Charge Separation

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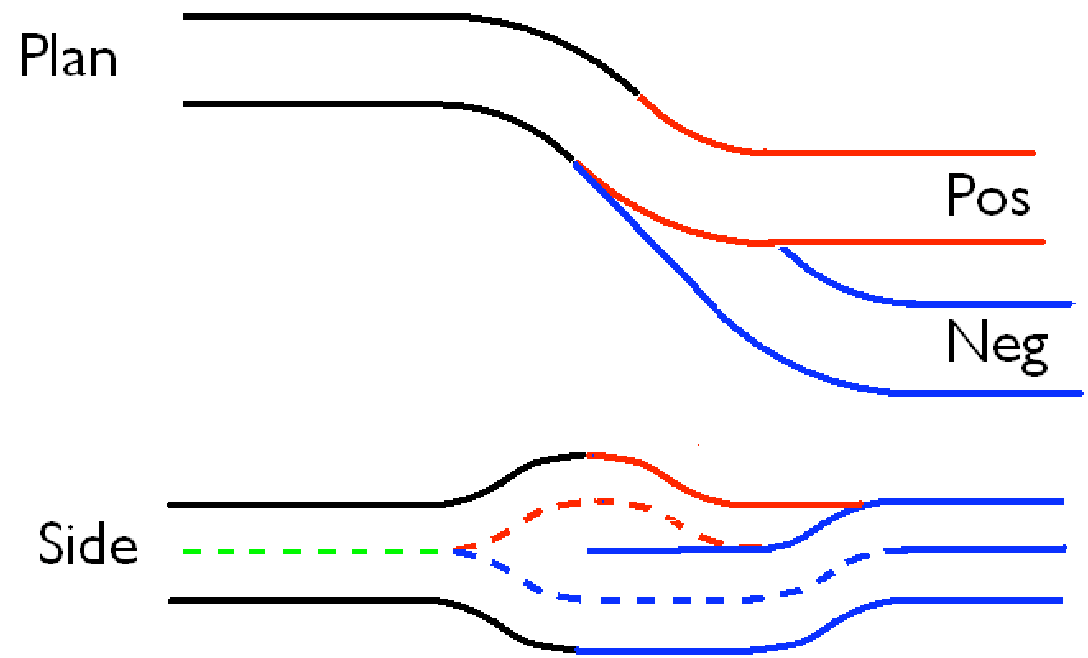
- Introduction
- Bent solenoid dynamics
- Design
- Examples from Gaussian sources
- Conclusion

Introduction

- The HCC and Guggenheim schemes need single charges
- Guggenheim 6D cooling gives better results for both NF and MC, compared with current front end linear cooling
- So, without Snakes, charge separation must be immediately after phase rotation
- A snake might allow initial cooling without charge separation that would
  - Save money
  - Make eventual charge separation easier
- The transverse and longitudinal emittances after phase rotation are ≈ 15 mm, and 50 mm, respectively - The long is huge!
- Snakes may not have enough acceptance
- so it is worth trying to design a charge separator working with these emittances

Charge Separation Method

Taken from Fernow



- Bend horizontally to generate vertical dispersion and charge separation
- Insert horizontal septum dividing upper positives from lower negatives
- Immediate reverse bend of upper positives, returning their vertical center to zero
- Linear transport of lower negatives to horizontally separate two signs
- Then reverse bend lower negatives, returning their vertical center also to zero

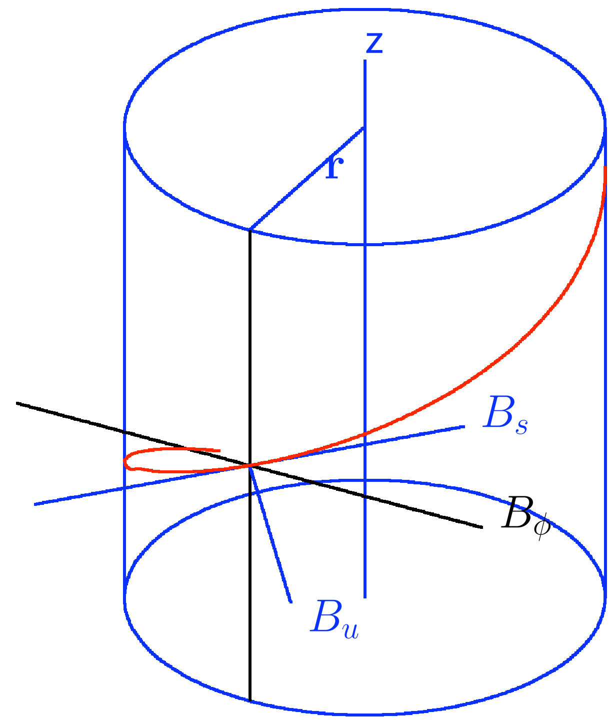
Since this involves bent solenoids recap "Bent Solenoid Dynamics"

Bent solenoid Dynamics

1) Momentum Drift

The field within a bent solenoid, far from its ends, is the same as that around a single vertical current I

$$B_\phi = I/r$$



For a trajectory to remain at a fixed radius r there has to be an inward bend of approximately that radius, i.e. there has to be a component of B at right angles to the trajectory. This condition is satisfied if the trajectory is an upward helix with angle α, giving a field perpendicular to the trajectory of $B_u = B_\phi \ \sin(\alpha)$.

The bending radius is then

$$r \ = \ \frac{[p_s \ c/e]}{B_\phi \sin(\alpha) \ c}$$

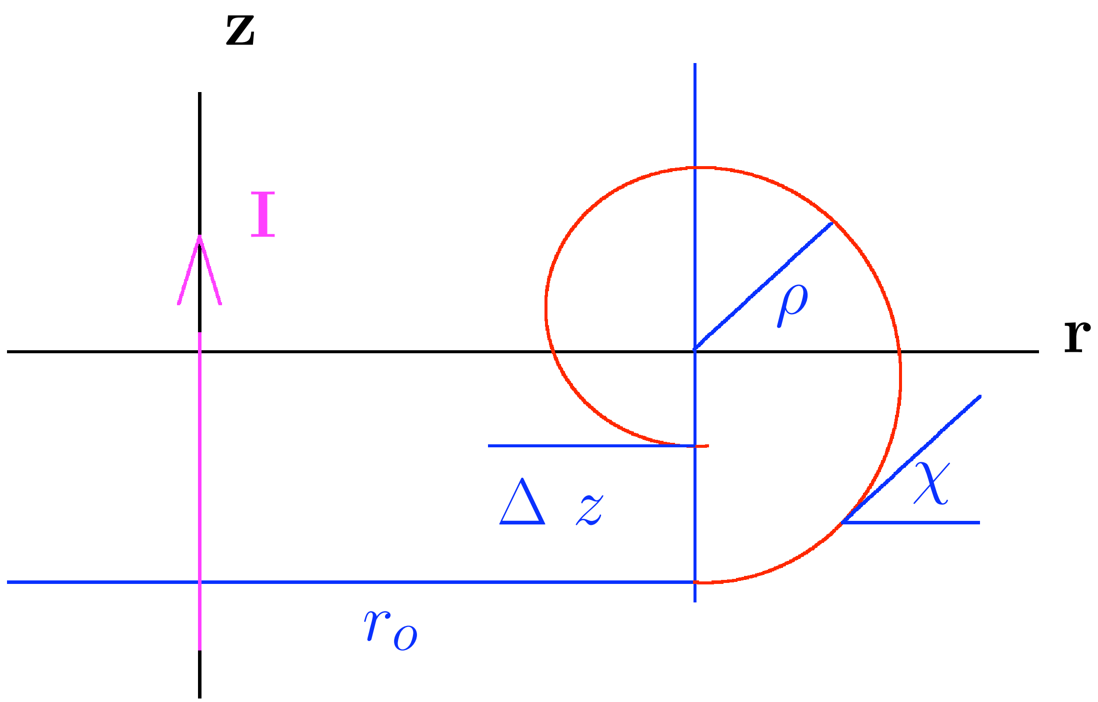
So approximately the upward helix angle $\alpha_{\rm mom}$ and Δz as function of $\theta = s/r$

$$\alpha_{\rm mom} \ = \ \frac{\Delta y}{s} \ = \ \frac{[p_\parallel c/e]}{r \ B_\phi \ c} \qquad\qquad \Delta z \ = \ \theta \ \frac{[p_\parallel c/e]}{B \ c}$$

2) Amplitude Drift

Particles with finite amplitudes around the reference will also drift upwards at approximately the same angle α but there are some aberrations. In particular, there is an additional amplitude dependent drift.

Consider now the situation with little or no momentum in the ϕ direction, but a relatively large initial radial momentum. The particle will remain perpendicular to the field lines and will, to first order, loop in a circle. But to second order, because the B_ϕ is not constant, the particle drifts upwards.



The bending radius ρ in the plane (x,r) including the fall off of B_ϕ with radius:

$$\rho \;=\; \rho_o \left(1 \;=\; \frac{\Delta r}{r_o}\right) \qquad \text{Where} \qquad \rho_o = \frac{[p_\perp c/e]}{cB_o}$$

Defining the track slope $\chi = \arctan(dz/dr)$, and
u the transverse distance along the track. So $du = \rho\; d\chi$, and $dr = du\;\sin(\chi)$

$$\text{To first order :} \qquad \Delta r \;=\; \rho_o\;\sin(\chi)$$

and to second order, including the change in ρ with r:

$$\Delta z \;=\; \int_o^{2\pi} \sin(\chi)\;\rho_o \left(1 \;+\; \frac{\rho_o\;\sin(\chi)}{r_o}\right)\; d\chi$$

$$= \left(\;0 \;+\; \frac{\rho_o^2}{r_o}\int_o^{2\pi} \sin^2(\chi)\; d\chi\right) \;=\; \frac{\pi\;\rho_o^2}{r_o}$$

$\alpha_{\mathrm{amp}} = \Delta z/\Delta s$ where Δs is the forward track distance for $\chi = 0 \longrightarrow 2\pi$

$$\Delta s \;\approx\; 2\;\pi\;\rho_o\;\frac{p_\|}{p_\perp}$$

giving $\quad \alpha_{\mathrm{amp}} \;\approx\; \left(\frac{[p_\| c/e]}{r_o\;B_o\;c}\right)\;\frac{1}{2}\left(\frac{p_\perp}{p_\|}\right)^2 \;=\; \frac{\alpha_{\mathrm{mom}}}{2}\;\left(\frac{p_\perp}{p_\|}\right)^2$

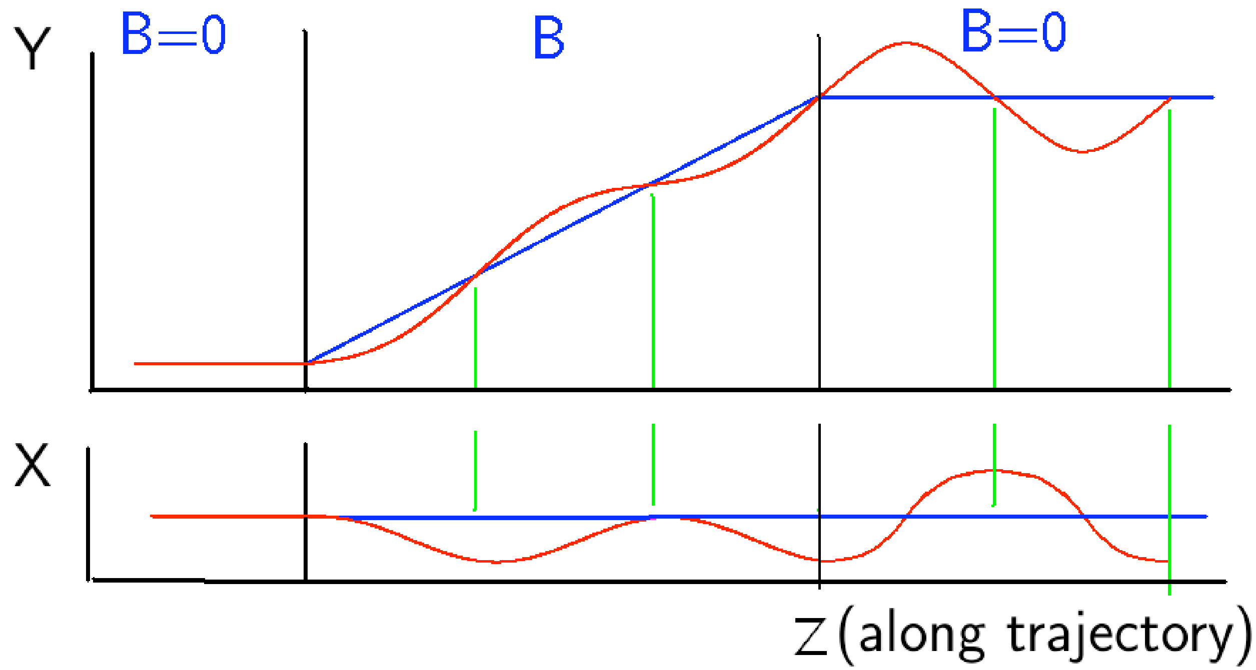
3) Discussion

- Both momentum drift and amplitude drift are, to some approximation, removed by reverse bends
- Amplitude drift reduction:
  - $p_\perp \propto 1/\sqrt{\beta} \propto \sqrt{B}$ while $p_\parallel$ will remain $\approx$ constant
  - so the relative amplitude drift $\propto p_\perp/p_\parallel$ will fall as B
  - at a lower enough B we can always make it negligible
- Dispersion relative to betatron beam size
  - Beam size $\sigma \propto \sqrt{\beta} \propto 1/\sqrt{B}$
  - While, for a given angle of turn $\theta = (s/r)$, the dispersion $D \propto 1/B$, so

$$\frac{\Delta y}{\sigma} \propto \frac{1/B}{1/\sqrt{B}} = \frac{1}{\sqrt{B}}$$

  - With a low enough B we eliminate the amplitude drift and improve the sharpness of the momentum selection
- But a low B requires a long bend (see below) and causes bunch length increases and distortion

4) Matching into Bent Solenoid



Inside a bent solenoid, without superimposed dipole, the equilibrium orbit is rising with an angle α. If this abruptly joins a straight solenoid, in which the equilibrium orbit is horizontal, then there will be a mismatch between them. If a dipole field is introduced over the bent portion of the solenoid, this can exactly remove the vertical drift so that the mismatch is removed.

The dipole field required is that field that will bend the reference particle with a curvature exactly equal to the bend of the solenoid:

$$B_{dipole} = \frac{[p_{\|}c/e]}{r\ c}$$

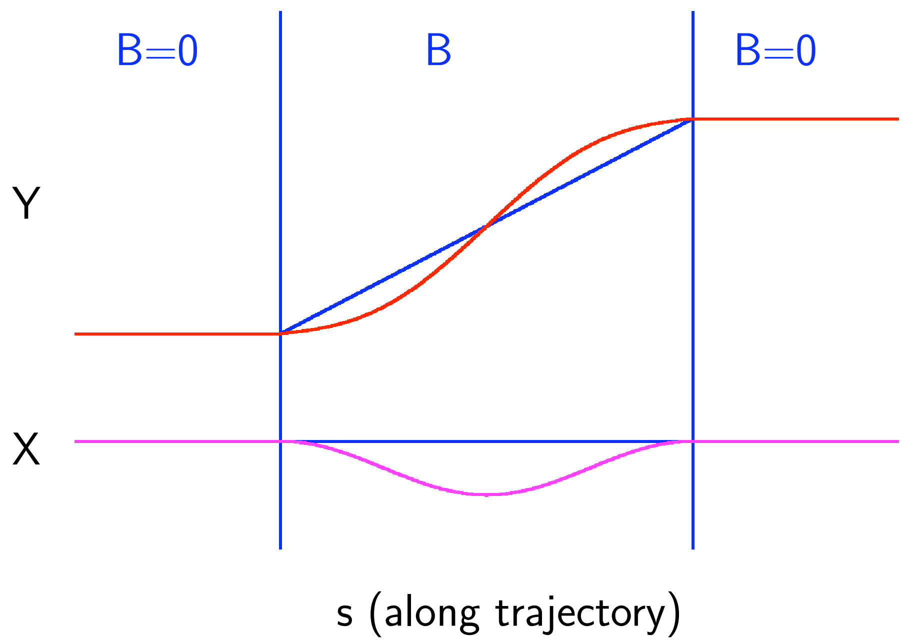
Matching, 2nd order, or when momentum drift needed

But the dipole can only remove the drift for one momentum. At other momenta there will still be an abrupt change in angle and a particle on the axis of the straight solenoid will mismatch into the bent portion and thus gain a finite betatron amplitude.

There are two ways of mitigating this problem. The first is to arrange that the length of the bend is an exact integer times the helix wavelength (=Larmor/2):

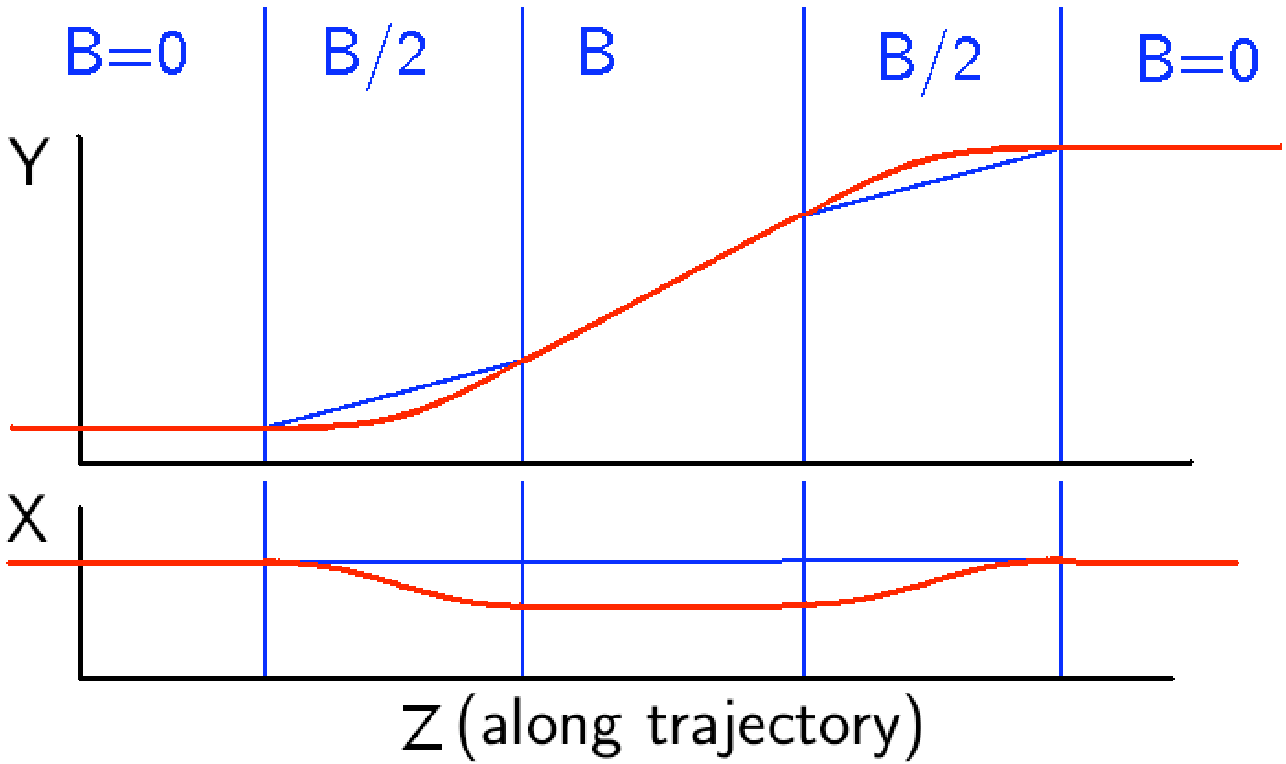
$$L \;=\; n\lambda_{\rm helix} \;=\; n\pi\beta_\perp \qquad\qquad (\text{remember } \beta_\perp \;=\; 2\beta_{\rm helix})$$

For n=1, this sets the minimum length of bend



Norem Matching

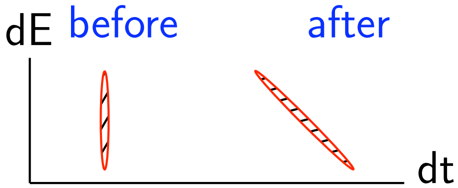
The second method (proposed by J. Norem) is to initiate the bend in two steps. For the first half helix wavelength ($L_{1/2} = \pi\beta_\perp/2$) the bend curvature (and corresponding dipole field if there is one) are set at half their final values. Only then is the full curvature and field applied. At the end of the bend, the field is similarly removed after a length$=L_{1/2}$ with half the curvature.



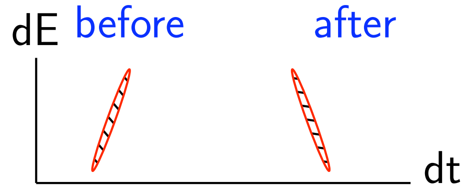
A suitable adiabatic onset of bending can have the same effect.

Charge Separation Design Criteria

- Match using $L_B = \lambda_\perp$ or Norem method
- Following the bends, there must be counter-bends
- There should be no rf until both are over
- Problem: bunch length growth over large distance without rf
  - The effect will be reduced by starting with left sloping phase space



As simulated



Preferable

  - $4\sigma_{ct}$ bunch length, without this, should probably be $< \lambda/3 \quad \sigma_{ct} < 18$ cm
- Choice of B and $\beta_\perp$
  - Min increase in long emittance requires short length $\rightarrow$ small $\beta_\perp$
  - Min increase in trans emittance requires small amplitude drifts: large $\beta_\perp$
  - These conflict
- But raising the energy always helps
  - dp/p reduced
  - more relativistic

Parameters for example at 230 MeV/c

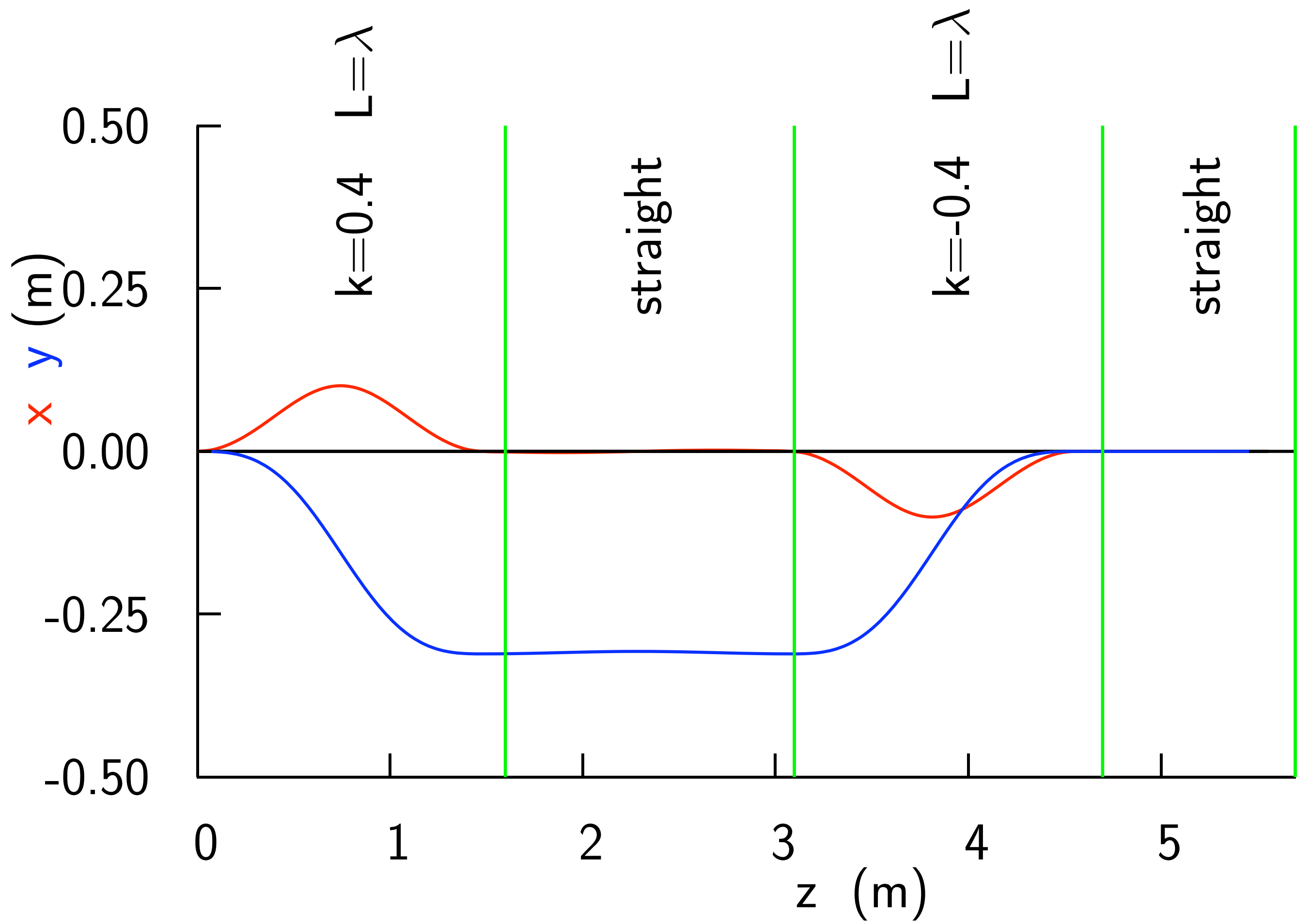
- () Parenthesized length is only for positives
- * These lengths were added for visualization - not included in emittance gains
- Mom: = 230 MeV/c i.e. $\approx$ as at end of phase rotation
- (files: bsx8c.f01 for negatives and bsx8m.f01 for positives
- Matching by $L = \lambda$
- $k = 1/\rho$ is curvature of solenoid
- $B_s = 3$ T is solenoid field throughout
  higher than Fernow's 1.9 T to keep short
- Input tracks are Gaussian distributions with:
  $\epsilon_\perp = 15\ \pi\ \mathrm{mm}$ (normalized)
  $\epsilon_\parallel = 40\ \pi\ \mathrm{mm}$ (normalized)
- Angular momentum added to remove Canonical:

$$[p_\phi c/e] = [p_\parallel c/e] - \frac{rc}{2}\, B_s$$

Len m	k m^{-1}	B_s T
1.6	0.4	3
(1.5)	0	3
1.6	-0.4	3
1.0*	0	3

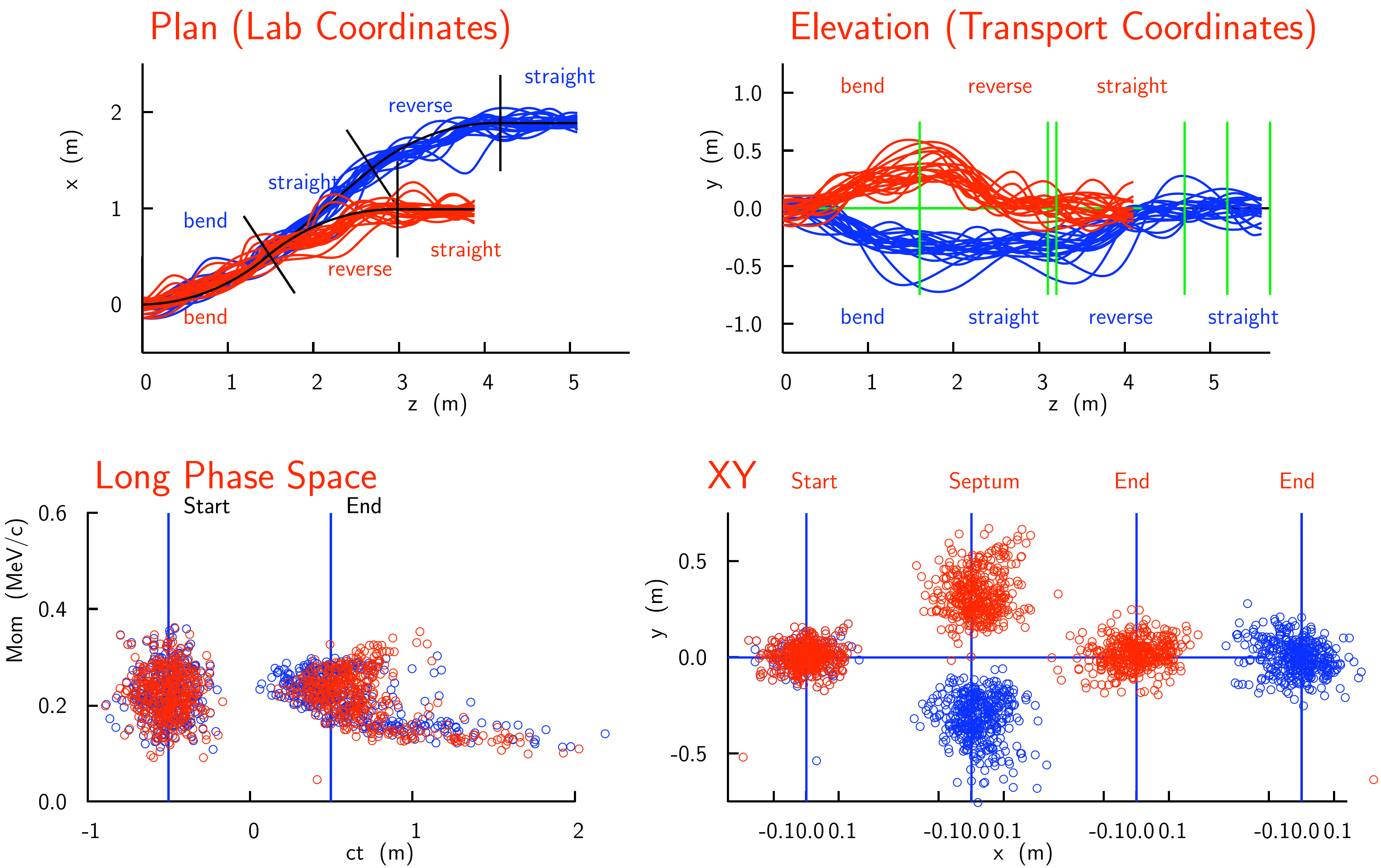
230 MeV/c example with $L = \lambda$ matching

Single track starting on axis

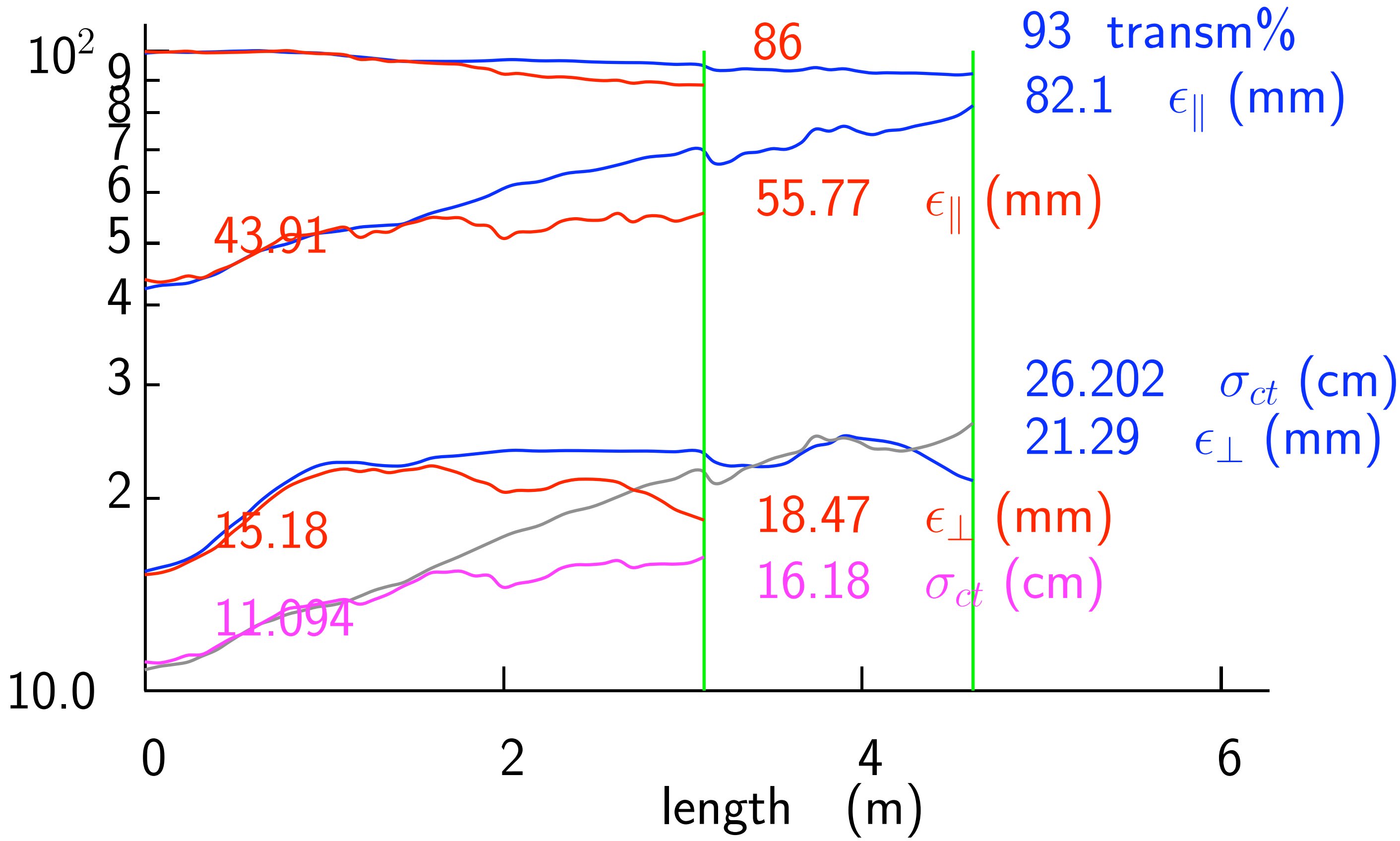


- Track makes significant departures from axis in x and y c.f sigma of beam is 1.8 cm
- but returns to the axis at the end

230 MeV/c example with $L = \lambda$ matching



Transm, Emits. & Bunch length



- $\sigma_{ct} = 21.2 >> 18$ cm too long compared with bunch spacing
- Severe distortion gives long emit increase of 60%
- Trans emit increased by 31%
- Transmission 89%
- Spacing between beams at end not enough for rf
- Unacceptable

Fixes for these problems

- Raising the energy has multiple advantages:
  - for the same long emittance dp/p is reduced
  - being more relativistic, bunch lengthening is less
  - with less bunch lengthening, the straight in the middle can be longer for greater separation
  - and it allows Norem matching with longer, but weaker, bends giving less amplitude drift and non-linearity
  - for the same trans emittance $p_\perp/p_\parallel$ is less giving less amplitude drift and non-linearity
- The higher the momentum used, the more extra rf needed
- But if 6D emittance is less blown up, less cooling rf is required
- Optimization needed

- Several simulations were done, first at 300 MeV/c and then 400 MeV/c
- Plots for one will be shown and results for the others summarized

Parameters for 5 examples

() Parenthesized length is only for positives

* These lengths were added for visualization - not included in emittance gains

$k = 1/\rho$ is curvature of solenoid B_s is solenoid field

	#1	#2	#3	#4
Mom:	230 MeV/c	300 MeV/c	400 MeV/c	400 MeV/c
Match:	$L = \lambda$	Norem	$L = \lambda$	Norem
File:	bsx8cm	bsx7em	bsx5dm	bsx9am

Len m	k m^{-1}	B_s T
1.6	0.4	3
(1.5)	0	3
1.6	-0.4	3
1.0*	0	3

Len m	k m^{-1}	B_s T
1.65	0.15	1.9
0.45	0.3	1.9
1.65	0.15	1.9
(2.0)	0	1.9
1.65	-0.15	1.9
0.45	-0.3	1.9
1.65	-0.15	1.9
2.0*	0	1.9

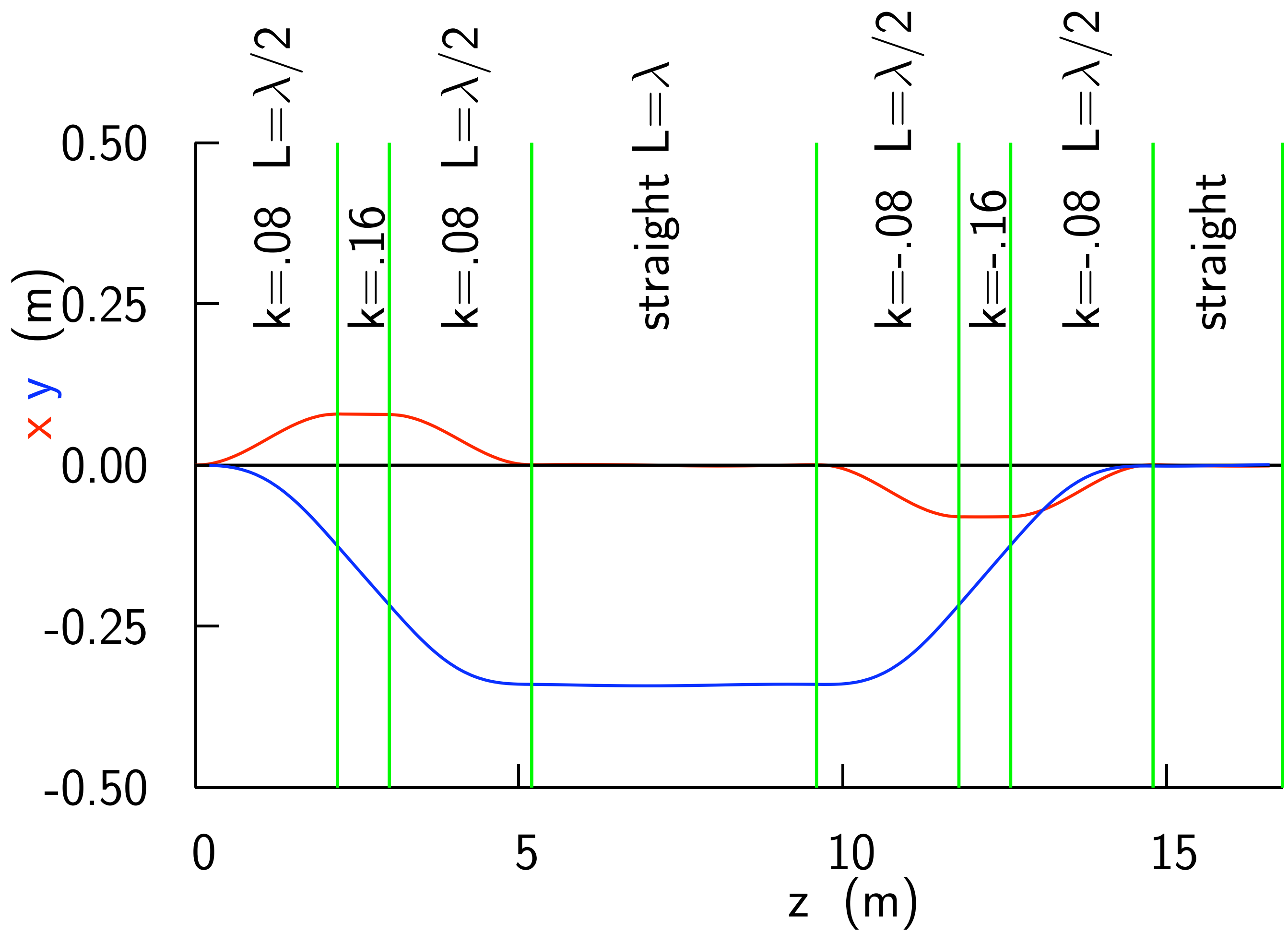
Len m	k m^{-1}	B_s T
4.4	0.11	1.9
(2.0)	0	1.9
4.4	-0.11	1.9
2.0*	0	1.9

Len m	k m^{-1}	B_s T
2.2	0.0807	1.9
0.8	0.1614	1.9
2.2	0.0807	1.9
(2.2)	0	1.9
2.2	-0.0807	1.9
0.8	-0.1614	1.9
2.2	-0.0807	1.9
2.0*	0	1.9

The #5 is the same as #4 but the stright in the middle is 4.4 m instead of 2.2 m. The file is bsx10am

400 MeV/c with Norem match & 2 m final spacing

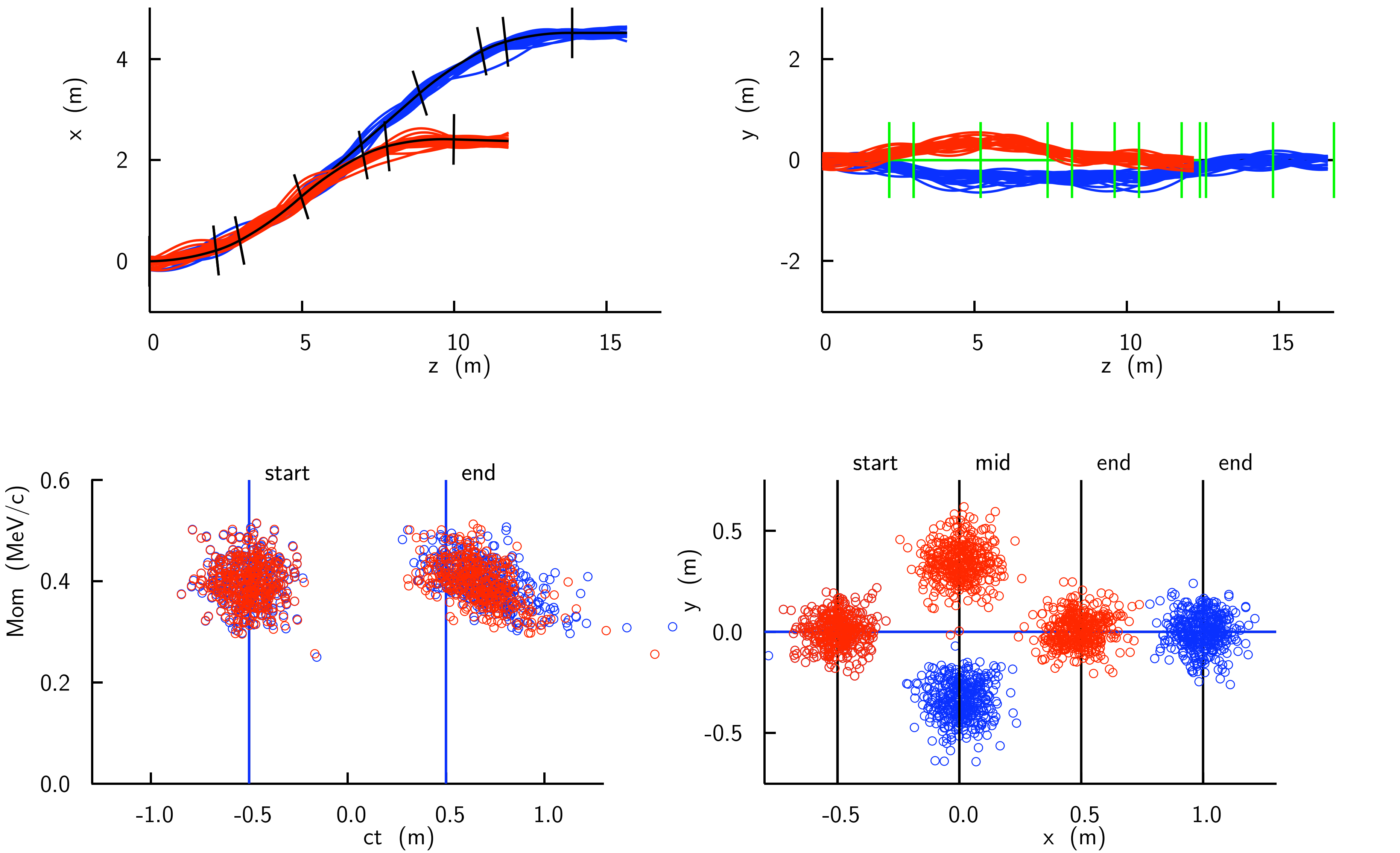
Single track starting on axis

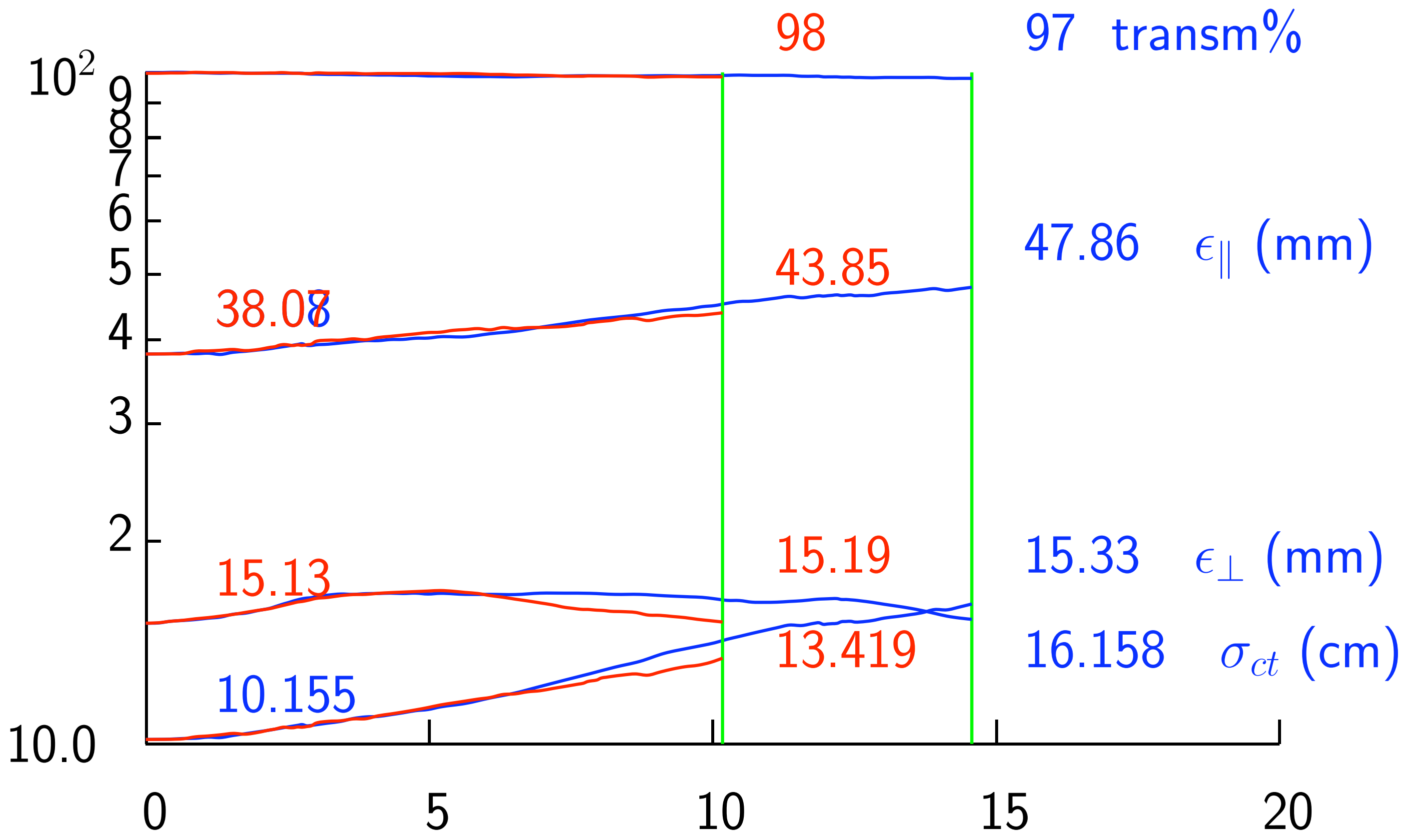


- Track makes significant departures from axis in x and y
- but returns to the axis at the end

400 MeV/c with Norem match and more spacing

Spacing now 2.13 m   was 1.06 m



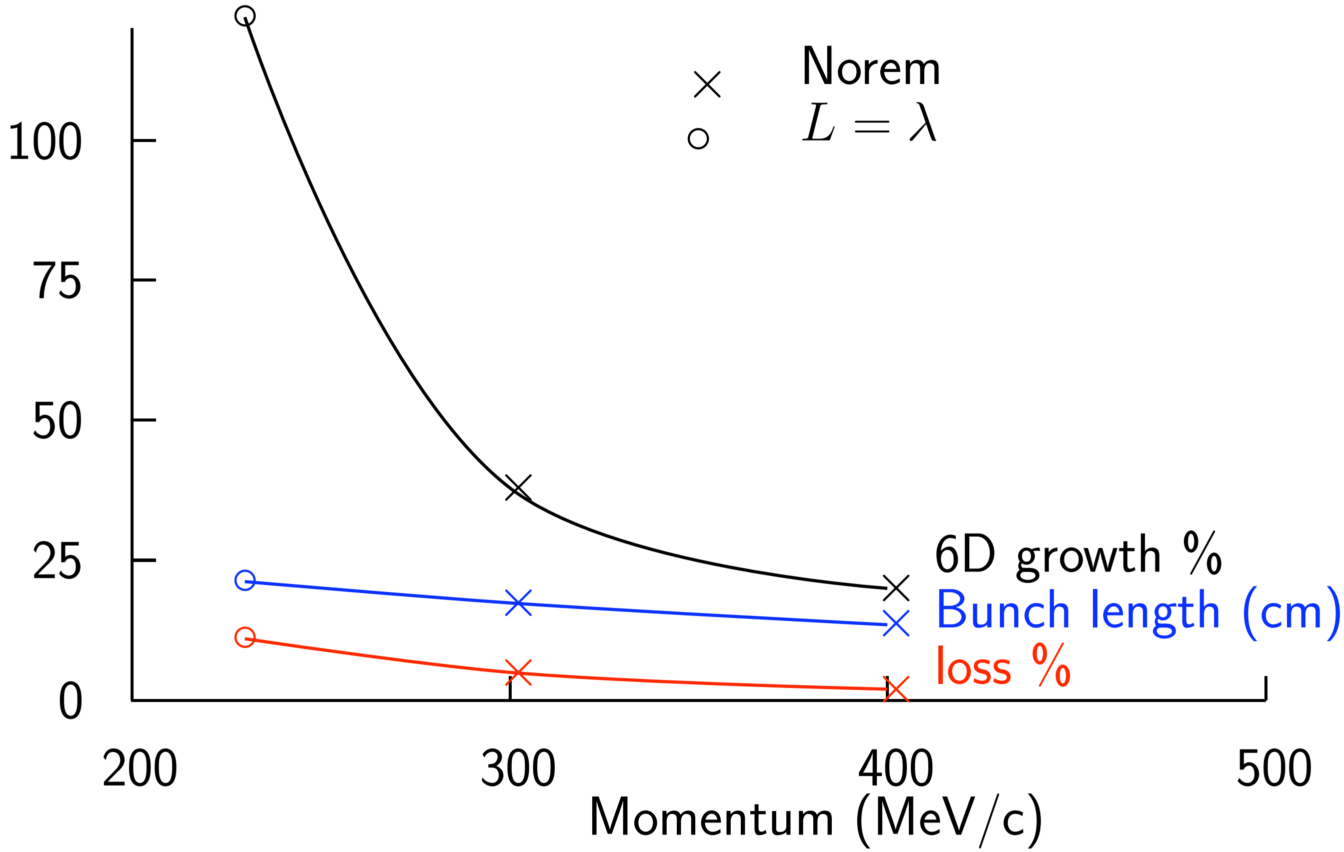


- ave $\sigma_{ct} = 14.8 < 18$ cm
- Long emit increased by 20%
- Trans emit increased by 1%
- Transmission 97.5%
- Acceptable

Summary of Simulated Performances

	Mom MeV/c	Match	spacing m		Init	Neg	Pos	ave. increase %
#1	230	$L = \lambda$	0.9	Trans emit (mm)	15.1	17.6	21.1	28%
				Long emit (mm)	43.9	59.7	82.1	61%
				sig ct (mm)	111	151	218	66%
				Transmission (%)	100	85	89	-13%
#2	300	Norem	1.1	Trans emit (mm)	15.1	15.4	16.7	6%
				Long emit (mm)	38.1	45.8	50.3	26%
				sig ct (mm)	102	162	187	66%
				Transmission (%)	100	95	95	-5%
#3	400	$L = \lambda$	0.9	Trans emit (mm)	15.1	15.8	17.0	9%
				Long emit (mm)	38.1	42.1	46.4	16%
				sig ct (mm)	102	123	141	29%
				Transmission (%)	100	97	99	-2%
#4	400	Norem	1.06	Trans emit (mm)	15.1	15.2	15.7	2%
				Long emit (mm)	38.1	43.8	44.9	16%
				sig ct (mm)	102	134	142	35%
				Transmission (%)	100	98	98	-2%
#5	400	Norem	2.13	Trans emit (mm)	15.1	15.2	15.3	1%
				Long emit (mm)	38.1	43.8	47.8	20%
				sig ct (mm)	102	134	162	45%
				Transmission (%)	100	98	97	-2.5%

Dependences on momentum



- Comparison of #3 and 4 shows clear impovement using Norem matching allowing weaker bends

Conclusions

- Bent solenoids can generate large dispersion from 'momentum drift', but can spoil emittance from 'amplitude drift'
- Match using $L = \lambda$, or Norem's match in and out of bends
- Reverse bending removes the dispersion and reduces 'amplitude drift' but only if there is no rf until after all bending

- The main problem is bunch length growth from long transport without rf
- To reduce bend lengths and thus bunch lengthening, we increased bending field (from Rick's 1.9 T to 3 T), but severe non-linearities increase 6D emittance by 117% and give 13% loss. Not acceptable
- Raising the momentum from 230 to 300 MeV helps because less dp/p, and more relativistic. With Norem matching and 1.9 T, 6D growth = 38% and 5% loss. Possibly acceptable
- Raising it further to 400 MeV gives very good results. With Norem matching: 6D growth of 24% and 2.5% loss Acceptable

- Norem matching, using more gentle bends, improves transverse emittance (3% vs. 9 % at 400 MeV/c) at a small loss of longitudinal (18 % vs. 16 % at 400 MeV/c) and greater bunch lengthening