UKQCD gauge field ensembles

- $24^3 \times 64 \times 16, \quad L \approx 2.7$ fm:

$a\, m_{\text{strange}}$	$a\, m_{\text{light}}$	m_π (MeV)
0.04	0.005	330
0.04	0.01	420
0.04	0.02	560
0.04	0.03	670

C. Allton et al., PRD **78**, 114509 (2008) [arXiv:0804.0473]

RBC / UKQCD gauge field ensembles

Ensembles with $a \approx 0.085$ fm, $am_{\text{res.}} \approx 0.0006$

- $32^3 \times 64 \times 16$, $L \approx 2.7$ fm:

$a\, m_{\text{strange}}$	$a\, m_{\text{light}}$	m_π (MeV)
0.03	0.004	300
0.03	0.006	350
0.03	0.008	400

Heavy-quark action: NRQCD

- unlike HQET, NRQCD can be used for systems with any number of b quarks. As an EFT, requires cut-off: $am_b > 1$.
- correct through order $\Lambda^2_{\mathrm{QCD}}/m_b^2$ (HL) and v^4_{rel} (HH)
- tree-level Symanzik improved, discretization errors are of order $\alpha_s a^2$
- tadpole improvement (using Landau gauge mean link) accounts for radiative corrections

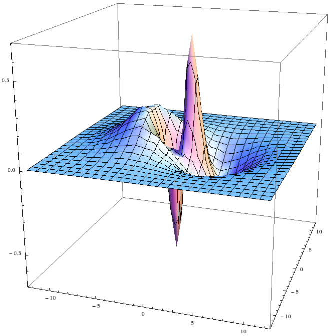
$$
\begin{aligned}
H_0 &= -\frac{1}{2m}\Delta^{(2)}, \\
\delta H &= -\frac{g}{2m}\boldsymbol{\sigma}\cdot\tilde{\boldsymbol{B}} + \frac{i\,g}{8m^2}\left(\boldsymbol{\Delta}^{\pm}\cdot\tilde{\boldsymbol{E}} - \tilde{\boldsymbol{E}}\cdot\boldsymbol{\Delta}^{\pm}\right) \\
&\quad -\frac{g}{8m^2}\ \boldsymbol{\sigma}\cdot\left(\tilde{\boldsymbol{\Delta}}^{\pm}\times\tilde{\boldsymbol{E}} - \tilde{\boldsymbol{E}}\times\tilde{\boldsymbol{\Delta}}^{\pm}\right) \\
&\quad -\frac{1}{8m^3}\ \left(\Delta^{(2)}\right)^2 + \ \delta H_{\mathrm{corr}}
\end{aligned}
$$

Bottomonium spectrum

Bottomonium: interpolating operators

- fix gauge configurations to Coulomb gauge, use "smearing" function $\Gamma(\boldsymbol{r})$, 2×2-matrix-valued in spinor space

$$O_\Gamma(\boldsymbol{p},t) = \sum_{\boldsymbol{x},\,\boldsymbol{x'}} \chi^\dagger(\boldsymbol{x},t)\,\Gamma(\boldsymbol{x}-\boldsymbol{x'})\,\psi(\boldsymbol{x'},t)\,e^{i\boldsymbol{p}\cdot(\boldsymbol{x}+\boldsymbol{x'})/2}$$



NB: choice of smearing only affects overlap with states, not their energies

Bottomonium: interpolating operators

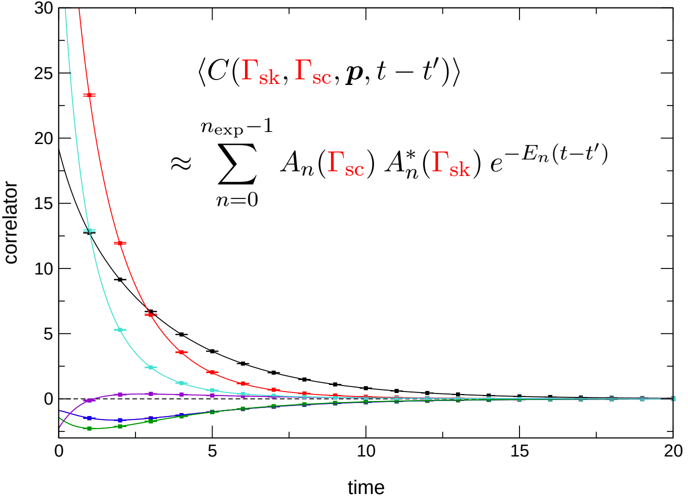
Name	L	S	J	P	C	$\mathcal{R}^{PC}$	$\Gamma(\boldsymbol{r})$
$\eta_b(nS)$	0	0	0	$-$	$+$	A_1^{-+}	$\phi_{nS}(\boldsymbol{r})$
$\Upsilon(nS)$	0	1	1	$-$	$-$	T_1^{--}	$\phi_{nS}(\boldsymbol{r})\,\sigma^i$
$h_b(nP)$	1	0	1	$+$	$-$	T_1^{+-}	$\phi_{nP}(\boldsymbol{r})\,r^i/r_0$
$\chi_{b0}(nP)$	1	1	0	$+$	$+$	A_1^{++}	$\phi_{nP}(\boldsymbol{r})\,(\boldsymbol{r}\cdot\boldsymbol{\sigma})/r_0$
$\chi_{b1}(nP)$	1	1	1	$+$	$+$	T_1^{++}	$\phi_{nP}(\boldsymbol{r})\,(\boldsymbol{r}\times\boldsymbol{\sigma})^i/r_0$
$\chi_{b2}(nP)$	1	1	2	$+$	$+$	T_2^{++}	$\phi_{nP}(\boldsymbol{r})\,(r^i\sigma^j+r^j\sigma^i)/r_0$
$\eta_b(nD)$	2	0	2	$-$	$+$	T_2^{-+}	$\phi_{nD}(\boldsymbol{r})\,r^ir^j/r_0^2$
$\Upsilon_2(nD)$	2	1	2	$-$	$-$	E^{--}	$\phi_{nD}(\boldsymbol{r})\,(r^ir^j\sigma^k-r^jr^k\sigma^i)/r_0^2$

$(i\neq j,\;\; k\neq j)$

State	$\phi(\boldsymbol{r})$
$1S$	$\exp[-\lvert\boldsymbol{r}\rvert/r_0]$
$2S$	$[1-\lvert\boldsymbol{r}\rvert/(2r_0)]\;\exp[-\lvert\boldsymbol{r}\rvert/(2r_0)]$
$3S$	$\left[1-2\lvert\boldsymbol{r}\rvert/(3r_0)+2\lvert\boldsymbol{r}\rvert^2/(27r_0^2)\right]\;\exp[-\lvert\boldsymbol{r}\rvert/(3r_0)]$
$1P$	$\exp[-\lvert\boldsymbol{r}\rvert/(2r_0)]$
$2P$	$[1-\lvert\boldsymbol{r}\rvert/(6r_0)]\exp[-\lvert\boldsymbol{r}\rvert/(3r_0)]$
$1D$	$\exp[-\lvert\boldsymbol{r}\rvert/(3r_0)]$

Multi-exponential Bayesian fitting

- matrix fits with multiple radial smearing functions (e.g. $1S$, $2S$ and $3S$) at source and sink



Multi-exponential Bayesian fitting

- actual fit parameters: $\ln(E_0)$, $A_0(\Gamma)$, and for $n > 0$

$$\begin{aligned} e_n &\equiv \ln(E_n - E_{n-1}), \\ B_n(\Gamma) &\equiv A_n(\Gamma)/A_0(\Gamma) \end{aligned}$$

- Bayesian fitting: $\chi^2 \to \chi^2 + \chi^2_{\text{prior}}$ with the Gaussian prior

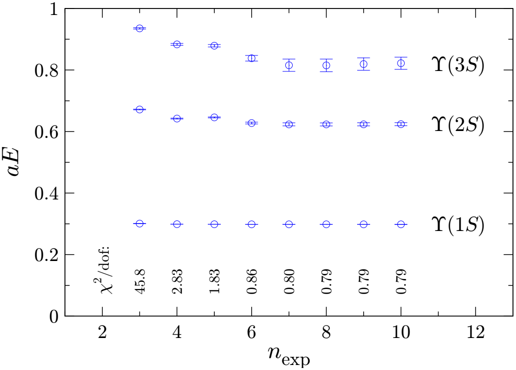
$$\chi^2_{\text{prior}} = \sum_i \frac{(p_i - \tilde{p}_i)^2}{\sigma^2_{\tilde{p}_i}}$$

- priors for low-lying states: central values from unconstrained fit at large t, width $= 10 \times$ error from fit
- priors for high-lying states (for $L = 24$ ensemble, lattice units):

$$\begin{aligned} \widetilde{e}_n = -1.4, &\quad \sigma_{\widetilde{e}_n} = 1, \\ \widetilde{B}_n(\Gamma) = 0, &\quad \sigma_{\widetilde{B}_n(\Gamma)} = 5 \end{aligned}$$

Multi-exponential Bayesian fitting

- increase n_{exp} until fit results stabilize



Tuning the b quark mass ($L = 24$)

- NRQCD energies do not include b rest mass $\Rightarrow$ use kinetic mass

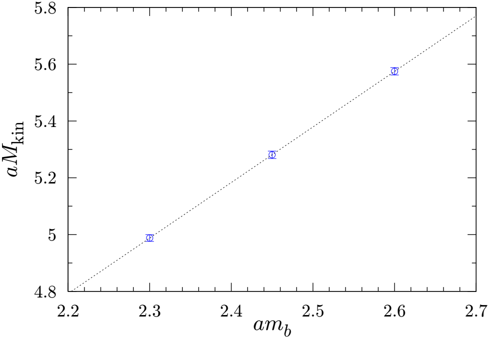
$$M_{\rm kin} \equiv \frac{\boldsymbol{p}^2 - [E(\boldsymbol{p}) - E(0)]^2}{2\,[E(\boldsymbol{p}) - E(0)]}$$

 for tuning

am_b	$aM_{\rm kin}(\eta_b)$	$\Upsilon(2S) - \Upsilon(1S)$ splitting
2.30	4.988(12)	0.3258(47)
2.45	5.281(13)	0.3242(46)
2.60	5.575(13)	0.3231(54)

- $\Upsilon(2S) - \Upsilon(1S)$ splitting very insensitive to am_b $\Rightarrow$ use to set lattice scale

Tuning the b quark mass ($L = 24$)



- linear fit $aM_{\mathrm{kin}} = A + B \cdot am_b$ gives

$$A = 0.489(25), \qquad B = 1.956(11).$$

- set M_{kin} equal to experimental value $9.389(5)$ GeV, solve for am_b. This gives

$$am_b = 2.514(36).$$

Speed of light ($L = 24$)

- test of lattice dispersion relation using "speed of light"

$$c^2 \equiv \frac{\left[E(\boldsymbol{p}) - E(0) + M_{\text{kin},1}\right]^2 - M_{\text{kin},1}^2}{\boldsymbol{p}^2}$$

$\boldsymbol{n}^2$	$aM_{\text{kin}}(\eta_b)$	c^2
1	5.450(17)	-
2	5.450(17)	1.00003(85)
3	5.450(18)	1.0001(16)
4	5.461(22)	0.9981(21)
5	5.457(20)	0.9987(24)
6	5.452(20)	0.9997(27)
8	5.454(22)	0.9993(35)
9	5.447(20)	1.0005(35)
12	5.445(21)	1.0009(42)

$(a\boldsymbol{p} = \boldsymbol{n} \cdot 2\pi/L)$

$\Rightarrow$ lattice dispersion relation is very close to continuum

Speed of light ($L = 32$)

$L = 32$, $a \approx 0.085$ fm ensemble: $am_b = 1.87$

$\boldsymbol{n}^2$	$aM_{\text{kin}}(\eta_b)$	c^2
1	4.1109(96)	-
2	4.111(10)	1.00008(31)
3	4.110(11)	1.00019(65)
4	4.113(12)	0.99939(97)
5	4.112(13)	0.9996(12)
6	4.111(14)	1.0000(16)
8	4.111(16)	0.9999(22)
9	4.109(18)	1.0004(26)
12	4.106(21)	1.0012(38)

PRELIMINARY

Bottomonium: radial and orbital excitations

Estimates of systematic errors due to lattice NRQCD action:
A. Gray et al., PRD 72, 094507 (2005) [arXiv:hep-lat/0507013]

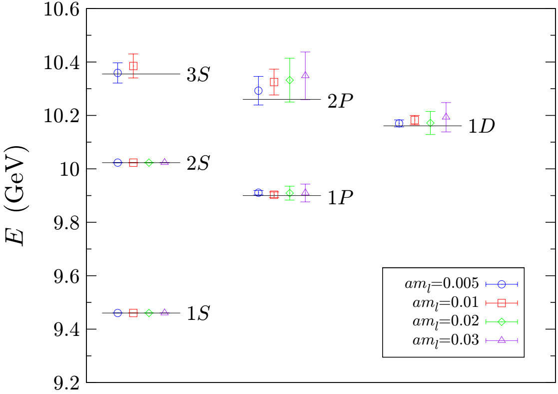
- $a = 0.12$ fm:

	$2S-1S$	$1P-1S$
relativistic	0.5%	1.0%
radiative	0.5%	1.7%
discretization	0.8%	3.2%
total	1.1%	3.8%

- $a = 0.09$ fm:

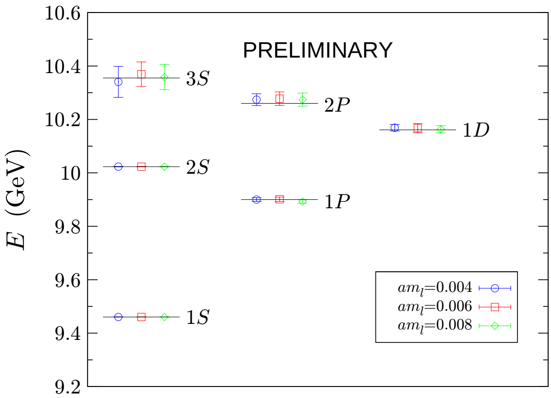
	$2S-1S$	$1P-1S$
relativistic	0.5%	1.0%
radiative	0.4%	1.3%
discretization	0.4%	1.3%
total	0.8%	2.1%

Bottomonium: radial and orbital excitations ($L = 24$)



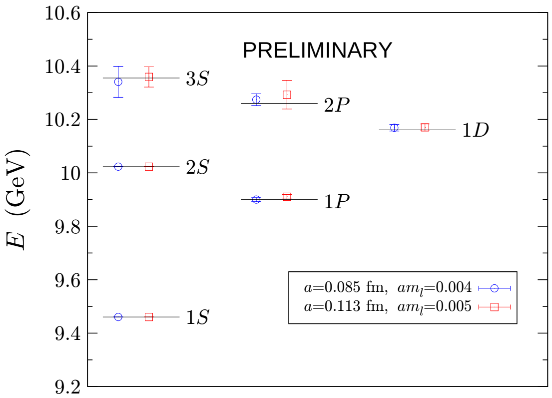
(1S and 2S used to determine energy shift and lattice spacing → not predictions here)

Bottomonium: radial and orbital excitations ($L = 32$)



(1S and 2S used to determine energy shift and lattice spacing → not predictions here)

Bottomonium: radial and orbital excitations: a-dependence



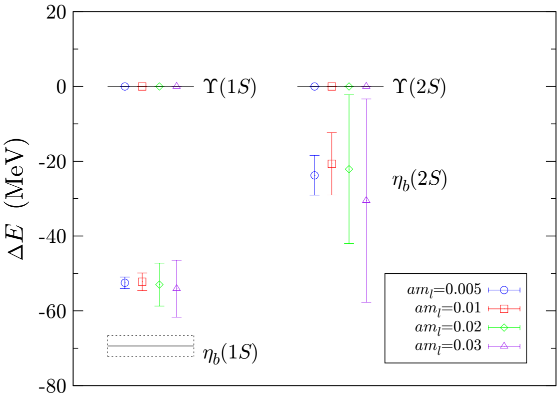
(1S and 2S used to determine energy shift and lattice spacing → not predictions here)

Bottomonium: spin-dependent energy splittings

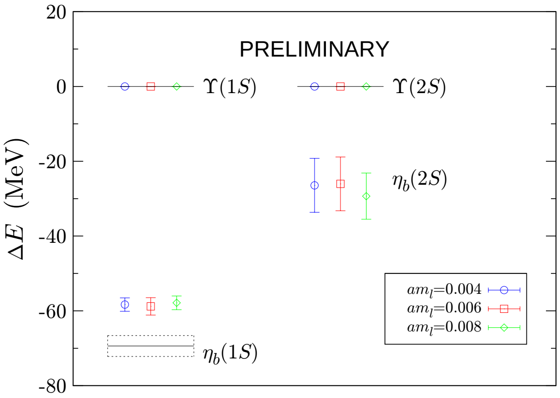
larger systematic errors expected here:

- relativistic errors $\mathcal{O}(v^2)$ ≈10%
- missing radiative corrections $\mathcal{O}(\alpha_s)$ ≈20% (but: tadpole improvement)
- discretization errors expected to be largest in S-wave hyperfine splitting (sensitive to very short distances)

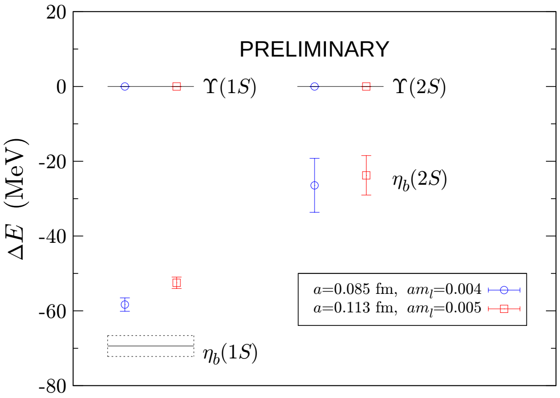
Bottomonium: S-wave hyperfine structure ($L = 24$)



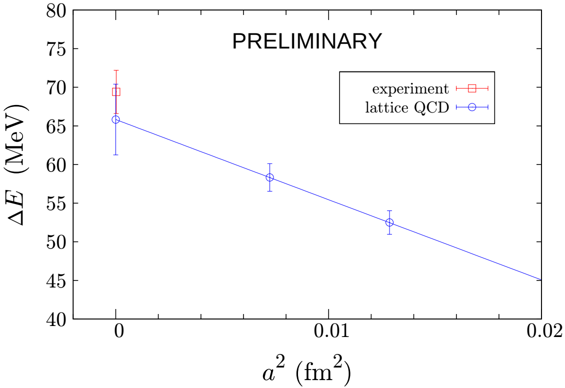
Bottomonium: S-wave hyperfine structure ($L = 32$)



Bottomonium: S-wave hyperfine structure: a-dependence



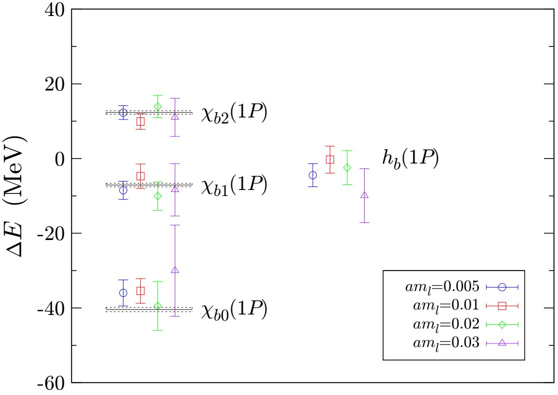
1S hyperfine splitting - continuum extrapolation



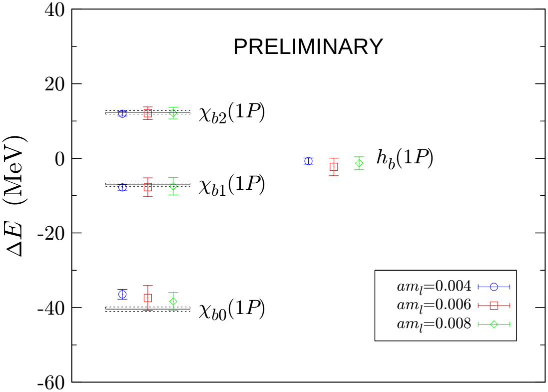
Preliminary result: $\Delta E(a=0) = 65.8 \pm 4.6$ (stat.) MeV

Experiment: 69.4 ± 2.4 MeV (average of BABAR 2008, 2009, CLEO 2009)

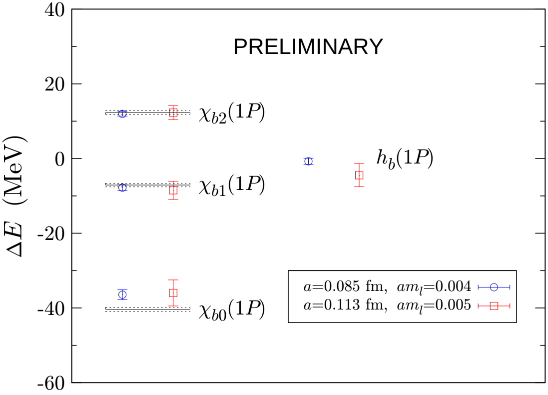
Bottomonium: P-wave spin splittings ($L = 24$)



Bottomonium: P-wave spin splittings ($L = 32$)



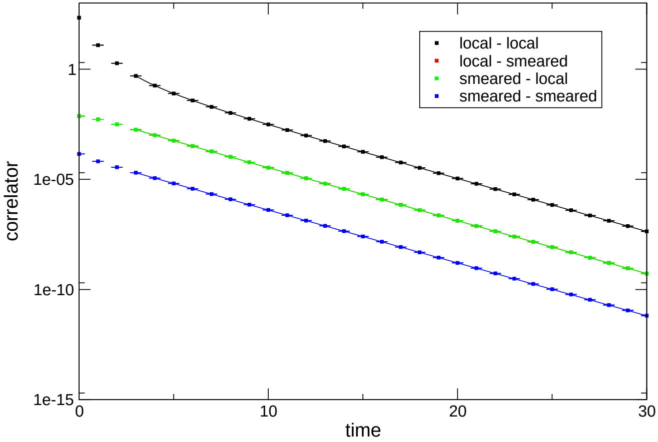
Bottomonium: P-wave spin splittings: a-dependence



Ω_{bbb}

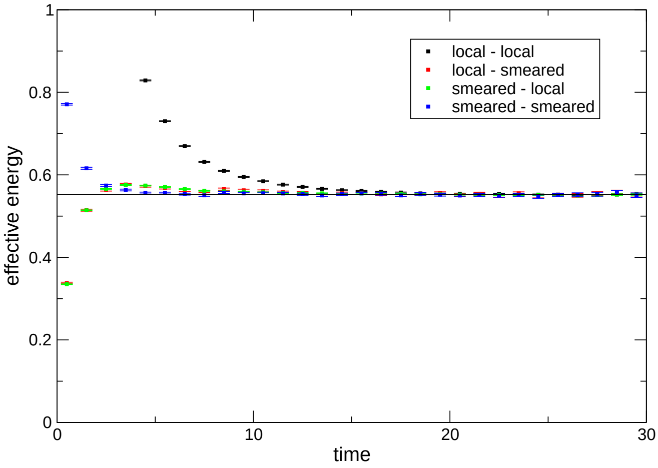
Fit of Ω_{bbb} matrix correlator ($L = 24$)

Interpolating operator: $\epsilon_{abc}\,(C\gamma_j)_{\beta\gamma}\,Q^a_\beta\,Q^b_\gamma\,Q^c_\alpha$, with or without Gaussian smearing of Q. Project correlator to spin $3/2$



The fit has $t_{\min} = 3$ and $n_{\exp} = 6$.

Effective energy plot of Ω_{bbb} matrix correlator $(L = 24)$



Determining the total mass

$$M_{\text{lat.}}^{\Omega_{bbb}} = E_{\text{lat.}}^{\Omega_{bbb}} + \frac{3}{2}\left(M_{\text{exp.}}^{\Upsilon} - E_{\text{lat.}}^{\Upsilon}\right)$$

Take lattice spacing from $\Upsilon(2S) - \Upsilon(1S)$ splitting.

Ω_{bbb} mass: preliminary results

a (fm)	L	am_l	am_b	$M_{\Omega_{bbb}}$ (GeV)
0.11	16	0.01	2.514	14.3733(42)
0.11	16	0.02	2.514	14.3718(98)
0.11	16	0.03	2.514	14.373(13)
0.11	24	0.005	2.45	14.3746(33)
0.11	24	0.005	2.514	14.3748(33)
0.11	24	0.005	2.60	14.3751(33)
0.11	24	0.01	2.514	14.3726(42)
0.11	24	0.02	2.514	14.378(10)
0.11	24	0.03	2.514	14.379(13)
0.08	32	0.004	1.87	14.3670(33)
0.08	32	0.006	1.87	14.3707(39)
0.08	32	0.008	1.87	14.3655(36)

(errors are statistical only)

Ω_{bbb} mass: comparison with continuum results

Reference	Method	$M_{\Omega_{bbb}}$ (GeV)
J.-R. Zhang, M.-Q. Huang, 2009	sum rules	$13.28(10)$
A. Bernotas, V. Simonis, 2008	bag model	14.276
P. Hasenfratz et al., 1980	bag model	14.30
Y. Jia, 2006	variational method	$14.37(08)$
This work, $L = 24$	lattice QCD	$14.3748(33)$
A. P. Martynenko, 2007	relativistic quark model	14.569

Conclusions

Conclusions:

- NRQCD on DW/Iwasaki lattices works very well for bottom spectroscopy
- Prediction of Ω_{bbb} mass seems to be robust

Current and future work:

- check/revise data analysis for bottomonium
- calculate bottomonium spectrum on $L = 16$ ensembles
- heavy-light calculations with DW light valence quarks
- finalize prediction of Ω_{bbb} mass
- compute spectrum of bbb excited states