Recent lattice progress on charmonia at finite temperature

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Outline

Basic ideas

- Production, spectroscopy and decays of heavy quarkonium are precision probes of QCD at zero temperature
- Maybe some related properties could also be useful at finite temperatures? [Matsui,Szatz '86]

- Physical obervable: dilepton production

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3 p} = \frac{5\alpha^2}{27\pi^2} \frac{1}{\omega^2 (e^{\omega/T} - 1)} \sigma_V(\omega, \vec{p}, T)$$

- Heavy quark diffusion constant

$$D = \frac{\pi}{3\chi^{00}} \lim_{\omega \to 0} \sum_{i=1}^{3} \frac{\sigma_V^{ii}(\omega, \vec{p} = 0, T)}{\omega}$$

- Spectral function:

$$\sigma(\omega) = \frac{D^{>}(\omega) - D^{<}(\omega)}{2\pi} = \frac{1}{\pi}\mathrm{Im} D_R(\omega)$$

- Correlation function:

$$G(\tau, T) = D^{>}(-i\tau)$$

$$G_H(\tau, T) = \sum_{\vec{x}} \left\langle J_H(\tau, \vec{x})\, J_H^{\dagger}(0, \vec{0}) \right\rangle$$

$$J_H(\tau, \vec{r}) = \bar{q}(\tau, \vec{r})\, \Gamma_H\, q(\tau, \vec{r})$$

Γ_H	$^{2S+1}L_J$	J^{PC}	$c\bar{c}$
γ_5	$^1S_0$	0^{-+}	η_c
γ_μ	$^3S_1$	1^{--}	J/ψ
1	$^3P_0$	0^{++}	χ_{c0}
$\gamma_5\gamma_\mu$	$^3P_1$	1^{++}	χ_{c1}

- Spectral reprentation:

$$G_H(\tau, T) = \int_0^{\infty} \mathrm{d}\omega\, K(\tau, \omega, T)\, \sigma_H(\omega, T)\,; \qquad K(\tau, \omega, T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh(\frac{\omega}{2T})}$$

Extract spf by Maximum Entropy Method (MEM) [Asakawa et al., 2001]

- Ill-posed problem: Inversion to extract $\sigma_H(\omega, T)$

$$G(\tau, T) = \int_0^\infty \mathrm{d}\omega \, \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh(\frac{\omega}{2T})} \, \sigma(\omega, T)$$

- MEM: find most probable $\sigma(\omega, T)$ which maximizes $P[\sigma|Gm]$

$$\text{Bayesian theorem :} \quad P[\sigma|Gm] \propto P[G|\sigma] \, P[m] = \exp(-\frac{\chi^2}{2} + \alpha S)$$

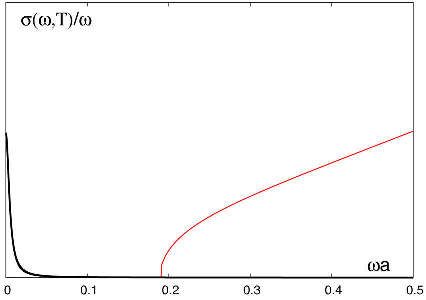
  - Shannon-Jaynes entropy
    $S = \int_0^\infty \left[\sigma(\omega) - m(\omega) - \sigma(\omega) \, log(\frac{\sigma(\omega)}{m(\omega)})\right] \mathrm{d}\omega$
  - $m(\omega)$, default model (DM), provides prior knowledge on $\sigma(\omega)$

- Results in principle should be independent of DMs
- Nothing beats good data in solving the ill-posed problem

1. free continuum spectral function

$$\sigma_H = \frac{N_c}{8\pi^2}\Theta(\omega^2 - 4m^2)\, \omega^2 \tanh(\frac{\omega}{4T}) \times\sqrt{1-(\frac{2m}{\omega})^2}\left[a_H + (\frac{2m}{\omega})^2\, b_H\right] + \frac{N_c}{3}\frac{T^2}{2} f_H\, \omega\delta(\omega)$$

2. zero mode contribution at $\omega \approx 0$ [Umeda 07]



$$\delta(\omega) \to \frac{1}{\pi}\frac{\eta}{\omega^2+\eta^2}$$

Aarts, Martinez-Resco 05,
Petreczky,Teaney 06

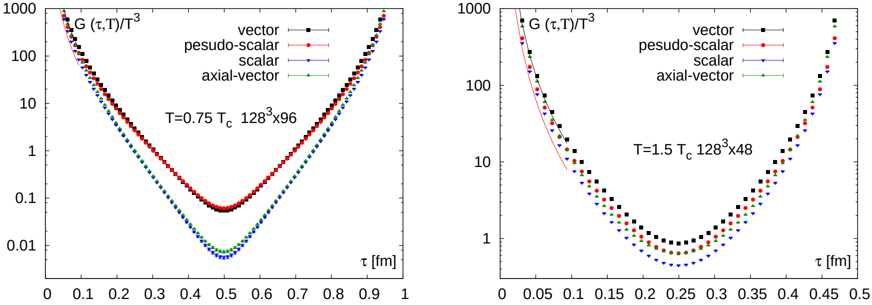
Lattice setup

- non-perturbatively clover improved Wilson fermions
- isotropic quenched lattice

β	κ	a [fm]	$N_\sigma^3 \times N_\tau$	T/T_c	# of conf.
7.793	0.13200	0.010	$128^3 \times 96$	0.75	132
			$128^3 \times 48$	1.5	471

- mass tuning: $M_{J/\psi}$ = 3.48(1) GeV, M_{η_c} = 3.35(1) GeV
- fine lattice: $m_c a \approx 0.0659 \ll 1$
- temporal extent: $\tau_{max} \approx 0.498$ fm (0.75 T_c)

charmonium temporal correlation function



- non-dengenerate states still at 1.5 T_c
- almost close to free correlators at very small separations
- largest distance 0.25 fm at 1.5 T_c due to restriction $\tau \leq 1/2T$
- only small distance regime (0.1-0.25 fm) relevant for thermal effects

Reconstructed correlator [Datta et al., PRD69(2004)094507]

$$G_{rec}(\tau, T; T') = \int_0^\infty \mathrm{d}\omega\ \sigma(\omega, T')\ \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

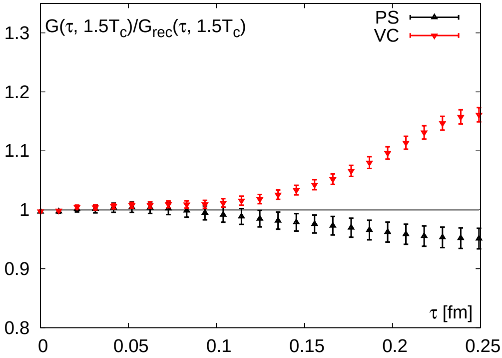
Deviation of $G(\tau, T)$ from $G_{rec}(\tau, T)$ indicates the medium modification

It is not necessary to reconstruct $\sigma(\omega, T')$ by Maximum Entropy Method to evaluate $G_{rec}(\tau, T)$ [Meyer, arXiv:1002.3343]

Exact relations:

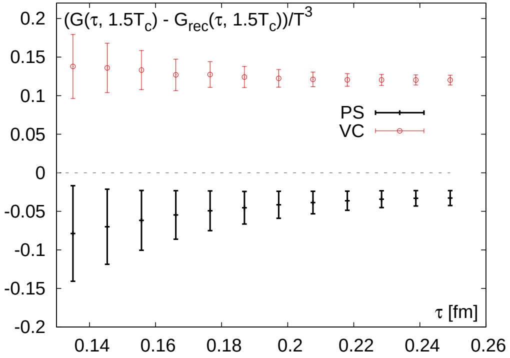
$$G_{rec}\left(\tau, T; \frac{1}{2}T\right) = G\left(t, \frac{1}{2}T\right) + G\left(\frac{1}{T} - t, \frac{1}{2}T\right).$$

Temperature dependence of charmonia



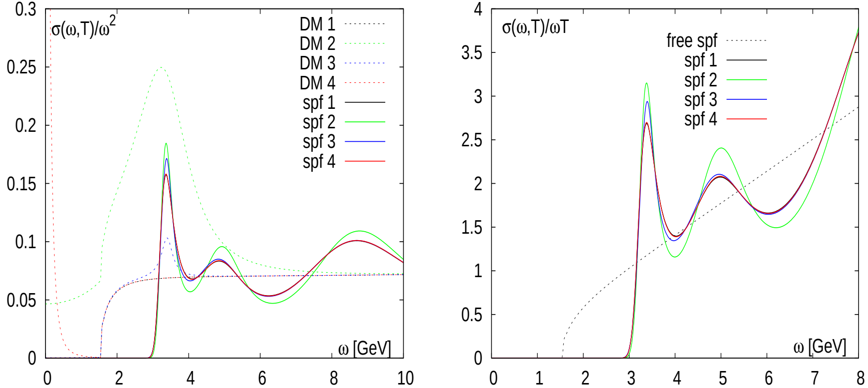
- G/G_{rec} for PS deviates from unity up to ≈5% at the largest distance
- G/G_{rec} for Vector deviates from unity up to ≈16% at the largest distance

$G(\tau, T) - G_{rec}(\tau, T)$



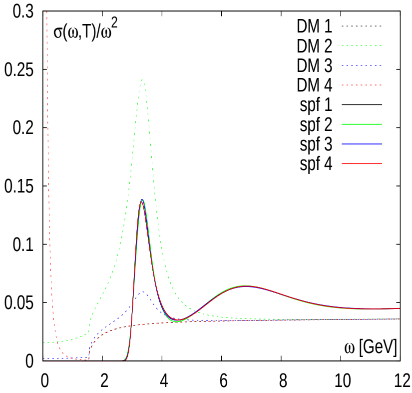
- $G(\tau, T) - G_{rec}(\tau, T)$ is nearly a τ independent constant at very large distance
- the curvature of $G(\tau, T) - G_{rec}(\tau, T)$ indicates something beyond free transport theory

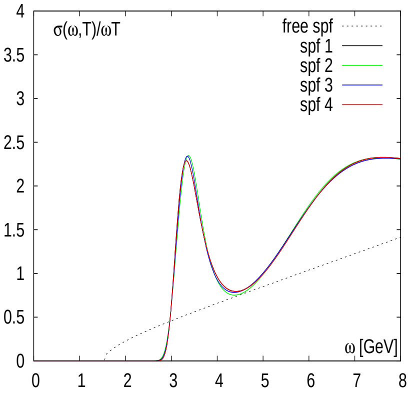
Vector spectral function at 0.75 T_c



- Small default model dependence
- Ground state remains robust
- Below T_c no zero mode contribution is found

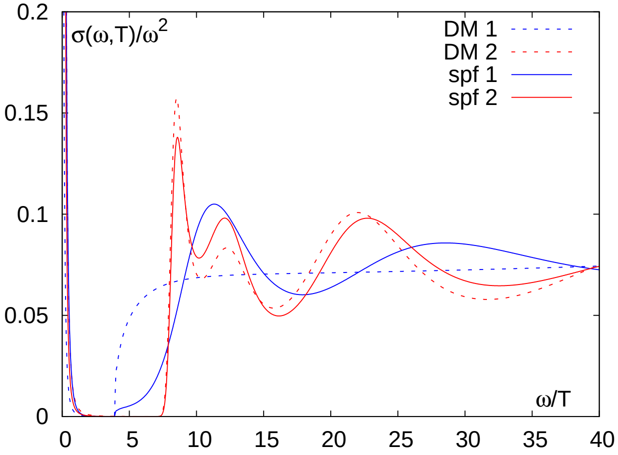
Pseudoscalar spectral function at 0.75 T_c





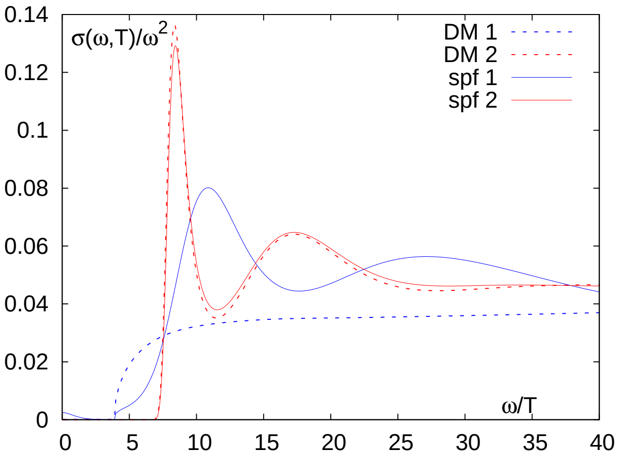
- Similar situation as that of the vector channel

Charmonium spectral functions at 1.5 T_c: Vector channel



- DM1 : free lattice spf plus transport peak around $\omega = 0$
- DM2 : spectral function obtained from MEM at 0.75 T_c plus transport peak around $\omega = 0$
- The fate of J/ψ at 1.5 T_c is not certain

Charmonium spectral functions at 1.5 T_c: PS channel



- DM1 : free lattice spf
- DM2 : spectral function obtained from MEM at 0.75 T_c
- The fate of η_c at 1.5 T_c is not certain

Summary & Outlook

- At 0.75 T_c, the ground state peaks of PS and VC channels are reliable and robust
- At 0.75 T_c, no transport peak is found in PS and VC channels
- At 1.5 T_c, J/ψ and η_c could either survive or melt
- non-zero momenta correlator
- QCD sum rule for heavy quark system could be helpful